SAND2021-2826C

Lyapunov control-inspired protocol for quantum optimization

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Background: Quantum approximate optimization algorithm (QAOA)

QAOA aims to achieve quantum speedup in the near term for discrete optimization

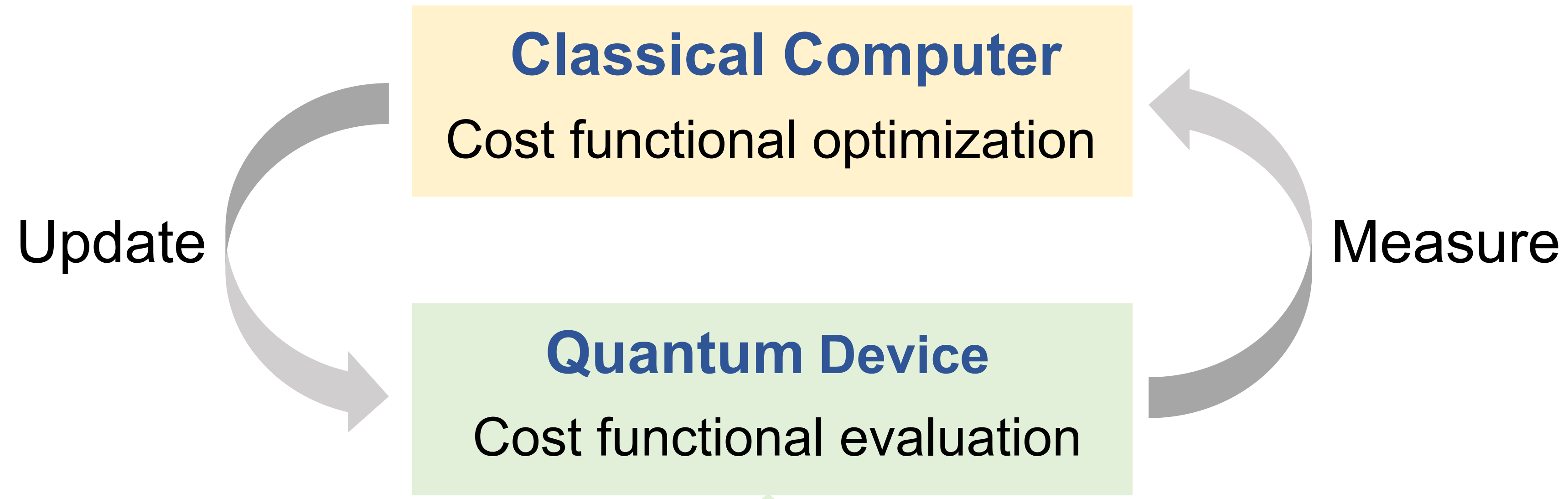
1. Map a discrete optimization problem to Ising "problem" Hamiltonian H_p, such that H_p's ground state encodes the optimization problem solution

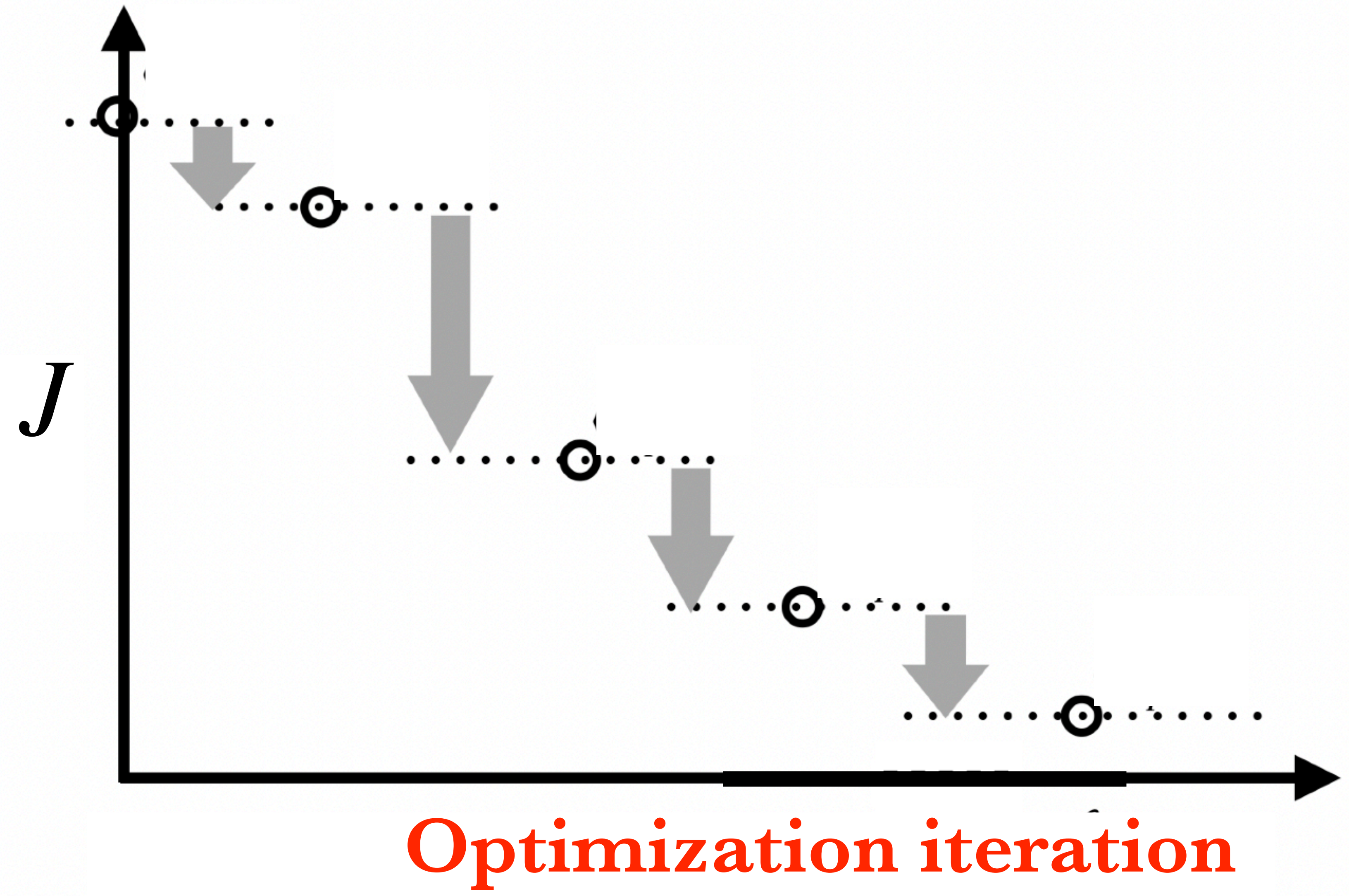
2. Iteratively optimize over a set of parameters $(\vec{\gamma}, \vec{\beta})$ in order to solve:

$$\min_{\vec{\gamma},\vec{\beta}} \langle \psi(\vec{\gamma}, \vec{\beta}) | H_p | \psi(\vec{\gamma}, \vec{\beta}) \rangle$$

E. Farhi, J. Goldstone, and S. Gutmann, arXiv:1411.4028 (2014).

Bottleneck: searching for the optimal $(\vec{\gamma}, \vec{\beta})$ parameters





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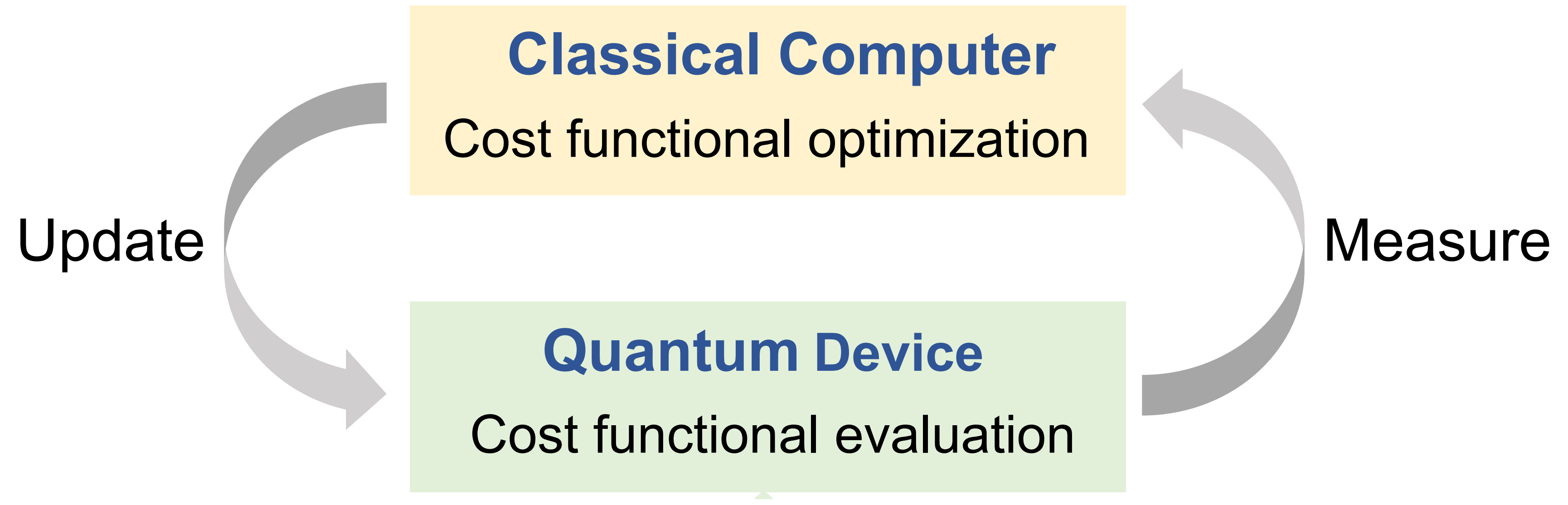
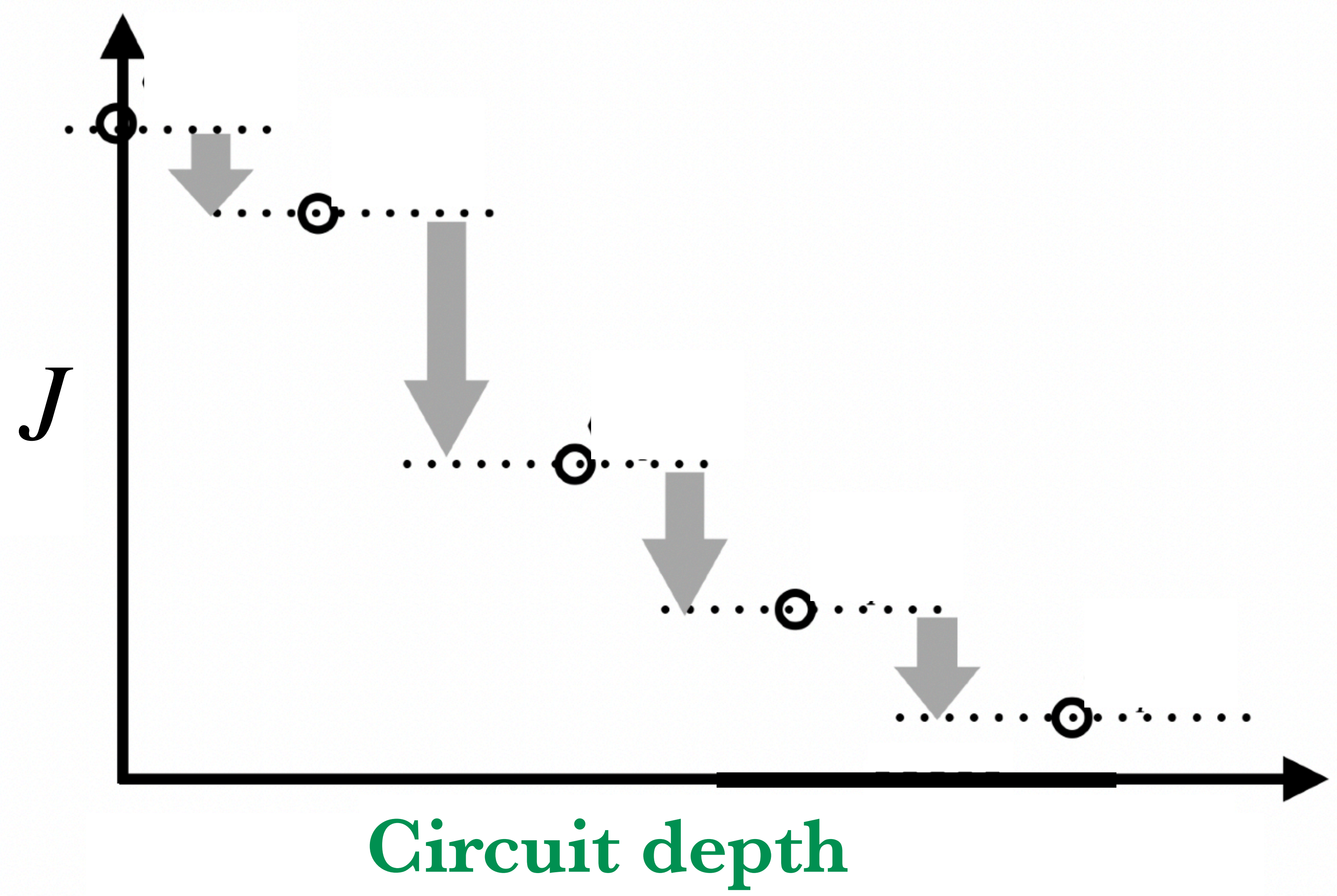
This work:
Introduce a feedback-based protocol that removes this bottleneck





Feedback control-inspired algorithm for quantum optimization

Consider a quantum system whose dynamics are governed by

$$|\dot{\psi}(t)\rangle = -iH(t)|\psi(t)\rangle \qquad \text{where } H(t) = H_p + H_d\beta(t)$$

Our objective is to design $\beta(t)$ to drive the system to a state $|\psi(t)\rangle$ that minimizes $J = \langle\psi(t)|H_p|\psi(t)\rangle$.

We design $\beta(t)$ to ensure that $\langle H_p\rangle(t) = \langle\psi(t)|H_p|\psi(t)\rangle$ decreases monotonically over time, by enforcing

$$\frac{d}{dt}\langle H_p\rangle(t) \leq 0, \quad \forall t$$

$$\frac{d}{dt}\langle H_p\rangle(t) = \frac{d}{dt}\langle\psi(t)|H_p|\psi(t)\rangle = i\langle\psi(t)|[H_d, H_p]|\psi(t)\rangle\beta(t)$$

Define $A(t) \equiv \langle\psi(t)|i[H_d, H_p]|\psi(t)\rangle$

Then, $\frac{d}{dt}\langle H_p\rangle(t) = A(t)\beta(t)$

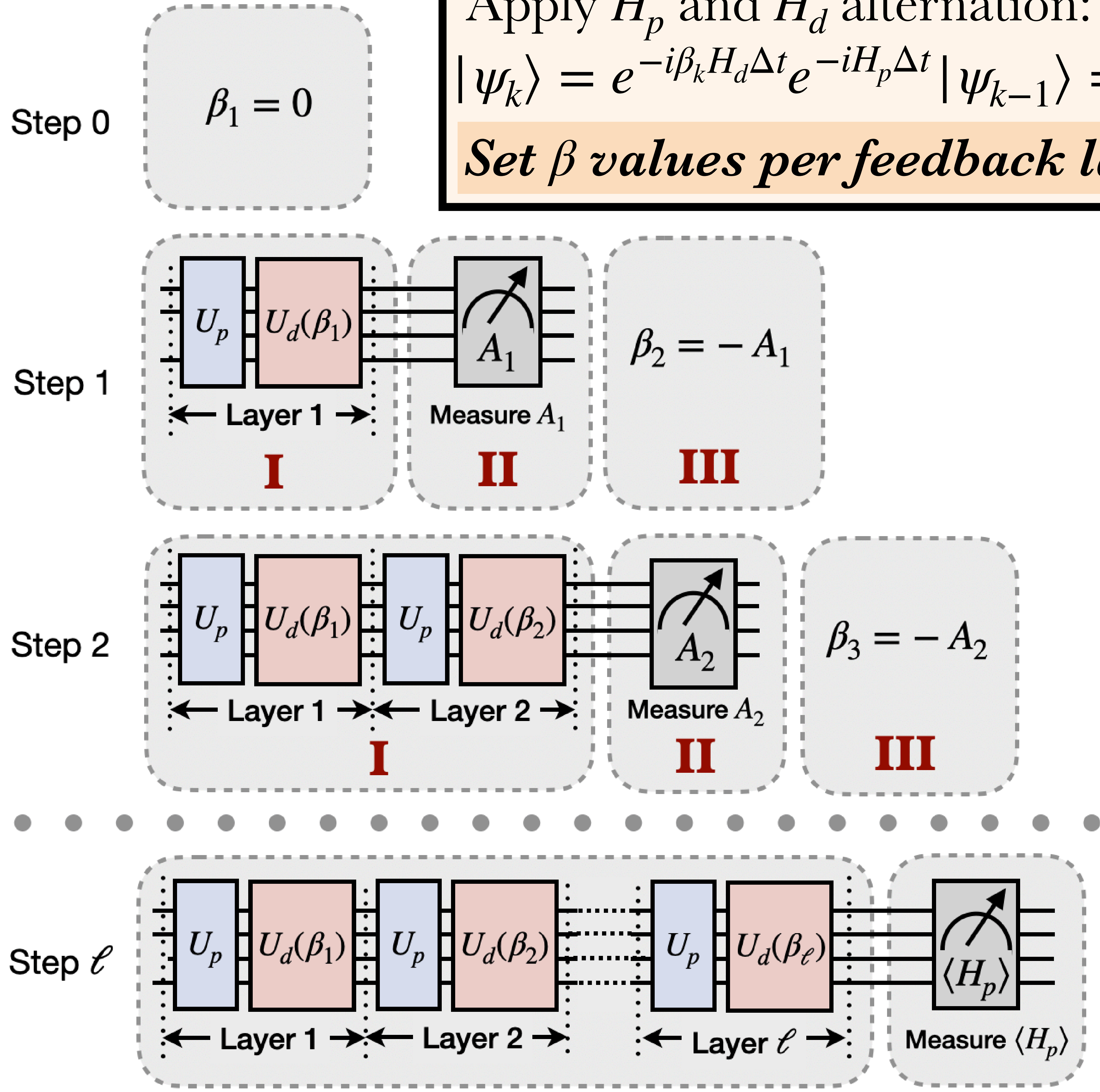
$$\beta(t) = -A(t) \Rightarrow \frac{d}{dt}\langle H_p\rangle(t) = -\left(A(t)\right)^2$$

Implementation:

Apply H_p and H_d alternation:

$$|\psi_k\rangle = e^{-i\beta_k H_d \Delta t} e^{-iH_p\Delta t}|\psi_{k-1}\rangle = U_d(\beta_k)U_p|\psi_{k-1}\rangle$$

Set β values per feedback law $\beta_k = -A_{k-1}$

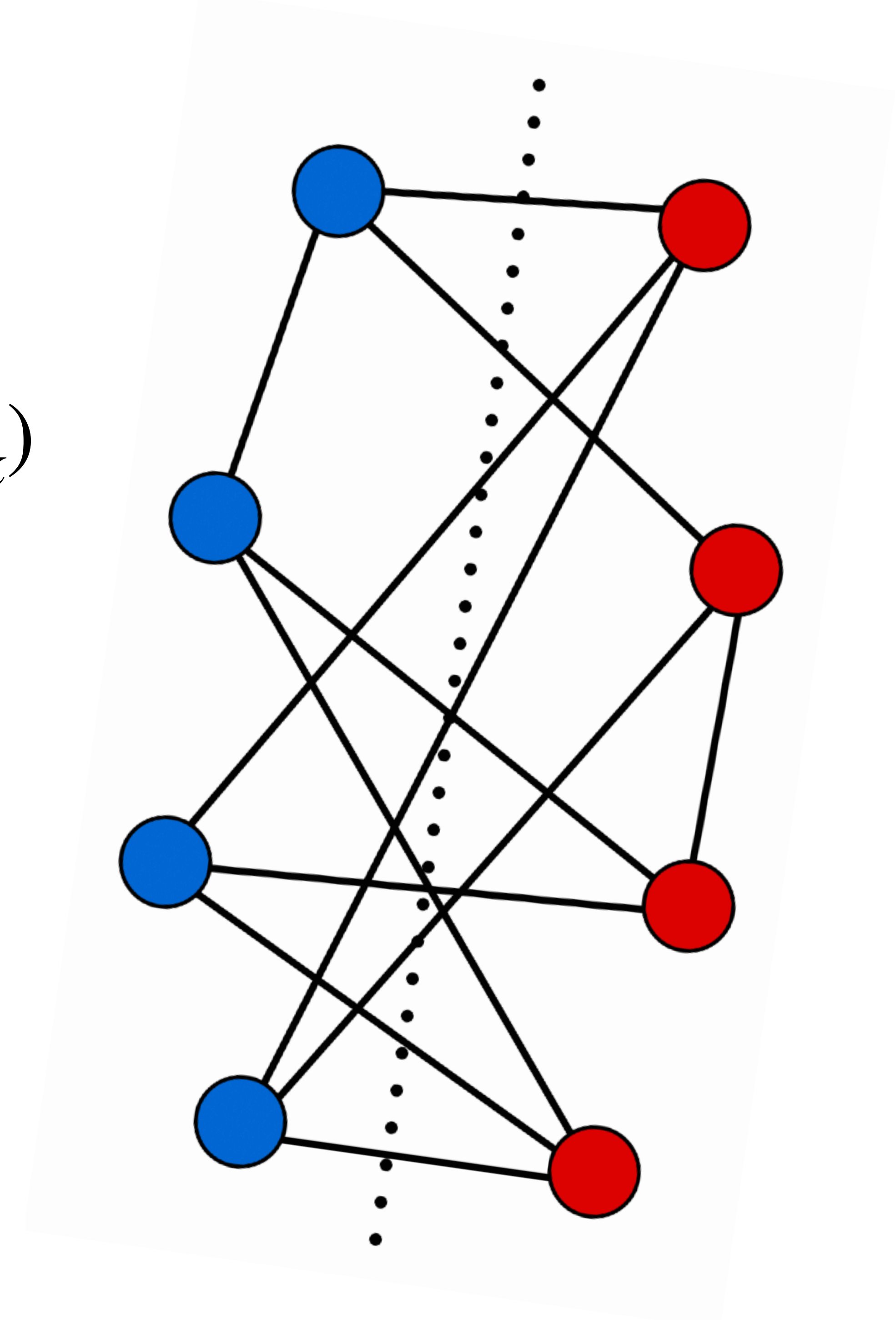


Numerical illustrations: Applications to MaxCut

The aim of MaxCut is to identify bipartition of a graph that maximizes the number of edges crossing the two sets.

We consider MaxCut on connected, 3-regular graphs, where $H_p = -\sum_{j,k} \frac{1}{2}(1 - Z_j Z_k)$

We map this to the task of searching for a state $|\psi\rangle$ that minimizes $\langle H_p \rangle(t)$, where the ground state of H_p, $|\psi_{min}\rangle$ encodes the global minimum solution



We consider two figures of merit: the probability PR of measuring $|\psi_{min}\rangle$, and the approximation ratio $AR = \dfrac{\langle H_p \rangle}{\min \langle H_p \rangle}$

QAOA sampling cost: $N_{samp} \geq mq(1 + 2\ell) = O(mq\ell)$

This work: $N_{samp} \leq m(1 + 2\ell(d + 1)) = O(md\ell)$

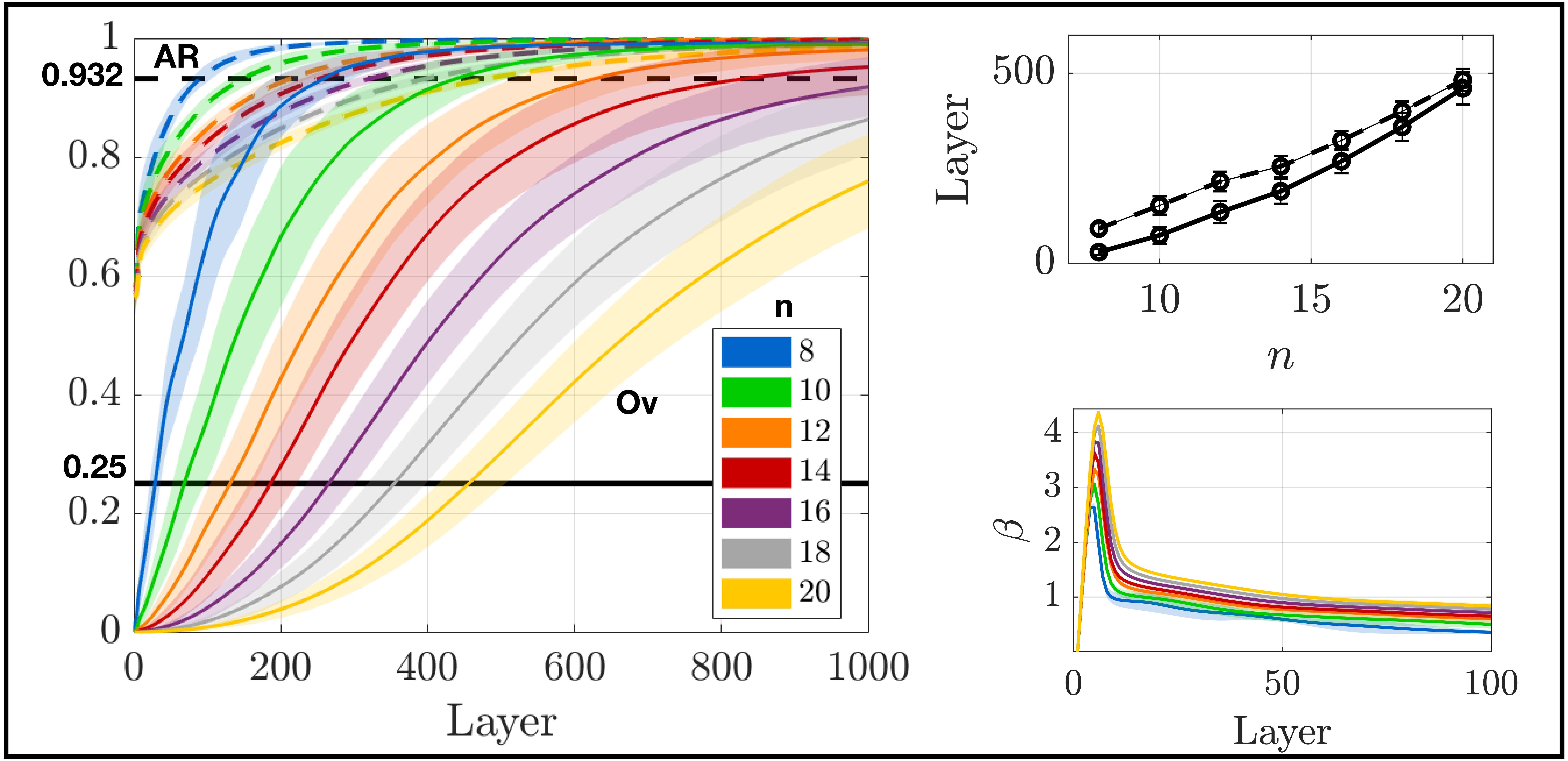
m: # samples to evaluate single expectation value

q: # classical optimization iterations

ℓ: # layers

d: degree of graph

Numerical illustrations: Applications to MaxCut



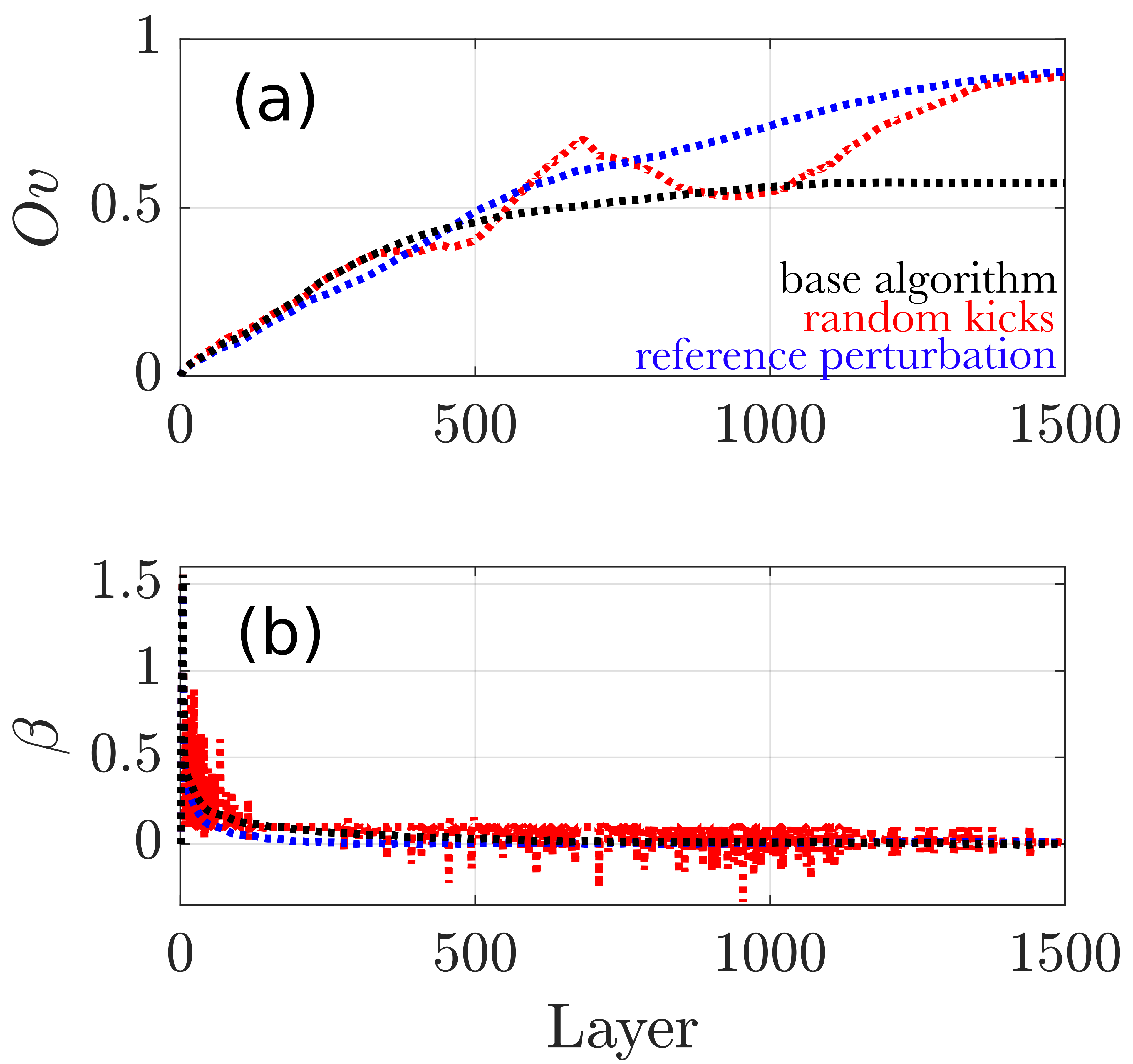
Numerical illustrations: Applications to MaxCut

We also considered weighted MaxCut on connected, 4-regular graphs, where

$$H_p = -\sum_{j,k} \frac{w_{jk}}{2}(1 - Z_j Z_k), \quad w_{jk} \in [0,1]$$

Discovered weighted instances where convergence of Ov was poorer compared with unweighted instances

Introduced two heuristic improvements to enhance performance of base algorithm for these instances: the addition of random kicks in β and the addition of a reference perturbation in β



Summary

- Introduced a **deterministic, feedback-based algorithm for quantum optimization**
- The use of feedback from qubit measurements **eliminates the need for classical optimization** to search for optimal parameters
- Presented numerical illustrations showing strong performance towards solving **MaxCut on regular graphs**
- Introduced **heuristics for enhancing performance** in difficult problem instances

Thank you

Matthew Grace

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