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Evaluation of Guardbanding Methods for Calibration and Product Acceptance

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ABSTRACT

The Primary Standards Lab employs guardbanding methods to reduce risk of false acceptance in calibration when test uncertainty ratios are low. Similarly, production agencies guardband their requirements to reduce false accept rates in product acceptance. The root-sum-square guardbanding method is recommended by PSL, but many other guardbanding methods have been proposed in literature or implemented in commercial software. This report analyzes the false accept and reject rates resulting from the most common guardbanding methods. It is shown that the root-sum-square method and the Dobbert Managed Guardband strategy are similar and both are suitable for calibration and product acceptance work in the NSE.

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ACRONYMS AND DEFINITIONS

Abbreviation	Definition
EOPR	End-Of-Period Reliability
ITP	In-Tolerance Probability
M&TE	Measuring and Test Equipment
NSE	Nuclear Security Enterprise
OOT	Out of Tolerance
PFA	Probability of False Accept
PFR	Probability of False Reject
PSL	Primary Standards Laboratory
RSS	Root Sum Square
TUR	Test Uncertainty Ratio

1. INTRODUCTION

The Primary Standards Lab (PSL) typically employs the root-sum-square (RSS) guardbanding method, where the guardband factor (gbf) is related to the Test Uncertainty Ratio (TUR) by $gbf = \sqrt{1 - 1/TUR^2}$. The relative acceptance limit (AL) is determined from the original relative specification limit (SL) and guardband factor: $AL = SL \times gbf$.

For TURs less than 4, this method achieves an approximately 0.6% global Probability of False Accept (PFA) when the Out-Of-Tolerance (OOT) rate of the units under test is 5%. The Probability of False Reject (PFR) ranges from 2% at a TUR of 4 to 18% at TUR of 1.5.

Other guardbanding methods have been proposed, such as those in Ref. [1] and Ref. [2]. These methods are sometimes encountered in third-party calibration software. For these methods to apply to calibrations at the PSL or product acceptance in the Nuclear Security Enterprise (NSE), they should achieve approximately the same false accept rates given the same situations. Additionally, any valid guardbanding method should result in less than 2% PFA as required by Z540.3 [3].

For any guardband method, the PFA calculation is a combination of the measurement uncertainty probability distribution (accounted for in the TUR) and the distribution of products being measured (whether Measuring and Test Equipment [M&TE] being calibrated or products being tested for acceptance). This product distribution can be estimated based on the OOT rate or the In-Tolerance Probability (ITP), which is 1-OOT rate. ITP is also referred to as the End-Of-Period Reliability (EOPR) in some literature, especially when discussing calibration intervals rather than product acceptance.

2. GUARDBAND METHODS

Common guardbanding methods are described here. In all these methods, the TUR is defined as

$$TUR = \frac{\pm\,Specification\ Limit}{\pm\,Measurement\ Uncertainty}$$

Where the measurement uncertainty is reported at a 95% level of confidence (k=2).

2.1. Root Sum Squares

The RSS method is recommended in the PSLM [4] and is one of two options allowed by General Requirements Document 9900000 [5].

$$gbf = \sqrt{1 - \frac{1}{TUR^2}}$$

2.2. 95% Measurement Uncertainty (U95)

The 95% Measurement Uncertainty method subtracts the expanded measurement uncertainty U^{95} from the specification limit ($AL = SL - U^{95}$). This method is the second option allowed by 9900000, and is the only available option for one-sided product requirements where the TUR cannot be calculated. In terms of TUR, it is equivalent to

$$gbf = 1 - \frac{1}{TUR}.$$

2.3. NCSLI RP-10

The U95 method is known to be overly conservative, so NCSLI's Recommended Practice 10 [6] suggested an alternative:

$$gbf = 1.25 - \frac{1}{TUR}.$$

2.4. Dobbert's Managed Guardband Strategy

Another method was proposed in Ref. [2]. This method aims to achieve the Z540.3 requirement of a PFA less than 2% for all combinations of TUR and OOT rate.

$$gbf = 1 - \frac{M_{2\%}}{TUR}$$
$$M_{2\%} = 1.04 - \exp\left(0.38\ln(TUR) - 0.54\right)$$

The $M_{2\%}$ expression was empirically fit to achieve a worst-case PFA of 2%.

2.5. Target False Accept

The final method defines a desired PFA value and solves the PFA integral (see Eq. 5-13 in Ref. [7]) for a guardband factor that achieves the desired PFA. This method, while it achieves exactly the desired PFA for any situation, requires numerical minimization techniques to solve for the limits on a double integral, and is thus far more complicated to implement and slower to execute.

3. DISCUSSION

In most calibration labs, including the PSL, uncertainties are set to achieve a 5% OOT rate (95% ITP). The effect of guardbanding methods on the PFA at this OOT rate is shown in Figure 3-1, calculated using Eq. 5-13 and 5-14 in Ref. [7]. The RSS and Dobbert methods are closely aligned, both with PFA near 0.6% and similar PFR for all TURs greater than the minimum allowable 1.5.

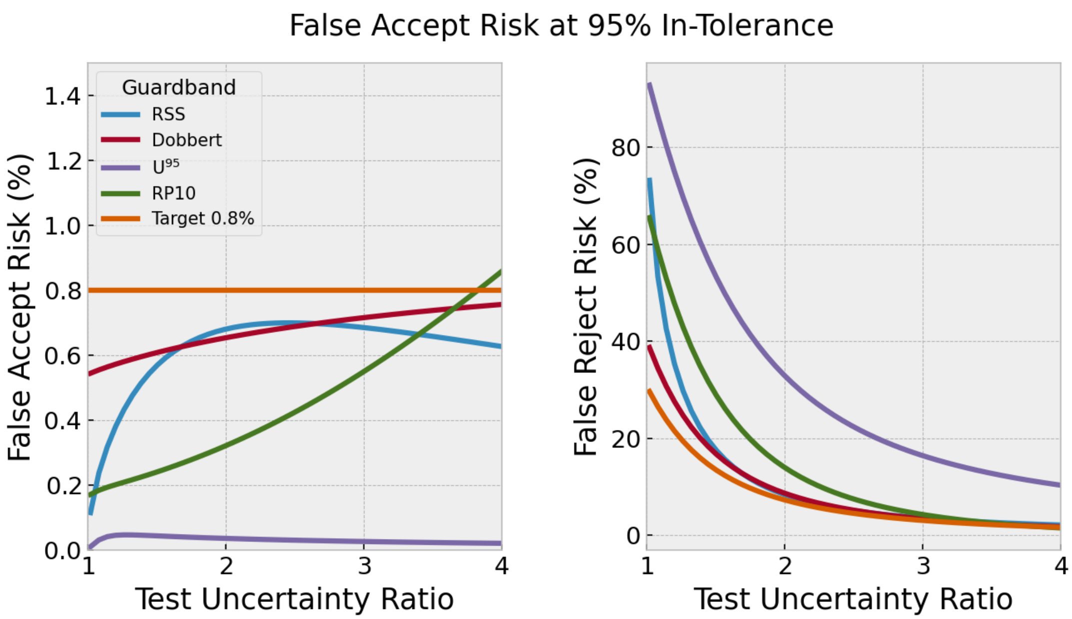

Figure 3-1. False accept/reject rates for different guardbanding methods at 95% ITP.

In product fabrication and acceptance, or when less reliable M&TE is being calibrated, the ITP is often lower than 95%. Figure 3-2 shows PFA and PFR plotted versus ITP for different TURs and guardbanding methods. Again, the RSS and Dobbert methods are similar for all situations. The U95 method reaches a much lower PFA, but at the cost of a significant PFR, reaching 50% for a TUR of 1.5. The RP10 method can exceed 2% PFA even with a TUR of 4 and offers no real advantages over the RSS method. The Target PFA method achieves exactly the desired PFA (in this case, 0.8%), but also comes at a cost of a higher PFR in most cases.

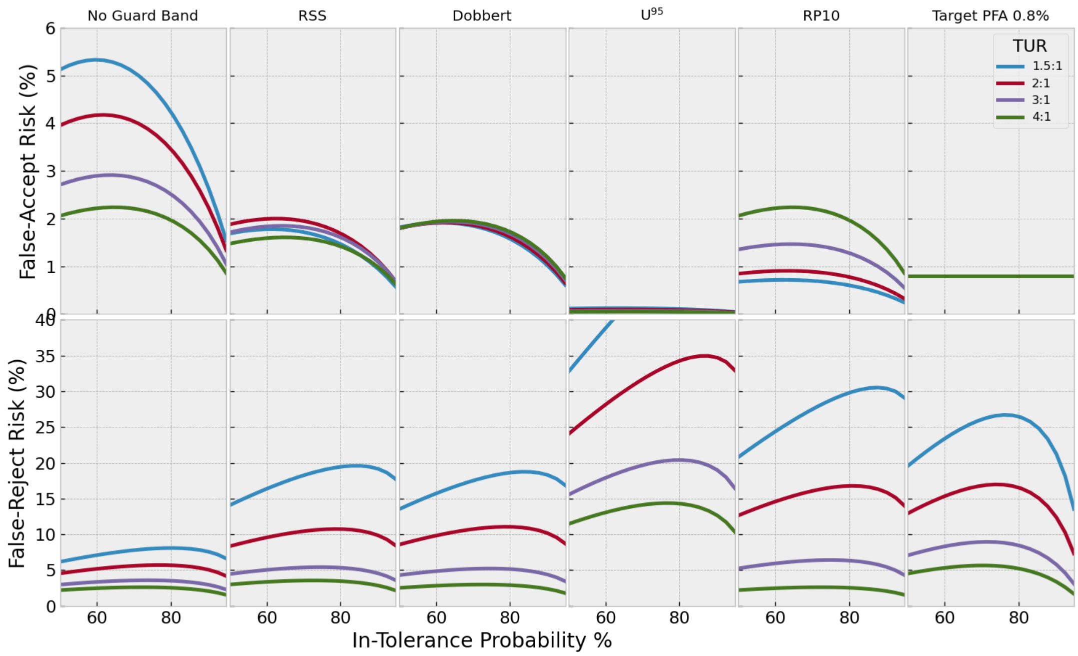

Figure 3-2. False accept/reject rates versus ITP at different TUR and guardband methods.

4. RECOMMENDATIONS

Based on Figure 3-1 and Figure 3-2, the RSS and Dobbert methods are very similar in their resulting false accept and reject rates. Either method should be acceptable for use in calibration work at PSL or for product acceptance in the NSE. The Dobbert method, while slightly more complicated to implement, has a small advantage with PFR rates when the ITP is lower than 95%. U95 and RP10 methods should be avoided because they can result in unreasonable false reject rates and have no advantages over the RSS or Dobbert methods. However, U95 is the only option for guardbanding one-sided limits when nothing can be assumed about the ITP or product distribution.

The Target PFA method may be a suitable guardband method for product acceptance if a very specific false accept rate is desired, but the ITP of the manufacturing process should be fully characterized to ensure the false reject rate is also acceptable. Alternatively, the coefficients used in the Dobbert method can be rederived to result in a different maximum PFA. While not trivial to derive, once in place this method is much simpler to calculate than the numerical minimization required for every acceptance limit calculation using the Target PFA method.

The Dobbert method should not be employed for any TUR greater than 4, because its equation results in an acceptance limit outside the specification limit—potentially accepting products that measure outside the requirement.

All these methods and risk calculations assume that the probability distributions involved (both test measurement distribution and product distribution) are normal and unbiased. See Ref. [8] for a discussion of how bias and nonnormalities can affect risk rates.

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