

Demand-adaptive Transit Design for Urban Transportation Hubs

Xinwu Qian, Ph.D.

Lyles School of Civil Engineering, Purdue University
550 Stadium Mall Dr, West Lafayette, IN, 47907
qian39@purdue.edu

Jiawei Xue

Lyles School of Civil Engineering, Purdue University
550 Stadium Mall Dr, West Lafayette, IN, 47907
xue120@purdue.edu

Zengxiang Lei

Lyles School of Civil Engineering, Purdue University
550 Stadium Mall Dr, West Lafayette, IN, 47907
lei67@purdue.edu

Juan Suarez

Lyles School of Civil Engineering, Purdue University
550 Stadium Mall Dr, West Lafayette, IN, 47907
jsuarezl@purdue.edu

Satish V. Ukkusuri, Ph.D.

Lyles School of Civil Engineering, Purdue University
550 Stadium Mall Dr, West Lafayette, IN, 47907
sukkusur@purdue.edu
(Corresponding Author)

Word Count: 6061 words + 3 figures \times 0 + 5 tables \times 250 = 7311 words

Submission Date: June 12, 2020

ABSTRACT

In this study, we proposed a novel three-stage framework for planning the optimal demand-adaptive transit (DAT) at urban transportation hubs. Given the potential trip demand and road traffic condition, the proposed framework sequentially generates the optimal set of candidate routes, combines the outgoing routes and incoming routes at the hub, and derives the optimal fleet size and corresponding route frequency under the fixed budget. In particular, we build the route generation algorithm which maximizes passenger demand coverage with travel time deviation constraint. And a heuristic algorithm is further developed which yields near-optimal operation routes for real-time demand. The fleet optimization problem is formulated to minimize the weighted cost of energy savings, operation cost and trip revenue. We conduct comprehensive numerical experiments for planning DAT with electric buses at JFK airport in NYC using NYC taxi and for-hire vehicle trip data and GoogleMap speed data. The results show the superior performance of the proposed route generation algorithm which is able to cover citywide passenger demand with only 61 DAT routes. The results also suggest that the proposed DAT planning framework may serve over 47% of existing taxi and FHV demand by operating 18 routes using the fleet of 62 electric buses.

Keywords: Transit planning, transportation hub, demand adaptive, route generation, fleet optimization, electric bus

1 INTRODUCTION

2 High-capacity public transit such as bus and metro serves as an affordable mobility solution to
 3 urban commuters. When properly planned and operated, public transit may significantly reduce
 4 commuters' dependency on private vehicles and plays a vital role of sustainable mobility system
 5 in dense populated urban areas (1). Unfortunately, the configuration and infrastructure of many
 6 existing public transit systems are no longer attractive to urban travelers with fast-changing mo-
 7 bility needs. One notable evidence is the recent prosperity of ride-hailing industry and the shift of
 8 passenger demand from public transit to for-hire vehicles (FHV). For instance, in New York City
 9 (NYC), the bus ridership declined by 1.3%, 5.1% and 5.8% from year 2016 to 2018 (2) while the
 10 number of FHV trips have increased by four times during the time (3). This has led to much higher
 11 road traffic, exacerbating already worse traffic congestion, emission and energy consumption levels
 12 in cities and poses emerging needs for better public transit systems.

13 One challenge associated with planning transit system is the trade-off between the acces-
 14 sibility and service coverage. It is often impractical for high-capacity urban transit such as fixed-
 15 route buses to provide (near) door-to-door service to meet the varying mobility needs of general
 16 public. Nevertheless, with flexible transit of adaptive service routes and schedule, it is possible to
 17 provide effective transit service for serving passengers sharing similar origins and destinations (4).
 18 One appealing and applicable scenario is the public transit rooted at urban transportation hubs
 19 such as airports and railway stations. These locations usually have high passenger volume and
 20 more number of passengers with similar travel routes to and from the transportation hubs. In par-
 21 ticular, we observe that over 25% of yellow taxi trips and 15% of FHV trips have either pick-up
 22 or drop-off at one of the airports in NYC during peak hours. This implies huge opportunity for
 23 public transit systems to reduce vehicle miles traveled and motivates us to investigate the design of
 24 demand-adaptive transit at urban transportation hubs.

25 In this study, we propose a novel three-stage framework for planning demand-adaptive
 26 transit (DAT) services at transportation hubs based on trip and traffic data. By demand adaptive,
 27 we referring to the transit service with temporally varying service route and trip schedule that
 28 is tailored to the mobility needs of passengers and the road traffic condition. We consider the
 29 proposed DAT to complement existing fixed route transit services and compete with on-demand
 30 mobility services such as taxis and FHVs. The primary goal of the planned DAT is to maximize
 31 passenger coverage and energy savings with minimum operation cost under the capital budget. To
 32 achieve this goal, the DAT planning consists of (1) routing generation based on potential passen-
 33 ger demand to and from transportation hubs and road traffic condition, (2) route combination for
 34 matching generated outgoing and incoming transit routes at hubs with similar demand level and
 35 (3) fleet optimization that maximizes the effectiveness of the DAT with fixed budget. To the best
 36 of our knowledge, this study is the first work that focuses on optimal transit planning at urban
 37 transportation hubs. And we demonstrate the effectiveness of the proposed framework with the
 38 case study of planning DAT at JFK airport, where the optimal planned DAT may serve over 47%
 39 of existing taxi and FHV passengers with 18 routes.

40 The rest of the study is organized as follows. The second section presents a brief review of
 41 the transit planning literature. The third section gives an overview of the research framework for
 42 DAT planning. The fourth section discusses the route generation problem for hub-based DAT and
 43 proposes an exact solution algorithm and a heuristic solution algorithm for obtaining optimal route
 44 set. We also present a heuristic route generation mechanism following a popular route generation
 45 mechanism in the literature. The fifth section presents the route combination problem for joining

planned routes to and from hubs. The sixth section develops the optimization problem for the optimal scheduling of the DAT given the generated route set and budget constraint. The seventh presents the numerical experiment on planning the DAT service at JFK airport in NYC. Finally, we conclude our study with major findings and future directions in the last section.

LITERATURE

Transit design problem is a well studied subject in the transportation field and readers may refer to (5–7) for more comprehensive reviews of related literature. As summarized by (6), the solution methodologies for the transit planning problem can be generalized in two main directions. The first direction divides the process of transit planning into route generation and frequency assignment. The second direction introduces the iterative route generation algorithm (RGA) where both route and frequency are calculated in an iterative process. Majority of the studies in the literature follows the first direction and various methods for route generation were proposed. Lampkin and Saalmans (8) developed the route generation method that started with 4 random control points and consecutively adds stops in between of these nodes based on travel time and number of passengers served by the route. Silman et al. (9) divided the city into zones and used a heuristic algorithm for calculating routes based on the minimization of walk time. Recently, Cipriano et al. (10) used Genetic Algorithms (GA) to generate candidate route sets which were evolved according to a travel time measure of fitness. Nikolic and Teodorovic (11) developed the RGA based on bee colony optimization, where some random routes got improved by modeling each route as a bee and modifying its route with a pheromone objective function. According to Farahani et al. (7), the transit design problem also involves various objective functions such as the minimization of energy consumption and the introduction of environmental effects in the economic assessment. Studies also focused on building models to search energy efficiency and environmental friendly routes as in Iliopoulou et al. (12) and Pternea et al. (13) developed the model for searching sustainable routes from both environmental and economics aspects and they used the continuous generation approach to find an optimal solution. Studies that follow the second direction include Drezner et al. (14), Fan and Wei (15), Chakroborty and Partha (16), where genetic algorithms were widely adopted to create routes and constantly evolve the routes on an iterative processes and links or control points were chosen as genes which may change in search to an optimal solution.

OVERALL FRAMEWORK

In this study, we follow the first transit planning framework as summarized in (6) by first generating the set of candidate routes and then framing the optimal route configurations. We propose a three-stage framework for developing hub-based DAT system as shown in Figure 1. Our framework is different from the existing studies in that an innovative route generation algorithm is proposed and is objective driven and deterministic. The overall framework takes the trip demand and road traffic condition as the inputs, and gives the fleet size and set of selected operation routes and their corresponding operation frequency under the budget constraint.

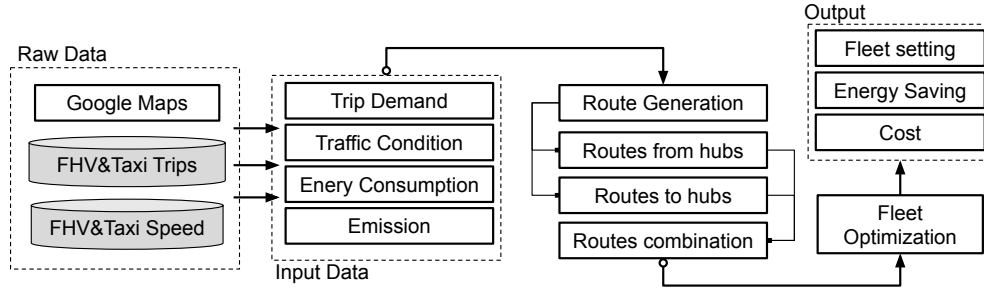


FIGURE 1: AEV Planning framework

In the first stage, we develop the route generation algorithm which builds the set of K **feasible** candidate routes that maximizes the coverage of potential passengers, where K is the user specified number. A route is considered as feasible if the trip time of all passengers is no larger than δ times the minimum trip time by taking alternative travel modes. Given the trip demand and road traffic condition, the route generation algorithm will first create two sets of feasible routes for incoming and outgoing trips at transportation hubs respectively.

The second step then identifies the optimal combination of the two sets of routes. And the route combination step focuses on balancing the passenger demand between two directions under vacant trip distance constraints.

The integrated routes will then serve as the input for the third stage problem, which is to optimize the fleet size and route service frequency. For the fleet optimization problem, our objective is to minimize the weighted cost of energy saving, trip revenue and operation expenditure under the fixed capital budget B . And the fixed capital budget restricts the number of available buses for operation. The fleet optimization also considers the rational choice of passengers between the developed DAT and alternative trip modes. We next describe the details of the three-stage framework.

ROUTE GENERATION

Problem description

As the first step of the sequential planning of hub-based transit, the route generation problem focuses on identifying the set of K candidate routes that yield the highest possible passenger coverage. We term this target set of K routes as the maximum coverage routes (MCR). We consider the study area consisted of $|V|$ zones and we denote $G(V, E, Q, W)$ as the network representation of the study area, with V being the set of zones, $e_{i,j} \in E$ denoting the shortest travel path between zone i and zone j , $q_i \in Q$ being the potential passenger demand (to hub or from hub) at zone i , and $w_{ij} \in W$ being the weight (travel time) on edge $e_{i,j}$. We make the following reasonable assumptions to assist framing the routing generation problem:

1. We assume DAT as a zone-based system and passenger demand of zone i will be covered by DAT if there exists a route that stops at i .
2. The DAT service will take the path with shortest travel time when travelling between two stops.
3. The shortest travel time should satisfy the triangular inequality constraint.
4. Passengers are time sensitive and will not ride DAT if the route travel time exceeds their expectation.

The first two assumptions state how we will consider DAT demand and travel time in this problem. The third constraint imposes a practical constraint on travel time, $t_{i,j} + t_{j,k} > t_{i,k}$, so that route travel time should strictly increase with the addition of intermediate stops. And the third assumption suggests that the all passengers served by DAT should be satisfied with the travel time by DAT over other alternative modes. For time sensitive passengers, we define a route k is feasible for passengers in zone i if:

$$t_i^k \leq \delta \text{mint}_i^m. \quad (1)$$

which suggests that the travel time of transit route $k \in \mathcal{R}_K$ for passengers at location i should be no greater than δ times the travel time of the mode that offers the lowest travel time. For instance, if we set $\delta = 1.5$, this may suggest that passengers will not use the DAT service if the travel time is 50% higher than the travel time of riding taxis. Based on this constraint, the route generation problem can be mathematically formulated an optimization problem:

$$\begin{aligned} & \text{maximize } \sum_{i=1}^{|V|} \mathbb{1}_i q_i \\ & \text{s.t. } t_i^k \leq \delta \text{mint}_i^m, \forall i, k \end{aligned} \quad (2)$$

- 1 where $\mathbb{1}_i$ is an indicator variable and takes the value of 1 if zone i is served by at least one of the
- 2 generated routes.
- 3 Problem 2 is a NP-hard problem. This is because we can reduce the longest path problem, a
- 4 known NP-hard problem, to Problem 2 under $K = 1$ case when δ is set to be $\frac{\sum_{i,j} t_{i,j}}{\min_{i,j} t_{i,j}}$. With such
- 5 δ , the travel time constraints are satisfied directly. And $K = 1$ is a special case of Problem 2,
- 6 demonstrating that it is indeed NP-hard.

7 Solution algorithm

8 Problem 2 is a difficult mixed integer linear programming problem and there are $|V|K$ binary
 9 variables and $|V|K$ constraints. And the problem may not be solved efficiently by using solvers
 10 due to the large search space and the complex interactions among the binary variables incurred by
 11 the travel time constraints. But the problem may be solved efficiently by utilizing the topological
 12 properties of the road network and the problem specific property that the generated routes have one
 13 end at the transportation hub.

14 First, denote \mathcal{R}_K as an optimal solution of problem 2 and the route $r_i \in \mathcal{R}_K$ as the ordered
 15 set of zones it traverses through, e.g. $r_i = \{v_1, v_2, v_3, \dots, v_{L_i}\}$ and $v_1, v_2, \dots, v_{L_i} \in V$. Then \mathcal{R}_K can
 16 be reduced to \mathcal{R}'_K of which for any $r_i, r_j \in \mathcal{R}'_K$, we have $r_i \cap r_j = \emptyset$. And we term \mathcal{R}'_K as the
 17 mutually disjoint MCR. To see this, we may assume that $r_i \cap r_j = S \neq \emptyset$ and we may remove
 18 the set of nodes in S from either r_i or r_j . This step will not affect the coverage of passengers as
 19 the removed nodes from one route is still served by the other route. In addition, the route after
 20 node removal is still feasible since the route travel time strictly decreases with the removal of an
 21 intermediate stop following the triangular inequality. Based on this property, we can therefore
 22 solve the route generation problem by sequentially and independently identifying route of highest
 23 passenger coverage and generating the mutually disjoint MCR.

On the other hand, finding the mutually disjoint route with maximum passenger coverage itself is still nontrivial. In the worst case, one may have to enumerate all possible routes in the

network and select the route with the highest served passengers. However, the time constraint in Problem 2 introduces a promising direction for reducing the search space for finding MCR. In particular, if we starting from the hub (denoted as h), the maximum passenger coverage path can be obtained by tracking the potential passenger coverage of the set of zones that are reachable from the hub. And we define **potential passenger coverage** of zone j as the largest possible passenger demand that can be covered by extending the route from the zone i to zone j . If we denote $\Gamma(i)$ as the operator for the potential passenger coverage of a route starting from location i , we can express the previous statement as:

$$\Gamma(i) = q_i + \max_{j \in REACH(i)} \Gamma(j) \quad (3)$$

1 In the equation, $REACH(i)$ represents the set of zones that are reachable from zone i , where j is
 2 said to be **reachable** from i the travel time of the extended route from i to j will not exceed the
 3 δ times the corresponding shortest travel time as shown in Problem 2. Instead of enumerating all
 4 routes and select the best one, we can therefore build the solution algorithm for finding maximum
 5 passenger coverage route based on these two properties recursively and the proposed hub-based
 6 MCR generation algorithm is presented in Algorithm 1.

7 While the complexity of the problem is still exponential, for real-world problems, the tri-
 8 angular inequality of travel time will help to reduce the size of the reachable set as the route being
 9 extended from the hub. The computation time is therefore much faster than its worst case perfor-
 10 mance and we also observe the exact algorithm to generate MCR efficiently based on our NYC
 11 case study. In case when the exact algorithm may not find the solution in a timely manner, we
 12 further observe the following two properties of the proposed algorithm which suggest the direction
 13 for developing an efficient heuristic algorithm.

14 **Proposition 1.** *The induced reachable set of a node is a subset of the reachable set of its prede-*
 15 *cessor node.*

16 **Proposition 2.** *The potential of a node is no larger than the sum of the potential of the nodes in its*
 17 *reachable set.*

Proposition 1 results from the triangular inequality constraint for travel time. Proposition 2 gives the upper bound of the maximum possible potential of a node since a route may not server more than the number of nodes in the current reachable set. Based on these two propositions, we can relax the Γ operator with a heuristic operator Γ^h as the sum of the node potential in its reachable set:

$$\Gamma(i) = q_i + \max_{j \in REACH(i)} \Gamma^h(j) \quad (4)$$

where

$$\Gamma^h(j) = \sum_{m \in REACH(j)} q_m \quad (5)$$

18 This step is equivalent to relax the $MAXP$ function in Algorithm 1 by replacing the recursion with
 19 the sum of penitential of nodes in the reachable set. In this regard, the computation time of the
 20 heuristic is $O(V^3)$ and we are able to find near-optimal MCR for large instances in polynomial
 21 time. And the relaxed $MAXP$ function is presented in Algorithm 2.

Algorithm 1 Hub-based Route Generation Algorithm

Input: $G = (V, A, Q, W)$ network of study area, K number of routes to be generated, h hub location, δ deviation threshold

Output: \mathcal{R}_K the set of candidate operation routes.

```

1: for  $i=1,2,\dots,K$  do
2:    $P_i \leftarrow 0$ 
3:    $L_i \leftarrow 0$ 
4:    $N_h \leftarrow REACH(h, V, Q, L_i, \delta)$ 
5:    $P_i, r_i \leftarrow MAXP(h, N_h, P_i, L_i, Q, W)$ 
6:    $Q \leftarrow UPDATEP(r_i, Q)$ 
7:   Add  $r_i$  to  $\mathcal{R}_K$ 
8: end for
9: return  $\mathcal{R}_K$ 

1: function REACH( $i, V, W, L, \delta$ )
2:    $N_i \leftarrow \emptyset$ 
3:   for  $j \in V$  do
4:     if  $L + w_{i,j} < \delta w_{j,j}$  then
5:        $N_i \leftarrow N_i \cup \{j\}$ 
6:     end if
7:   end for
8:   return  $N_i$ 
9: end function

1: function MAXP( $i, N, P_i, L_i, V, Q, W$ )
11:  if  $|N| = 1$  then return  $q_j, j$ 
12:  else if  $|N| = \emptyset$  then return  $0, \emptyset$ 
13:  else
14:     $P \leftarrow 0, r = \emptyset$ 
15:    for  $j \in N$  do
16:       $N_j \leftarrow REACH(j, V, W, L_i + w_{i,j}, \delta)$ 
17:       $p, r' \leftarrow MAXP(j, N_j, P_i + d_i, L_i + w_{i,j}, V, Q, W)$ 
18:      if  $p > P$  then
19:         $P \leftarrow p, r \leftarrow r' \cup i$ 
20:      end if
21:    end for
22:  end if
23:  return  $P, r$ 
24: end function

25: function UPDATEP( $r, Q$ )
26:  for  $i \in r$  do
27:     $q_i \leftarrow 0$ 
28:  end for return  $Q$ 
29: end function

```

Algorithm 2 Heuristic MAXP

```

1: function HEURISTIC-MAXP( $i, N, P_i, L_i, V, D, W$ )
2:   if  $|N| = 1$  then return  $D_j, j$ 
3:   else if  $|N| = \emptyset$  then return  $0, \emptyset$ 
4:   else
5:      $P \leftarrow 0, r = \emptyset$ 
6:     for  $j \in N$  do
7:        $N_j \leftarrow \text{REACH}(j, V, W, L_i + w_{i,j}, \delta)$ 
8:        $p \leftarrow \sum_{k \in N_j} q_k$ 
9:       if  $p > P$  then
10:         $p \leftarrow P, r \leftarrow r' \cup \{i, j\}$ 
11:       end if
12:     end for
13:   end if
14: return  $P, r$ 
15: end function

```

1 Heuristic route generation

2 In addition to the developed algorithm for MCR generation, we also introduce a shortest-path
3 based heuristic route generation algorithm that is popular in the literature (17) and we compare the
4 effectiveness between MCR and the heuristic route set. The main idea of the heuristic algorithm
5 is to first generate a large collection of candidate routes and the final set of operation routes is
6 determined by pruning the candidate routes based on user-specified thresholds and through the
7 optimization problem. Following this philosophy, the proposed heuristic approach in this study
8 consists of three major steps including initial route generation, feasible route expansion and route
9 pruning. The output of the heuristic algorithm is the set of candidate transit routes for optimization
10 and we next discuss briefly the steps of the heuristic approach.

11 Initial route generation

The initial candidate routes are created by first finding k -shortest routes from each zone i to and
from hubs using k -shortest path algorithm (18). This ensures the route to be expanded and com-
bined in the next steps will not deviate too much from the passengers' shortest travel route and
therefore may meet the time sensitivity constraint in a heuristic manner. The process results in the
heuristic candidate path set R^h with $k|V|$ candidate routes and we also keep track of the zones that
each $r \in R^h$ traverses through and denote it as N^r . Since there may be multiple candidate routes
that stop at zone i , we then perform passenger demand assignment based on the travel time of each
shortest route following the logit assignment:

$$P_i^r = \frac{e^{-t_i^r}}{\sum_{r' \in R_i^h} e^{-t_i^{r'}}}, \forall i \in V, r \in R_i^h, \quad (6)$$

where t_i^r is the route travel time if passengers at zone i take route r and R_i^h is the set of heuristically
generated routes that traverse from zone i . We can therefore measure the relative importance of

each zone and each route as zone weight W_i and route weight W^r :

$$W_i = \sum_{j \in V} \sum_{r \in R_j^h} \mathbb{1}_i^r q_j P_j^r, \quad W^r = \sum_{i \in N^r} W_i \quad (7)$$

1 where $\mathbb{1}_i^r$ denotes whether route r travels by zone i .

2 *Candidate route expansion*

3 The candidate routes generated in the previous step are the set of k -shortest routes from zone i ,
 4 and each pair path in R^h may be further combined to create new routes that extend the coverage of
 5 passenger demand. We consider expanding two routes $r_1, r_2 \in R^h$ following two rules: 1) if r_1 is a
 6 sub-route of r_2 where $N_{r_1}^h \subseteq N_{r_2}^h$ and 2) if r_1 and r_2 may share common link segments. The route
 7 expansion is then performed based on crossover operation over the actual link segments of the two
 8 routes following Algorithm 3. And the weight of the expanded route is also calculated by the zone
 9 weight W_i .

Algorithm 3 candidate routes extension

Input: $r_1 = \{l_1, l_2, \dots, l_L\}, r_2 = \{p_1, p_2, \dots, p_P\}, N_{r_1}^h, N_{r_2}^h$.

Output: Route expansion algorithm.

```

1:  $G \leftarrow \emptyset$ 
2: for  $l_i \in r_1, p_j \in r_2$  do
3:   if  $l_i = p_j$  then
4:     if  $N_{r_1}^h \subseteq N_{r_2}^h$  then
5:        $\bar{r} \leftarrow \{l_1, l_2, \dots, l_i, p_{j+1}, \dots, p_P\}$ 
6:        $G \leftarrow G \cup \bar{r}$ 
7:     else
8:        $\bar{r}_1 = \{l_1, l_2, \dots, l_i, p_{j+1}, \dots, p_P\}$ 
9:        $\bar{r}_2 = \{p_1, p_2, \dots, p_j, l_{i+1}, \dots, l_L\}$ 
10:       $G \leftarrow G \cup \{\bar{r}_1, \bar{r}_2\}$ 
11:    end if
12:  end if
13: end for
14: return  $G$ 
```

10 *Route pruning*

11 The set of candidate routes may grow rapidly after the route expansion step with new routes gener-
 12 ated through the crossover operations. On the other hand, directly using this large candidate route
 13 set may prevent the optimization problem from identifying the set of optimal operation routes due
 14 to the large search space. Meanwhile, not all routes in the expanded set are practical in real-world
 15 applications and there may exist many routes that are similar. To eliminate unnecessary routes, we
 16 next conduct route pruning based on three metrics.

17 The first metric is route circuitry computed as total route length divided by the euclidean distance
 18 between the start and end location. With user-specified length and circuitry threshold l_{thd} and C_{thd} ,
 19 we choose routes with length longer than l_{thd} , and circuitry smaller than C_{thd} and get the new R^h .

The second metric is the subset relationship. If a route is a sub-route of the other route with smaller length, then the shorter length will be discarded. Accordingly, we find the following set of routes and remove them from the R_h :

$$\{r_1 | \exists r_2 \in R^h, N_{r_1}^h \subseteq N_{r_2}^h, L_{r_1} < L_{r_2}\} \quad (8)$$

The last metric is based on route similarity and it is measured as the summation of the nearest distance between each pair of individual link segments over the total link segments of the two routes as:

$$S(r_1, r_2) = \frac{\sum_{l \in r_1} \min_{p \in r_2} \text{dist}(l, p) + \sum_{p \in r_2} \min_{l \in r_1} \text{dist}(p, l)}{L_{r_1} + L_{r_2}} \quad (9)$$

where L_r is the length of route r and $\text{dist}(p, l)$ is the euclidean distance between the center of two road segments p and l respectively. The similarity index quantifies the spatial adjacency of two routes. To prune similar routes, we first sort the weight of the routes in R^h in descending order as:

$$W_{i=1} \geq W_{i=2} \geq W_{i=3} \geq \dots \geq W_{i=|R^h|} \quad (10)$$

We then remove the following set of routes from R^h based on route similarity threshold S_{thd} :

$$\{r_i | \exists 1 \leq j \leq i-1, S(r_i, r_j) < S_{thd}\} \quad (11)$$

- 1 In this manner, it removes the repetitive routes with lower weight (importance) and only keeps
- 2 those with higher passenger coverage. And we can adjust the number of routes to keep for the
- 3 optimization problem by changing the threshold values in these three metrics.

4 Route combination

5 The proposed route generation algorithms can be applied separately to obtain MCRs for passengers
 6 traveling to and from transportation hubs. Denote R_K^d as the MCR for serving passengers traveling
 7 to hubs and R_K^o as the MCR for serving passengers traveling from hubs, the next step is to connect
 8 R_K^d and R_K^o to form round routes and therefore reduce the amount of vacant mileage.

9 The route combination is conducted following two rules: 1) routes to and from hubs with
 10 similar demand level should be combined and 2) the connecting distance between the end of two
 11 routes should not exceed the route distance. The first rule helps to balance the service frequency
 12 between two combined routes and the second rule is set to avoid excessive vacant travel distance to
 13 combine two routes of similar demand level but are dist anted from each other. Based on these two
 14 settings, we develop a bipartite matching problem as a solution approach for optimally combining
 15 routes in R_K^d and R_K^o . In particular, we denote g_{r_1, r_2} as the gap between the demand level of route
 16 $r_1 \in R_K^o$ and $r_2 \in R_K^d$. Moreover, if the connecting travel time between the two routes exceeds the
 17 shorter route travel time between the two routes, g_{r_1, r_2} is set to an arbitrary large value to denote
 18 infeasible combination. Consequently, the optimal route combination can be obtained by solving

1 the following optimization problem:

$$\begin{aligned}
& \text{minimize } \sum_{r_1} \sum_{r_2} x_{r_1, r_2} g_{r_1, r_2} \\
& \text{s.t. } \sum_{r_1} x_{r_1, r_2} = 1, \forall r_2 \in R_K^d \\
& \quad \sum_{r_2} x_{r_1, r_2} = 1, \forall r_1 \in R_K^o \\
& \quad x_{r_1, r_2} \in \{0, 1\}
\end{aligned} \tag{12}$$

2 The problem is equivalent to the minimum weight perfect bipartite matching problem and can be
3 solved efficiently using the Hungarian algorithm. Readers may refer to (19) for implementation
4 details.

5 Fleet optimization

TABLE 1: Table of notation

Parameter	Descriptions
\mathcal{R}	Set of candidate routes.
q_i	Total induced passenger arrival rate at location i .
\bar{q}_i^k	Served passenger arrival rate at location i by transit route k .
t_i^k	Total travel time between i and hub using transit route k .
$t_{j,i}$	In vehicle travel time between zone i and zone j .
t_l	Loading (unloading) time per unit passenger.
w^k	Expected waiting time for route k .
e_i	Energy consumption per passenger for trips from location i to transportation hub.
x^k	Frequency over route k , number of vehicles per hour, $k \in \mathcal{R}$.
C	Maximal capacity of the selected electric bus.
s_i^k	Indicator variable and $s_i^k = 1$ if route k stops at zone i . 0 otherwise.
d_k	Travel length of route k .
t_s	average stopping time per stop.
c_c	Capital cost per vehicle.
c_o	Operational cost per vehicle per kilometer.
c_r	Cost per ride.
B	Total project budget.

6 Given the set of candidate routes, the optimization problem aims at planning the service
7 frequency x_k of each routes and hence the fleet setting which maximizes the energy savings and
8 service revenue while minimizing operation and capital cost. The mathematical notion used in the
9 optimization problem is shown in Table 1. We consider the realistic scenario where passengers
10 have access to both planned DAT service and other trip modes. And their choices depend on
11 the utility they perceived (travel time and trip cost) following the classical discrete choice model.
12 The travel time of a passenger at location i using route k consists of the transit operation time

- 1 between each pair of stations, the expected waiting time at the location and the cumulative loading
- 2 (unloading) time before arriving at her final destination:

$$t_i^k = w^k + \sum_{j \in \mathcal{P}_i^k} (t_{j-1,j} + t_l q_j) \quad (13)$$

- 3 where \mathcal{P}_i^k is the set of stops proceeding to i on route k . We can further write $\bar{t}_i^k = \sum_{j \in \mathcal{P}_i^k} t_{j-1,j}$
- 4 (constant value) to represent the total segment travel time to reach the transportation hub if the
- 5 passenger ride route k at location i .

The objective function consists of three components. The energy saving of the DAT can be computed as the energy cost (in monetary value) of the served passengers if they use their original trip mode:

$$F_{\text{energy}} = \sum_{i \in V} \sum_{k \in \mathcal{R}} e_i q_i^k \quad (14)$$

- 6 where e_i measures the energy cost of the alternative mode associated with each passenger for
- 7 commuting to hubs at location i .

The trip revenue of the DAT service can be written as:

$$F_{\text{revenue}} = c_r \sum_{i \in V} \sum_{k \in \mathcal{R}} q_i^k \quad (15)$$

- 8 where a flat rate c_r is considered in this study.

Finally, the operation cost of DAT can be expressed as:

$$F_{\text{operation}} = c_o \sum_{k \in \mathcal{R}} x^k d^k \quad (16)$$

- 9 where $\sum_{k \in \mathcal{R}} x^k d^k$ measures the total operation distance per hour for the proposed DAT service.
- 10 Given the travel time and the objective function, the optimization problem follows

maximize $F_{\text{energy}} + F_{\text{revenue}} + F_{\text{operation}}$

$$t_i^k = \bar{t}_i^k + \frac{1}{2x^k} + \sum_{j \in \mathcal{P}_i^k} t_l q_j$$

$$q_i^k = \frac{Q_i^k e^{c_1^{\text{transit}} t_i^k + c_2^{\text{transit}} c_r + b^{\text{transit}}}}{\sum_{m \in \mathcal{M}} e^{c_1^M t_i^M + c_2^M c_i^M + b^M}}$$

$$\sum_i q_i^k s_i^k \leq C x^k, \quad \forall k \in \mathcal{R}$$

$$x^k (1 - x^k) \leq 0$$

$$c_c \sum_k x_k T_k \leq B$$

$$x^k \geq 0, \quad \forall k \in \mathcal{R}$$

- 12 The first constraint is the calculation of average travel time from zone i to hub h using route k ,
- 13 where w^k is computed as the expected headway of route k as $\frac{1}{2x^k}$. The second constraint models

the choice made by passengers for selecting between DAT and other modes based on trip cost and travel time. The third constraint states that the capacity provided by each route should be higher than the induced passenger demand. The fourth constraint restricts that there should be at least one trip if we plan to operate route k . Finally, the fifth constraint sets the budget for purchasing buses for serving the planned routes. The optimization problem is a difficult nonlinear programming problem, and we solve the problem by iteratively relaxing the nonlinear constraints using sequential quadratic programming approach (20). We omit the reformulation details here due to the page limit. While the problem may only be solved to local optimal solution, we use a cross-validation approach which creates a candidate set of starting points and we report the best solution identified from the set of starting points.

RESULTS

Experiment setup

We choose NYC as the study area and demonstrate the effectiveness of the proposed DAT framework by developing the DAT service at JFK airport. To quantify the modeling parameters, we collect information from publicly available datasets including road information, geographical subdivisions of the city, public transit information and demand of taxi and for hire vehicles (FHV). The network of our study area is built from the taxi zone shapefile of NYC which has 263 taxi zones across 5 major boroughs. The State Island is not included due to low demand level (less than 0.5% of total demand to hubs such as LGA, JFK and Penn Station) which reduces the total number of zones to 234. And the route generation problem is therefore to find the MCR over the 234 zones.

For the numerical experiments, we consider the proposed DAT to compete with FHV and yellow taxis for potential passengers. In 2018, FHV serve an average of 550,000 trips per day (only workdays taken into account) and the demand for yellow taxis is around 300,000. Among those trips, approximately 30,670 trips have either their origins or destinations at the JFK airport (3.5%) and we use the weekday's hourly FHV and taxis' passenger demand associated with the JFK airport in each taxi zone in 2018 (21) as the total potential demand for the DAT service. We summarize the parameter setting for the numerical experiments in Table 2. We set the parameters in the discrete choice model following the results of the passengers' behavior study at the airport (22). And the mean trip cost and travel time for FHV and taxis can be calculated directly from the trip data.

TABLE 2: Model parameters

Parameter	Descriptions	Value	Reference
δ	Distance threshold	1.5	-
e	Energy saving per passenger for trip (measured as price per gallon gasoline)	3	-
C	Capacity of one electric bus	40	-
c_c	Capital cost per electric bus	850,000	(23)
c_o	Operational cost (electricity cost + maintenance cost)	0.42\$/mile (0.10+0.32)	(23)
c_r	Cost per ride	\$ 10	-

We consider the energy saving of DAT as the total energy cost of the served passengers if they take taxis and FHVs. And the energy saving of each taxi trips are calculated as the product of trip distance and the fuel consumption multiplier estimated by COPERT model (24):

$$F = \frac{217 + 0.253V + 0.00965V^2}{1 + 0.096V - 0.000421V^2} \quad (18)$$

where V is the average trip speed (km/h) that are calculated from the taxi trip data.

For the bus configuration, we consider the fleet of pure electric buses and we find that Proterra, Inc. and BYD, Inc. are the two major suppliers of electric buses. Based on their specification data (25, 26), we choose the electric bus of battery capacity 100 kWh and the estimated charging cost is 0.10 \$/mile. A recent electric bus report (23) presented that the purchase price of a normal electric bus is between \$800,000 and \$900,000 and the maintenance cost is about three times of the electricity cost. We therefore assume that the capital price is \$850,000 per bus and the maintenance cost is 0.32 \$/mile. The typical electric bus has the capacity of 40 seats. Finally, we consider each purchased electric bus has the lifetime of 12 years (23) and each bus will operate for 12 hours per day. And we set the budget constraint in the experiments as \$1000/h which approximately equals a fleet of 62 buses.

Generated path

Given the input parameters, we next evaluate the performance of the route generation algorithms on the resulting coverage of passenger demand. In particular, we evaluate the increment in passenger coverage by increasing the number of routes generated by the route generation algorithms and the results are shown in Figure 2(a). Among the three algorithms (MCR with exact solution, MCR with heuristic solution and heuristic routes), the MCR with exact solution gives the best performance where all demand from JFK airports can be covered by 61 mutually disjoint routes with travel time constraints satisfied for all passengers. Moreover, we observe that over 40% of passengers may be covered by operating as few as 9 routes. This implies the effectiveness of MCR generation to identify most valuable operation routes and indicates great potential for operating DAT at transportation hubs. Moreover, the MCR with heuristic solution also results in high quality candidate routes and its performance is on par with the exact algorithm especially on the first 5 paths generated. But the solution of MCR heuristic gets worse with increasing number of paths and the passenger coverage may be 20% lower than the results from the exact solution. On the other hand, the heuristically generated routes are observed to have the worst performance as compared to the results from two other MCR generation algorithms and the passenger coverage is 50-70% of that of the exact solution for MCR. This states the ineffectiveness of the routes generation mechanisms proposed in the literature, which is primarily a general purpose route generation approach and may fail to obtain solutions for hub-based transit effectively. Finally, the computational time for obtaining the MCR exact solution is 59.3 seconds and is 15.7 seconds for MCR with heuristic solution respectively. This demonstrates the efficiency of the exact algorithm even for real-world problems, nevertheless, the computation time will increase significantly if the threshold for passenger travel time is relaxed to 2 or larger. For $\delta = 2$, it takes 10 hours and 35 minutes for the exact algorithm to find the exact optimal solution while the computational time for heuristic is 63.7 seconds. In this case, the size of the reachable set may not drop quickly and the computational time for obtaining the heuristic solution will be barely affected and becomes the better option. Based on these results, we choose the set of 9, 18 and 37 routes generated by each of these algorithms as

- 1 the input for the fleet optimization problem which cover approximately 43%, 67% and 87% of taxi
- 2 and FHV passengers respectively as shown in Figure 2(b).

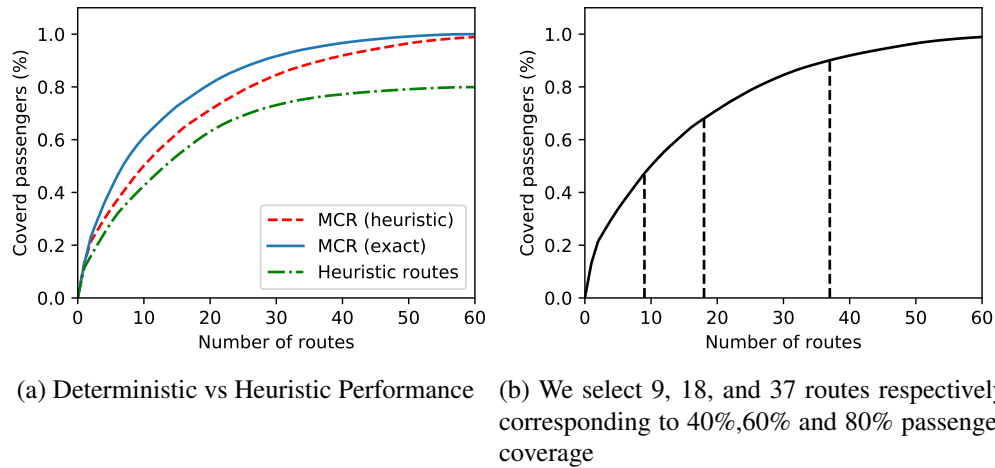


FIGURE 2: Comparison of heuristic approach and deterministic algorithm

3 In addition to the passenger coverage, we also visualize the routes generated by the algo-
 4 rithms for four different time periods of the day and the results are shown in Figure 3. These routes
 5 are generated based on the passenger demand and traffic condition at the particular time of the day
 6 to reflect the value of being demand adaptive. And we visualize the top three routes of largest
 7 passenger coverage after the route combination process. As we can see in the figures, for maxi-
 8 mum passenger coverage, the generated routes share similar philosophy by serving the zones in
 9 Manhattan of high passenger demand but also visit several intermediate locations in Queens along
 10 the route under the travel time constraint. And the incoming and outgoing routes at transportation
 11 hubs are observed to be well paired to form the round loops based on their trip demand and con-
 12 nection distance. The first route remains largely the same across different time of day, by visiting
 13 two zones near the JFK airport and then goes directly into mid-Manhattan areas for dropping off
 14 passengers from the hub and picking up passengers to the hub. And the other two routes are found
 15 to complement the first routes by visiting locations at middle to lower Manhattan and middle to
 16 upper Manhattan areas. And there are some subtle differences for these two routes depending on
 17 time of day. For instance, there are no stops in upper-east side of Manhattan, which is primarily of
 18 residential areas, during morning peak hours. And this region is covered with drop-off stops during
 19 evening peak hours and pick-up stops during late night hours which may be because of business
 20 travelers returning home and leaving for trips for the next business day.

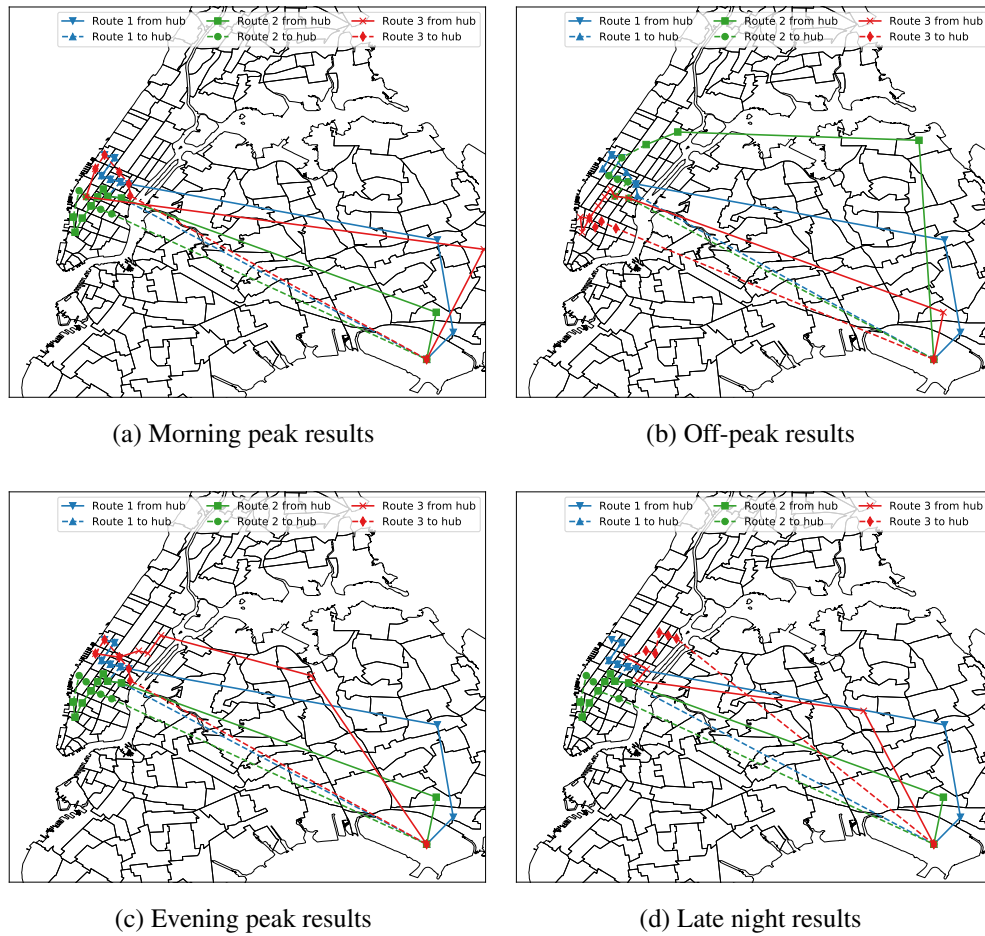


FIGURE 3: Generated path. For route based algorithm, the paths from hub and to hub are overlapping.

1 Optimized fleet

2 We use the set of candidate routes generated by the three different route generation approaches
 3 as the input for the fleet optimization problem. And we conduct experiments for travel demand
 4 under five different scenarios: morning peak hours (7:00-9:00), off-peak hours (11:00-13:00),
 5 evening peak hours (17:00-19:00), late night hours (21:00-23:00), and daily average (average over
 6 24 hours). The results for comparing different route generation approaches are presented in Table 3
 7 and the results for different time of day using MCR algorithm are shown in Table 4 and Table 5.

8 As shown in Table 3, by operating only 9 routes, the planned DAT at JFK with the routes
 9 generated by MCR exact algorithm may serve 36.8% of the passengers from taxis and FHVVs. And
 10 the service rate may reach 45.53% and 46.41% respectively if we use the 18 and 37 candidate routes
 11 respectively, under the same budget constraint. On the other hand, the planned DAT with MCR
 12 heuristic algorithm and the heuristically generated routes have consistently worse performance as
 13 compared to the MCR exact approach. For all three approaches, the optimization results for 9
 14 candidate routes yield the capital expenditure being lower than the proposed budget constraint,
 15 indicating the room for scheduling service for more routes. And the budget constraint is binding

for 18 and 37 candidate routes scenarios, with 37 routes resulting in higher objective function values. But there are only minor differences between the results of 18 routes and 37 routes, where the optimal planned DAT may serve 0.9% more passengers and achieve 2.02% higher objective function value under 37 candidate routes than 18 candidate routes (using MCR exact). This again demonstrates the quality of the MCR approach where the algorithms are able to precisely identify the most effective set of candidate routes for the fleet optimization problem. And there is a huge gap between the objective function value as well as the served number of passengers between heuristically generated routes and other two approaches. We therefore conclude that the type of route generation mechanism are not effective for hub-based DAT planning and we exclude it from further evaluations.

While the set of candidate routes generated by MCR are based on the maximum potential of the route set, we also observe that the optimally planned DAT is able to retain majority of the passengers. For instance, the 9 candidate routes may cover up to 43% of total passengers and the planned DAT based on the 9 candidate routes eventually serve 36.84% of total passengers or equivalently satisfy 85.7% of the covered passengers considering their rational choice. And with the budget constraint, the planned DAT based on 18 candidate routes may still satisfy 68% of the covered passengers. These results confirm the effectiveness of our proposed DAT planning framework and also suggest the feasibility of the DAT as a sustainable and more eco-friendly mobility solution at uber transportation hubs. And we conclude that our proposed route generation algorithm and the fleet optimization approach are able to generate high quality DAT solutions and the framework may even meet the needs of real-time DAT planning at transportation hubs based on our MCR heuristic algorithm.

Finally, the effectiveness of the proposed DAT planning framework can be further validated by the results during different time of the day. Note that the travel demand at transportation hubs is largely affected by the flight or train schedules and the trip needs to and from hubs at different time of day are therefore very different. In these experiments we consider only the case with 18 candidate routes and the optimal planned DAT is found to have consistent performances across all these scenarios. The planned DAT is observed to serve the highest number of passengers during PM peak (reflected by the highest revenue) which suggests the trip origins and destinations to and from hubs during this time period are more similar. On the other hand, the late night period is the scenario with the lowest number of passengers and lowest percentage of passengers served. And this is due to the fact that the demand between the incoming and outgoing routes are more unbalanced than other time periods and such scenario is indeed inefficient for any type of transit systems. But even under this scenarios, we still find the planned DAT to serve over 42% of the total passengers with the MCR exact approach and this result and the performances of MCR heuristic are found to be close to the exact solutions.

TABLE 3: Results for different size of candidate routes (Daily Average)

	Routes (selected / candidates)	Objective function	Passenger served (%)	Energy saving (\$ / hour)	Revenue (\$ / hour)	Operation cost (\$ / hour)	Capital cost (\$ /hour)	Diff(%)
MCR (exact)	9 / 9	4359.30	36.84	497.8	4227.9	366.4	754.6	-
	18 / 18	5329.92	45.53	602.7	5211.3	484	1000	-
	25 / 37	5437.43	46.41	612.5	5311.3	486.3	1000	-
MCR (heuristic)	9 / 9	3549.39	30.6	408.9	3502.2	361.7	670.1	19.6
	18 / 18	4568.02	40.05	528.4	4583.7	544.1	1000	14.3
	26 / 37	4605.53	40.34	526.7	4617.4	538.6	1000	15.3
Heuristic	9 / 9	1553.95	14.98	169.9	1670.1	286.1	716.1	64.4
	18 / 18	2437.68	23.02	270.4	2634.5	467.2	991.2	54.3
	34 / 37	3767.15	34.14	431.4	390.7	571.3	1000	30.7

TABLE 4: Result for different time period (MCR heuristic)

Scenario	Objective function	Served passengers (%)	Energy saving (\$ / hour)	Revenue (\$ / hour)	Operation cost (\$ / hour)	Capital cost (\$ / hour)
Daily Average	4568.02	40.05	528.4	4583.7	544.1	1000
AM Peak	4591.03	41.46	525.1	4597.3	531.3	1000
PM Peak	5923.28	42.96	676.8	5783.6	537.1	1000
Off Peak	5390.52	44.53	613.7	5311.0	534.1	1000
Night Period	4178.82	38.69	490.3	4210.3	521.7	1000

TABLE 5: Result for different time period (MCR exact)

Scenario	Objective function	Served passengers (%)	Energy saving (\$ / hour)	Revenue (\$ / hour)	Operation cost (\$ / hour)	Capital cost (\$ / hour)
Daily Average	5329.92	45.53	602.7	5211.3	484.0	1000
AM Peak	5185.30	45.84	585.2	5082.1	482.0	1000
PM Peak	6541.57	46.77	733.7	6296.4	488.5	1000
Off Peak	5778.96	47.24	649.0	5634.4	504.4	1000
Night Period	4636.11	42.26	527.5	4598.3	489.7	1000

1 CONCLUSION

2 In this study, we present the three-stage framework for optimal planning of DAT at urban trans-
3 portation hubs. The proposed framework consists of route generation, route combination, and fleet
4 optimization. And we develop both efficient and effective route generation algorithms to generate
5 high quality candidate route set for the fleet optimization problem. To demonstrate the perfor-
6 mance of our proposed solution, we conduct comprehensive numerical experiments for planning

optimal DAT at JFK airport in NYC. To best capture real-world settings, we use NYC taxi and FHV data to calibrate the potential demand and use GoogleMap API to obtain the corresponding road traffic information. The results highlight the effectiveness of the proposed DAT service which may serve 47% of the passengers originally riding taxis and FHVs. The results also demonstrate the quality of the candidate routes generated by our MCR algorithm and the consistency of the planned DAT to reach optimal performance across different time of the day. For future studies, the MCR heuristic algorithm can be modified to take the spatial properties of planar graph to further tighten the solution obtained so that close to optimal solutions can be found more efficiently. And the proposed framework can be further validated in other countries and under larger numerical experiments with more refined zone configurations.

AUTHOR CONTRIBUTIONS

The authors confirm contribution to the paper as follows: study conception and design: X. Qian, J. Xue, S.V. Ukkusuri; data collection: J. Xue, Z. Lei, J. Suarez; analysis and interpretation of results: X. Qian, J. Xue, Z. Lei, J. Suarez; draft manuscript preparation: X. Qian, J. Xue, Z. Lei, J. Suarez, S.V. Ukkusuri. All authors reviewed the results and approved the final version of the manuscript.

ACKNOWLEDGEMENT

This work is partly funded by the U.S. Department of Energy, Office of Energy Efficiency and Renewable Energy, under Award Number DE-EE0008524. The authors are solely responsible for the findings in this paper.

REFERENCES

- [1] Ming Zhang. Can transit-oriented development reduce peak-hour congestion? *Transportation Research Record*, 2174(1):148–155, 2010.
- [2] Metropolitan Transportation Authority. New York City transit ridership trend, 2018.
- [3] New York City Taxi and Limousine Commission and Department of Transportation. Improving efficiency and managing growth in New York’s for-hire vehicle sector, 2019.
- [4] Federico Malucelli, Maddalena Nonato, and Stefano Pallottino. Demand adaptive systems: some proposals on flexible transit. In *Operational research in industry*, pages 157–182. Springer, 1999.
- [5] Valérie Guihaire and Jin-Kao Hao. Transit network design and scheduling: A global review. *Transportation Research Part A: Policy and Practice*, 42(10):1251–1273, 2008.
- [6] Konstantinos Kepaptsoglou and Matthew Karlaftis. Transit route network design problem. *Journal of transportation engineering*, 135(8):491–505, 2009.
- [7] Reza Zanjirani Farahani, Elnaz Miandoabchi, Wai Yuen Szeto, and Hannaneh Rashidi. A review of urban transportation network design problems. *European Journal of Operational Research*, 229(2):281–302, 2013.
- [8] W Lampkin and PD Saalmans. The design of routes, service frequencies, and schedules for a municipal bus undertaking: A case study. *Journal of the Operational Research Society*, 18(4):375–397, 1967.
- [9] Lionel Adrian Silman, Zeev Barzily, and Ury Passy. Planning the route system for urban buses. *Computers & operations research*, 1(2):201–211, 1974.
- [10] Ernesto Cipriani, Stefano Gori, and Marco Petrelli. Transit network design: A procedure and

- an application to a large urban area. *Transportation Research Part C: Emerging Technologies*, 20(1):3–14, 2012.
- [11] Miloš Nikolić and Dušan Teodorović. Transit network design by bee colony optimization. *Expert Systems with Applications*, 40(15):5945–5955, 2013.
- [12] Christina Iliopoulou, Ioannis Tassopoulos, Konstantinos Kepaptsoglou, and Grigorios Beligiannis. Electric transit route network design problem: Model and application. *Transportation Research Record*, 2019.
- [13] Moschoula Pternea, Konstantinos Kepaptsoglou, and Matthew G Karlaftis. Sustainable urban transit network design. *Transportation Research Part A: Policy and Practice*, 77:276–291, 2015.
- [14] Zvi Drezner and Said Salhi. Using hybrid metaheuristics for the one-way and two-way network design problem. *Naval Research Logistics (NRL)*, 49(5):449–463, 2002.
- [15] Wei Fan. *Optimal transit route network design problem: Algorithms, implementations, and numerical results*. The University of Texas at Austin, 2004.
- [16] Partha Chakroborty. Genetic algorithms for optimal urban transit network design. *Computer-Aided Civil and Infrastructure Engineering*, 18(3):184–200, 2003.
- [17] Fabio Pinelli, Rahul Nair, Francesco Calabrese, Michele Berlingerio, Giusy Di Lorenzo, and Marco Luca Sbodio. Data-driven transit network design from mobile phone trajectories. *IEEE Transactions on Intelligent Transportation Systems*, 17(6):1724–1733, 2016.
- [18] Jin Y Yen. Finding the k shortest loopless paths in a network. *management Science*, 17(11):712–716, 1971.
- [19] Harold W Kuhn. The hungarian method for the assignment problem. *Naval research logistics quarterly*, 2(1-2):83–97, 1955.
- [20] Paul T Boggs and Jon W Tolle. Sequential quadratic programming for large-scale nonlinear optimization. *Journal of computational and applied mathematics*, 124(1-2):123–137, 2000.
- [21] 2018 NYC yellow taxi trip record data, accessed May, 2019. Available online at <https://www1.nyc.gov/site/tlc/about/tlc-trip-record-data.page>.
- [22] Hyuk-Jae Roh. Mode choice behavior of various airport user groups for ground airport access. *The Open Transportation Journal*, 7(1), 2013.
- [23] Judah Aber. Electric bus analysis for new york city transit. *Columbia University, New York*, 2016.
- [24] Leonidas Ntziachristos, Dimitrios Gkatzoflias, Chariton Kouridis, and Zissis Samaras. Copert: a european road transport emission inventory model. In *Information technologies in environmental engineering*, pages 491–504. Springer, 2009.
- [25] Inc Proterra. Low-floor electric transit buses, accessed on July, 2019. Available online at <https://www.proterra.com/vehicles/>.
- [26] BYD North America. Driving the future, accessed on July, 2019. Available online at <https://en.byd.com/bus/>.