

Lightning Talk : Capillary Pressure Relationships

SFWST Annual Meeting
Tuesday May 18, 2021 (11:00-12:30)

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Capillary Pressure Non-linearities

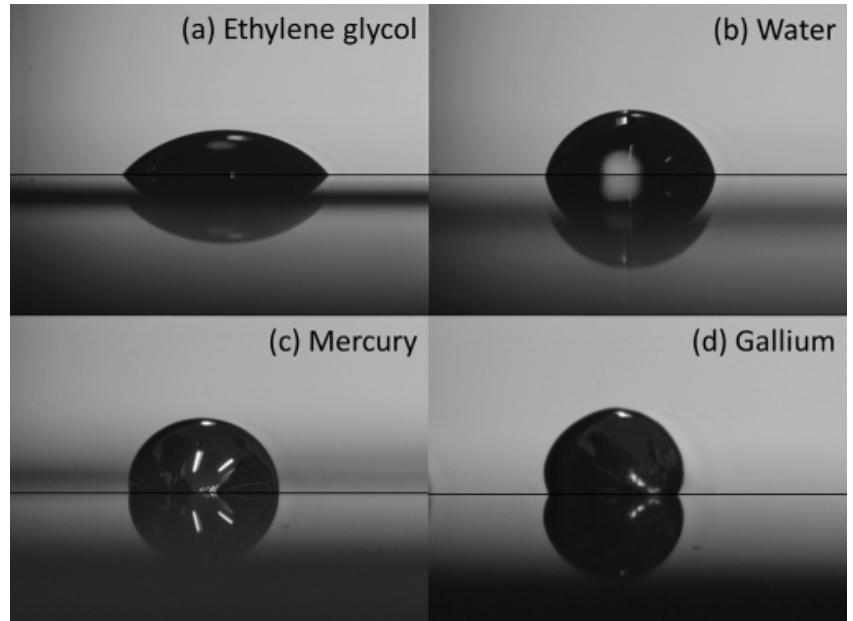
Capillary “pressure” is a form of “matrix potential”, the free energy of the fluid in a matrix relative to the bulk fluid.

$$\frac{P_c(S_w)}{\rho} = \psi_{matrix} = \psi_{adhesion} - \psi_{cohesion}$$

It can be positive or negative.

Multiple non-linear aspects of one function:

1. Hysteresis $S_w(A_{ad}, A_{co})$
2. Adhesion $\psi_{adhesion}(A_{ad})$
3. Cohesion $\psi_{cohesion}(A_{co})$



[1] Guisbiers, Arscott, and Snyders (2012). “An accurate determination of the surface energy of solid selenium” *Appl. Phys. Let.* 101

Hysteresis and Cohesion

Saturation is **convenient**, but **imperfect** state variable for matrix potential.

There is no **one-to-one** mapping of saturation to interfacial areas.

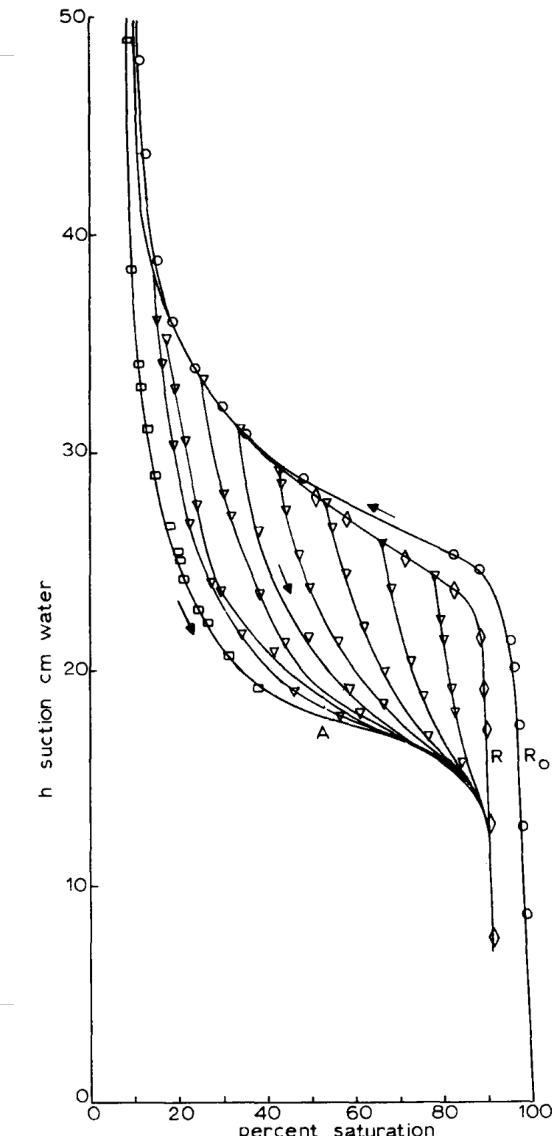
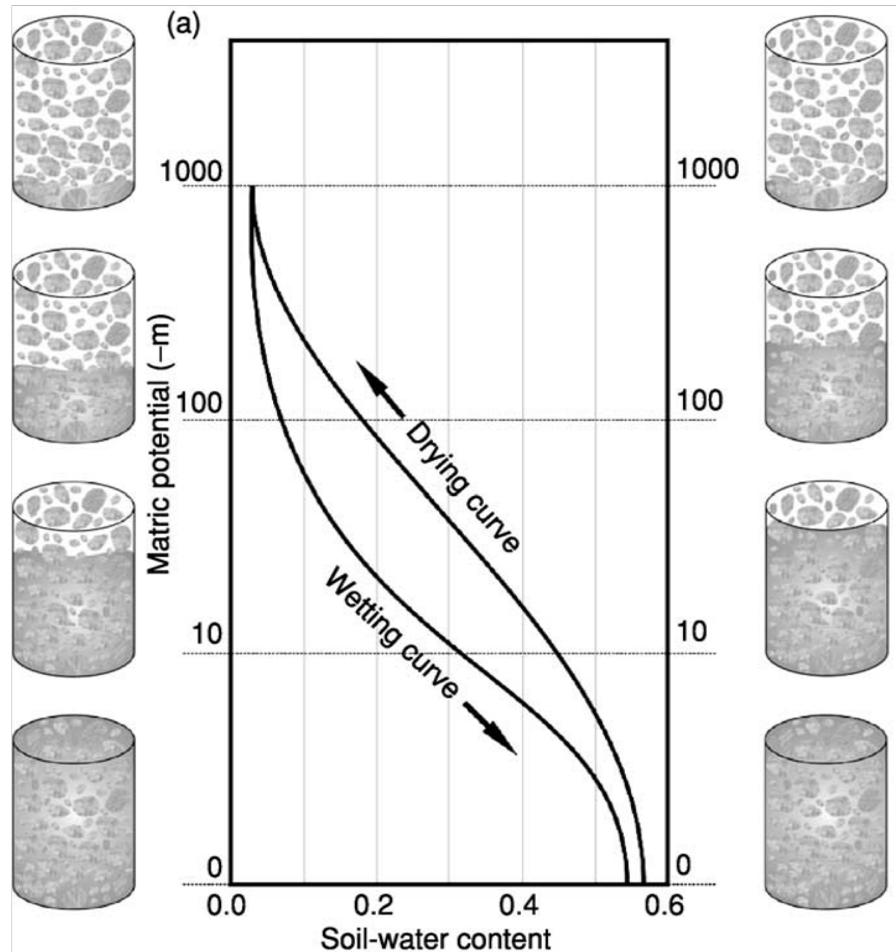
Different **interfacial areas** are created during imbibition and draining.

Small pores have large adhesion areas but very small cohesive areas

First filled by wetting fluids (H_2O)

Last filled by non-wetting fluids (Hg)

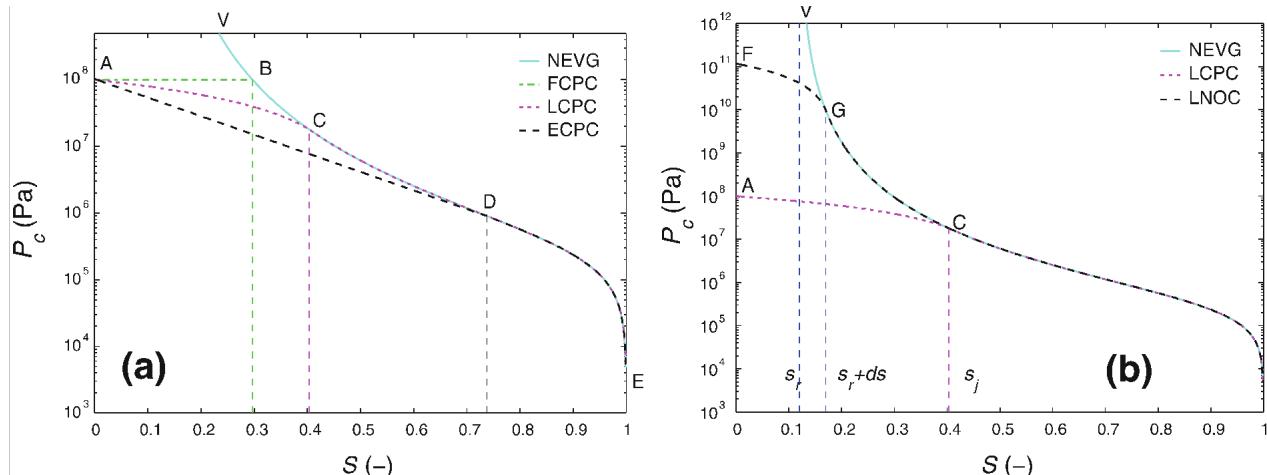
Matrix potential increases dramatically at the “irreducible” or “residual saturation...



[1] Tuller and Or (2005). “Water retention and characteristic curve.” *Encyclopedia of Soils in the Environment*. Elsevier.

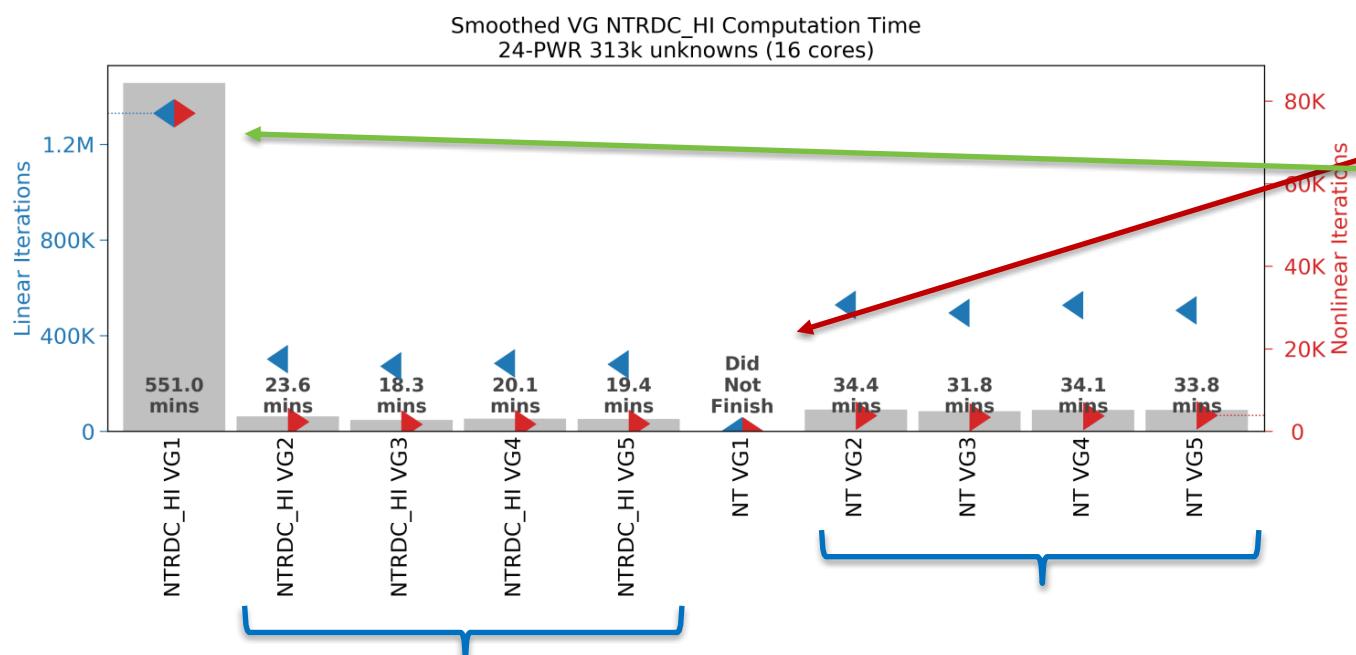
[2] Morrow and Harris (1965). “Capillary equilibrium in porous materials.” *SPE J.* 5. 15-24

Unsaturated Behavior



What happens to "capillary pressure" at below "irreducible"?
Van Genuchten is undefined.

[1] Sun et al (2010). "Modeling Thermal-Hydrologic Processes for a Heated Fractured Rock System: Impact of a Capillary-Pressure Maximum." *Transp Porous Med* 83



Currently developing approaches to model this regime.

1. Solver convergence

Ordinary **Newton (NT)** solvers are unstable at cusps (VG1).
Newton Trust Region Dogleg-Cauchy (**NTRDC**) is stable.
(Development by Heeho Park)

2. Unsaturated extensions to van Genuchten

"All of the above" approach to compare models
Smooth, piecewise extensions perform much better.

Can these extensions be physically justified?

Adsorption limits maximum “capillary pressure”

- Matrix potential energy is that in excess of liquefaction

$$-\frac{\Delta P_c}{\rho}(S_w = 0) = \Delta H_{ad}(0) - \Delta H_{liq}$$

- BET theory supports interpolation

- ΔH Monolayer > ΔH Multilayer > ΔH Bulk Condensate
- “Capillary pressure” diminishes with increasing saturation

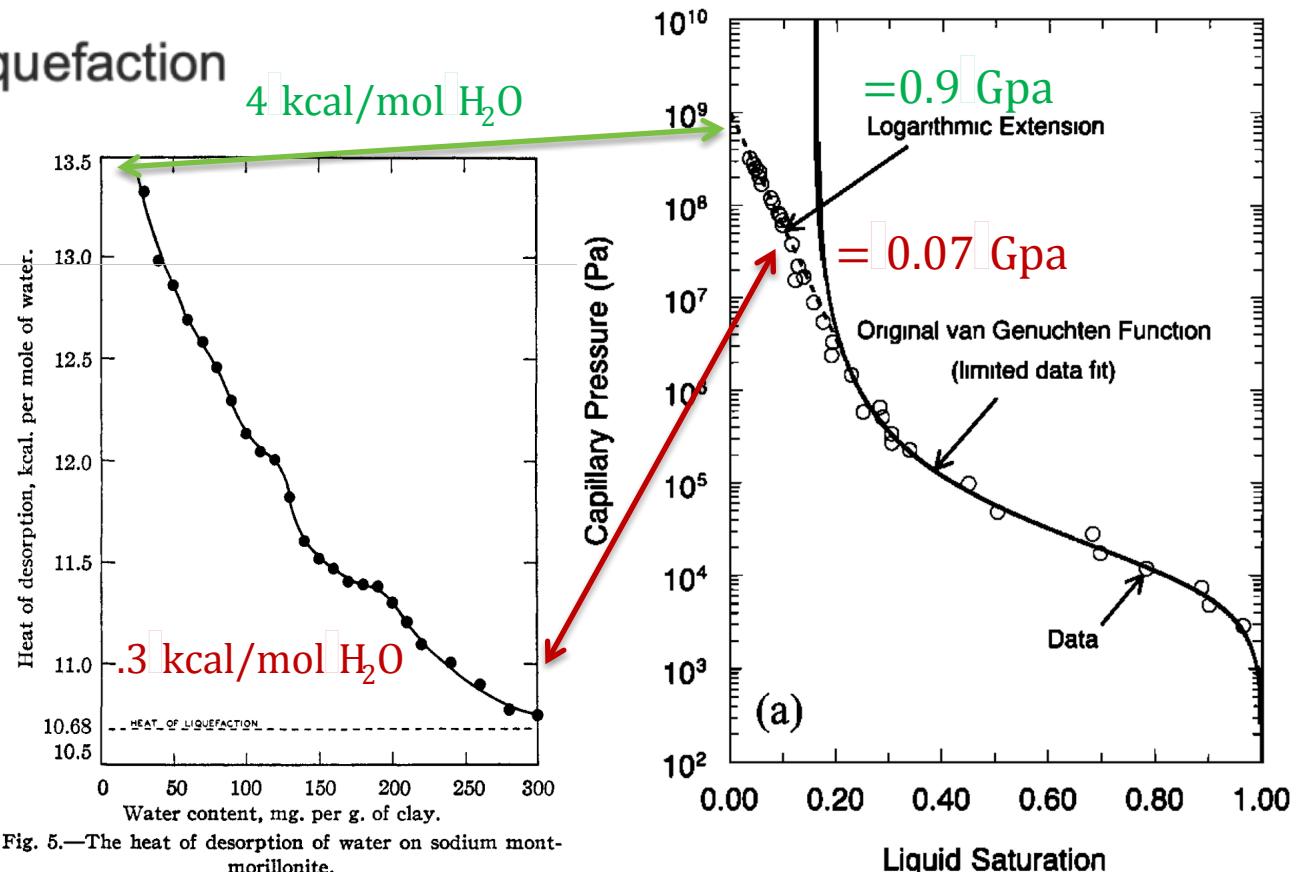
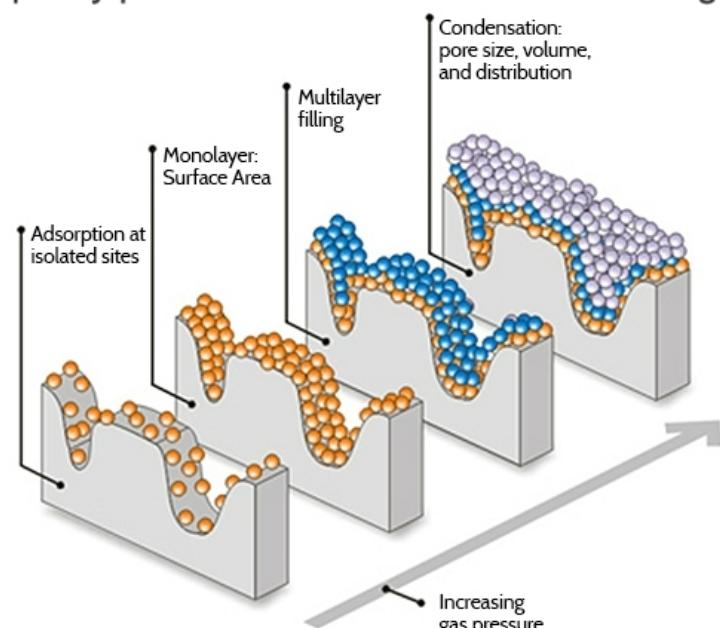


Fig. 5.—The heat of desorption of water on sodium montmorillonite.

- A maximum with interpolated slope is justified

[1] Image from micromeritics.com

[2] Mooney, Keenan, and Wood (1952). “Adsorption of Water Vapor by Montmorillonite. I. Heat of Desorption and Application of BET Theory” *J Am Chem Soc* 74, 6.

[3] Webb (2000). “A simple extension of two-phase characteristic curves to include the dry region” *Water Resource Res* 36, 6.