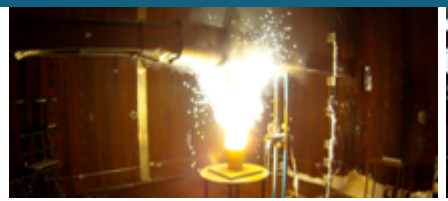
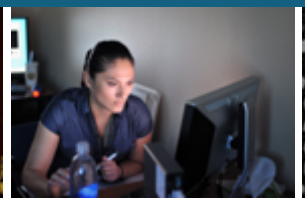
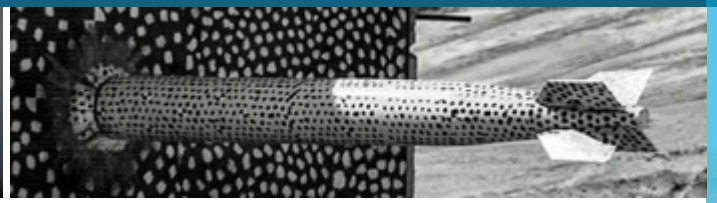
SAND2021-1928C

Sandia National Laboratories

An Analog Preconditioner for Solving Linear Systems







Ben Feinberg[*], Ryan Wong[†], T. Patrick Xiao[*], Christopher H. Bennett[*], Jacob N. Rohan[‡], Erik G. Boman[*], Matthew J. Marinella[*], Sapan Agarwal[*], Engin Ipek[§]

[*] Sandia National Laboratories

[†] University of Rochester

[‡] University of Texas at Austin

[§] Qualcomm

Sandia National Laboratories is a multimission laboratory managed and operated by National Technology & Engineering Solutions of Sandia, LLC, a wholly owned subsidiary of Honeywell International Inc., for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-NA0003525.

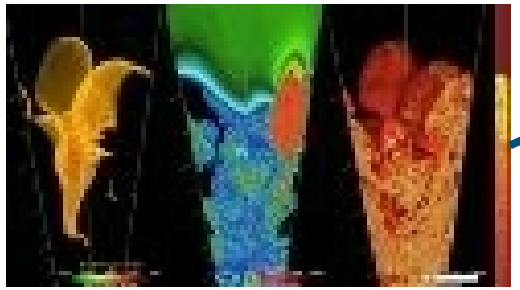
Linear Algebra is Everywhere

- Linear algebra is at the heart of computational science and engineering.

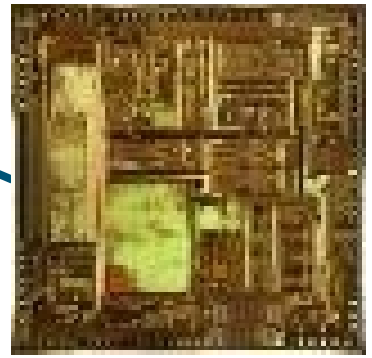
Structural Modeling

Climate Modeling

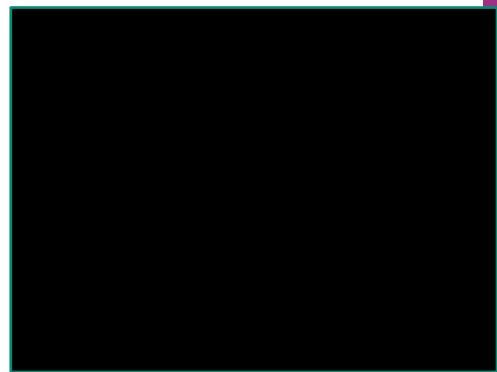
$$A\vec{x} = \vec{b}$$



Computational Fluid Dynamics



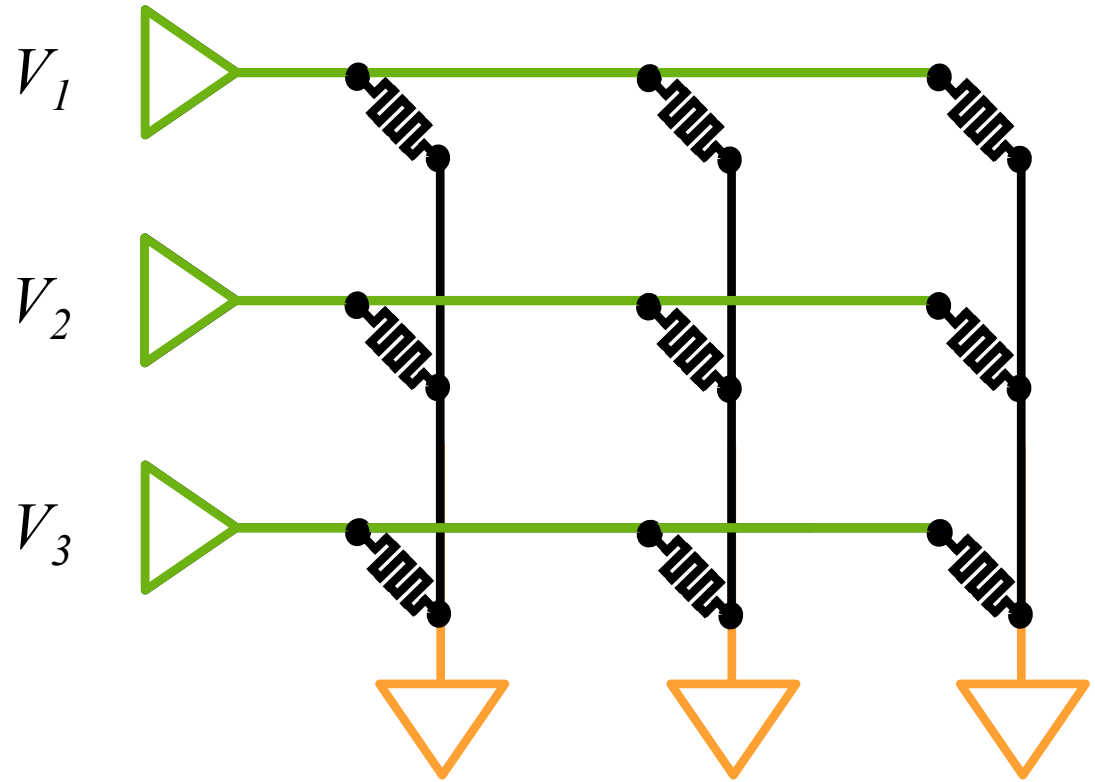
Circuit Simulation



In Situ Matrix Vector Multiplication

- Implementing analog matrix vector multiplication in a resistive mesh.

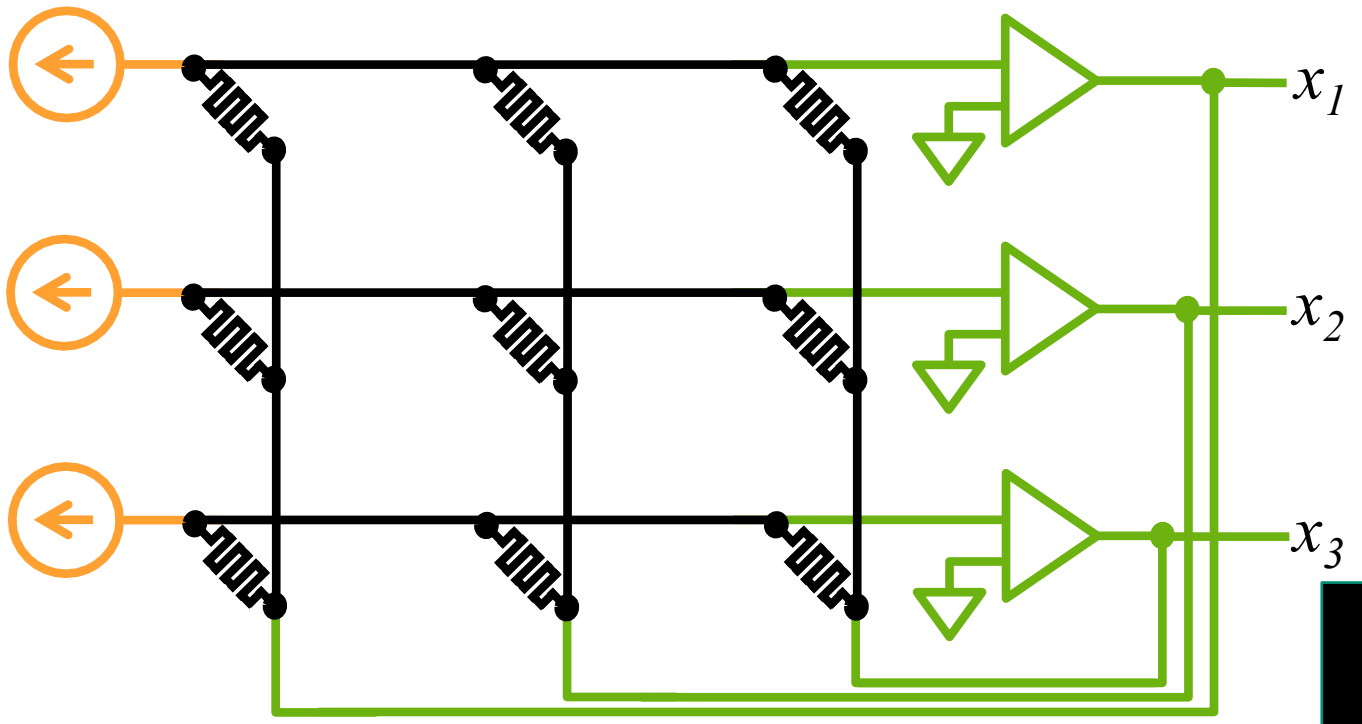
$$A\vec{x} = \vec{y}$$



In Situ Matrix Solver

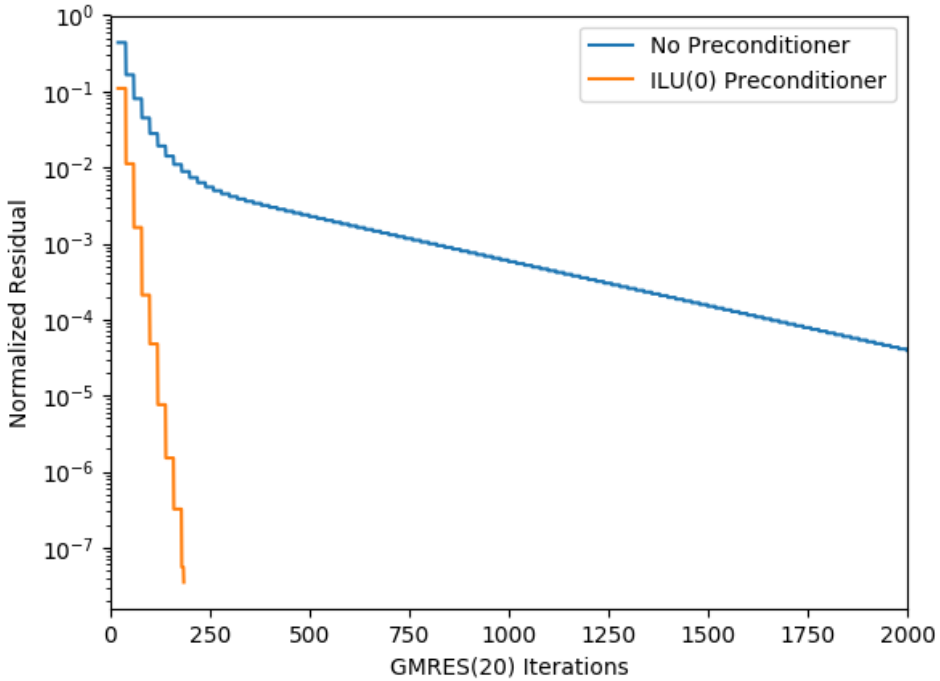
- Implementing an analog solution to a system of linear equations in a resistive mesh.

$$A\vec{x} = \vec{b} = 0$$



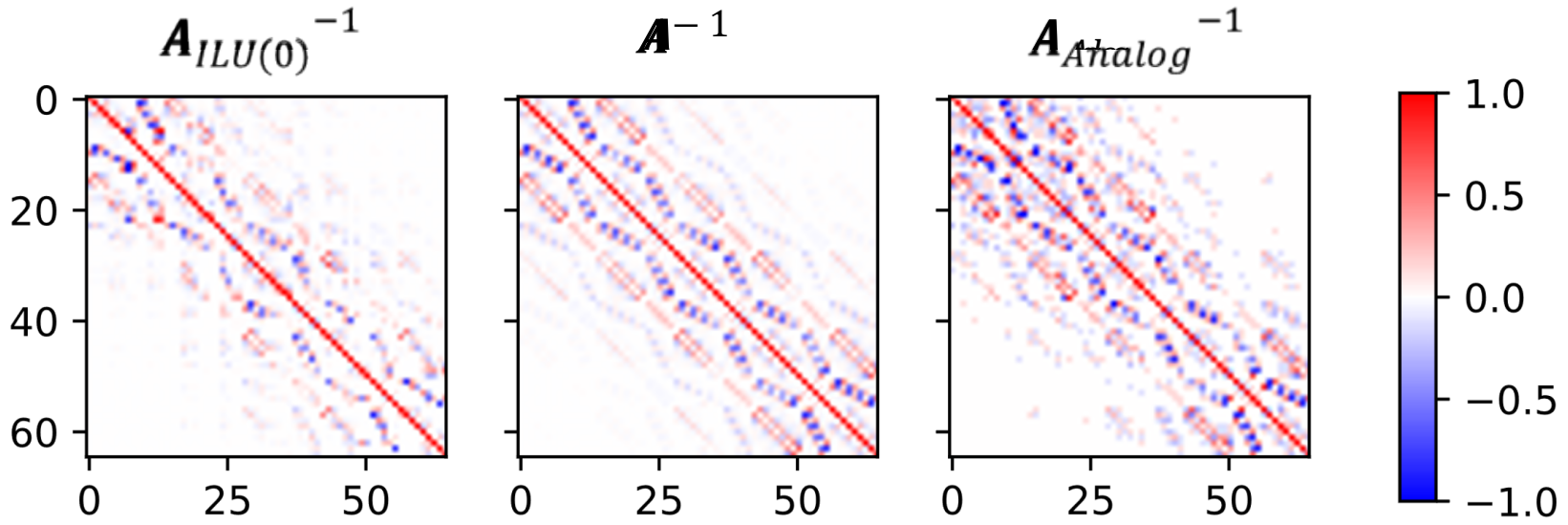
Preconditioning

- Preconditioning improves the rate of convergence for iterative solvers. Harder problems may require preconditioners to converge at all.
- Preconditioning is typically implemented by approximately solving a system of linear equations each iteration.
- More complex preconditioners can lead to faster convergence, at the cost of additional preconditioner complexity.
- Common algorithms:
  - **Incomplete LU Factorization (ILU)**
  - Algebraic Multigrid (AMG)



Comparison of convergence rates with and without preconditioning on the HB/sherman01 computational fluid dynamics matrix.

ILU vs Analog Preconditioning



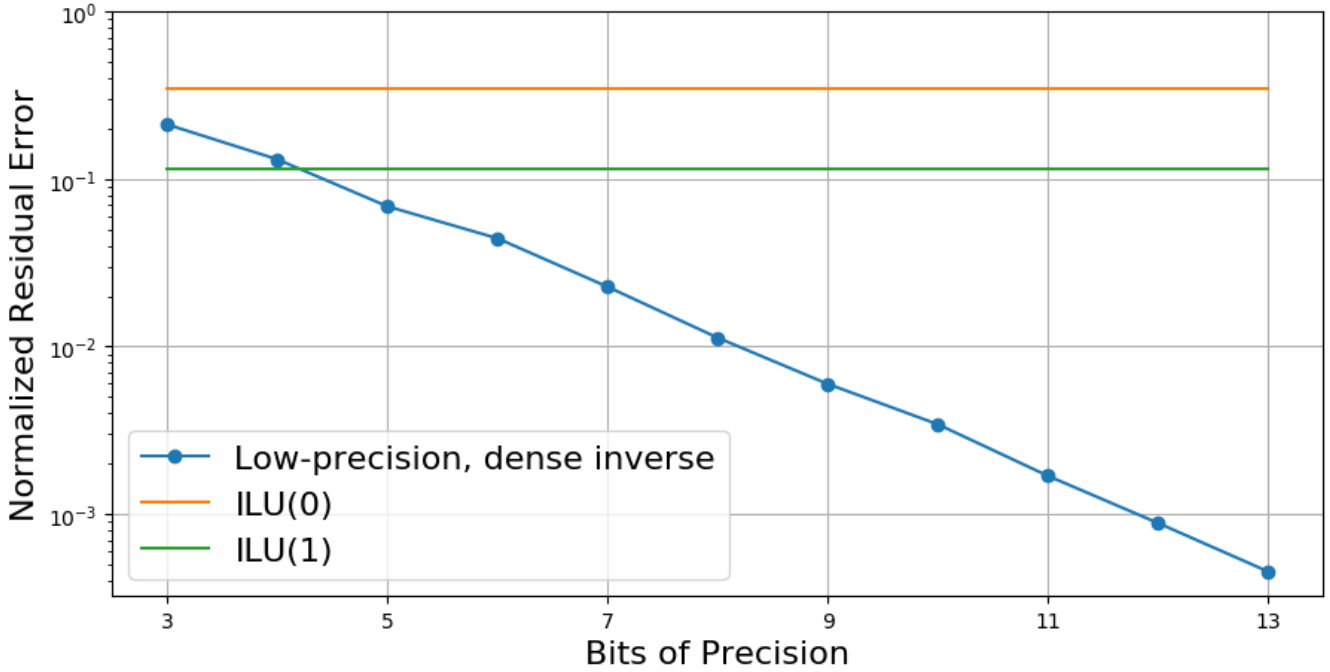
Incomplete LU Factorization:

- High precision sparse approximation
- Nonzero values are represented with high precision floating point
- Approximation can be improved by increasing the density of the approximation

Analog Solver:

- Low precision dense approximation
- Matrix elements are represented using low precision fixed point
- Approximation can be improved b increasing the bit precision

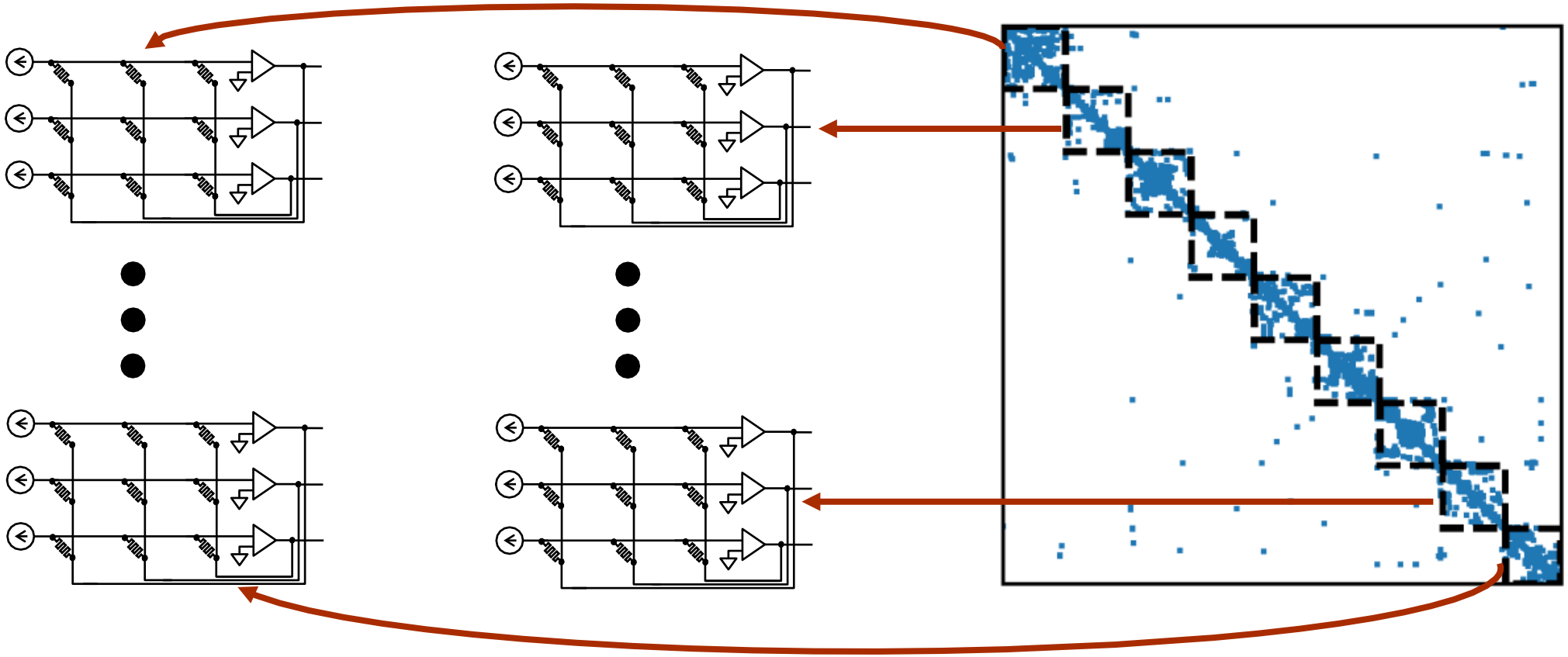
ILU vs Analog Preconditioning



Comparison of approximation accuracy between ILU and a low-precision, dense inverse as a function of bits using a 1024x1024 Wathen matrix. Does not include circuit non-idealities.

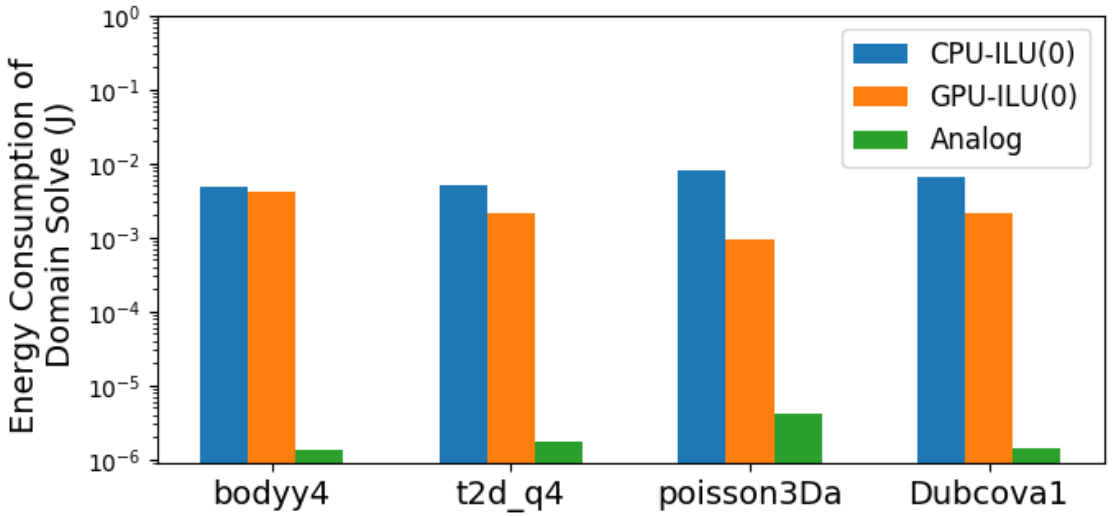
- A low-precision, dense inverse can be a better approximation than ILU using very few bits.

Analog Domain Decomposition Preconditioning

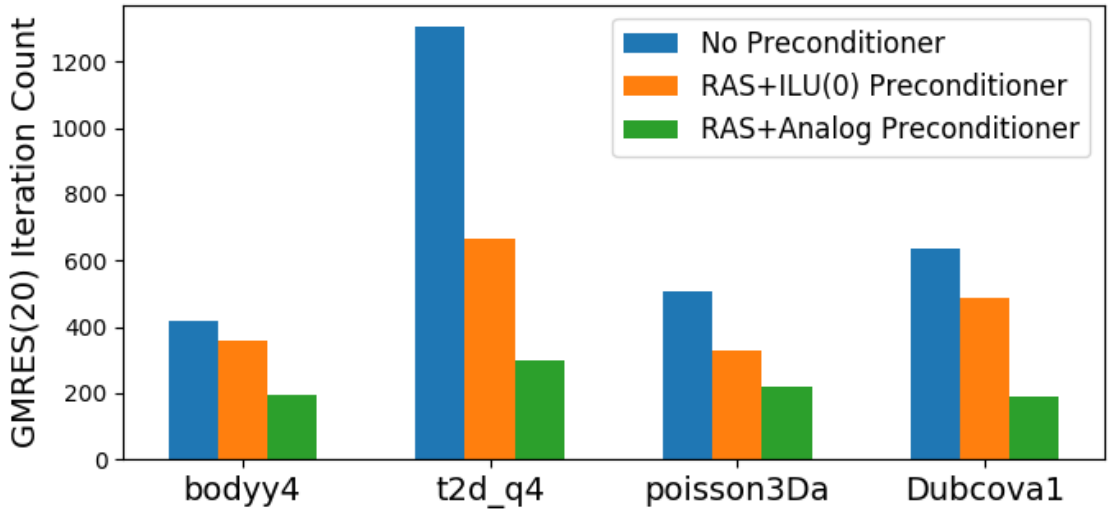


- Each domain (submatrix) is assigned to an *in situ* solver array.
- *In situ* solves can directly replace the domain solve minimizing changes to existing software.

Benefits of Analog Preconditioning

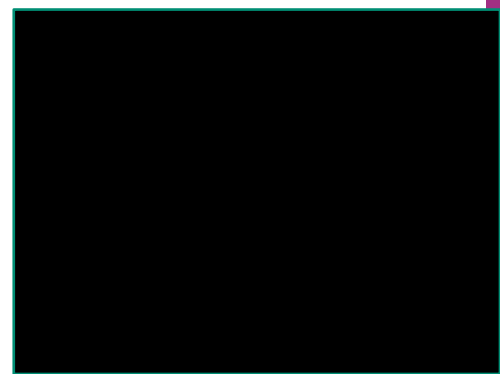


Domain Solve Energy Consumption

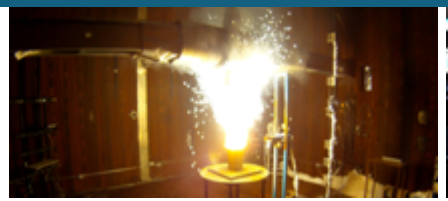


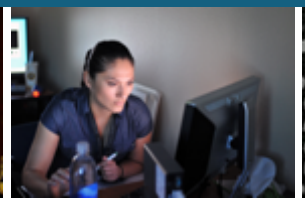
GMRES(20) Iteration Count

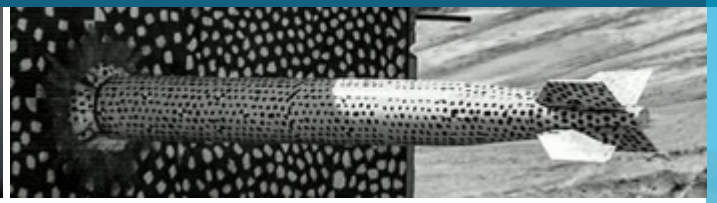
- Reduces average domain solve energy consumption by **1025x** and worst-case domain solve runtime by **105x**.
- RAS+Analog Preconditioning can reduce the total iteration count of iterative method nearly 50% over RAS+ILU(0).



An Analog Preconditioner for Solving Linear Systems







Ben Feinberg[*], Ryan Wong[†], T. Patrick Xiao[*], Christopher H. Bennett[*], Jacob N. Rohan[‡], Erik G. Boman[*], Matthew J. Marinella[*], Sapan Agarwal[*], Engin Ipek[§]

[*] Sandia National Laboratories

[†] University of Rochester

[‡] University of Texas at Austin

[§] Qualcomm

Sandia National Laboratories is a multimission laboratory managed and operated by National Technology & Engineering Solutions of Sandia, LLC, a wholly owned subsidiary of Honeywell International Inc., for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-NA0003525.