IEEE TRANSACTIONS ON
SMART GRID

Networked Microgrids Planning Through Chance Constrained Stochastic Conic Programming

Xiaoyu Cao, *Student Member, IEEE,* Jianxue Wang, *Senior Member, IEEE,* and Bo Zeng, *Member, IEEE*

***Abstract*—This paper presents a chance constrained stochastic conic program model for networked microgrids planning. Under a two-stage optimization framework, we integrate a multi-site microgrids investment problem and two sets of operational problems that correspond to the grid-connected and islanding modes, respectively. To handle the uncertain nature of renewable energy generation and load variation, as well as the contingent islanding caused by external disruptions, stochastic scenarios are employed to capture randomness and a joint chance constraint is introduced to control the operational risks. A second-order conic program (SOCP) formulation is also utilized to accurately describe the AC optimal power flow (OPF) in operational problems. As the resulting mixed integer SOCP model is computationally difficult, we customize the bilinear Benders decomposition with non-trivial enhancement techniques to deal with practical instances. Numerical results on 5- and 69-bus networked microgrids demonstrate the effectiveness of the proposed planning model and the superior performance of our solution algorithm.**

***Index Terms*—Networked microgrids, multi-site resource planning, chance constrained stochastic program, second-order conic program, bilinear Benders decomposition.**

I. Introduction

The growing proliferation of microgrids motivates their interconnection to make a more reliable, secure, and resilient network near the customer-side [1]–[3]. Networking a few neighboring microgrids enables them to share the generation, storage, and reserve resources, which prompts the whole community to operate in a more economical and efficient way. Also, each individual microgrid can benefit from the reliability improvement due to the backup of others, which ensures the power-supply continuity in case of emergency events, e.g., utility contingencies or natural disasters [4], [5]. To achieve the full strength of the aforementioned advantages, we believe that a networked microgrids system needs to be properly configured and analytically studied.

As mentioned in [6]–[8], the ownership of microgrids could belong to the utility (e.g., grid operator and government), local community, electricity retailers, end consumers, or a hybrid of above. According to the ownership, the microgrids planning can be implemented in a centralized or decentralized way. In this study, we consider that the microgrids are owned by a sole stakeholder (e.g., the utility) and planned in a centralized manner. We note that the centralized planning situation exists in many practical systems, e.g., some utility microgrids with a single owner in U.S. [1], [9], Europe [10], and China [11]. Under a centralized environment, the main task of networked microgrids planning (NMP) is to optimize the siting and sizing of multiple interconnected microgrids in a distribution network, aiming to maximize the total investment and operational benefits with guaranteed system performances, e.g., reliability, flexibility, and efficiency.

Comparing to the single microgrid planning, which has been heavily studied in literatures [12]–[14], the planning issues of networked microgrids are much more complicated. Essentially, the planning problems of networked microgrids, which fall into a multi-site resource planning category, must follow the system-wide power balance principle by including complex power flow representations. Also, it should consider relevant reactive power, voltage, and congestion issues. As a result, the NMP is actually a non-linear combinatorial optimization problem, which could be very challenging for a practical network. But for the single-site system, the network complications can be ignored and the resulting planning formulation is drastically simpler. Another critical issue for NMP is to manage the multi-source uncertainties associated with the internal dynamics and the external circumstances of microgrids. On one hand, the intermittency of renewable energy generation as well as the inaccuracy of load forecast within microgrids bring non-trivial uncertainties into the planning data [14]. On the other hand, the uncertainties of external disruptions, e.g., the forced or scheduled maintenance of upstream grid, may drive the entire networked system to transit from normal grid-connected mode to islanded operation mode [15]. Since the needs for islanding (a salient feature of microgrids) could lead to costly investment, it is necessary to make a trade-off between the cost-effectiveness and risk-immunity in planning decisions.

With the aforementioned challenges, it demands for strong tools to analytically consider the impact of network issues and multi-source uncertainties in NMP problems, which, however, have not yet been fully addressed in the current literatures. Many of the existing studies, e.g., [16]–[23], have concerned the internal uncertainties, e.g., intermittent generation and variable load. Ref [16] proposed a probabilistic minimal-cut based approach for the interconnection planning of multiple microgrids considering the stochastic output of distributed energy resources (DERs). Ref [17] studied the meteorological data analysis of renewable energy generation to support the

This work was supported in part by the Key Research and Development Program of Shaanxi (2017ZDCXL-GY-02-03). The work of Xiaoyu Cao was supported in part by the China Scholarship Council (201706280137). The work of Bo Zeng was supported in part by National Science Foundation under grant CMMI 1635472 and in part by DoE under grant OE0000842

Xiaoyu Cao and Jianxue Wang are with the School of Electrical Engineering and Shaanxi Key Laboratory of Smart Grid, Xi'an Jiaotong University, Xi'an, Shaanxi 710049 China (e-mail: cxykeven2013@stu.xjtu.edu.cn; jxwang@mail.xjtu.edu.cn.)

Bo Zeng is with the Department of Industrial Engineering and the Department of Electrical and Computer Engineering, University of Pittsburgh, Pittsburgh, PA 15106 USA (e-mail: bzeng@pitt.edu).

Digital Object Identifier: 10.1109/TSG.2019.2908848

cooperative planning of interconnected microgrids. Ref [18] presented a two-stage stochastic program (SP) model to co-optimize the investment plans of microgrids, generators, and transmission lines under uncertain contingencies and load growth. Ref [19], [20] adopted the heuristic-based SP methods to handle the randomness of DERs' output and load in NMP problems. Ref [21]–[23] developed the bi-level program frameworks to decide the sectioning and configuration scheme of microgrids in a distribution network, while the uncertainties were captured by scenario-based methods and robust optimization respectively. Nevertheless, the uncertainties of external disruptions, which may cause microgrids islanding and further challenge the system performance, were often neglected in the existing literatures [12], [24]. Moreover, due to the non-linearity and non-convexity introduced by network representations (e.g., AC power flow equations), many of the current NMP formulations can only be solved by heuristic approaches [19]–[23], which, however, generally do not guarantee the global optimality of their solutions. The aforementioned research gaps motivate our exploration on more realistic modeling and analytical computation tools for the configuration planning of networked microgrids.

In this paper, we capture the sequential and interdependent microgrids' investment and operation decisions using a two-stage framework, and thus formulate the NMP as a two-stage chance constrained stochastic conic program. Our formulation incorporates the multi-site investment scheme at the first-stage and the dual-mode (i.e., grid-connected and islanding modes) operational models at the second-stage. Based on two sets of operational problems, we combine the SP and chance constrained program (CCP) to address the multi-source uncertainties: 1) the SP is applied to manage the internal generation and load uncertainties under grid-connected mode; and 2) the CCP is included to ensure the feasibility of islanded operation subject to external uncertainties, which provides a trade-off scheme to balance the cost-benefit and the immunity against operational risks. Also, the actual operation of multi-microgrid network is captured by the non-linear branch flow model [25]. We mention that this power flow model actually can be convexified into a computationally friendly SOCP, which ensures its solution's global optimality with respect to our original planning formulation under mild conditions [26]–[28]. Moreover, to handle the challenging mixed-integer SOCP formulation of the proposed planning problem, we customize the bilinear Benders decomposition method [14], [29] with strong duality and make non-trivial enhancements through the techniques of *Jensen's inequalities* and *Pareto-optimal cuts*, which yields a strong computational capacity.

Comparing to the current literatures, our main contributions can be summarized as:

1) A holistic NMP model is presented to consider the multi-site microgrids investment and dual-mode network operations.

2) An integrated chance constrained stochastic framework is proposed to manage the multi-source uncertainties associated with different operational modes of networked microgrids.

3) An exact and efficient decomposition algorithm is developed to analytically solve the proposed mixed-integer SOCP formulation.

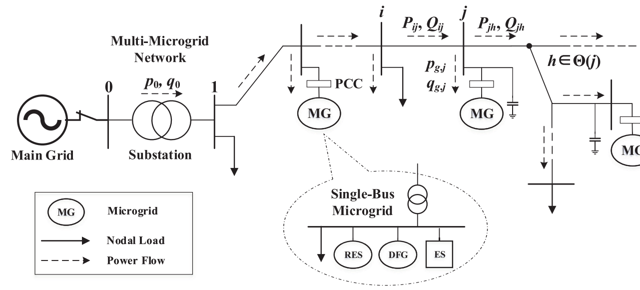


Fig. 1. Conceptual Structure of Networked Microgrids

The remainder of this paper is organized as below. Section 2 formulates the chance constrained stochastic NMP model. Section 3 presents details of the enhanced bilinear Benders decomposition method. Section 4 shows the results of numerical tests. Finally, conclusions are drawn in Section 5.

II. Problem Formulation

The conceptual architecture of networked microgrids is shown in Fig. 1. The multi-microgrid distribution network is assumed to hold a radial topology with buses indexed by $j \in \Omega$ and branches indexed by $(i, j) \in E$. The substation bus is indexed by $j = 0$. The buses with microgrids integration (via the points of common coupling (PCC)) are indexed by $j \in M$, and the other buses are indexed by $j \in \Omega/M$. The (unique) sending end and the receiving ends of bus j are indexed by i and $h \in \Theta(j)$. Also, we regard each individual microgrid as a single-bus system even though it could have inner network structure at a lower voltage class. Note that these inner structures can be considered through extended network modeling, but are not in the scope of this study.

Under these preliminaries, we formulate a two-stage chance constrained stochastic NMP model. The centralized investment decisions are made at the first stage to deploy the microgrids on regional network as well as to decide the sizing plan of DERs for each microgrid. Then, two sets of operational SOCPs are included at the second stage to coordinately dispatch the networked microgrids under grid-connected and islanding modes, respectively. Without loss of generality, the candidate DERs refer to renewable energy sources (RES), dispatchable fuel generators (DFG), and energy storage devices (ES) [14].

A. 1st-Stage Problem: Investment Decisions

The first-stage problem is to minimize the annualized cost of microgrids investment before the realization of uncertain factors, which can be written as:

$$\min \sum_{j\in\Omega} \sum_{k\in\{\text{res},\text{dfg}\}} \phi_k IC_k \bar{p}_k X_{k,j} + \sum_{j\in\Omega} \sum_{k\in\text{es}} \phi_k (EC_k \bar{e}_k + PC_k \bar{p}_k) X_{k,j} \quad (1)$$

$$\phi_k = \frac{R(1+R)^{L_k}}{(1+R)^{L_k} - 1}, \quad \forall k \in \{\text{res}, \text{dfg}, \text{es}\} \quad (2)$$

$$s.t.\ z_j \underline{X}_{k,j} \le X_{k,j} \le z_j \overline{X}_{k,j}, \forall k \in \{\text{res}, \text{dfg}, \text{es}\}, \forall j \in \Omega \quad (3)$$

$$\sum_{j\in\Omega} z_j \leq N_{\mathrm{mg}} \tag{4}$$

$$X_{k,j} \in \mathbb{Z}_+, z_j \in \{0,1\}, \quad \forall k \in \{\mathrm{res}, \mathrm{dfg}, \mathrm{es}\}, \forall j \in \Omega \tag{5}$$

As shown in (1), the annualized investment cost is evaluated by multiplying the capital cost of candidate DERs with their capital recovery factors (ϕ_k), as defined in (2). Also, the capacities of candidate DERs are constrained by (3) to reflect the limitations due to geographical, financial, and environmental conditions, and the cardinality constraint in (4) introduces an upper bound (N_{mg}) on the total number of microgrid sites.

Remark 1. *Note that the optimal microgrids siting is actually a complicated combinatorial problem [19], [30], [31]. For the sake of illustration, a conceptual siting model is adopted as in* (1)-(5) *with several of the practical factors, e.g., financial and geographical issues. Moreover, our siting model can be easily extended by incorporating other practical considerations (e.g., by adding linear or conic constraints) as well as the preference of system planners (e.g., by fixing part of the siting variables).*

B. 2nd-Stage Operational Problem in Grid-Connected Mode

The first set of operational problem is to perform the coordinated scheduling of networked microgrids given a finite set of stochastic scenarios $S = \{\xi_s | s = 1, 2, \ldots, N_s\}$, which are defined under typical days $D(s)$ to capture the uncertainties of RES generation (i.e., $\delta_k^t(\xi_s)$) and load variation (i.e., $p_{l,j}^t(\xi_s)$ and $q_{l,j}^t(\xi_s)$). Each scenario follows a probability π_s. The grid-connected scheduling problem (GSP) under scenario $s \in S$ can be written as:

$$\min \; C_{\mathrm{om}}^s + C_{\mathrm{pt}}^s + C_{\mathrm{loss}}^s + C_{\mathrm{lc}}^s \tag{6}$$

$$C_{\mathrm{om}}^s = \hbar \sum_{t\in D(s)} \sum_{j\in M} \sum_{k\in\{\mathrm{res,dfg,es}\}} OC_k \bar{p}_k X_{k,j} \Delta_t + \hbar \sum_{t\in D(s)} \sum_{j\in M} \sum_{k\in \mathrm{dfg}} FC_k p_{k,j}^{s,t} \Delta_t \tag{7}$$

$$C_{\mathrm{pt}}^s = \hbar \sum_{t\in D(s)} \sum_{j\in M} \rho^t p_{g,j}^{s,t} \Delta_t \tag{8}$$

$$C_{\mathrm{loss}}^s = \hbar \sum_{t\in D(s)} \sum_{(i,j)\in E} \vartheta r_{ij} \ell_{ij}^{s,t} \Delta_t \tag{9}$$

$$C_{\mathrm{lc}}^s = \hbar \sum_{t\in D(s)} \sum_{j\in\Omega} (\varsigma_{\mathrm{p}} p_{\mathrm{lc},j}^{s,t} + \varsigma_{\mathrm{q}} q_{\mathrm{lc},j}^{s,t}) \Delta_t \tag{10}$$

$s.t.$

$$\sum_{k\in\{\mathrm{res,dfg}\}} p_{k,j}^{s,t} + \sum_{k\in \mathrm{es}} (d_{k,j}^{s,t} - c_{k,j}^{s,t}) + p_{g,j}^{s,t} = p_{l,j}^t(\xi_s) - p_{\mathrm{lc},j}^{s,t}, \quad \forall j \in M, \forall t \tag{11}$$

$$\sum_{k\in \mathrm{res}} q_{k,j}^{s,t} + q_{c,j} + q_{g,j}^{s,t} = q_{l,j}^t(\xi_s) - q_{\mathrm{lc},j}^{s,t}, \quad \forall j \in M, \forall t \tag{12}$$

$$0 \leq p_{k,j}^{s,t} \leq \delta_k^t(\xi_s) \bar{p}_k X_{k,j}, \quad \forall k \in \mathrm{res}, \forall j \in M, \forall t \tag{13}$$

$$|q_{k,j}^{s,t}| \leq \sqrt{\bar{s}_k^2 - \bar{p}_k^2} X_{k,j}, \quad \forall k \in \mathrm{res}, \forall j \in M, \forall t \tag{14}$$

$$\underline{p}_k X_{k,j} \leq p_{k,j}^{s,t} \leq \bar{p}_k X_{k,j}, \quad \forall k \in \mathrm{dfg}, \forall j \in M, \forall t \tag{15}$$

$$0 \leq d_{k,j}^{s,t} \leq \bar{p}_k X_{k,j}, \quad \forall k \in \mathrm{es}, \forall j \in M, \forall t \tag{16}$$

$$0 \leq c_{k,j}^{s,t} \leq \bar{p}_k X_{k,j}, \quad \forall k \in \mathrm{es}, \forall j \in M, \forall t \tag{17}$$

$$e_{k,j}^{s,t+1} = e_{k,j}^{s,t} + \eta_k c_{k,j}^{s,t} - d_{k,j}^{s,t}/\eta_k, \; \forall k \in \mathrm{es}, \forall j \in M, \forall t \tag{18}$$

$$\underline{e}_k X_{k,j} \leq e_{k,j}^{s,t} \leq \bar{e}_k X_{k,j}, \quad \forall k \in \mathrm{es}, \forall j \in M, \forall t \tag{19}$$

$$\sum_{t\in D(s)} (\eta_k c_{k,j}^{s,t} - d_{k,j}^{s,t}/\eta_k) = 0, \quad \forall k \in \mathrm{es}, \forall j \in M \tag{20}$$

$$P_{ij}^{s,t} - r_{ij}\ell_{ij}^{s,t} - \sum_{h\in\Theta(j)} P_{jh}^{s,t} = \begin{cases} p_{g,j}^{s,t}, \; \forall j \in M, \forall t \\ p_{l,j}^{s,t} - p_{\mathrm{lc},j}^{s,t}, \; \forall j \in \Omega/M, \forall t \end{cases} \tag{21}$$

$$Q_{ij}^{s,t} - x_{ij}\ell_{ij}^{s,t} - \sum_{h\in\Theta(j)} Q_{jh}^{s,t} = \begin{cases} q_{g,j}^{s,t}, \; \forall j \in M, \forall t \\ q_{l,j}^{s,t} - q_{\mathrm{lc},j}^{s,t}, \; \forall j \in \Omega/M, \forall t \end{cases} \tag{22}$$

$$v_i^{s,t} - v_j^{s,t} = 2(r_{ij}P_{ij}^{s,t} + x_{ij}Q_{ij}^{s,t}) - (r_{ij}^2 + x_{ij}^2)\ell_{ij}^{s,t}, \quad \forall (i,j) \in E, \forall t \tag{23}$$

$$\ell_{ij}^{s,t} v_i^{s,t} = (P_{ij}^{s,t})^2 + (Q_{ij}^{s,t})^2, \quad \forall (i,j) \in E, \forall t \tag{24}$$

$$\underline{U}_j^2 \leq v_j^{s,t} \leq \overline{U}_j^2, \quad \forall j \in \Omega, \forall t \tag{25}$$

$$0 \leq \ell_{ij}^{s,t} \leq \overline{I}_{ij}^2, \quad \forall (i,j) \in E, \forall t \tag{26}$$

$$-\overline{P}_{\mathrm{sub}} \leq p_0^{s,t} = \sum_{k\in\Theta(0)} P_{0k}^{s,t} \leq \overline{P}_{\mathrm{sub}}, \quad \forall t \tag{27}$$

$$-\overline{Q}_{\mathrm{sub}} \leq q_0^{s,t} = \sum_{k\in\Theta(0)} Q_{0k}^{s,t} \leq \overline{Q}_{\mathrm{sub}}, \quad \forall t \tag{28}$$

The second-stage objective (6) contains the operation and maintenance (O&M) cost (C_{om}^s), power transaction cost (C_{pt}^s), network loss cost (C_{loss}^s), and load reduction cost (C_{lc}^s), which are evaluated through (7)-(10) respectively. In (7), the fixed O&M cost is proportional to DERs installation, while its varying part refers to the fuel cost of DFGs. Eq. (8) calculates the total cost incurred by the power transactions between the microgrids cluster (as a whole) and the main grid [32], [33]. The network loss and load curtailment are penalized by specific cost factors as in (9) and (10). All these cost terms are evaluated on a daily basis and then scaled to derive yearly values by a factor $\hbar$.

Eqs. (11)-(20) represent the operational constraints of each individual microgrid. Constraints (11)-(12) ensure the active and reactive power balance within each microgrid. The microgrids are supplied by both the DERs and the external network. The power transactions through the PCCs could be positive (power procurement status) or negative (power selling status). Note also that the reactive power can be supplied by existing shunt capacitors as well as the power electronic interface (e.g., converter) equipped with RES [34]. Additionally, the upper restrictions on load reduction variables $p_{\mathrm{lc},j}^{s,t}$ and $q_{\mathrm{lc},j}^t$ can be relaxed since they are heavily penalized in the objective function. In (13), the active power output of RES is calculated as the product of its installed capacity and time-varying factor $\delta_k^t(\xi_s)$, which mainly depends on weather conditions (e.g., solar irradiation and ambient temperature). The reactive power served by the converters of RES is constrained by (14). The generation constraint of DFGs is given in (15). The power and energy states of ES are constrained by (16)-(20). Note that in (20), the energy state of ES is kept as its initial value after a daily charge-discharge cycle.

Eqs. (21)-(28) denote the operational constraints of multi-microgrid network (as shown in Fig. 1). A nonlinear branch flow model is defined by (21)-(24). With a recoverable angle relaxation [28], we only need to deal with the magnitude representations of these power flow equations. The square of magnitudes of nodal voltage and branch current are constrained by (25) and (26). The exchanged active/reactive power between networked microgrids and up-stream grid are constrained by (27) and (28), respectively. Besides, the reference voltage level at substation bus is specified as U_0.

Remark 2. *Since* (21)-(24) *introduce the non-convexity to our mixed integer nonlinear planning formulation, its global optimality cannot be guaranteed. To convexify* (21)-(24)*, we relax the equalities in* (24) *to derive a set of rotated second-order conic constraints:*

$$(P_{ij}^{s,t})^2 + (Q_{ij}^{s,t})^2 \leq \ell_{ij}^{s,t} v_i^{s,t}, \quad \forall (i,j) \in E, \forall t, \forall s \tag{29}$$

As a result, we obtain a convex and computationally friendly SOCP relaxation, which includes (6)-(23)*,* (25)-(28) *and* (29) *(denoted by* ***GSP-r****), to the original* ***GSP****. Moreover, through the sufficient conditions presented in [28], [35], this SOCP relaxation is exact and its convexity guarantees a global optimal solution to the original GSP.*

C. 2nd-Stage Operational Problem in Islanding Mode

The second set of operational problem is to guarantee the feasibility of islanding under external disruptions. The capacity sufficiency for islanded operation is validated based on a finite set of scenarios $I = \{\omega_n | n = 1, 2, \dots, N_I\}$, which are defined under τ-hour islanding periods [15]. Each islanding scenario n occurs with a probability π_n. According to *IEEE Standard 1547.4* [36], the major concern for microgrids islanding is to maintain the reliability and operational security. As the economic target is less critical for the islanded operation, we only require that the operational cost (Λ_n), which includes the O&M cost, network loss cost, and load reduction cost, is no larger than an expected value by imposing the cost-bound constraints. Note that the power transaction cost, which originates from the exchanged power with main grid, is excluded when computing Λ_n. Hence, the islanding validation problem (IVP-r) under scenario $n \in I$ can be written as:

$$\Lambda_n = C_{\text{om}}^n + C_{\text{loss}}^n + C_{\text{lc}}^n \leq \bar{\Lambda} \tag{30}$$

$$\Xi^{n,t}\{(11) - (23), (25) - (26), (29)\}, \quad \forall t \in \tau(n) \tag{31}$$

$$p_0^{n,t} = 0, \ q_0^{n,t} = 0, \quad \forall t \in \tau(n) \tag{32}$$

where the cost-bound constraint is given in (30). In (31), the operational constraints in GSP-r are re-defined in constraints set $\Xi^{n,t}$ under each islanding scenario. Besides, the boundary conditions for islanded operation are clarified in (32)

Since the load reduction is penalized in Λ_n, the variables that slack constraints set $\Xi^{n,t}$ are actually restricted by our cost-bound constraints. In this regard, the cost-bound requirements in (30) could become too restrictive for some very adversarial scenarios (i.e., with high risks of load reduction). The full consideration of cost-bound may force the planners to generate and implement a highly capacitated and costly planning scheme. To achieve a trade-off between the cost-benefit and the immunity against islanding risks, we impose a chance constraint on (30) so that it is allowed to be relaxed with a pre-defined small probability. Note also that the chance constraint of (30) involves multi-variate integration, which makes it very difficult to be represented by a closed-form expression. Hence, we follow the strategies in [29] to express the chance constraints by a scenario-based bilinear formulation as below.

$$\Lambda_n(1 - u_n) \leq \bar{\Lambda}, \quad \forall n \in I \tag{33}$$

$$\sum_{n \in I} \pi_n u_n \leq \varepsilon \tag{34}$$

Remark 3. *Constraints* (33)-(34) *ensure that the cost-bound constraint is satisfied with a probability greater than or equal to $1 - \varepsilon$, where ε is the risk tolerance level. Note that binary variable u_n is utilized to indicate that the full requirements of islanding scenario n is imposed or not. Clearly, when $u_n = 0$, the cost-bound constraint under islanding scenario n must be satisfied, and when $u_n = 1$, otherwise. The total probability of islanding scenarios that can be partially deactivated is restricted by the joint chance constraint* (34)*.*

Actually, the chance constraints (33)-(34) *can be equivalently interpreted as value-at-risk (VaR) constraints [37], [38], which perform as the risk measure. Hence, by adjusting the parameter ε, the chance constraints can be applied to quantitatively control the operational risks of microgrids islanding.*

D. Full Formulation of Networked Microgrids Planning

Combining (1)-(5) with (6)-(23), (25)-(29) and (31)-(34), we obtain the full formulation of two-stage chance constrained stochastic NMP model. For convenience, its compact matrix form is given as below.

$$\zeta = \min \ c^T x + \sum_{s \in S} \pi_s d^T y_s \tag{35}$$

$$s.t. \ Ax \leq b \tag{36}$$

$$F y_s = f_s, \quad \forall s \in S \tag{37}$$

$$W_s x + R y_s \geq v, \quad \forall s \in S \tag{38}$$

$$\|G_m y_s\|_2 \leq g_m^T y_s, \quad \forall s \in S, \forall m \in \mathcal{L}_s \tag{39}$$

$$L_0 y_s \geq l_0, \quad \forall s \in S \tag{40}$$

$$x \in \mathbb{Z}_+, y_s \in \mathbb{R}, \quad \forall s \in S \tag{41}$$

$$q^T y_n (1 - u_n) \leq \bar{\Lambda}, \quad \forall n \in I \tag{42}$$

$$H y_n = h_n, \quad \forall n \in I \tag{43}$$

$$V_n x + K y_n \geq w, \quad \forall n \in I \tag{44}$$

$$\|B_m y_n\|_2 \leq r_m^T y_n, \quad \forall n \in I, \forall m \in \mathcal{L}_n \tag{45}$$

$$\sum_{n \in I} \pi_n u_n \leq \varepsilon, \quad \forall n \in I \tag{46}$$

$$y_n \in \mathbb{R}, u_n \in \{0, 1\}, \quad \forall n \in I \tag{47}$$

The objective function is abstracted in (35). The vector x represent the first-stage variables, while y_s and y_n denote the second-stage variables defined under stochastic and islanding scenarios. The investment decisions are constrained by (36), corresponding to constraints (3)-(4). Constraints (37)-(38) denote the linear equalities and inequalities in GSP-r, i.e., (11)-(23), (25), (26). Constraint (39) stands for the conic constraints in (29), where m is the index of such constraints. Constraint (40) represents constraints (27)-(28), which are only for grid-connected operation. Constraints (42) and (46) denote the joint chance constraints in IVP-r, i.e., (33) and (34). Constraints (43)-(45) correspond to the rest constraints in IVP-r.

III. Enhanced Bilinear Benders Decomposition Method for Stochastic Mixed Integer SOCP

We observe that the chance constrained stochastic mixed integer SOCP in (35)-(47) is a large-scale non-convex formulation, which is very challenging for professional solvers. To address such computational challenges, we follow the strategy in [14], [29] to develop and customize a decomposition method, i.e., the bilinear Benders decomposition method, that can significantly improve our solution capacity. Also, two enhancement techniques are adopted for better performance.

The framework of classical Benders decomposition for stochastic linear programming (LP) involves a master problem and a set of subproblems. The latter one is actually the dual of the second-stage problem in every scenario. Note that, different from LP, it is not always the case that an SOCP formulation has the strong duality, which actually could be the case for the popular AC OPF SOCP formulation (as revealed by the numerical study in [39]). Note that if the strong conic duality fails, there is no guarantee that Benders decomposition for mixed-integer SOCP leads to exact solutions. However, to the best of our knowledge, although conic dual problem has been utilized for algorithm development (e.g., [40]–[42]), this issue was rarely mentioned in any publication. Based on an analytical study in [39], our second-stage AC OPF problems are observed to fit a sufficient condition that guarantees the strong duality of SOCP formulation, which thus ensures the exactness of our bilinear Benders reformulation.

1) Feasibility-Check Subproblem: We first define a subproblem ($\mathbf{SP}_*$) to check the feasibility of first-stage solution (denoted by $\hat{x}$) with respect to the chance constraints (42) and (46) under islanding scenarios. $\mathbf{SP}_*$ is formulated as below.

$$\mathbf{SP}_* : \Delta_f = \min \sum_{n\in I} t_n \tag{48}$$

$$s.t.\ t_n \geq \hat{\Upsilon}_n(1-u_n) - \bar{\Lambda}, \quad \forall n \in I \tag{49}$$

$$\sum_{n\in I} \pi_n u_n \leq \varepsilon \tag{50}$$

$$t_n \geq 0, u_n \in \{0,1\}, \quad \forall n \in I \tag{51}$$

where Υ_n is to denote the operational cost from the islanding scenario n, which can be attained by computing the cost-minimization counterpart of IVP-r as in the following.

$$\hat{\Upsilon}_n = \min_{y_n} \{q^T y_n :\ (43)-(47)\}, \quad \forall n \in I \tag{52}$$

Remark 4. *As mentioned in [14] that if $\Delta_f = 0$, we can conclude that the current $\hat{x}$ is a feasible solution towards constraints* (42) *and* (46)*. Otherwise, a different first stage investment plan should be generated in the next iteration.*

2) Optimality Subproblems: Next, two types of optimality subproblems are defined based on our operational SOCP problems. The first type $\mathbf{SP}^s_{\mathrm{A}}$, which corresponds to the GSP-r under each scenario $s \in S$, is to minimize the recourse cost $d^T y_s$ subject to constraints (37)-(40) for a given $\hat{x}$. The dual form of $\mathbf{SP}^s_{\mathrm{A}}$ is given as follows.

$$\mathbf{SP}^s_{\mathrm{A}} : \tilde{\Phi}_s = \max\ \mu_s^T f_s + \lambda_s^T(v - W_s\hat{x}) + \theta_s^T l_0 \tag{53}$$

$$s.t.\ F^T\mu_s + R^T\lambda_s + L_0^T\theta_s + \sum_{m\in\mathcal{L}_s}(G_m^T\gamma_s^m + \sigma_s^m g_m) = d \tag{54}$$

$$\|\gamma_s^m\|_2 \leq \sigma_s^m, \quad \forall m \in \mathcal{L}_s \tag{55}$$

$$\mu_s, \gamma_s^m \in \mathbb{R},\ \lambda_s, \theta_s, \sigma_s^m \in \mathbb{R}_+, \quad \forall m \in \mathcal{L}_s \tag{56}$$

where $\mu_s, \lambda_s, \theta_s$ are the dual variables of constraints (37), (38) and (40), while γ_s^m, σ_s^m are the dual variables of conic constraints for all $m \in \mathcal{L}_s$ in (39) . Φ_s is the operational cost from the stochastic scenario s. Note that GSP-r is bounded and also feasible since (37)-(40) can be slacked by load reduction variables. The dual solution of $\mathbf{SP}^s_{\mathrm{A}}$ provides a set of Benders optimality cuts OC^s_{A} as in (63).

Another type of optimality subproblem is needed to ensure the feasibility of (42)-(46) under each islanding scenario $n \in I$. Following an idea presented in [14] to deal with such constraint, we consider the left-hand-side of (42) and make use of its associated optimality cuts to achieve the feasibility. The second type of optimality subproblem ($\mathbf{SP}^n_{\mathrm{B}}$) is defined as following, which is in the dual form of (52):

$$\mathbf{SP}^n_{\mathrm{B}} : \tilde{\Upsilon}_n = \max\ \chi_n^T h_n + \psi_n^T(w - V_n\hat{x}) \tag{57}$$

$$s.t.\ H^T\chi_n + K^T\psi_n + \sum_{m\in\mathcal{L}_n}(B_m^T\nu_n^m + \kappa_n^m r_m) = q \tag{58}$$

$$\|\nu_n^m\|_2 \leq \kappa_n^m, \quad \forall m \in \mathcal{L}_n \tag{59}$$

$$\chi_n, \nu_n^m \in \mathbb{R},\ \psi_n, \kappa_n^m \in \mathbb{R}_+, \quad \forall m \in \mathcal{L}_n \tag{60}$$

Similarly, the dual solution of $\mathbf{SP}^n_{\mathrm{B}}$ (i.e., $\chi_n, \psi_n, \nu_n^m, \kappa_n^m$) yields another set of optimality cuts OC^n_{B} as in (65).

Remark 5. *Note that the primal forms of $\mathbf{SP}^s_{\mathrm{A}}$ and $\mathbf{SP}^n_{\mathrm{B}}$, i.e.,* (37)-(40) *and* (52)*, can be equivalently expressed as the **OPF-SOCP** in [39]. Then, due to the inclusion of load reduction variables (with relaxed upper bounds), the condition C1-(d) in [39] holds for $\mathbf{SP}^s_{\mathrm{A}}$ and $\mathbf{SP}^n_{\mathrm{B}}$, which thus ensures the strong duality of our SOCP subproblems.*

3) Investment Master Problem: Combining (35), (36), (42), and (46) with the Benders optimality cuts OC^s_{A} in (63) and OC^n_{B} in (65), the master problem ($\mathbf{MP}$) (in j-th iteration) of our networked microgrids planning model can be defined as:

$$\mathbf{MP} : O = \min\ c^T x + \sum_{s\in S}\pi_s\Phi_s \tag{61}$$

$$s.t.\ Ax \leq b \tag{62}$$

$$\Phi_s \geq \tilde{\Phi}_{s,i} - \hat{\lambda}_{s,i}^T W_s(x - \hat{x}),\ \forall s \in S, i = 1,\ldots,j-1 \tag{63}$$

$$\Upsilon_n(1-u_n) \leq \bar{\Lambda}, \quad \forall n \in I \tag{64}$$

$$\Upsilon_n \geq \tilde{\Upsilon}_{n,i} - \hat{\psi}_{n,i}^T V_n(x - \hat{x}), \forall n \in I, i = 1,\ldots,j-1 \tag{65}$$

$$\sum_{n\in I}\pi_n u_n \leq \varepsilon \tag{66}$$

$$x \in \mathbb{Z}_+,\ \Phi_s \in \mathbb{R}, \quad \forall s \in S \tag{67}$$

$$\Upsilon_n \in \mathbb{R}_+, u_n \in \{0,1\}, \quad \forall n \in I \tag{68}$$

where j is the counter of current iteration.

Remark 6. (i) *Note that the bilinear structure in* (64)-(66) *corresponds to the chance constrained formulation* (42)-(46). *When* $u_n = 1$, (64) *is always satisfied, which means that the corresponding* OC^n_{B} *in* (65) *is deactivated. Otherwise,* OC^n_{B} *is enforced in* $\mathbf{MP}$.
(ii) *The bilinear inequalities in* (64) *can be easily linearized through McCormick linearization method [29]. Consequently,* $\mathbf{MP}$ *is converted into a mixed-integer linear program (MILP). Actually,* (64)-(65) *are strengthened bilinear Benders cuts developed in [14], which renders* $\mathbf{MP}$ *readily computable for state-of-the-art MILP solvers. Then, the linearized* $\mathbf{MP}$ *can be strengthened by adding two sets of cuts* OC^s_{A} *and* OC^n_{B} *iteratively until the optimality condition is satisfied.*

Following the decomposition scheme in (53)-(68), the detailed procedures of customized bilinear Benders decomposition method are outlined as below:

- **Step 1.** Set $LB = -\infty$, $UB = +\infty$, $j = 0$, $OC^{s,j}_{\mathrm{A}} = OC^{n,j}_{\mathrm{B}} = \emptyset$; Set the gap threshold e;
- **Step 2.** Compute the master problem $\mathbf{MP}_j$;
  • If $\mathbf{MP}_j$ is infeasible, terminate the algorithm and *report the infeasibility of the original problem*;
  • Otherwise, derive an optimal solution $(\hat{x}, \hat{u})$ and objective value $\hat{O}_j$, then update $LB = \hat{O}_j$ and $j = j + 1$;
- **Step 3.** For $\hat{x}$, compute $\mathbf{SP}^s_{\mathrm{A}}$ for every $s \in S$; Get the dual solutions and generate the cuts $OC^{s,j}_{\mathrm{A}}$;
- **Step 4.** For $\hat{x}$, compute $\mathbf{SP}^n_{\mathrm{B}}$ for every $n \in I$; Get the dual solutions and generate the cuts $OC^{n,j}_{\mathrm{B}}$;
- **Step 5.** Compute $\mathbf{SP}_*$ to get the feasibility gap Δ_f; If $\Delta_f = 0$, evaluate the primal objective value $\hat{\zeta}$ in (35) and update $UB = \min\{UB, \hat{\zeta}\}$;
- **Step 6.** Get the optimality gap $\Delta_o = |(UB - LB)/LB|$;
  • If $\Delta_o \le e$, converge to current $\hat{x}$, then go to **Step 7**;
  • Otherwise, $\mathbf{MP}_j \leftarrow \mathbf{MP}_{j-1} \bigcup_{s=1}^{N_s} OC^{s,j}_{\mathrm{A}} \bigcup_{n=1}^{N_I} OC^{n,j}_{\mathrm{B}}$, then go to **Step 2**;
- **Step 7.** Report the optimal solution $\hat{x}$.

□ **Enhancement Techniques**: We note that those two different sets of subproblems (subject to stochastic and islanding scenarios), which are of different natures, generate complicated interactions and incur a large number of Benders iterations in our computation. Hence, on top of the aforementioned customization of bilinear Benders procedure, we have designed and implemented two enhancement techniques to achieve a stronger computational performance:

- One is to generate and adopt more useful Benders cuts, i.e., *Pareto-Optimal cuts* [43]–[45], from computing the subproblems with core-point based reformulation. Also, the core points are updated following the methods in [44] to further improve the computation efficiency.
- Another one is to strengthen the master problem by creating and including a virtual scenario through *Jensen's inequality* [29], [46]. Although such inclusion increases the size and computational complexity of the master problem, the augmented master problem should be a tighter relaxation to the original formulation.

TABLE I
MAJOR PARAMETERS OF CANDIDATE DERS

Renewable Energy and Dispatchable Fuel Generators								
Label	Type	Nom. Cap. (kW)	Min Output (kW)	Capital Cost ($/kW)	O&M Cost ($/kW/h)	Fuel Cost ($/kW/h)	Life Time (yr)	
PV	Solar Panel	120	0	1800	0	0	15	
MT	Micro Turbine	60	6	800	0.030	0.153	10	
Energy Storage Devices								
Label	Type	Power Cap. (kW)	Energy Cap. (kWh)	Power Cost ($/kW)	Energy Cost ($/kWh)	O&M Cost ($/kW/h)	Efficiency	Life Time (yr)
BB	Battery Bank	100	200	250	200	0.004	0.90	8

Actually, with those two enhancements, as demonstrated in our numerical study in Section IV, the computational capability of proposed bilinear Benders decomposition algorithm is greatly improved by reducing the necessary Benders iterations significantly before convergence.

IV. NUMERICAL RESULTS

The proposed two-stage chance constrained stochastic NMP model and bilinear Benders decomposition algorithm with enhancements are first verified on a 5-bus illustrative networked microgrids system. Then, our method is further tested on a more complex microgrids structure based on IEEE 69-bus distribution system to prove its scalability. For demonstration, we choose photovoltaic (PV) panels, micro turbines, and battery banks to represent RES, DFG, and ES, respectively. Table I presents the major parameters of candidate DERs. Table II lists other essential parameters (partly from the published data of U.S. Energy Information Administration). The time-varying patterns of PV generation and load demand are captured by daily operating curves in 10,000 scenarios. To make a trade-off between accuracy and computational efficiency, the k-means clustering method is applied to generate the reduced scenario sets. All the algorithm development and computations, including our bilinear Benders decomposition, are made by CPLEX in MATLAB environment on a laptop computer with Intel Core i7-7820HQ 2.90GHZ processer.

A. 5-Bus Test System

The 5-bus networked microgrids system is shown in Fig. 2. The proposed dual-mode planning model (35)-(47), as denoted by CC_SP, is solved given $\bar{\Lambda} = \$300,000$, $\varepsilon = 0.10$, $\tau =$ 8h, $(N_s, N_I) = (80, 80)$. To demonstrate the effectiveness of CC_SP, we set up two benchmark cases (i.e., DT and SP) as in Table III. The optimal solutions and objective values of DT, SP, and CC_SP are presented in Fig. 2-(a), (b), (c), respectively. We observe that the DT solution invests 3 microgrids at bus 1, 2, 3, which are neighboring to the substation bus. Even though such siting scheme enables an easy access of microgrids to gain power-selling revenues, the remote users at bus 4, 5 could be vulnerable to the prevailing uncertainties. In contrast, the SP solution moves MG_B from bus 2 to bus 5 to improve the

TABLE II
TECH-ECONOMICAL PARAMETERS FOR PLANNING

Parameter	Description	Value
R	Discount rate	0.04
ϑ	Cost Coefficient of Power Loss (\$/kWh)	0.05
ς_p/ς_q	Penalty Cost Factors of Load Reduction	20/20
ρ^t	On-Peak Electricity Price (\$/kWh)	0.193
	Partial-Peak Electricity Price (\$/kWh)	0.138
	Off-Peak Electricity Price (\$/kWh)	0.083
U_0	Reference Voltage of Distribution Network (kV)	10
$[\underline{U},\overline{U}]$	Allowable Range of Voltage Magnitudes (kV)	[9,11]
$\overline{I}$	Upper Bound of Current Magnitudes (A)	250
$\overline{P}_{\text{sub}}$	Active Power Limit of Substation (MW)	5.00
$\overline{Q}_{\text{sub}}$	Reactive Power Limit of Substation (MVar)	3.10

TABLE III
NOTATIONS OF PLANNING CASES

Notation	Definition
DT	Deterministic NMP model
SP	Stochastic NMP model (i.e., (35)-(41) that excludes islanding mode)
CC_SP	Chance constrained stochastic NMP model (i.e., (35)-(47))

reliability of remote users. Also, the total storage capacity is added from 3.00MW to 4.40MW to handle the randomness of PV generation. Compared to the SP solution, CC_SP adopts a similar siting scheme while reinforcing the capacity plan of DERs to further hedge the external uncertainties. The MT installation is increased from 0.66MW to 1.08MW, while more storage units are deployed at the remote buses to ensure the power supply continuity during islanded operation.

Then, the rationality of CC_SP solution is verified through Monte Carlo simulation (MCS) given different islanding probabilities (ρ). Note that, when ρ=5%, the actual operational cost of CC_SP solution (ε=0.10) is expected to be 277.7k\$ (<300.0k\$), which is significantly lower than SP (685.1k\$) and DT (1806.0k\$). We also notice that the biased evaluation for operational cost in DT solution may lead to overly optimistic investment in RES units, which renders larger investment cost (724.1k\$) than CC_SP (601.7k\$). Other performance metrics attained by MCS include expected network loss (ENL) and loss of power supply probability (LPSP) [47]. Table IV shows detailed results of performance evaluation. In all instances, the CC_SP solution demonstrates higher reliability and efficiency levels than DT. Particularly, when ρ=10%, our CC_SP can reduce the LPSP and ENL by 72.4% and 42.4%. Given the same condition, the CC_SP solution also shows obvious advantage over SP, which reduces the LPSP by 54.6% due to its dual-mode consideration. In Table IV, we also investigate the impact of risk tolerance level ε on CC_SP solutions. The increase of ε from 0.10 to 0.20 leads to an increasing LPSP (in most cases) and a falling ENL, along with the decline of investment cost. Hence, by solving CC_SP via a

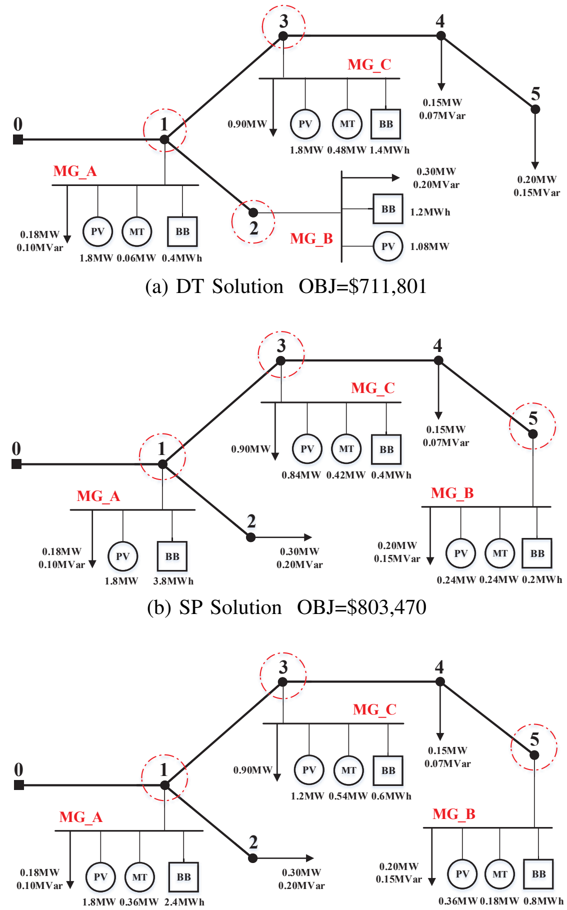


Fig. 2. Planning Solutions: 5-Bus System

TABLE IV
PERFORMANCE EVALUATION OF PLANNING SOLUTIONS: 5-BUS SYSTEM

Solution		Inv. Cost (k\$)	ρ=0 ENL (MW)	ρ=0 LPSP	ρ=5% ENL (MW)	ρ=5% LPSP	ρ=10% ENL (MW)	ρ=10% LPSP
CC_SP	ε=0.10	601.7	276.5	0.34%	273.5	0.44%	271.3	0.97%
	ε=0.15	538.9	272.3	0.42%	270.1	0.55%	268.1	1.04%
	ε=0.20	529.1	256.0	0.48%	254.1	0.61%	251.7	0.94%
SP		527.0	257.5	0.42%	254.5	0.97%	251.0	1.54%
DT		724.1	510.2	2.45%	495.9	3.01%	470.6	3.51%

varying ε, we can obtain different trade-offs between the cost-effectiveness and the actual operational performance under uncertainties. The above observations demonstrate the validity of proposed planning model, which provides a flexible scheme to manage the multi-source uncertainties in NMP problem.

Finally, the computational capability of enhanced bilinear Benders decomposition method (EBD) is tested by comparing to the basic bilinear Benders decomposition (BD) and the direct use of CPLEX (CPX). Table V exhibits the test results under different sizes of scenario sets (N_s, N_I). The objective values, iteration numbers, solution times (in minutes), termination gaps are recorded in columns “OBJ”, “itr”, “min”, and

TABLE V
COMPUTATIONAL TEST: 5-BUS SYSTEM

(N_s,N_I)	EBD OBJ (k$)	EBD itr	EBD min	EBD gap	BD OBJ (k$)	BD itr	BD min	BD gap	CPX OBJ (k$)	CPX min	CPX gap
(5,5)	795	12	0.93	0.42%	797	25	1.63	0.10%	798	0.31	<0.5%
(10,10)	805	9	0.99	0.35%	805	17	1.66	0.40%	804	6.22	<0.5%
(20,20)	812	8	1.41	0.41%	810	17	2.73	0.31%	810	6.74	<0.5%
(40,40)	817	11	4.55	0.13%	817	17	5.77	0.27%	816	64.69	<0.5%
(80,80)	821	10	10.18	0.46%	824	14	10.70	0.33%	858	T	5.63%
(120,120)	823	9	15.93	0.26%	824	14	16.52	0.17%	/	T	N/A

"gap", respectively. If any problem is terminated due to the time limit of *480 mins*, its solution time will be marked by "T". Also, the gap threshold is set as 0.5%. In case where the gap report is unavailable, it will be recorded by "N/A". When (N_s, N_I) =(5,5), it can be seen that all the methods can efficiently solve the problem. With the growth of scenario size, however, both BD methods (even without enhancements) perform much faster than CPX, which can mostly reduce the solution time by 97.88%. When (N_s, N_I)=(120,120), our EBD reaches the optimality condition in less than 16 mins, while CPX fails to derive any feasible solution in 8 hours with no gap available. We also notice that, for those can be solved by both CPX and EBD, the difference between objective values is maintained below 0.33% (<0.5%). Hence, the proposed algorithm holds similar precision to commercial solver but drastically improves the computational efficiency.

B. 69-Bus Test System

To evaluate our planning model and solution algorithm on practical-scale systems, we further test a 69-bus networked microgrids. The planning solution of 69-bus system is shown in Fig. 3. We observe that the microgrid with largest DER installation (MG_D) is located near the substation bus to pursue higher cost benefits. Also, several smaller microgrids (MG_C, MG_E, MG_G, MG_H) are deployed near the far-end buses, so as to improve the reliability level and reduce the network loss. Table VI exhibits the results of computational tests on 69-bus network. We notice that the EBD is capable to address all the instances using at most 420 mins (for intractable 120-scenario instance), which shows a clear superiority over CPX. Furthermore, EBD outperforms its basic form since the iteration numbers are significantly reduced by applying the enhancement techniques. For those can be solved by both methods, our enhancements reduce the iteration number by more than 60.0%, and thus saving the computation time by 56.3%-73.5%. When (N_s, N_I)=(80,80) or (120,120), EBD is still applicable while BD can only report a low quality solution with a very large optimality gap. Together with our observations on 5-bus test system, we can conclude that the proposed algorithm has a strong scalable capacity to solve a practical NMP problem with numerous stochastic and islanding scenarios.

V. CONCLUSION AND DISCUSSION

This paper proposes a two-stage chance constrained stochastic conic program model to address the networked microgrids planning problem concerning the multi-site investment, dual-mode operations, multi-source uncertainties, and non-linear power flow representations. Moreover, the bilinear Benders decomposition method is customized with two enhancement techniques to analytically solve the challenging mixed-integer SOCP formulation. Numerical studies are conducted to verify the proposed planning method on 5- and 69-bus networked microgrids systems. Some key observations and insights from our numerical results are listed as below:

1) **Importance of dual-mode operational modeling**: We notice that the consideration on dual-mode operations has clearly influenced the siting and sizing decisions of networked microgrids. So, the solution of our CC_SP model can provide an informative guidance for the planning and long-term dual-mode operations of networked microgrids.
2) **Effectiveness of chance constrained stochastic conic formulation**: We observe that the planning solution of our CC_SP model achieves a significantly lower operational cost as well as higher reliability and energy efficiency levels than the benchmark cases. Moreover, by adjusting the risk parameter, our model yields a flexible and practical scheme to support real systems with trade-offs between the cost-effectiveness and the risk-hedging capability for islanded operations.
3) **Strong scalable capacity of enhanced bilinear Benders decomposition**: Our enhancements on bilinear Benders decomposition method demonstrate a superior computational capacity to its basic form and the direct use of a professional commercial solver, which makes it applicable for networked microgrids planning in practical distribution systems.

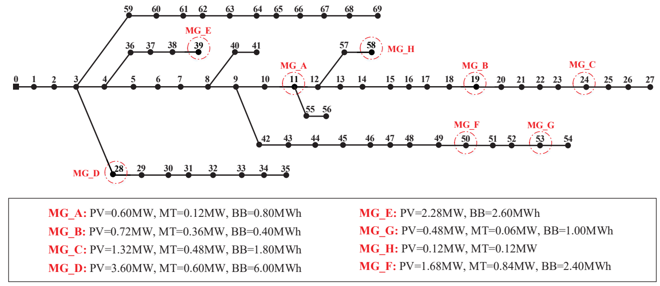


Fig. 3. Planning Solution: 69-Bus System

TABLE VI
COMPUTATIONAL TEST: 69-BUS SYSTEM

(N_s,N_I)	EBD OBJ (k$)	EBD itr	EBD min	EBD gap	BD OBJ (k$)	BD itr	BD min	BD gap	CPX OBJ (k$)	CPX min	CPX gap
(5,5)	1619	38	37.59	0.49%	1618	95	141.63	0.49%	1619	47.22	<0.5%
(10,10)	1638	30	31.44	0.47%	1632	83	93.38	0.46%	1633	42.20	<0.5%
(20,20)	1655	26	52.58	0.22%	1655	69	143.61	0.47%	1657	480.00	<0.5%
(40,40)	1659	23	118.09	0.49%	1655	59	269.96	0.35%	1654	369.75	<0.5%
(80,80)	1680	19	251.03	0.36%	1691	47	T	1.89%	/	T	N/A
(120,120)	1667	17	420.09	0.46%	2176	30	T	47.69%	/	T	N/A

In our future work, the proposed method will be extended in several aspects, e.g., microgrids planning under a deregulated environment and a multi-stage framework.

References

[1] M. Shahidehpour, Z. Li, S. Bahramirad, Z. Li, and W. Tian, "Networked microgrids: Exploring the possibilities of the IIT-Bronzeville grid," *IEEE Power & Energy Magazine*, vol. 15, no. 4, pp. 63–71, 2017.

[2] Z. Li, M. Shahidehpour, F. Aminifar, A. Alabdulwahab, and Y. Al-Turki, "Networked microgrids for enhancing the power system resilience," *Proceedings of the IEEE*, vol. 105, no. 7, pp. 1289–1310, 2017.

[3] Z. Wang, B. Chen, J. Wang, M. M. Begovic, and C. Chen, "Coordinated energy management of networked microgrids in distribution systems," *IEEE Trans. Smart Grid*, vol. 6, no. 1, pp. 45–53, 2015.

[4] Z. Wang, B. Chen, J. Wang, and C. Chen, "Networked microgrids for self-healing power systems," *IEEE Trans. Smart Grid*, vol. 7, no. 1, pp. 310–319, 2016.

[5] A. Hussain, V.-H. Bui, and H.-M. Kim, "A resilient and privacy-preserving energy management strategy for networked microgrids," *IEEE Trans. Smart Grid*, vol. 9, no. 3, pp. 2127–2139, 2018.

[6] A. Hirsch, Y. Parag, and J. Guerrero, "Microgrids: A review of technologies, key drivers, and outstanding issues," *Renewable & Sustainable Energy Reviews*, vol. 90, pp. 402–411, 2018.

[7] E. M. Gui, M. Diesendorf, and I. MacGill, "Distributed energy infrastructure paradigm: Community microgrids in a new institutional economics context," *Renewable & Sustainable Energy Reviews*, vol. 72, pp. 1355–1365, 2017.

[8] C. Marnay, S. Chatzivasileiadis, C. Abbey, R. Iravani, G. Joos, P. Lombardi, P. Mancarella, and J. von Appen, "Microgrid evolution roadmap," in *Smart Electric Distribution Systems and Technologies (EDST), 2015 International Symposium on*. IEEE, 2015, pp. 139–144.

[9] D. T. Ton and M. A. Smith, "The US department of energy's microgrid initiative," *The Electricity Journal*, vol. 25, no. 8, pp. 84–94, 2012.

[10] EU Community Research and Development Information Service, "More microgrids," https://https://cordis.europa.eu/project/rcn/75085_en.html, 2011.

[11] X. Wu, X. Yin, Q. Wei, Y. Jia, and J. Wang, "Research on microgrid and its application in china," *Energy & Power Engineering*, vol. 5, no. 4, pp. 171–176, 2013.

[12] A. Khodaei, S. Bahramirad, and M. Shahidehpour, "Microgrid planning under uncertainty," *IEEE Trans. Power Syst.*, vol. 30, no. 5, pp. 2417–2425, 2015.

[13] E. Hajipour, M. Bozorg, and M. Fotuhi-Firuzabad, "Stochastic capacity expansion planning of remote microgrids with wind farms and energy storage," *IEEE Trans. Sustain. Energy*, vol. 6, no. 2, pp. 491–498, 2015.

[14] X. Cao, J. Wang, and B. Zeng, "A chance constrained information-gap decision model for multi-period microgrid planning," *IEEE Trans. Power Syst.*, vol. 33, no. 3, pp. 2684–2695, 2018.

[15] A. Khodaei, "Microgrid optimal scheduling with multi-period islanding constraints," *IEEE Trans. Power Syst.*, vol. 29, no. 3, pp. 1383–1392, 2014.

[16] L. Che, X. Zhang, M. Shahidehpour, A. Alabdulwahab, and A. Abusorrah, "Optimal interconnection planning of community microgrids with renewable energy sources," *IEEE Trans. Smart Grid*, vol. 8, no. 3, pp. 1054–1063, 2017.

[17] H. Wang and J. Huang, "Cooperative planning of renewable generations for interconnected microgrids," *IEEE Trans. Smart Grid*, vol. 7, no. 5, pp. 2486–2496, 2016.

[18] A. Khayatian, M. Barati, and G. Lim, "Integrated microgrid expansion planning in electricity market with uncertainty," *IEEE Trans. Power Syst.*, 2017.

[19] S. Mojtahedzadeh, S. N. Ravadanegh, and M.-R. Haghifam, "Optimal multiple microgrids based forming of greenfield distribution network under uncertainty," *IET Renew. Power Gen.*, vol. 11, no. 7, pp. 1059–1068, 2017.

[20] F. S. Gazijahani and J. Salehi, "Stochastic multi-objective framework for optimal dynamic planning of interconnected microgrids," *IET Renew. Power Gen.*, vol. 11, no. 14, pp. 1749–1759, 2017.

[21] S. Manshadi and M. Khodayar, "Expansion of autonomous microgrids in active distribution networks," *IEEE Trans. Smart Grid*, 2017.

[22] F. S. Gazijahani and J. Salehi, "Optimal bi-level model for stochastic risk-based planning of microgrids under uncertainty," *IEEE Trans. Industr. Inform.*, 2017.

[23] ——, "Robust design of microgrids with reconfigurable topology under severe uncertainty," *IEEE Trans. Sustain. Energy*, 2017.

[24] H. E. Farag and E. F. El-Saadany, "Optimum shunt capacitor placement in multimicrogrid systems with consideration of islanded mode of operation," *IEEE Trans. Sustain. Energy*, vol. 6, no. 4, pp. 1435–1446, 2015.

[25] M. E. Baran and F. F. Wu, "Optimal capacitor placement on radial distribution systems," *IEEE Trans. Power Del.*, vol. 4, no. 1, pp. 725–734, 1989.

[26] R. A. Jabr, "Radial distribution load flow using conic programming," *IEEE Trans. Power Syst.*, vol. 21, no. 3, pp. 1458–1459, 2006.

[27] S. H. Low, "Convex relaxation of optimal power flow-Part I: Formulations and equivalence," *IEEE Trans. Contr. of Network Syst.*, vol. 1, no. 1, pp. 15–27, 2014.

[28] M. Farivar and S. H. Low, "Branch flow model: Relaxations and convexification-Part I," *IEEE Trans. Power Syst.*, vol. 28, no. 3, pp. 2554–2564, 2013.

[29] B. Zeng, Y. An, and L. Kuznia, "Chance constrained mixed integer program: Bilinear and linear formulations, and Benders decomposition," *Technical Report*, 2014.

[30] P. S. Georgilakis and N. D. Hatziargyriou, "Optimal distributed generation placement in power distribution networks: Models, methods, and future research," *IEEE Trans. Power Syst.*, vol. 28, no. 3, pp. 3420–3428, 2013.

[31] B. R. Pereira, G. R. M. da Costa, J. Contreras, and J. R. S. Mantovani, "Optimal distributed generation and reactive power allocation in electrical distribution systems," *IEEE Trans. Sustain. Energy*, vol. 7, no. 3, pp. 975–984, 2016.

[32] Z. Wang, B. Chen, J. Wang, J. Kim, and M. M. Begovic, "Robust optimization based optimal DG placement in microgrids," *IEEE Trans. Smart Grid*, vol. 5, no. 5, pp. 2173–2182, 2014.

[33] A. Parisio, C. Wiezorek, T. Kyntäjä, J. Elo, K. Strunz, and K. H. Johansson, "Cooperative MPC-based energy management for networked microgrids," *IEEE Trans. Smart Grid*, vol. 8, no. 6, pp. 3066–3074, 2017.

[34] V. Kekatos, G. Wang, A. J. Conejo, and G. B. Giannakis, "Stochastic reactive power management in microgrids with renewables," *IEEE Trans. Power Syst.*, vol. 30, no. 6, pp. 3386–3395, 2014.

[35] L. Gan, N. Li, U. Topcu, and S. Low, "Exact convex relaxation of optimal power flow in radial networks," *IEEE Trans. Automatic Control*, vol. 60, no. 1, pp. 72–87, 2015.

[36] "IEEE guide for design, operation, and integration of distributed resource island systems with electric power systems," *IEEE Std 1547.4-2011*, pp. 1–54, July 2011.

[37] S. Sarykalin, G. Serraino, and S. Uryasev, "Value-at-risk vs. conditional value-at-risk in risk management and optimization," in *Tutorials in Operations Research*. INFORMS, 2008, pp. 270–294.

[38] P. Krokhmal, M. Zabarankin, and S. Uryasev, "Modeling and optimization of risk," *Surveys in Operations Research and Management Science*, vol. 16, no. 2011, pp. 49–66, 2011.

[39] X. Cao, J. Wang, and B. Zeng, "A study on the strong duality of conic relaxation of AC optimal power flow in radial networks," https://arxiv.org/abs/1807.08785, 2018.

[40] C. Lin, W. Wu, B. Zhang, B. Wang, W. Zheng, and Z. Li, "Decentralized reactive power optimization method for transmission and distribution networks accommodating large-scale DG integration," *IEEE Trans. Sustain. Energy*, vol. 8, no. 1, pp. 363–373, 2017.

[41] M. Nick, R. Cherkaoui, and M. Paolone, "Optimal planning of distributed energy storage systems in active distribution networks embedding grid reconfiguration," *IEEE Trans. Power Syst.*, vol. 33, no. 2, pp. 1577–1590, 2018.

[42] Z. Yuan and M. R. Hesamzadeh, "A modified Benders decomposition algorithm to solve second-order cone AC optimal power flow," *IEEE Trans. Smart Grid*, 2018.

[43] T. L. Magnanti and R. T. Wong, "Accelerating Benders decomposition: Algorithmic enhancement and model selection criteria," *Operations Research*, vol. 29, no. 3, pp. 464–484, 1981.

[44] N. Papadakos, "Practical enhancements to the Magnanti-Wong method," *Operations Research Letters*, vol. 36, no. 4, pp. 444–449, 2008.

[45] L. Kuznia, B. Zeng, G. Centeno, and Z. Miao, "Stochastic optimization for power system configuration with renewable energy in remote areas," *Annals of Operations Research*, vol. 210, no. 1, pp. 411–432, 2013.

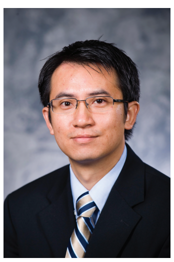
[46] A. Madansky, "Inequalities for stochastic linear programming problems," *Management Science*, vol. 6, no. 2, pp. 197–204, 1960.

[47] X. Cao, J. Wang, and Z. Zhang, "Multi-objective optimization of preplanned microgrid islanding based on stochastic short-term simulation," *International Transactions on Electrical Energy Systems*, vol. 27, no. 1, pp. 1–16, 2017.

Xiaoyu Cao (S'17) received the B.S. degree in electrical engineering from North China Electric Power University, Baoding, China, in 2013. He is currently pursuing the Ph.D. degree at the School of Electrical Engineering in Xi'an Jiaotong University, Xi'an, China. His current research interests include microgrid planning and scheduling, and generation expansion planning.

Jianxue Wang (M'11) received the B.S., M.S., and Ph.D. degrees in electrical engineering from Xi'an Jiaotong University, Xi'an, China, in 1999, 2002, and 2006, respectively.

He is currently a Professor at the School of Electrical Engineering in Xi'an Jiaotong University. His current research interests include microgird planning and schdeuling, power system planning and scheduling, and electricity market.



Bo Zeng (M'11) received the Ph.D. degree in industrial engineering from Purdue University, West Lafayette, IN, USA. He currently is an Associate Professor with the Department of Industrial Engineering, and the Department of Electrical and Computer Engineering, University of Pittsburgh, Pittsburgh, PA, USA. His research interests include polyhedral study and algorithms for stochastic and robust mixed integer programs, coupled with applications in power and logistics systems. He is also a member of IISE and INFORMS.