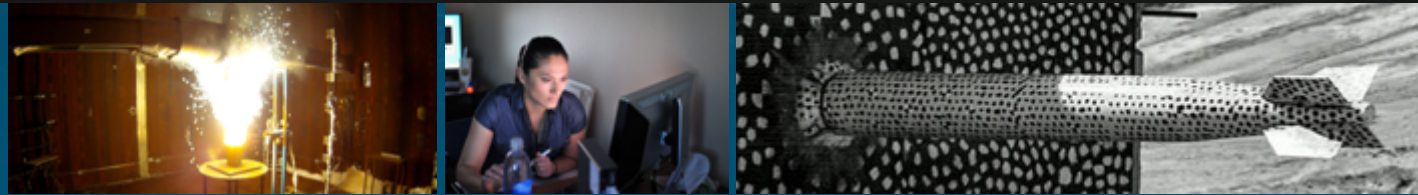




# Optimization-based Design for Additive Manufacturing



## CONTRIBUTORS

Miguel Aguilo, Brett Clark, Kyle Johnson, Joshua Robbins, Ryan Viertel

## PRESENTED BY

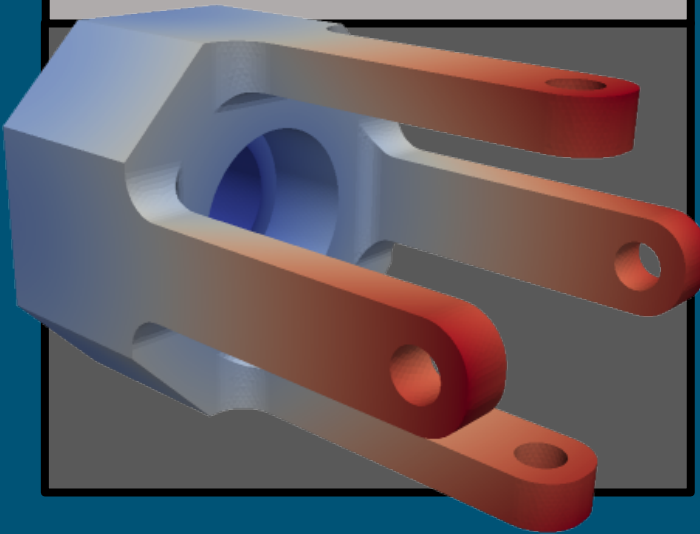
Joshua Robbins



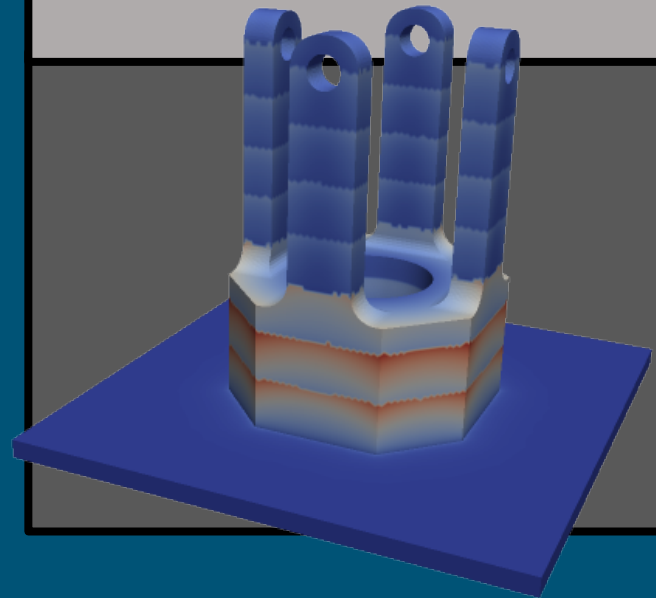
# Design for AM

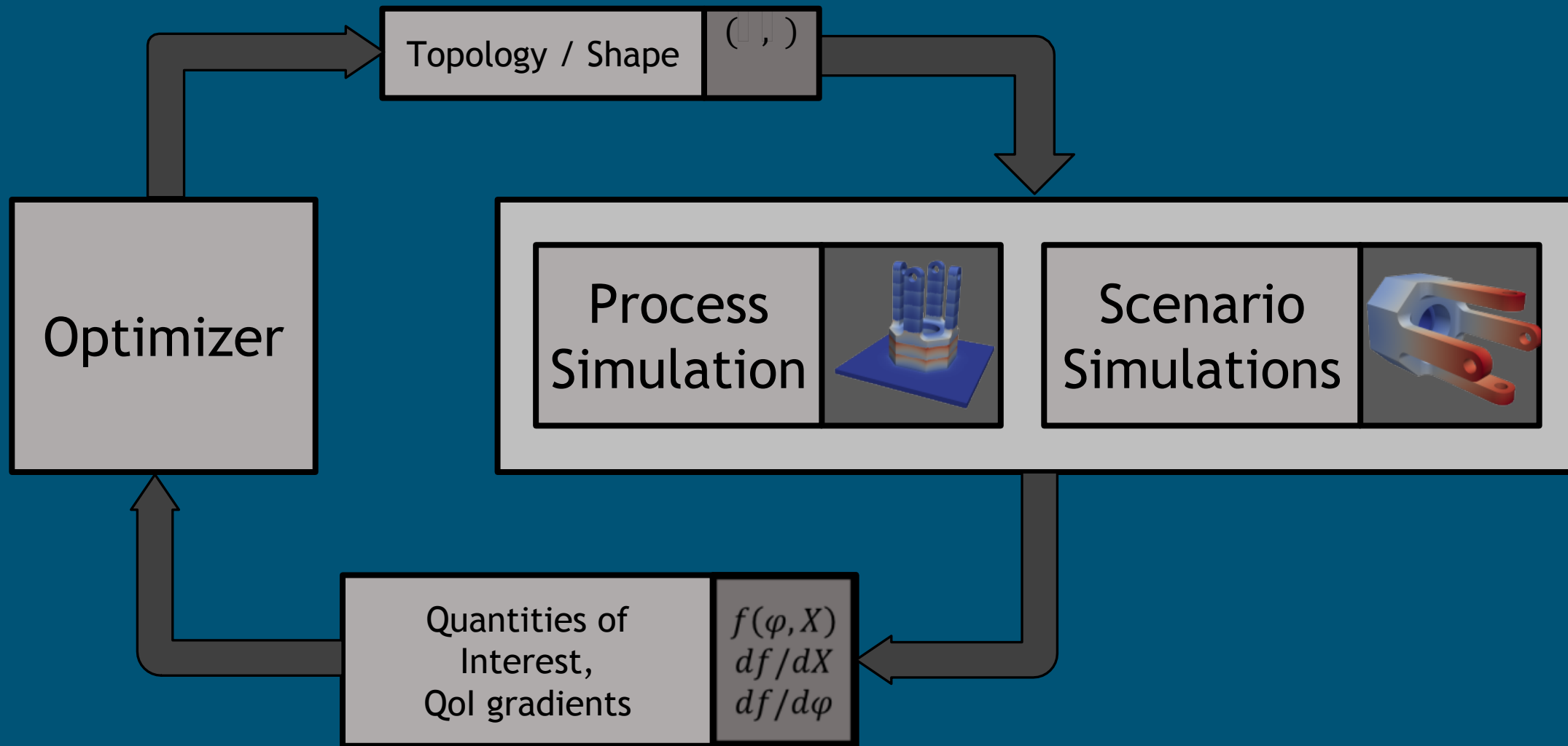


Performance



Manufacturability







# Process-aware Shape/Topology Optimization

**Problem:**  $\min_{\varphi, X} f(\varphi, X, U(\varphi, X), S(\varphi, X), \mathcal{U}(\varphi, X))$

**Process solutions:**  $U(\varphi, X) = \{u_I(\varphi, X) \mid I = 1 \dots n_I\}$   
 $S(\varphi, X) = \{s_I(\varphi, X) \mid I = 1 \dots n_I\}$

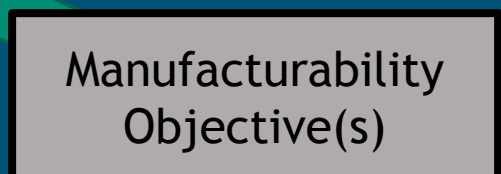
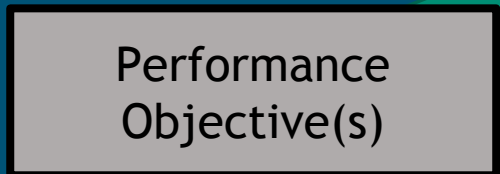
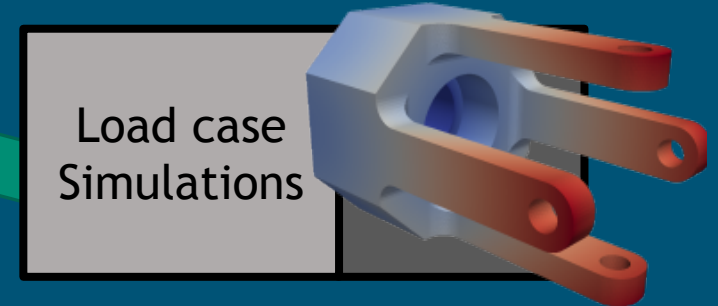
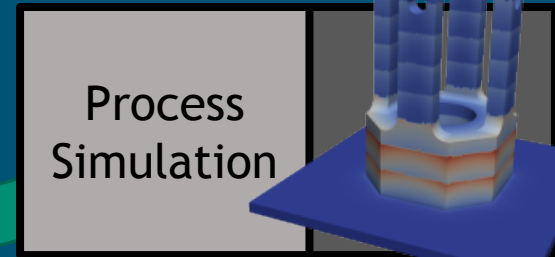
**Load case solutions:**  $\mathcal{U}(\varphi, X) = \{u_I(\varphi, X) \mid I = 1 \dots n_c\}$

**Process:**  $g_I(\varphi, X, u_I, s_{I-1}) = 0$  for  $I = 1 \dots n_I$

**Load cases:**  $c_J(\varphi, X, u_{n_I}, s_{n_I}, u_J) = 0$  for  $J = 1 \dots n_c$

**Linear constraint:**  $h(\varphi, X) \leq 0$

**Objective:**  $f = \sum_{K=1}^{n_c} \alpha_K f_K(\varphi, X, u_K(\varphi, X)) + \alpha_m f_m(\varphi, X, U(\varphi, X), S(\varphi, X))$





# Process-aware Shape/Topology Optimization

$$g_I(\varphi, X, u_I, s_{I-1}; C, \varepsilon^{IS}) = 0 \quad \text{for } I = 1 \dots n_I$$

$n_I$ : Number of sequence steps

Inherent strain method (elastic):

$$g_I = \begin{Bmatrix} R^I \\ H^I \end{Bmatrix}$$

Small strain  
Elastostatics:

$$R^I = F_{int}^I(\varphi, X, u_I, s_{I-1}; C, \varepsilon^{IS}) - F_{ext}^I$$

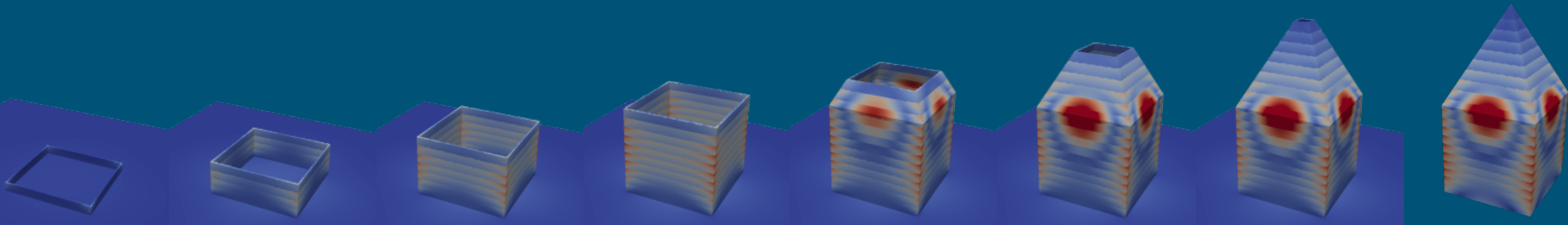
State update:

$$H^I = s_I - s_{I-1} - B(u_I)$$

( , )

Forward Solution (What is the manufacturability of the current design?)

$$f_m(\varphi, X, U(\varphi, X), S(\varphi, X))$$



# Process-aware Shape/Topology Optimization



$$\hat{f}_m = f_m + \sum_{k=1}^n \lambda^{kT} R^k + \sum_{k=1}^n \mu^{kT} H^k$$

$$\begin{aligned} \frac{d\hat{f}_m}{d\varphi} &= \frac{\partial f_m}{\partial \varphi} \\ &+ \sum_{k=1}^n \left( \frac{\partial f_m}{\partial u^k} \frac{\partial u^k}{\partial \varphi} + \frac{\partial f_m}{\partial c^k} \frac{\partial c^k}{\partial \varphi} \right) \\ &+ \sum_{k=1}^n \lambda^{kT} \left( \frac{\partial R^k}{\partial \varphi} + \frac{\partial R^k}{\partial u^k} \frac{\partial u^k}{\partial \varphi} + \frac{\partial R^k}{\partial c^{k-1}} \frac{\partial c^{k-1}}{\partial \varphi} \right) \\ &+ \sum_{k=1}^n \mu^{kT} \left( \frac{\partial H^k}{\partial \varphi} + \frac{\partial H^k}{\partial u^k} \frac{\partial u^k}{\partial \varphi} + \frac{\partial H^k}{\partial c^k} \frac{\partial c^k}{\partial \varphi} + \frac{\partial H^k}{\partial c^{k-1}} \frac{\partial c^{k-1}}{\partial \varphi} \right) \end{aligned}$$

# Process-aware Shape/Topology Optimization



$$\begin{aligned} \frac{d\hat{f}_m}{d\varphi} &= \frac{\partial f_m}{\partial \varphi} + \sum_{k=1}^n \left( \lambda^{kT} \frac{\partial R^k}{\partial \varphi} + \mu^{kT} \frac{\partial H^k}{\partial \varphi} \right) \\ &+ \left( \frac{\partial f_m}{\partial c^n} + \mu^{nT} \frac{\partial H^n}{\partial c^n} \right) \frac{\partial c^n}{\partial \varphi} \\ &+ \sum_{k=1}^n \left( \frac{\partial f_m}{\partial u^k} + \lambda^{kT} \frac{\partial R^k}{\partial u^k} + \mu^{kT} \frac{\partial H^k}{\partial u^k} \right) \frac{\partial u^k}{\partial \varphi} \\ &+ \sum_{k=1}^{n-1} \left( \frac{\partial f_m}{\partial c^k} + \mu^{kT} \frac{\partial H^k}{\partial c^k} + \mu^{k+1T} \frac{\partial H^{k+1}}{\partial c^k} + \lambda^{k+1T} \frac{\partial R^{k+1}}{\partial c^k} \right) \frac{\partial c^k}{\partial \varphi} \end{aligned}$$

# Process-aware Shape/Topology Optimization

(last step)

$$\frac{\partial f_m}{\partial c^n} + \mu^{nT} \frac{\partial H^n}{\partial c^n} = 0$$

$$\frac{\partial f_m}{\partial u^n} + \lambda^{nT} \frac{\partial R^n}{\partial u^n} + \mu^{nT} \frac{\partial H^n}{\partial u^n} = 0$$

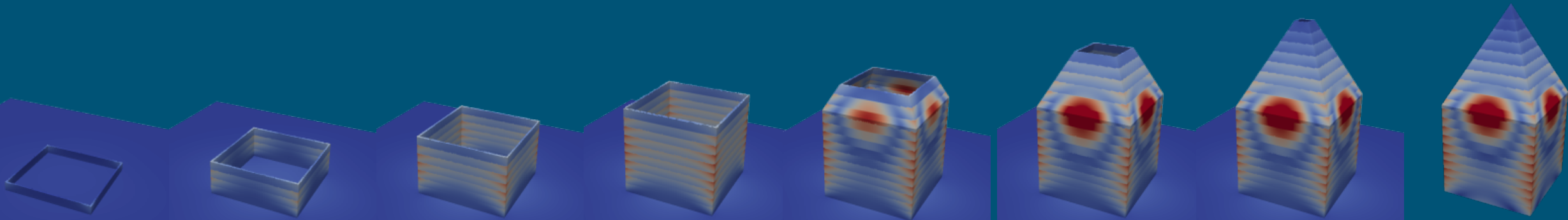
(kth step),  $k = n - 1, \dots, 2, 1$

$$\frac{\partial f_m}{\partial c^k} + \mu^{kT} \frac{\partial H^k}{\partial c^k} + \mu^{k+1T} \frac{\partial H^{k+1}}{\partial c^k} + \lambda^{k+1T} \frac{\partial R^{k+1}}{\partial c^k} = 0$$

$$\frac{\partial f_m}{\partial u^k} + \lambda^{kT} \frac{\partial R^k}{\partial u^k} + \mu^{kT} \frac{\partial H^k}{\partial u^k} = 0$$

← Adjoint Solution (How does the manufacturability change with design changes?)

$$f_m(\varphi, X, U(\varphi, X), S(\varphi, X))$$







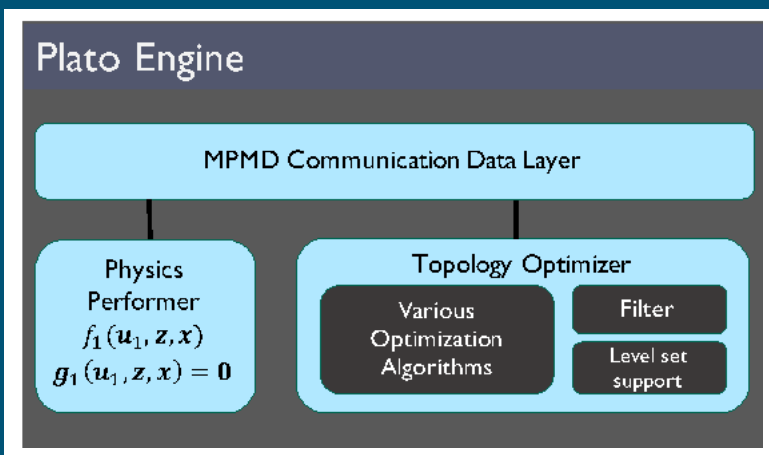
# What is Plato?

Plato is a suite of tools for applying optimization to engineering problems with an emphasis on optimization-based design.



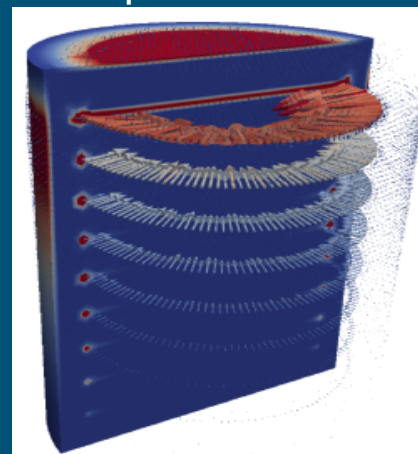
## Plato Engine

Open-source architecture for orchestrating MPMD optimization problems



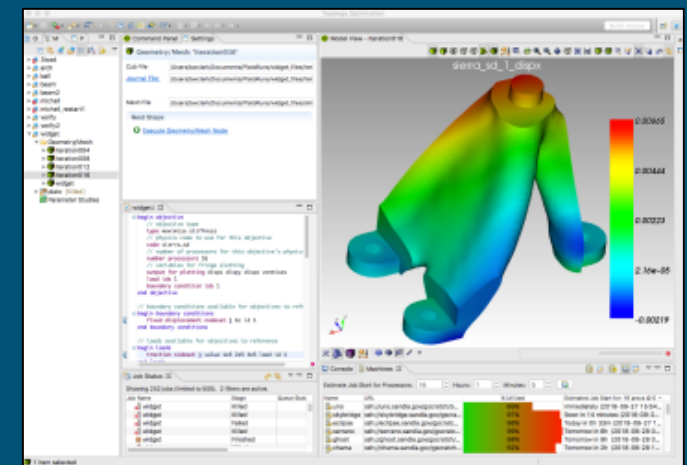
## Plato Analyze

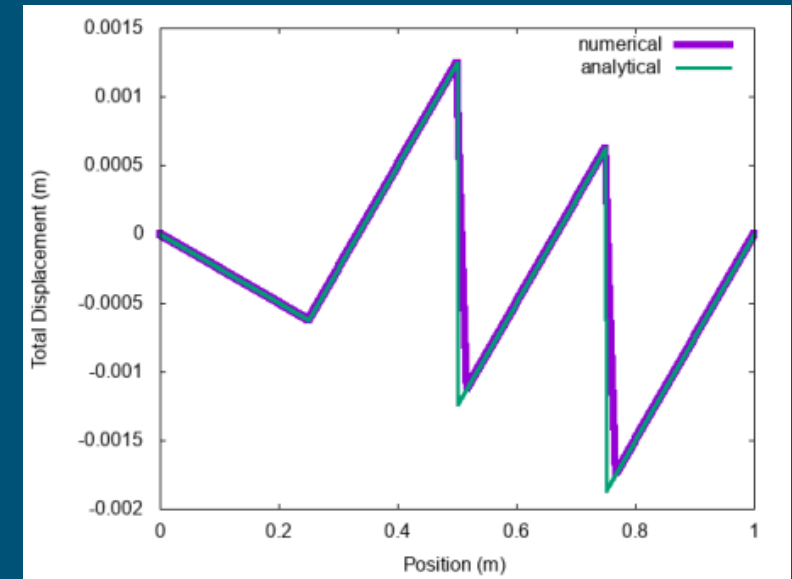
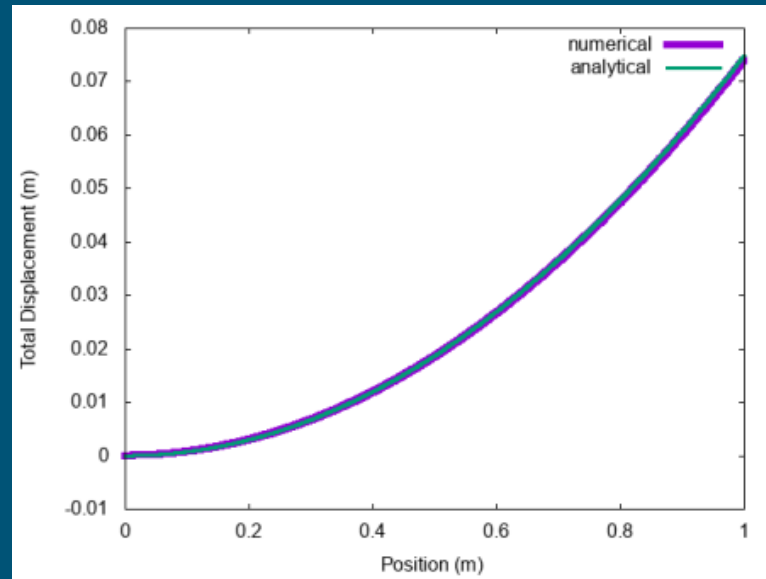
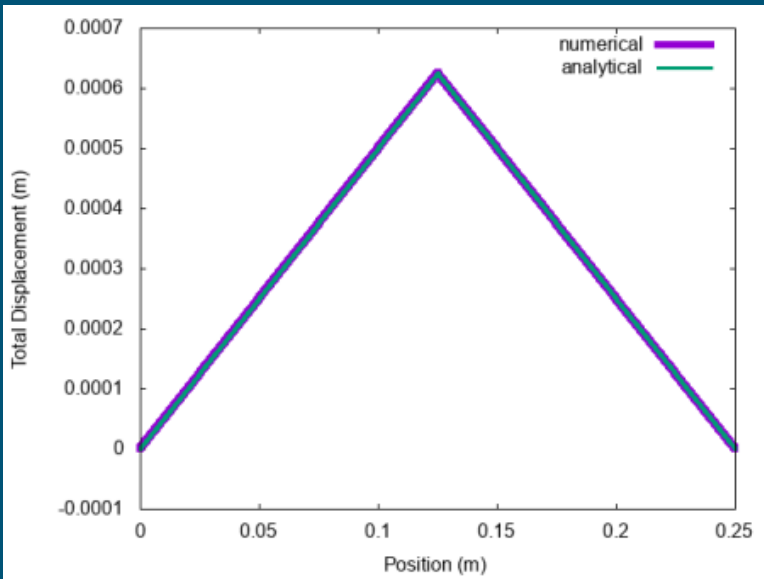
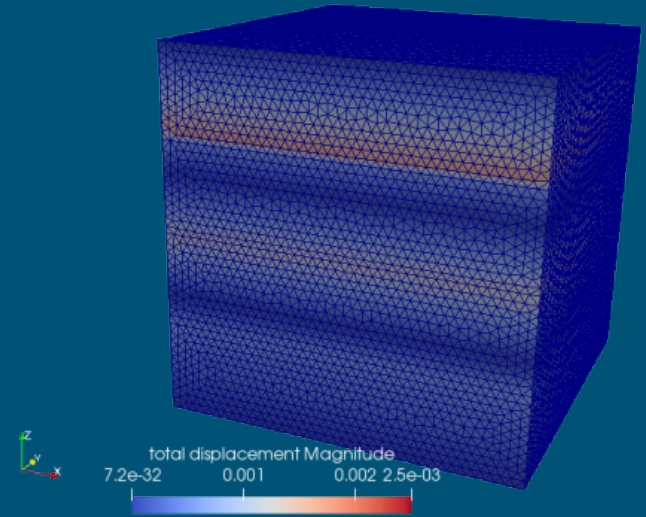
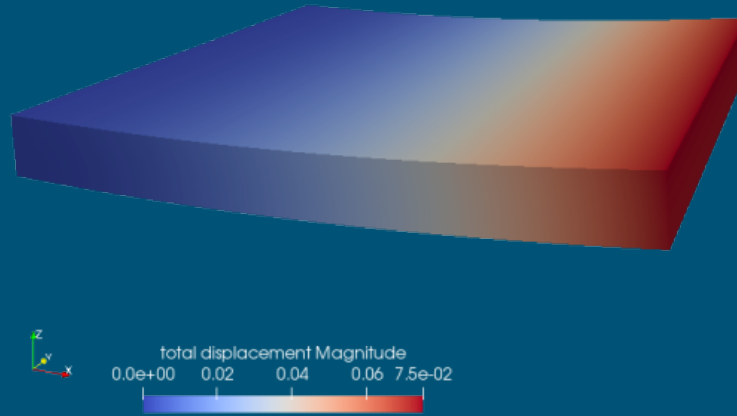
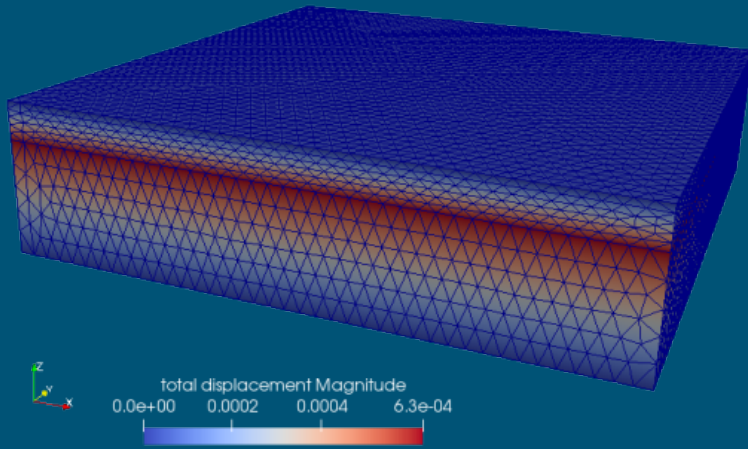
Open-source FEA code architected specifically for performance portability and use in gradient-based optimization

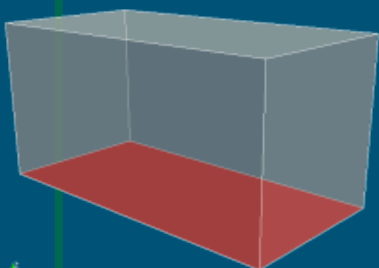


## Plato v2.3

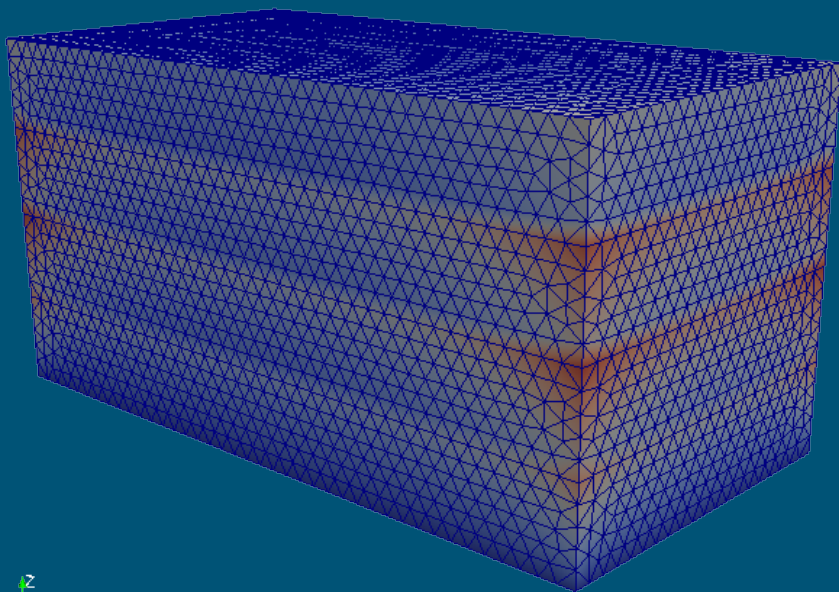
GUN-licensed Plato product including Plato-SAW GUI, Sierra/SD and some support for using Plato Analyze



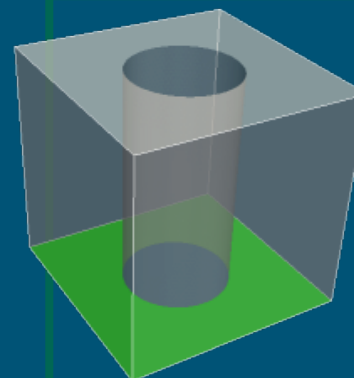




Topology sensitivities: Randomly perturb the nodal densities and compare the predicted vs compute gradients vs step size.



Step Size	Grad'*Step	FD Approx	abs(Error)
1.00000000e-01	1.03614525e-03	1.03614525e-03	2.38499368e-12
1.00000000e-02	1.03614525e-03	1.03614530e-03	4.42565318e-11
1.00000000e-03	1.03614525e-03	1.03614497e-03	2.85467809e-10
1.00000000e-04	1.03614525e-03	1.03614326e-03	1.98971396e-09



Shape sensitivities: Compute the sensitivities, increment shape parameters, and compute new objective.

Objective value:  
1.29526388939

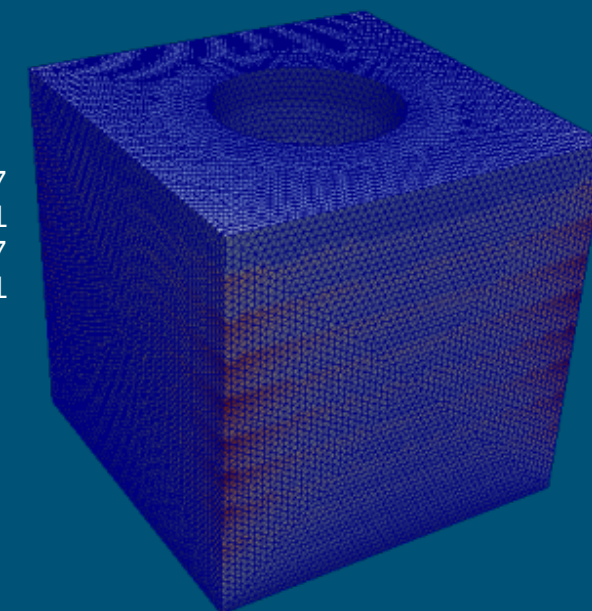
Shape sensitivities:  
X width: 1.180865479582657  
Y width: 2.153667356794931  
Z width: 2.154590003921607  
Radius: -4.397825196763741

Step size: 0.01

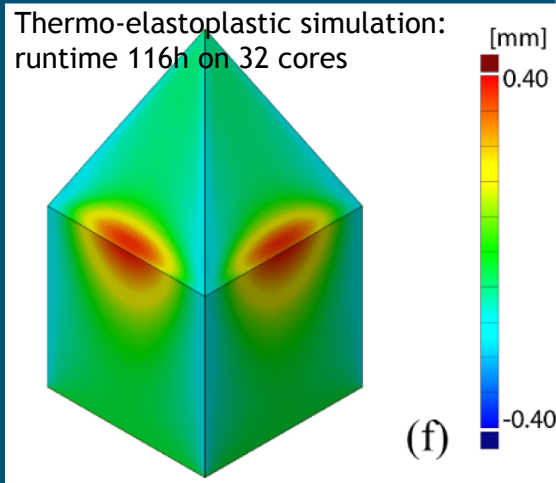
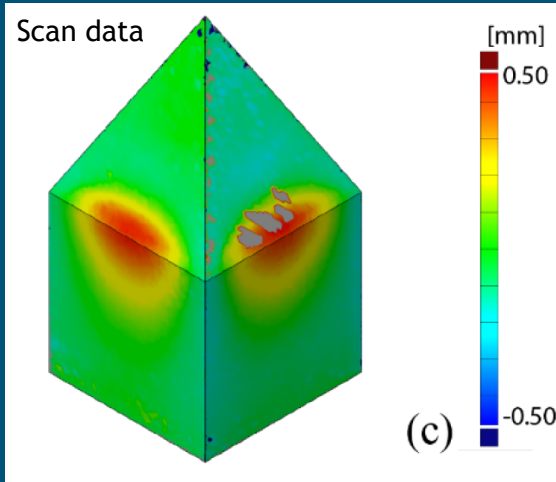
Predicted new value:  
1.3475894287252181

Computed new value:  
1.3500506130617436

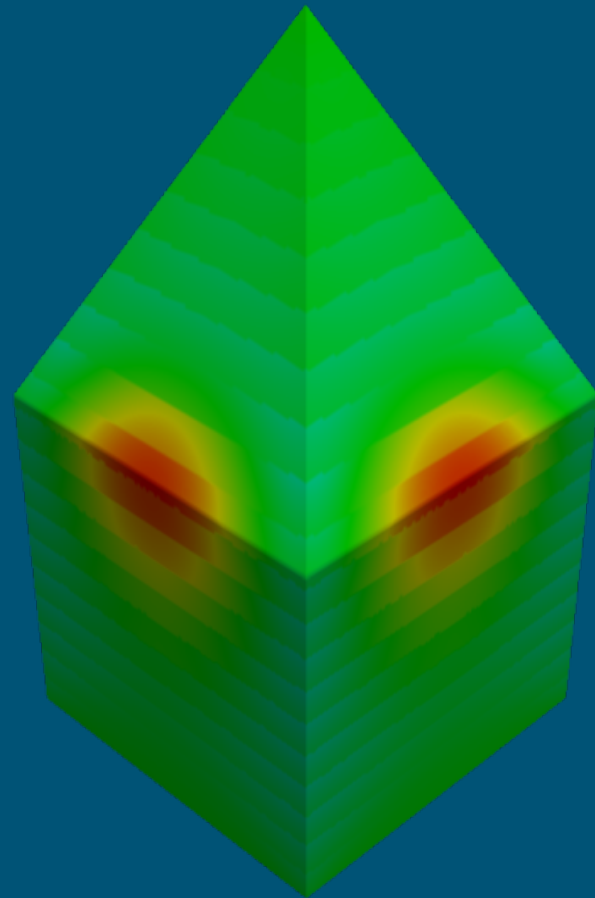
Relative error:  
0.001900141242789151



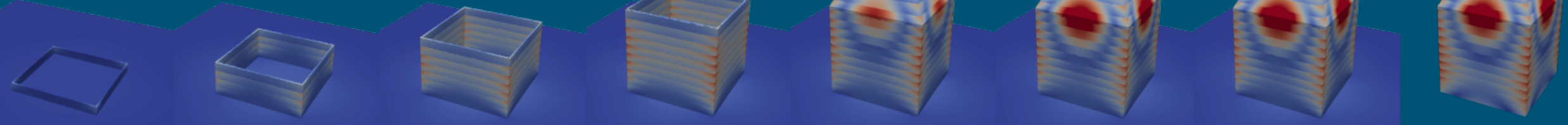
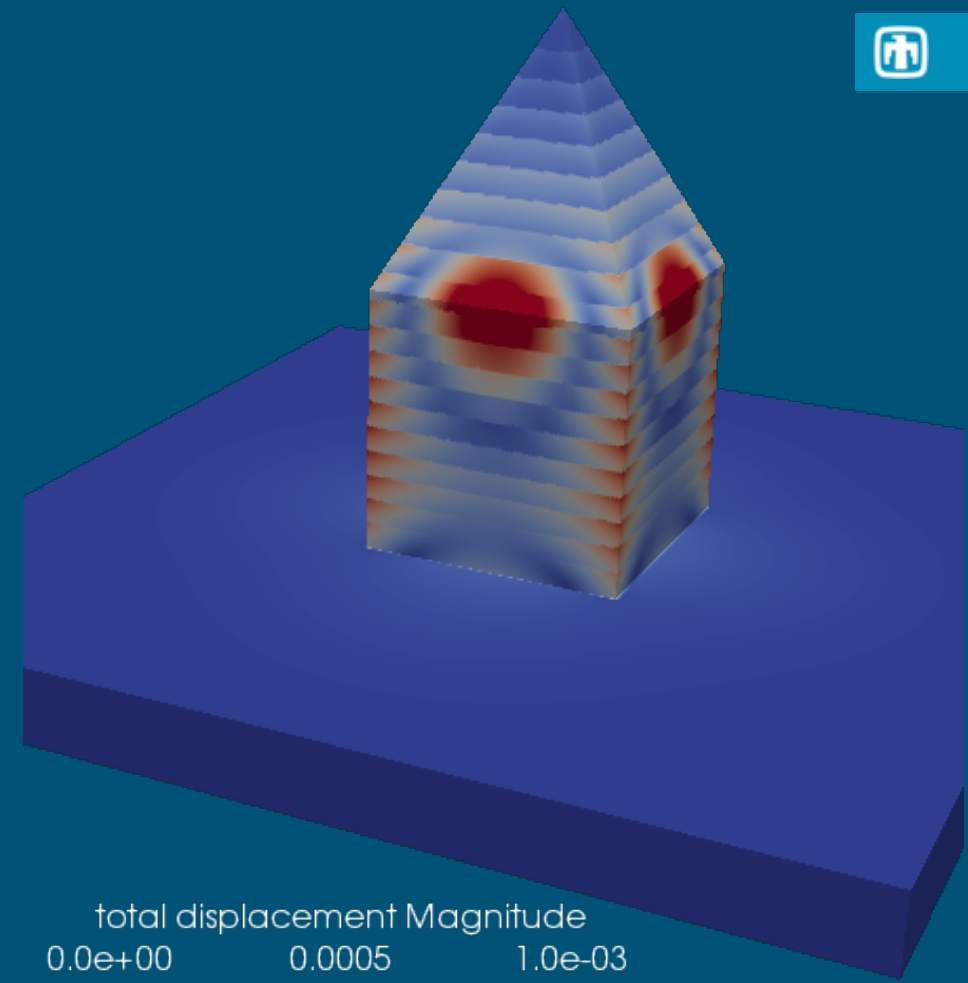
# Testing, Verification, and Validation



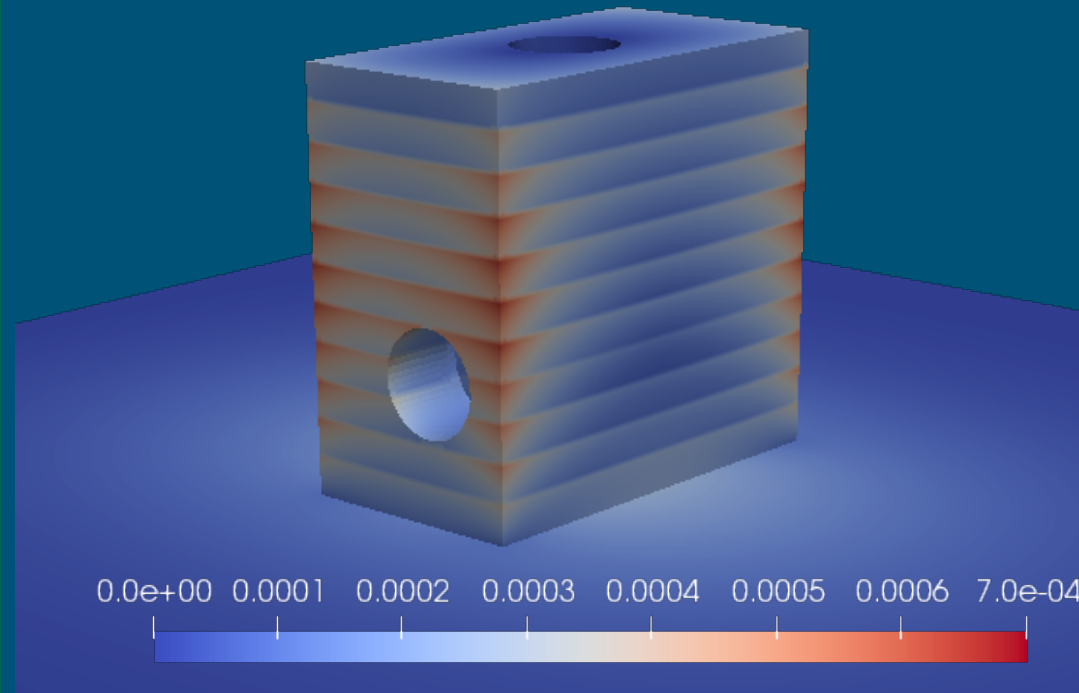
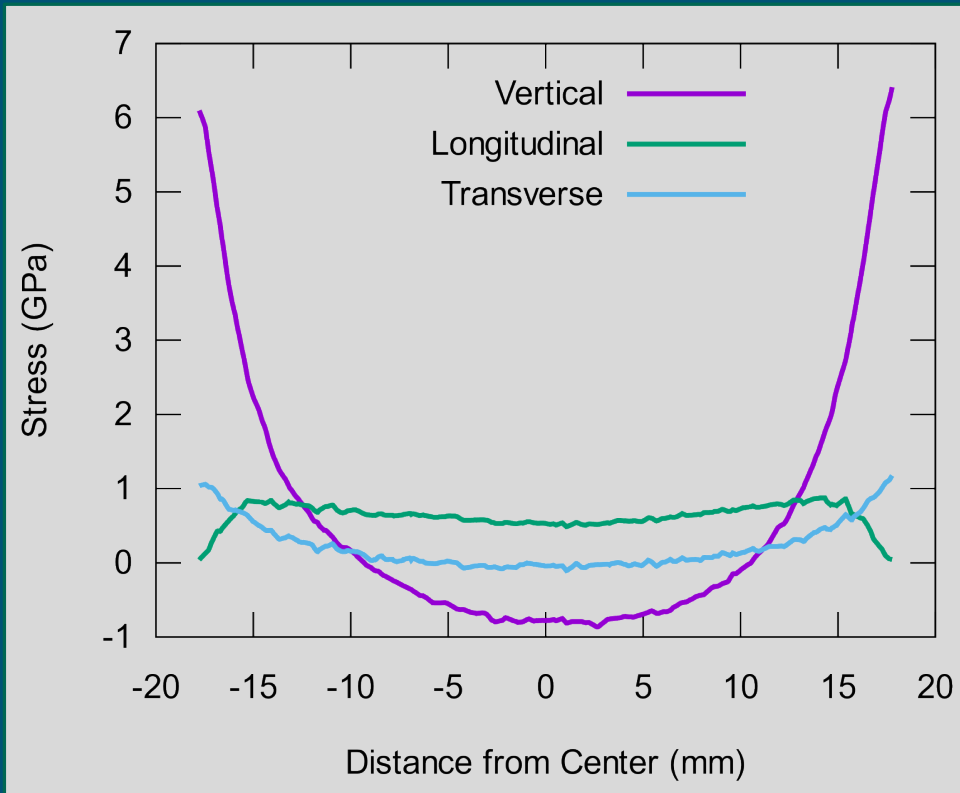
J Gordon et al. (2021) Journal of Materials Engineering and Performance (submitted)



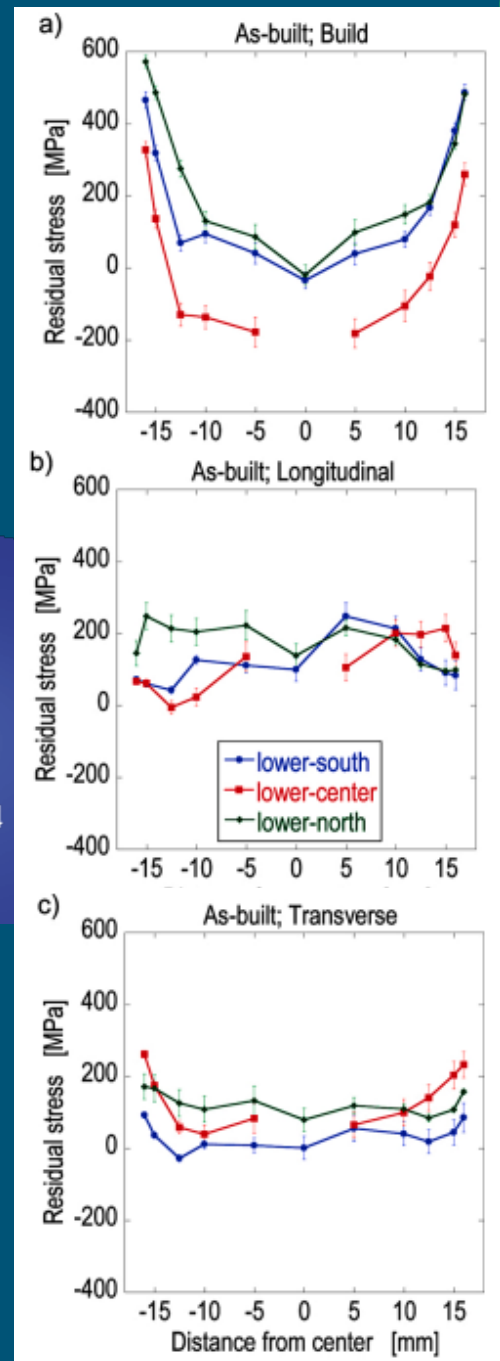
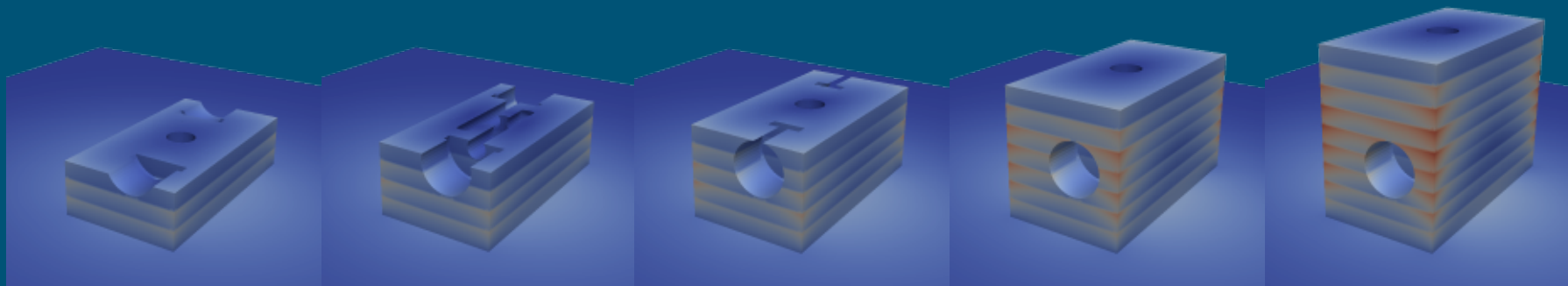
Elastic inherent strain: runtime 5m on RTX 5000 mobile GPU



# Testing, Verification, and Validation

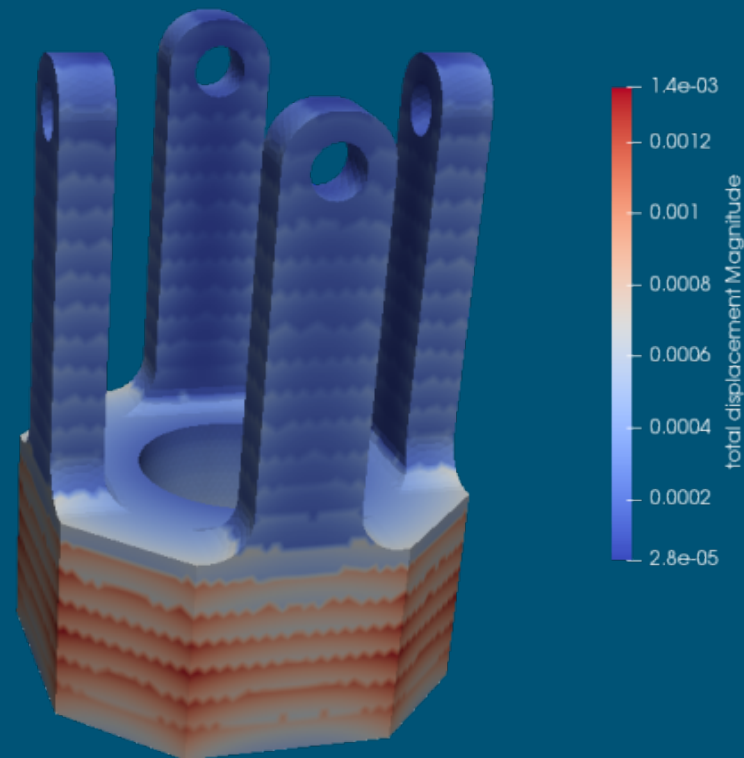
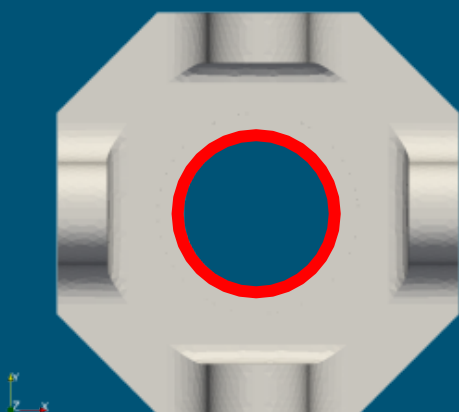


Elastic inherent strain: runtime 2m on RTX 5000 mobile GPU



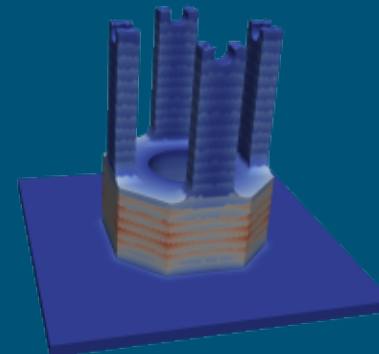
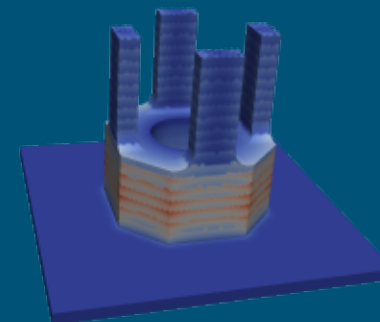
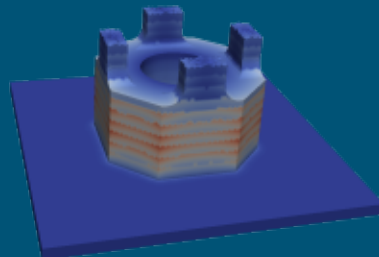
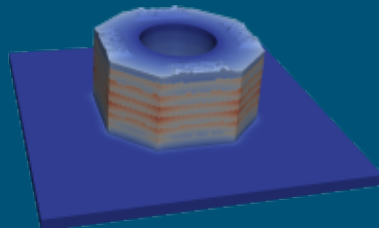
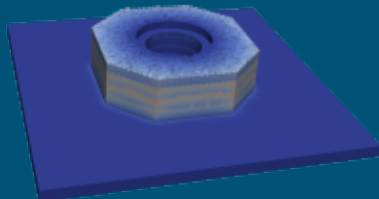
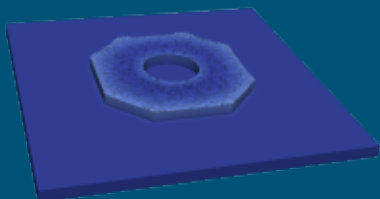


## Process Aware Example

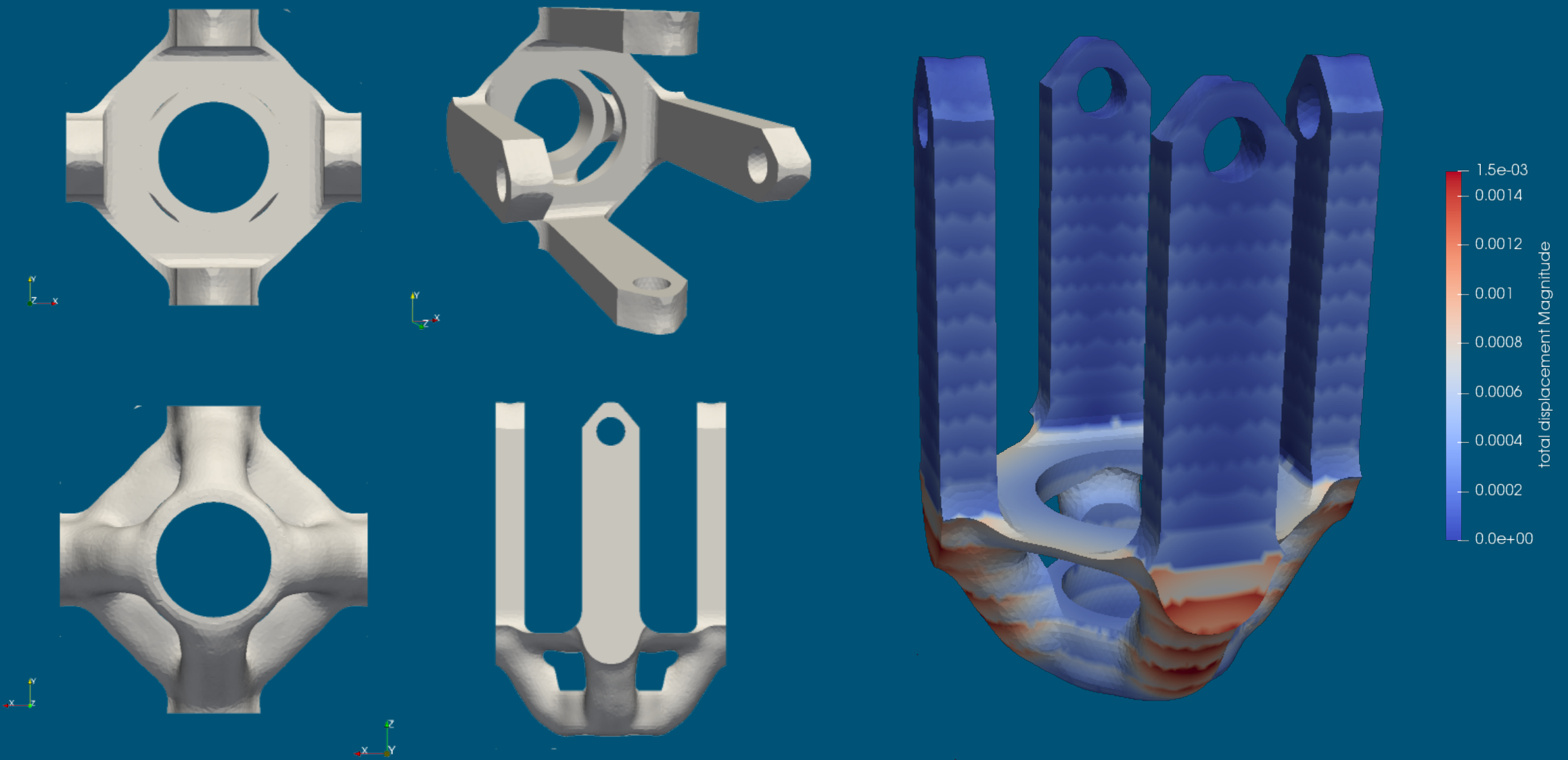


Performance: Compliance

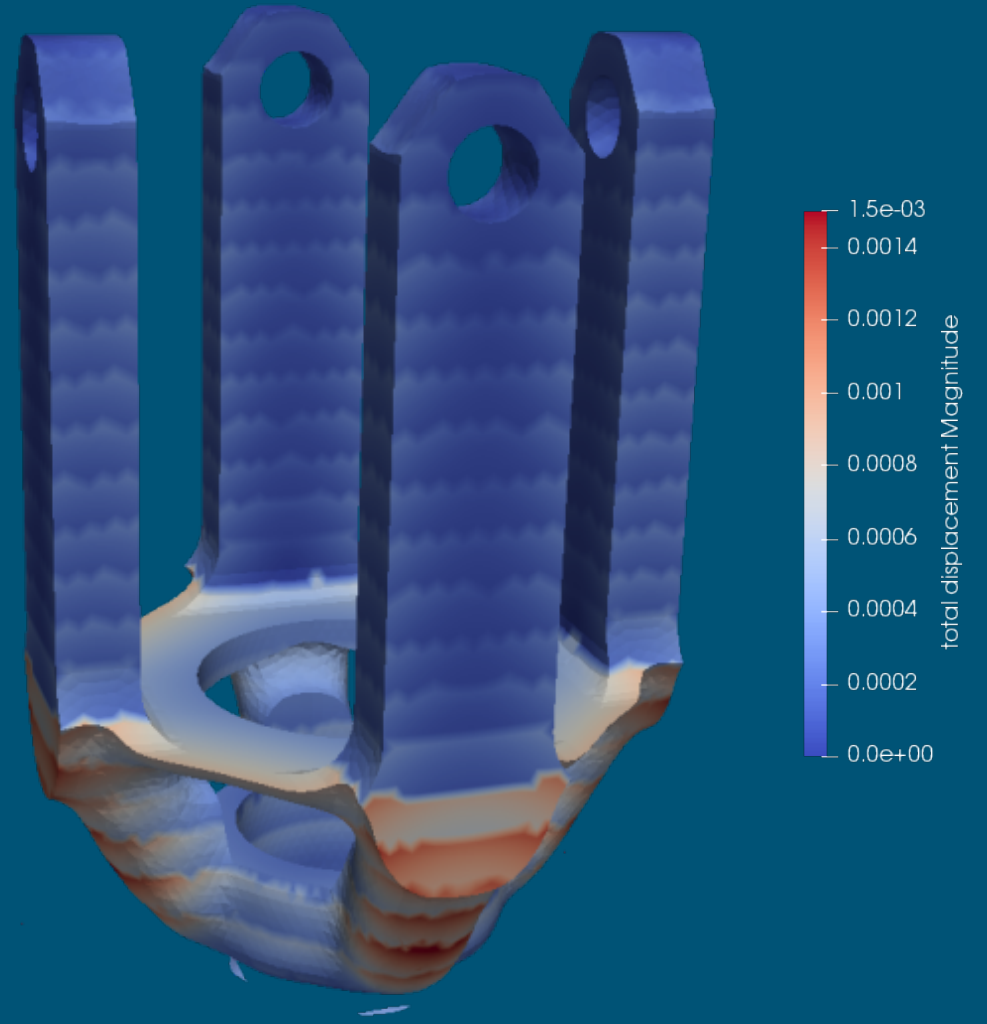
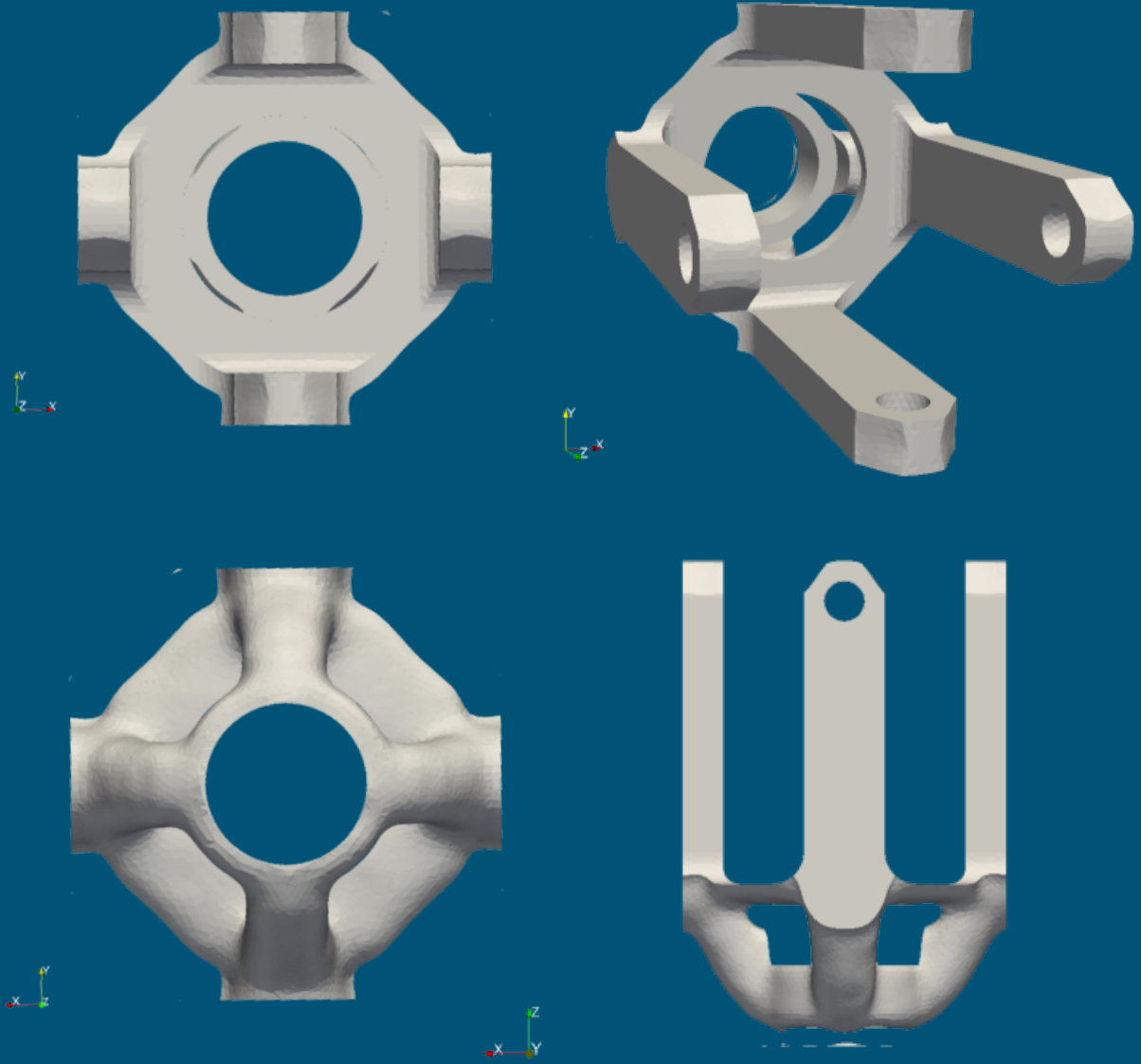
Manufacturability: Residual energy



Manufacturability priority: 0.01  
Displacement: 5.236 mm  
Residual Energy: 4124 KJ  
Deflection: -0.0405 mm

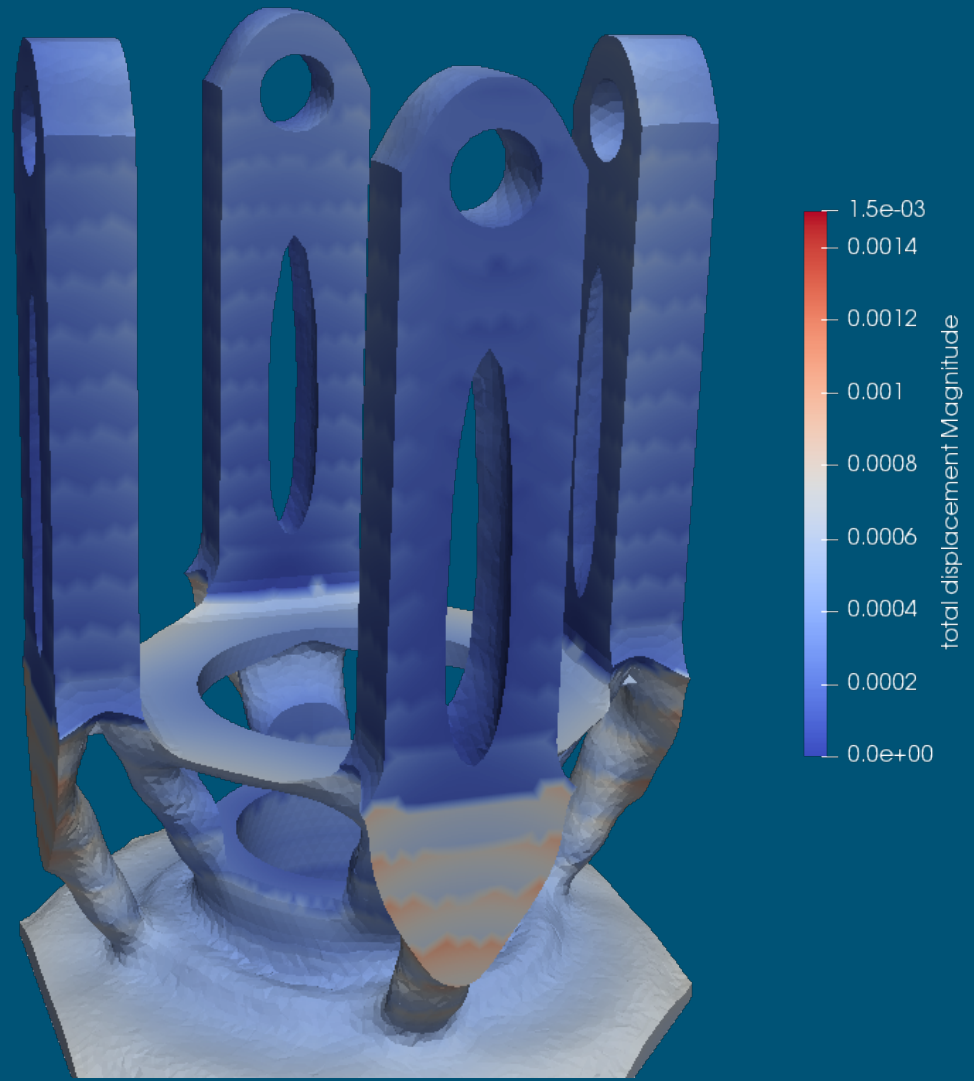
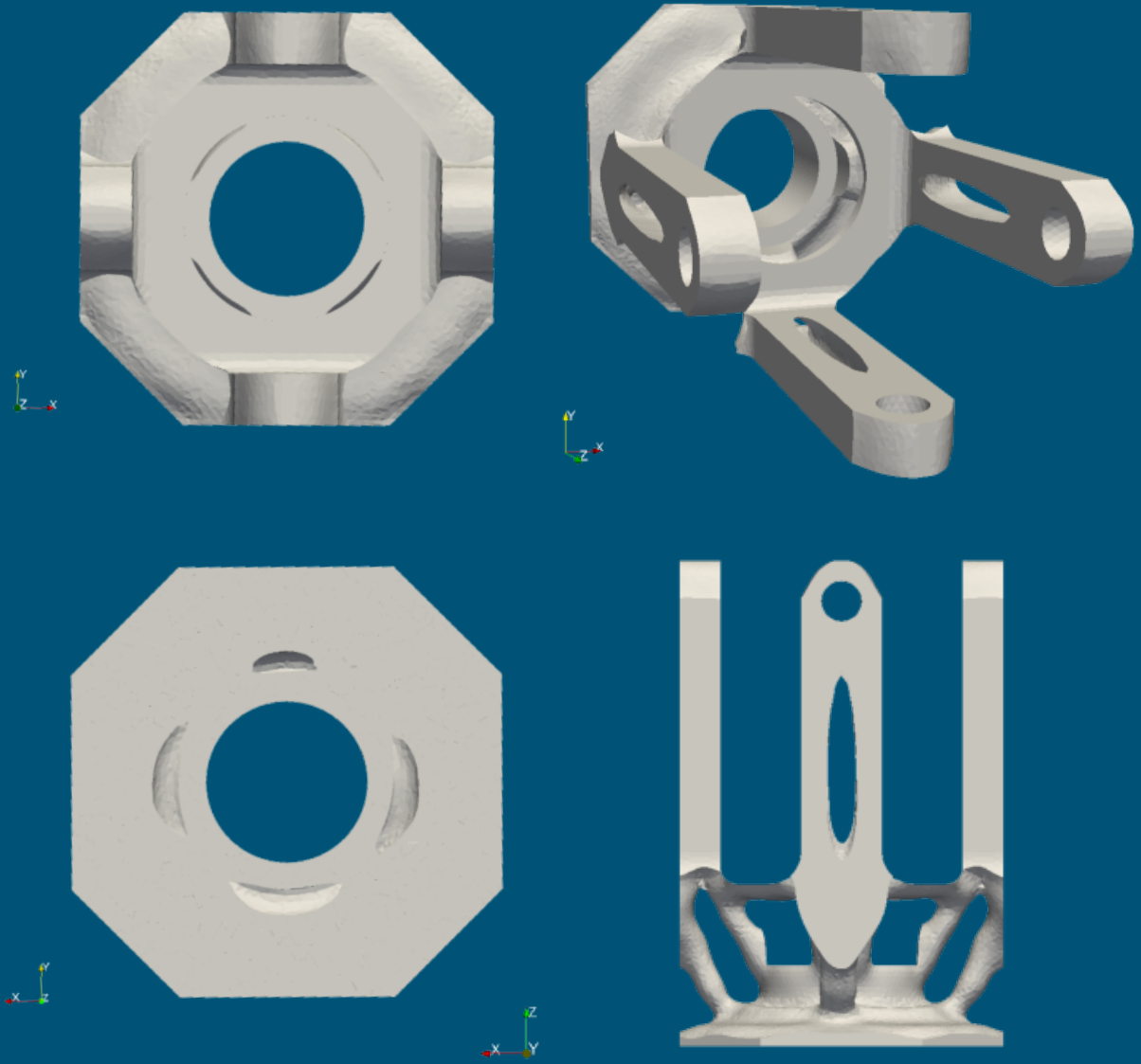


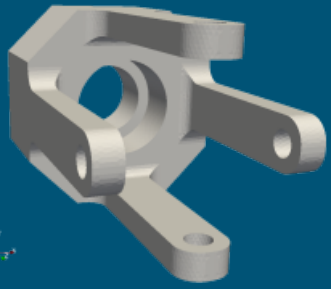
Manufacturability priority: 0.3  
Displacement: 5.551 mm (+6.0%)  
Residual Energy: 3720 KJ (-9.8%)  
Deflection: -0.029 mm (-28.4%)





Manufacturability priority: 0.5  
Displacement: 10.8 mm (+106.3%)  
Residual Energy: 1645 KJ (-60.0%)  
Deflection: 0.155 mm (-484.0%)





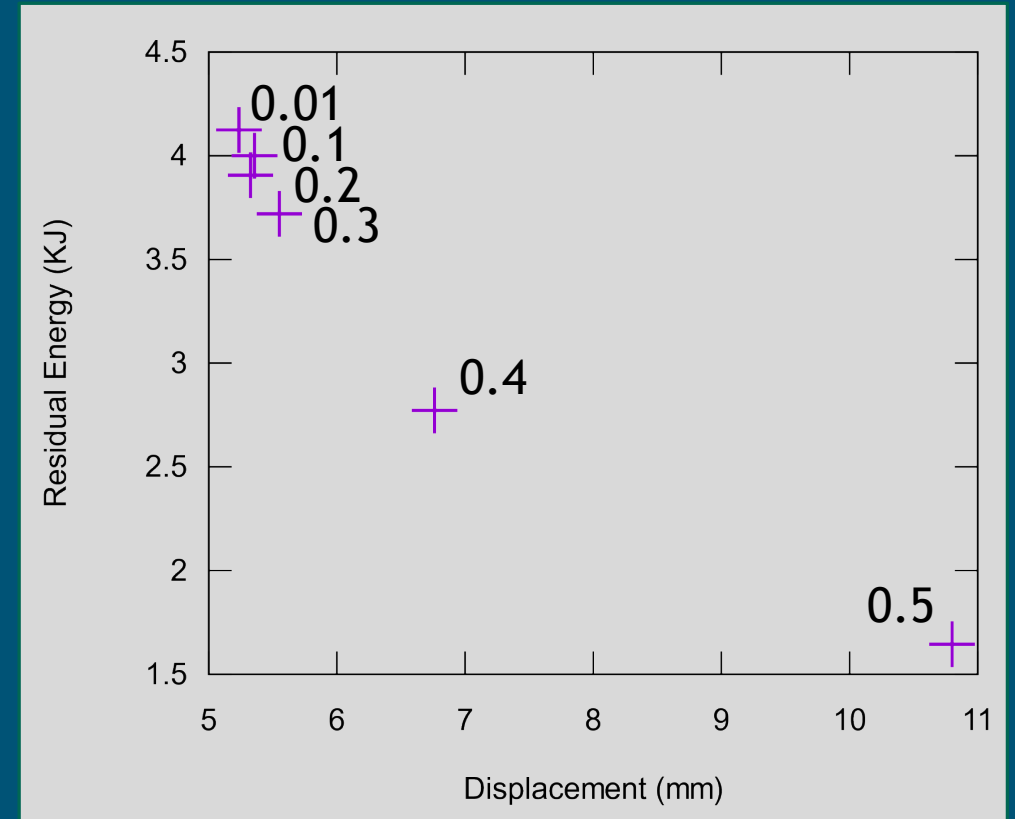
## Process Aware Example

### Observations:

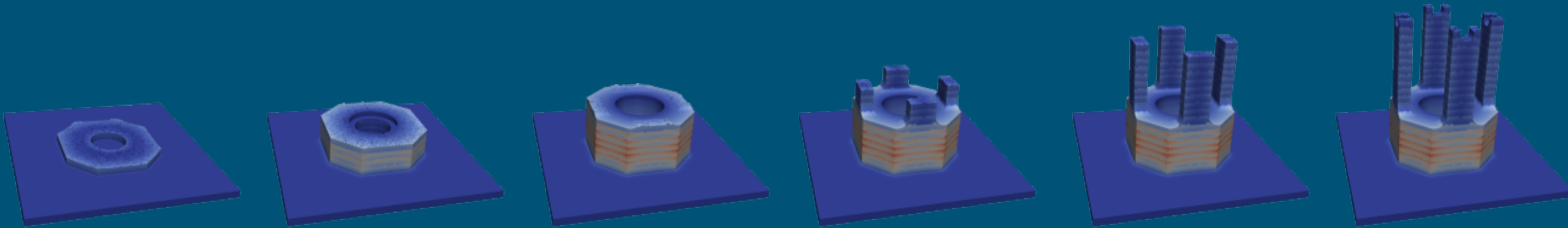
1. The run-time for process-aware TO with elastic inherent strain is practical. Roughly 90 minutes on a mobile GPU (RTX 5000, 0.5 Tflops).
2. Significant decreases in residual energy are associated with sparser structure.

### Next Steps:

1. More criteria (displacement, stress p-norm)
2. Plasticity (needs hatching)
3. Differentiable support structure
4. Thermo-plasticity



Trade-off between compliance and residual energy. Point labels are the priority assigned to residual energy.



Thank you!

