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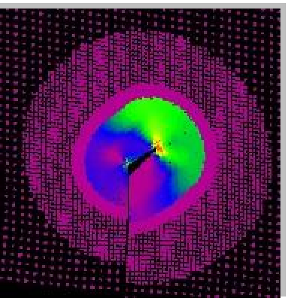

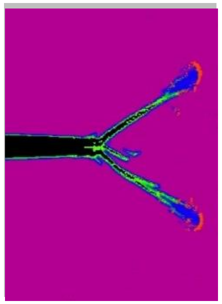

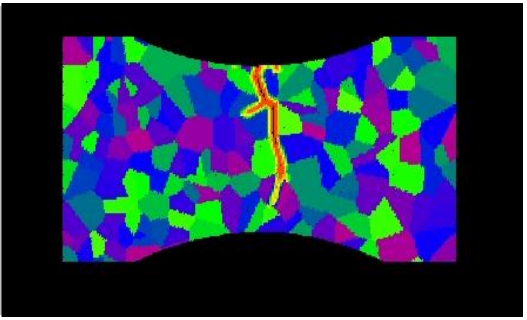

Combined Lagrangian and Eulerian approaches in peridynamic material modeling

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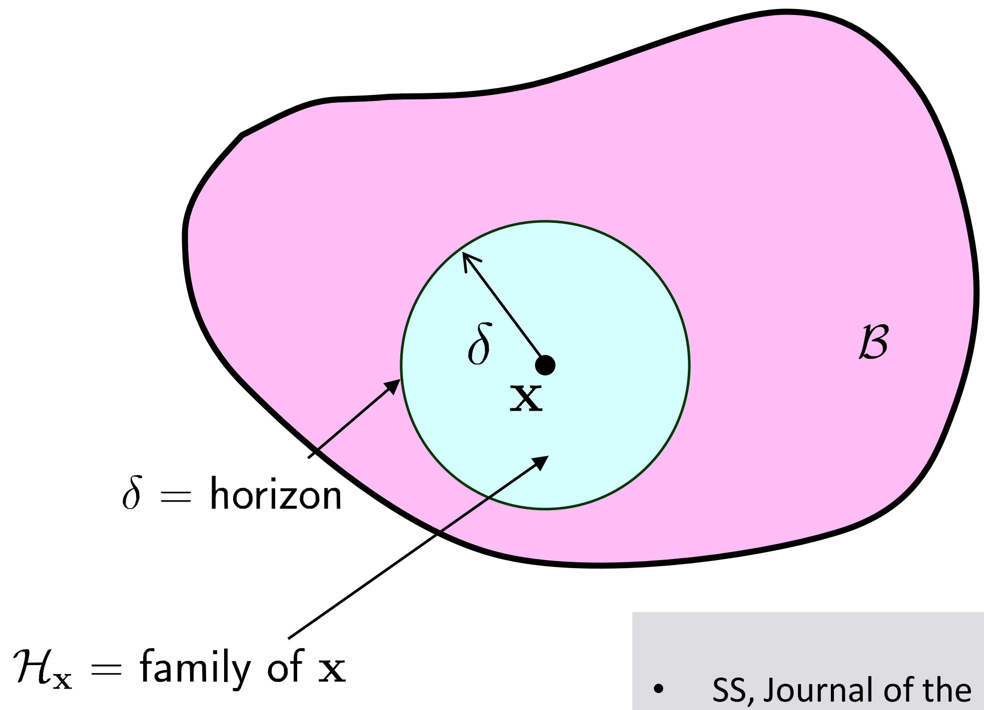
Outline

- Peridynamics background
- Eulerian vs. Lagrangian material models
  - Combining the two
- Applications
  - Contact and friction
  - Postfailure response
  - Soft materials
  - Bird strike

Peridynamics basics: Horizon and family

- Any point $\mathbf{x}$ interacts directly with other points within a distance δ called the "horizon."
- The material within a distance δ of $\mathbf{x}$ is called the "family" of $\mathbf{x}$, $\mathcal{H}_{\mathbf{x}}$.

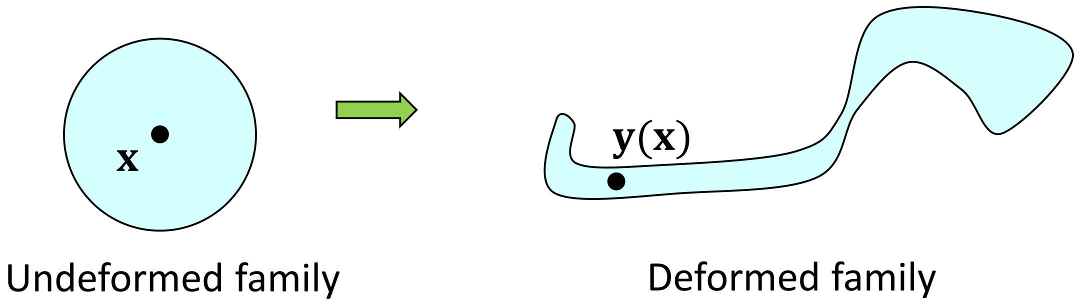


General references

- SS, Journal of the Mechanics and Physics of Solids (2000)
- SS and R. Lehoucq, Advances in Applied Mechanics (2010)
- Madenci & Oterkus , *Peridynamic Theory & Its Applications* (2014)

Motivation

- The traditional form of peridynamics is Lagrangian.
  - Material models refer explicitly to a reference (undeformed) configuration.
  - Good assumption for solids if there isn't too much deformation
  - This is not well-suited to fluids and large deformations.



  - What about materials that have both solid-like and fluid-like behavior?

States:
Objects that keep track of families

- A *state* is a mapping whose domain is a family.

$$\underline{A}\langle\xi\rangle = \text{something}$$

  where ξ is a bond in a family $\mathcal{H}$.

- Famous states: Deformation state...

$$\underline{\mathbf{Y}}[\mathbf{x}]\langle\mathbf{q}-\mathbf{x}\rangle = \mathbf{y}(\mathbf{q}) - \mathbf{y}(\mathbf{x}) = \text{deformed image of the bond}$$

  Force state...

$$\underline{\mathbf{T}}[\mathbf{x}]\langle\mathbf{q}-\mathbf{x}\rangle = \mathbf{t}(\mathbf{q},\mathbf{x}) = \text{force density within a bond}$$

- Dot product of states $\underline{A}$ and $\underline{B}$:

$$\underline{A} \bullet \underline{B} = \int_{\mathcal{H}} \underline{A}\langle\xi\rangle \underline{B}\langle\xi\rangle \, d\xi.$$

Thermodynamic form of a peridynamic material model

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SS & Lehoucq, Adv Appl Mech (2010)
Oterkus, Madenci & Agwai, JMPS (2014)

- First law expression:

$$\dot{\varepsilon} = \underline{\mathbf{T}} \bullet \underline{\mathbf{Y}} + r + h$$

where ε is the internal energy density, r is the source rate, h is the rate of heat transport.

- Second law expression:

$$\theta\dot{\eta} \geq r + h$$

where θ is the temperature and η is the entropy.

- Free energy:

$$\psi = \varepsilon - \theta\eta.$$

- Assume a material model of the form

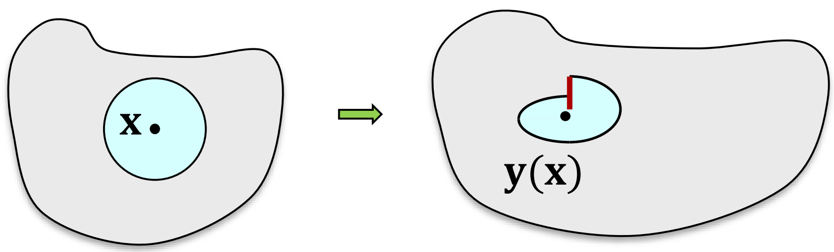
$$\psi(\underline{\mathbf{Y}}, \theta)$$

- First + second laws imply (through Coleman-Noll or similar method):

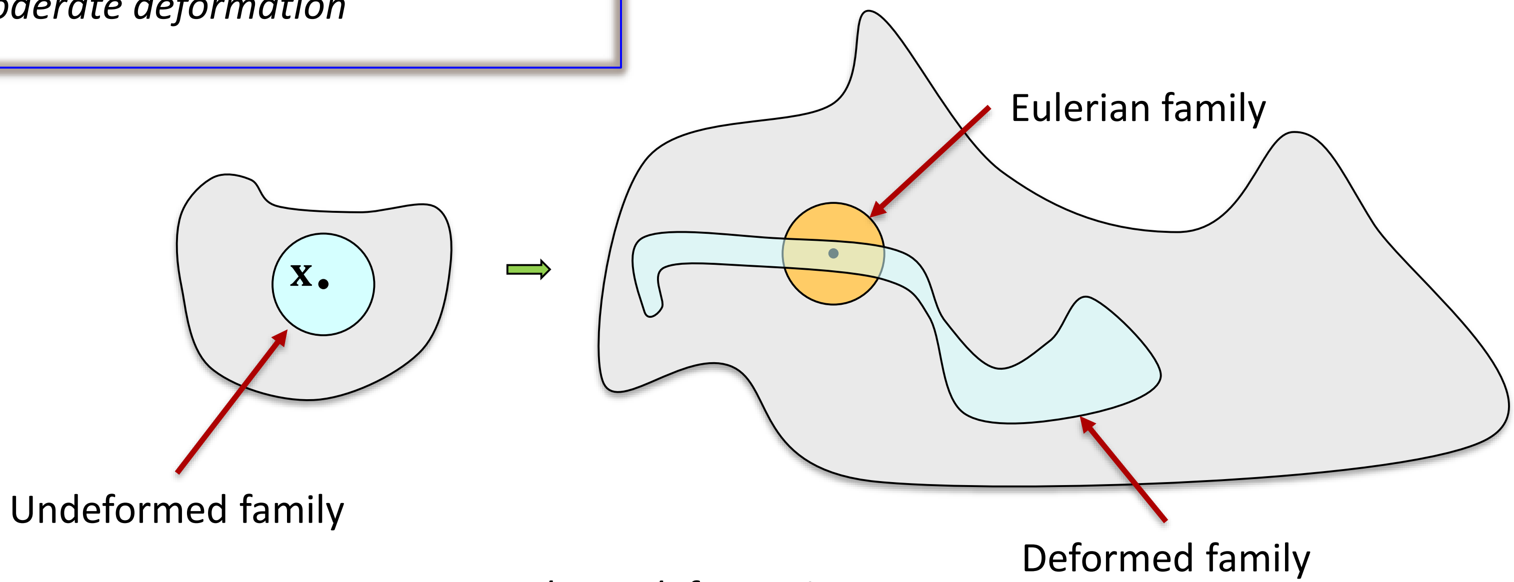
$$\underline{\mathbf{T}} = \psi_{\underline{\mathbf{Y}}}, \qquad \eta = -\psi_{\theta}.$$

For fluids, we'd like to apply the horizon in the deformed configuration

- Points interact only with other points within δ in the Eulerian family.



Moderate deformation



Very large deformation

Effectively Eulerian material models

- A Lagrangian material model involves both the undeformed and deformed bond vectors. Example:

This term makes the model Lagrangian

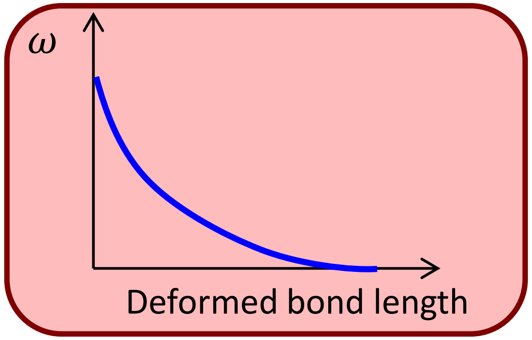
$$\underline{\mathbf{T}}\langle\xi\rangle = (|\underline{\mathbf{Y}}\langle\xi\rangle| - |\xi|)\frac{\underline{\mathbf{Y}}\langle\xi\rangle}{|\underline{\mathbf{Y}}\langle\xi\rangle|}.$$

- An Eulerian material model has bond forces that depend only on the deformed bond vectors. Example:

$$\underline{\mathbf{T}}\langle\xi\rangle = |\underline{\mathbf{Y}}\langle\xi\rangle|^{-n}\frac{\underline{\mathbf{Y}}\langle\xi\rangle}{|\underline{\mathbf{Y}}\langle\xi\rangle|},$$

$n > 0$.

Eulerian model for a fluid



- Define a nonlocal density by

$$\rho = \rho_0 \int_{\mathcal{B}} \omega(|\underline{\mathbf{Y}}\langle\boldsymbol{\xi}\rangle|)\, dV_{\boldsymbol{\xi}}$$

where ρ_0 is the reference density and ω is a weighting function such that $\int \omega = 1$. Integration is in the reference configuration.

- Compute the pressure from

$$p = -\frac{1}{\rho^2}\frac{\partial \psi}{\partial \rho}.$$

where ψ is the free energy density.

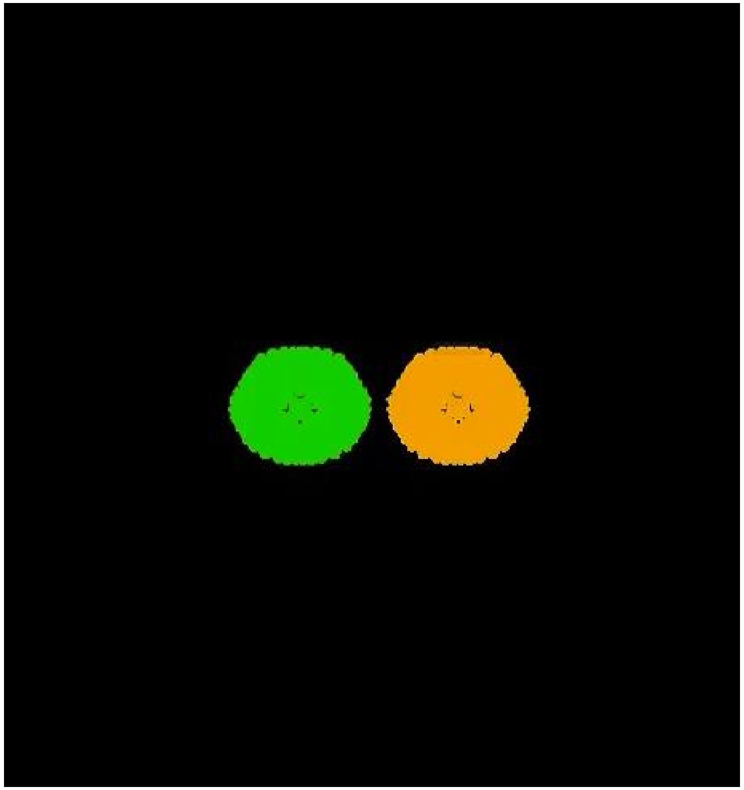
- The force state is found from the Frechet derivative of ψ to be

$$\underline{\mathbf{T}}\langle\boldsymbol{\xi}\rangle = \frac{\partial \psi}{\partial \underline{\mathbf{Y}}} = \frac{\partial \psi}{\partial \rho}\frac{\partial \rho}{\partial \underline{\mathbf{Y}}} = \frac{p\omega'(\boldsymbol{\xi})}{\rho^2}\frac{\underline{\mathbf{Y}}\langle\boldsymbol{\xi}\rangle}{|\underline{\mathbf{Y}}\langle\boldsymbol{\xi}\rangle|}.$$

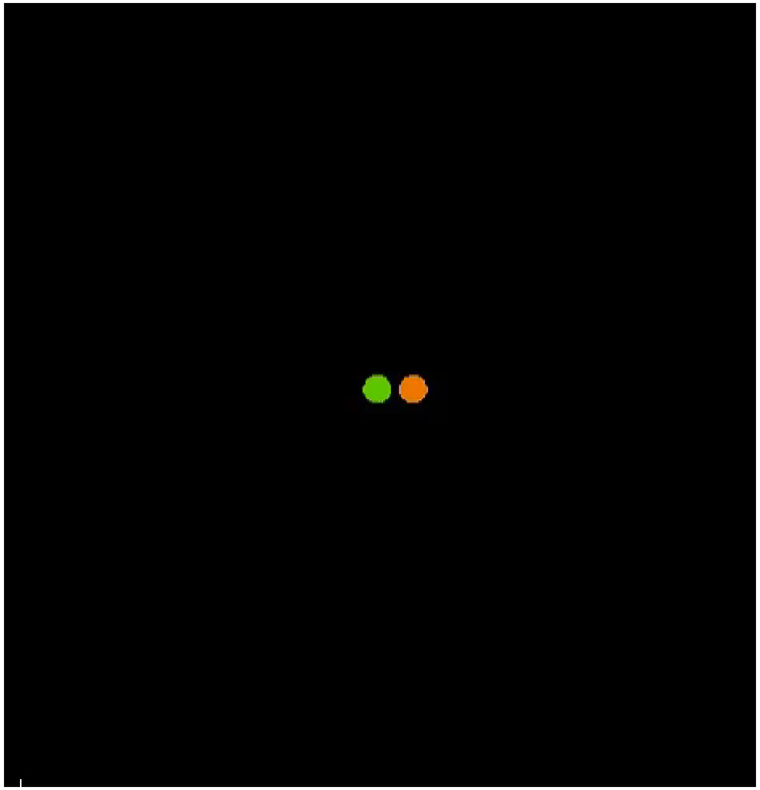
Surface tension examples

- Two droplets collide.
- Mie-Gruneisen EOS is used.

Low impact velocity



High impact velocity



Combining Lagrangian and Eulerian response in a single material model

- We'd like to model both fluid-like and solid-like response in the same material model.

- Combine the two as a linear combination of force states:

$$\underline{\mathbf{T}} = \beta(p)\underline{\mathbf{T}}^E + (1-\beta(p))\underline{\mathbf{T}}^L$$

  where $\underline{\mathbf{T}}^E$ and $\underline{\mathbf{T}}^L$ are the Eulerian (fluid-like) and Lagrangian (solid-like) contributions respectively.

- $\beta(p)$ is a pressure-dependent interpolation parameter, $0 \leq \beta \leq 1$.

- Example: EOS & bond-based:

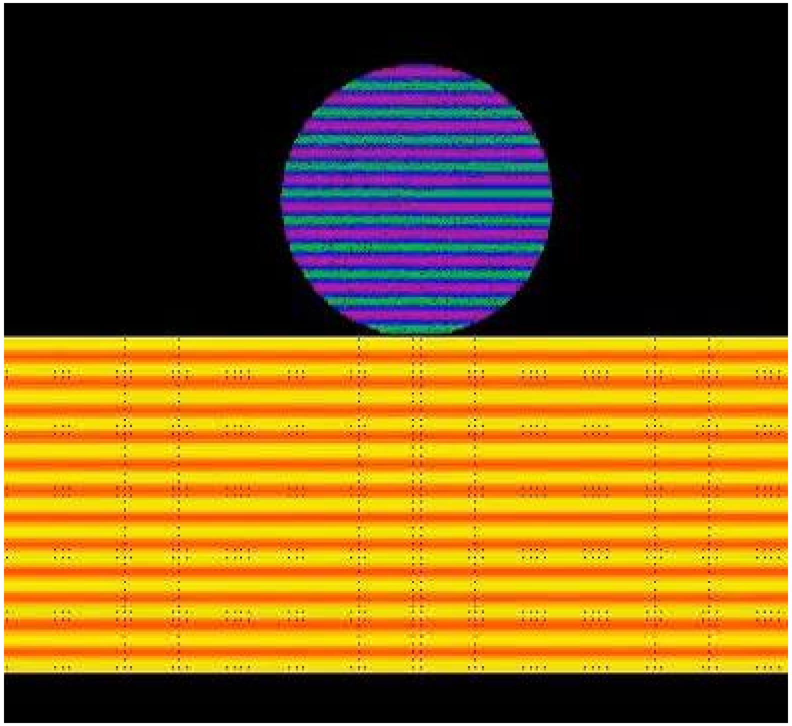
$$\underline{\mathbf{T}}\langle\xi\rangle = \left(\frac{\beta(p)p\omega'(\xi)}{\rho} + (1-\beta(p))C(\xi)(|\underline{\mathbf{Y}}\langle\xi\rangle| - |\xi|)\right)\frac{\underline{\mathbf{Y}}\langle\xi\rangle}{|\underline{\mathbf{Y}}\langle\xi\rangle|}$$

Friction and contact as Eulerian forces

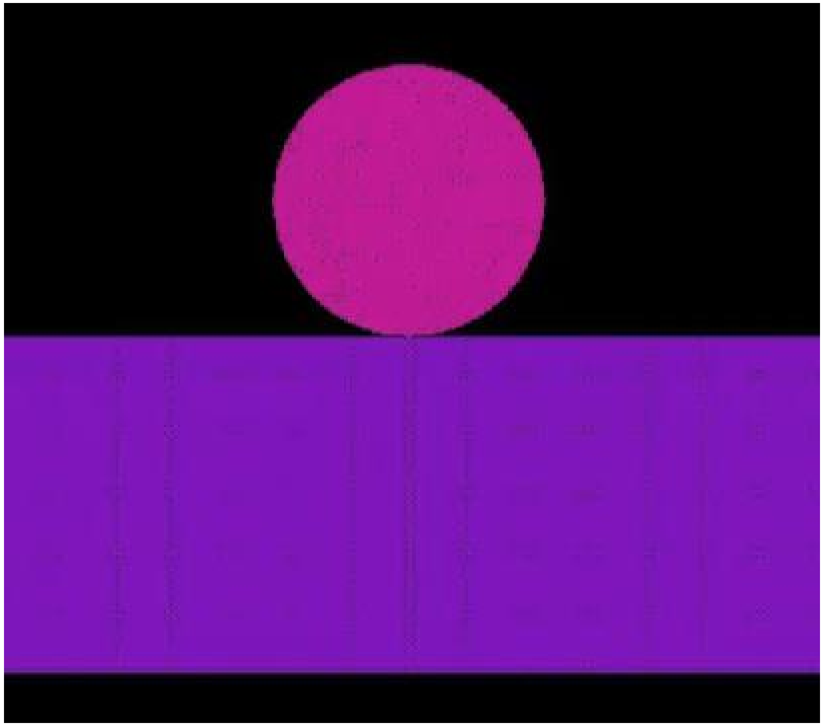
$$\underline{\mathbf{T}}^{\text{friction}}\langle\xi\rangle = F\,\frac{\underline{\mathbf{Y}}\langle\xi\rangle}{|\underline{\mathbf{Y}}\langle\xi\rangle|}\,\text{sgn}(\dot{\underline{\mathbf{Y}}}\langle\xi\rangle)$$

where F is the frictional bond force.

VIDEOS

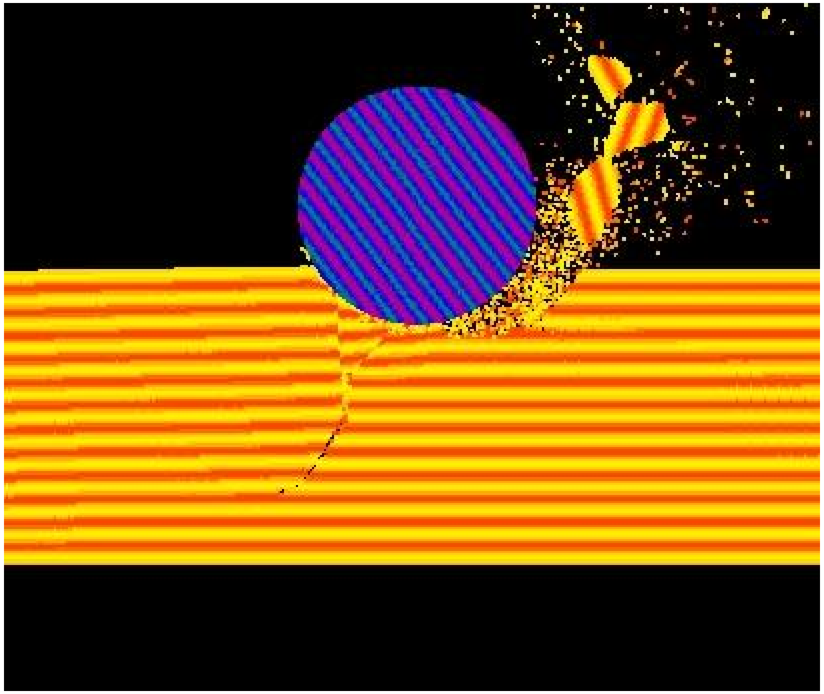


Material deformation

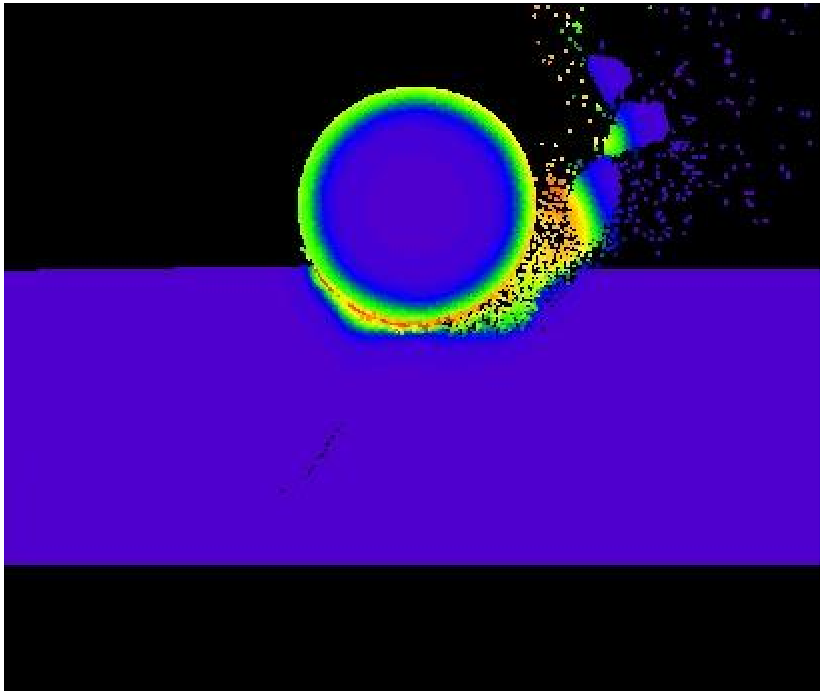


Damage

Frictional forces contribute to material failure

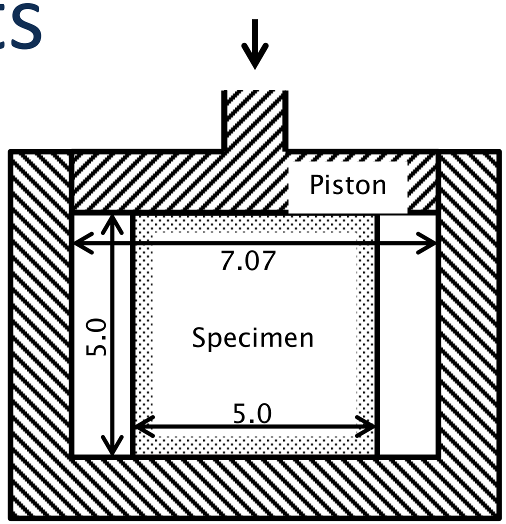


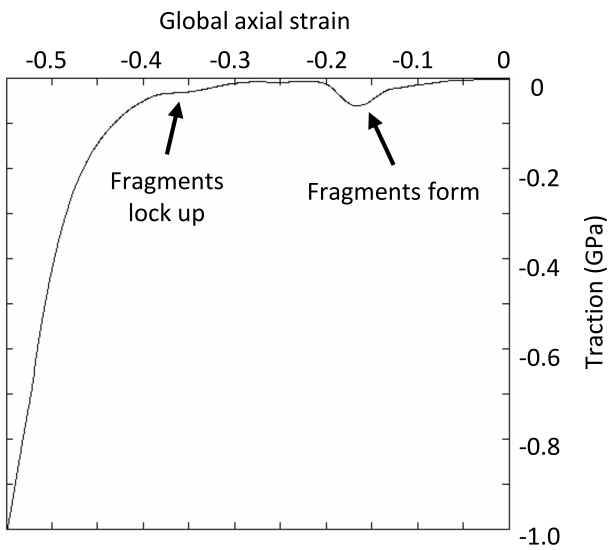
Material deformation

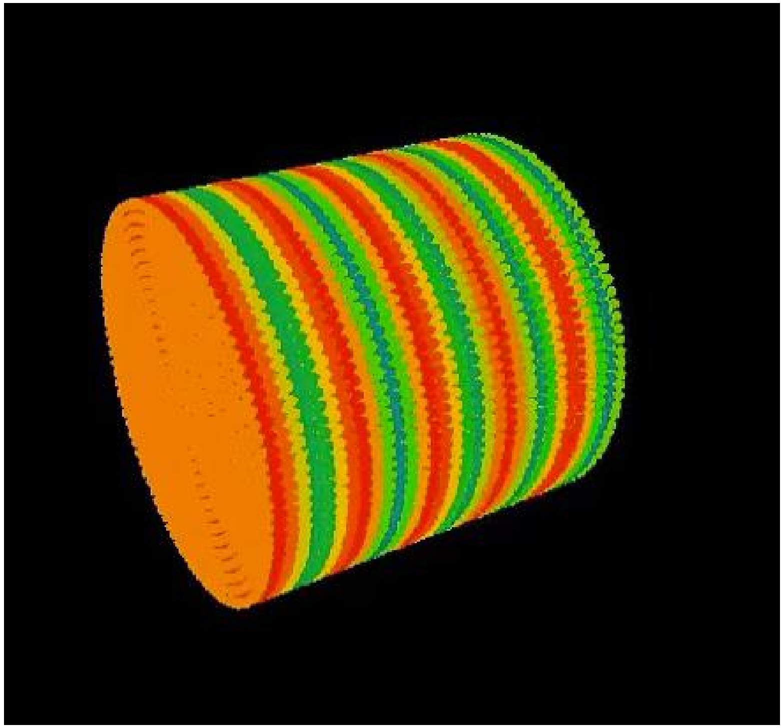


Temperature

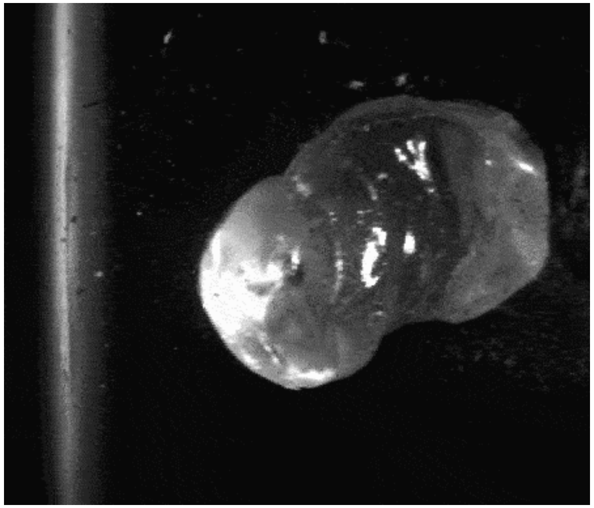
Fragmentation and recompression of fragments



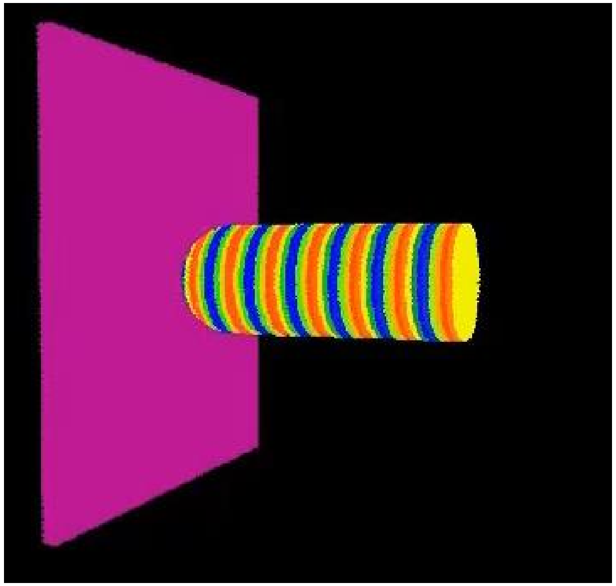




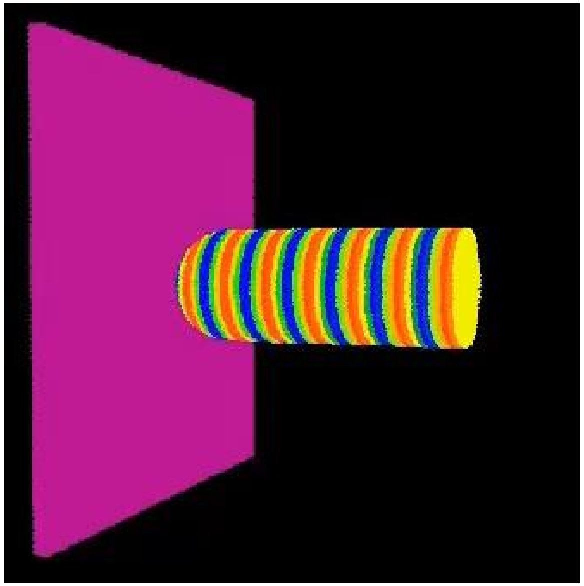
Bird strike simulant (gelatin)



Typical test
(credit: Arthur Core)



Meshless PD



Meshless PD
with bond damage

Summary

- An Eulerian material model is a special case of a Lagrangian model that doesn’t explicitly involve undeformed bonds.
  - Can assume that the horizon cuts off interactions in the deformed configuration.
  - Search for neighbors in each time step (changes over time).
- Otherwise, Eulerian and Lagrangian parts of a model are the same and can be combined arbitrarily.