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BIFURCATION ANALYSIS OF A DUFFING OSCILLATOR WITH A MULTI-SEGMENTED FREEPLAY NONLINEARITY

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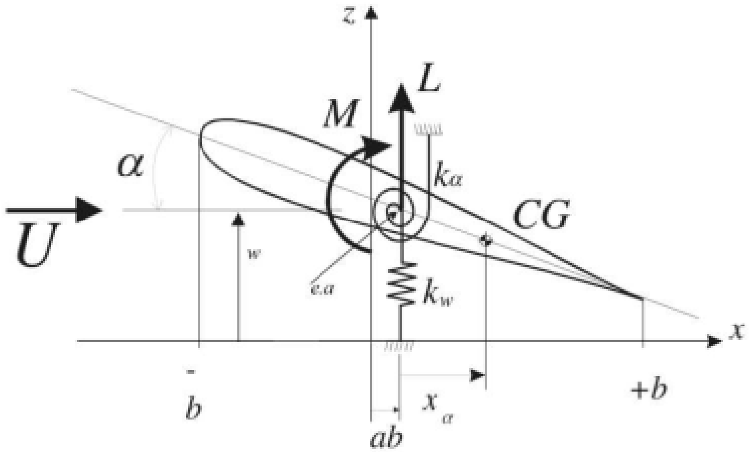
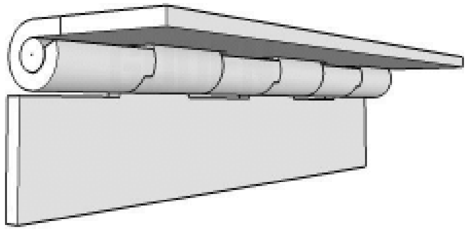
Outline

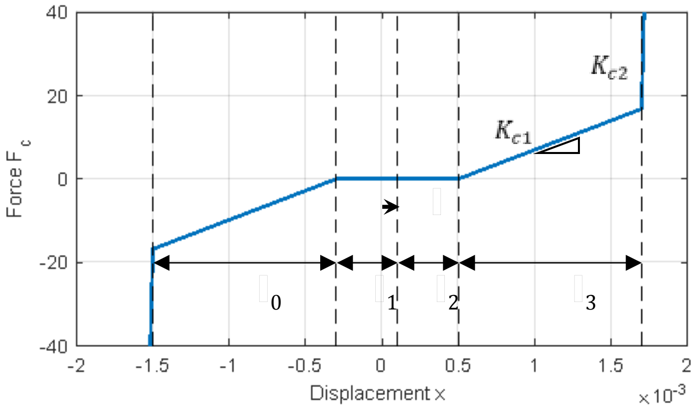
- ❑ Background and motivation
- ❑ Numerical model and solution methods
- ❑ Results of simple freeplay (for later comparison)
- ❑ Effects of multi-segmented freeplay on system physics
  - ➢ Changing outer gap size
  - ➢ Changing contact stiffness (route to impact)
  - ➢ Changing freeplay asymmetry
  - ➢ Changing forcing magnitude
- ❑ Conclusions

Background and motivation

- Contact/impact behavior is very common in engineering
  - E.g., Freeplay: gap between parts intermittently contacting
  - Often induces complex nonlinear behavior and grazing bifurcations
  - Multi-segmented freeplay: not 1, but 2 stiffness transitions in the contact model
- In this work: a system with multi-segmented freeplay
  - Determine the effects on bifurcations and the physics of the system







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H. A. Guden Co., Inc., 2021, "More Operations : Stop Hinges." https://www.guden.com/customize/Continuous-Hinges/More-Operations/Stop-Hinges
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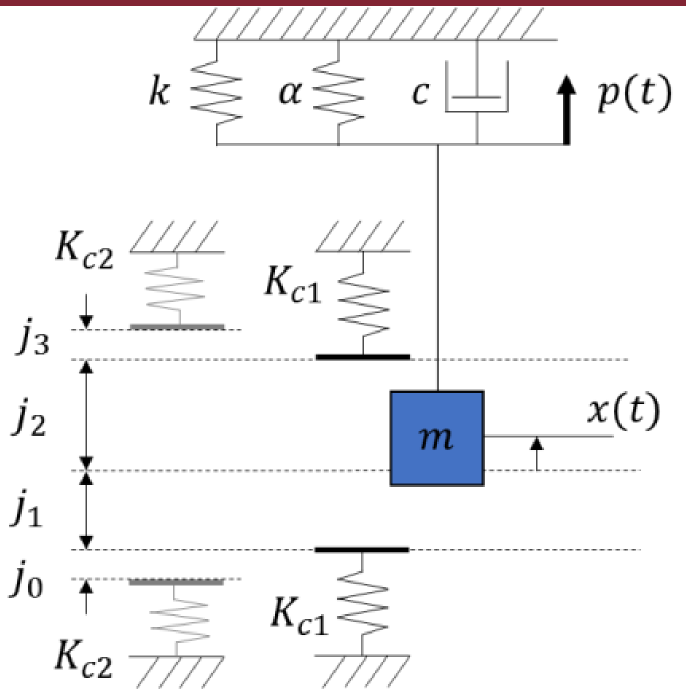
Numerical model and solution methods

❑ Forced Duffing oscillator with multi-segmented freeplay

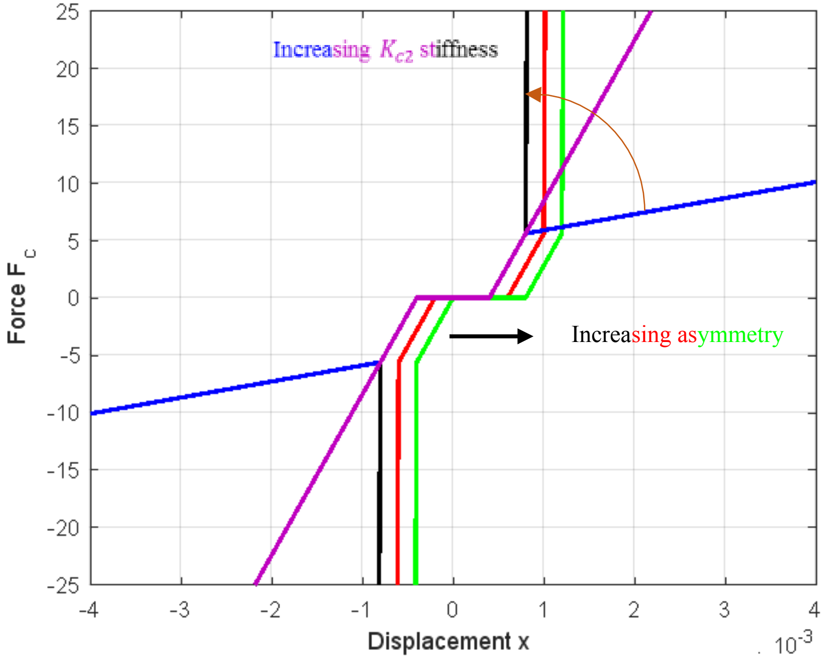
$$\ddot{x} + 2\omega_n \zeta \dot{x} + \omega_n^2 x + \frac{\alpha}{m} x^3 + \frac{F_c}{m} = \frac{p}{m} cos(\omega t)$$

$$F_c = \begin{cases} K_{c2}(x + j_0 + e) + K_{c1}(j_1 - j_0), & x < -j_0 + e \\ K_{c1}(x + j_1 - e), & -j_0 + e < x < -j_1 + e \\ 0, & -j_1 + e \le x \le j_2 + e \\ K_{c1}(x - j_2 - e), & j_2 + e < x < j_3 + e \\ K_{c2}(x - j_3 - e) + K_{c1}(j_3 - j_2), & x > j_3 + e \end{cases}$$



❑ Notes:

- Inner contact stiffness is soft ($K_{c1} = 1.4 * 10^4 N/m$)
- Outer contact stiffness will primarily be hard in this work ($K_c = 1.4 * 10^6 N/m$)
- No complex segment asymmetries are studied; only overall asymmetry/eccentricity parameter e (thus, $j_0 = j_3, j_1 = j_2$)



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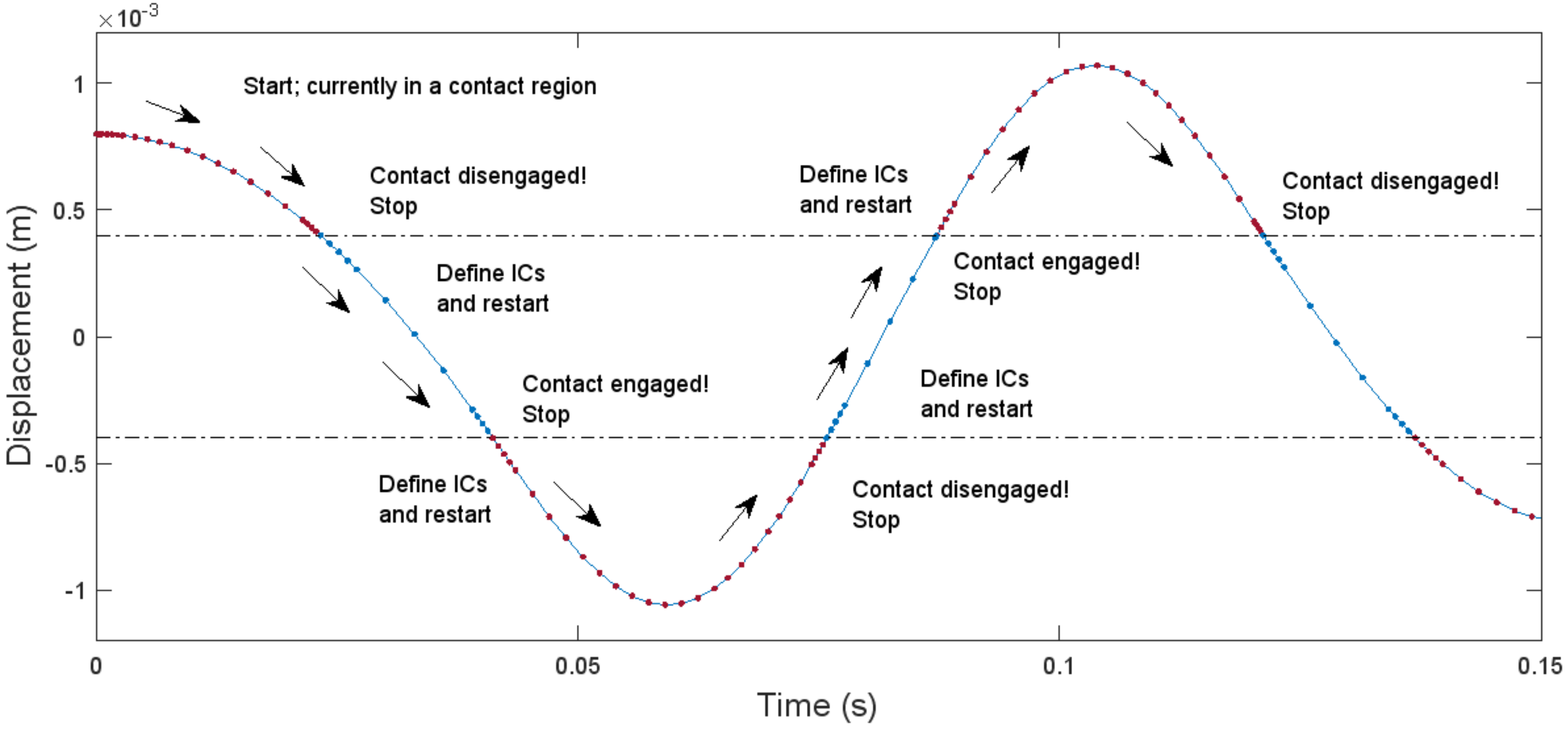
Numerical model and solution methods

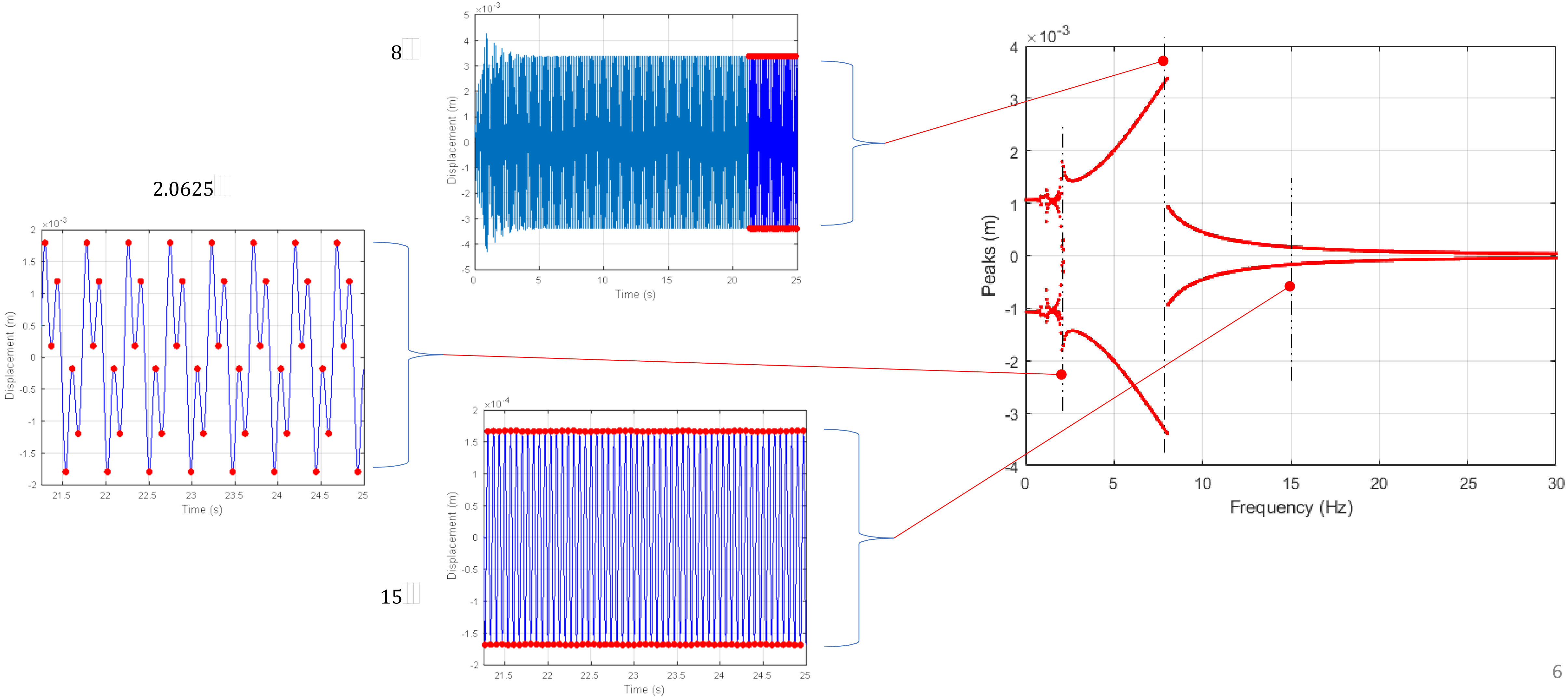
Experiments	Finite element analysis (FEA)	Numerical/reduced-order modeling	Analytical methods
You run tests	You simulate parts with computers	You approximately solve system ODEs with computers	You solve the ODEs exactly
Most realistic	Highly realistic	Reasonably realistic	Perfect, but only for idealized situations
Most expensive	Can be expensive (software licenses, supercomputers, etc.)	Fairly inexpensive (cheaper software, like Matlab®)	Least expensive
Most time-consuming	Ranges seconds - months	Fairly low time cost	Ranges fast - impossible

- Matlab® ode45 with *Event Location* is used for reduced -order modeling of contact systems
- A timestep is always forced at every instance of contact to ensure accuracy



B.E. Saunders, R. Vasconcellos, R.J. Kuether, A. Abdelkefi, "Importance of event detection and nonlinear characterization of dynamical systems with discontinuity boundary," AIAA 2021-1499., AIAA SciTech 2021 Forum., virtual, January 11-15 & 19-21, 2021.

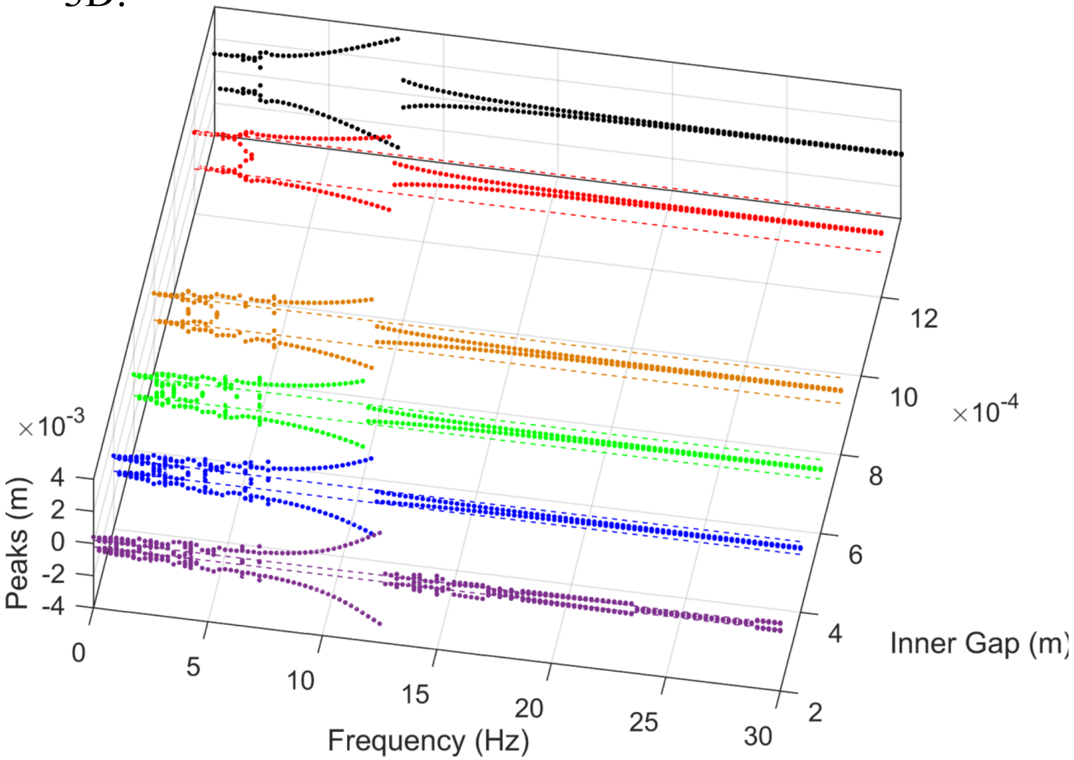
❑ Explaining bifurcation diagrams



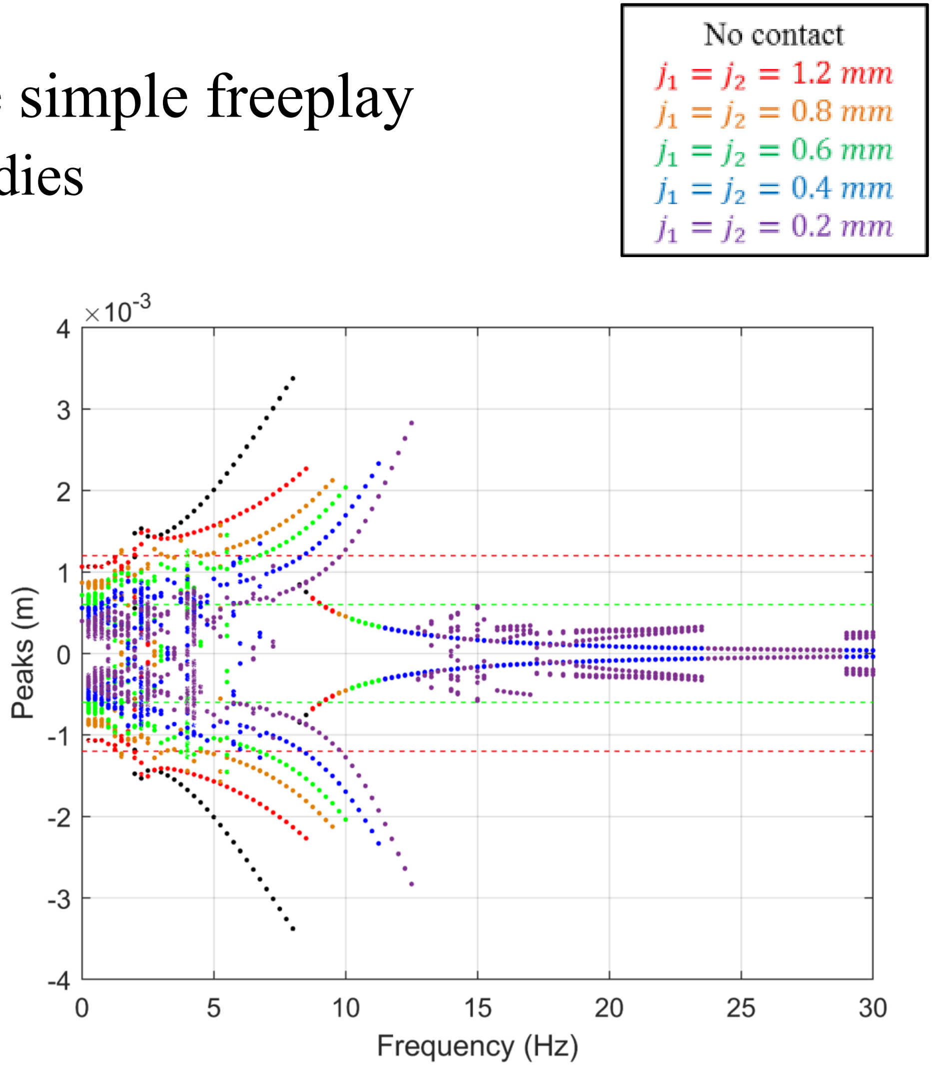
Results of simple freeplay

- Outer contact-force segments are neglected to give simple freeplay
  - Decide on a large and a small *inner* gap for later studies

3D:

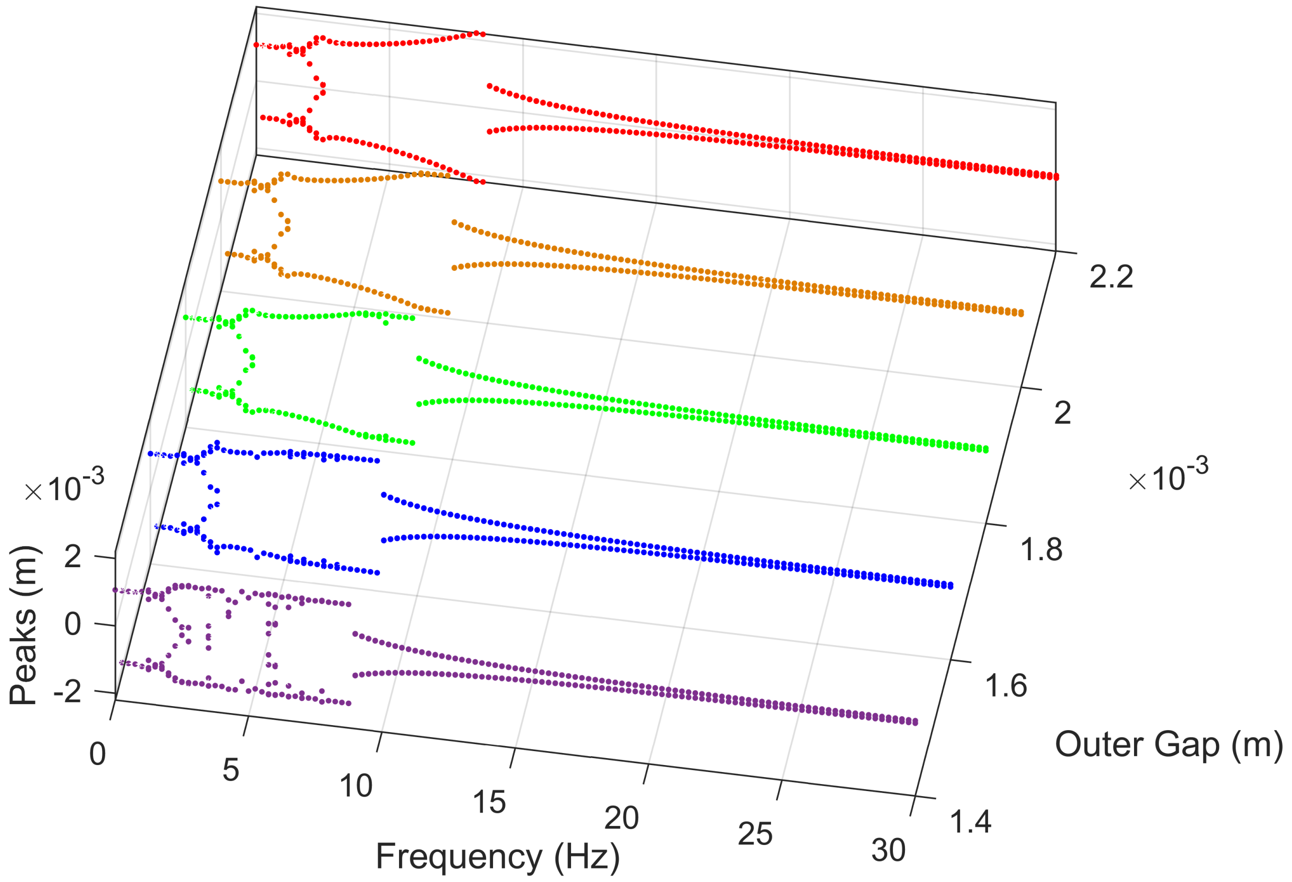


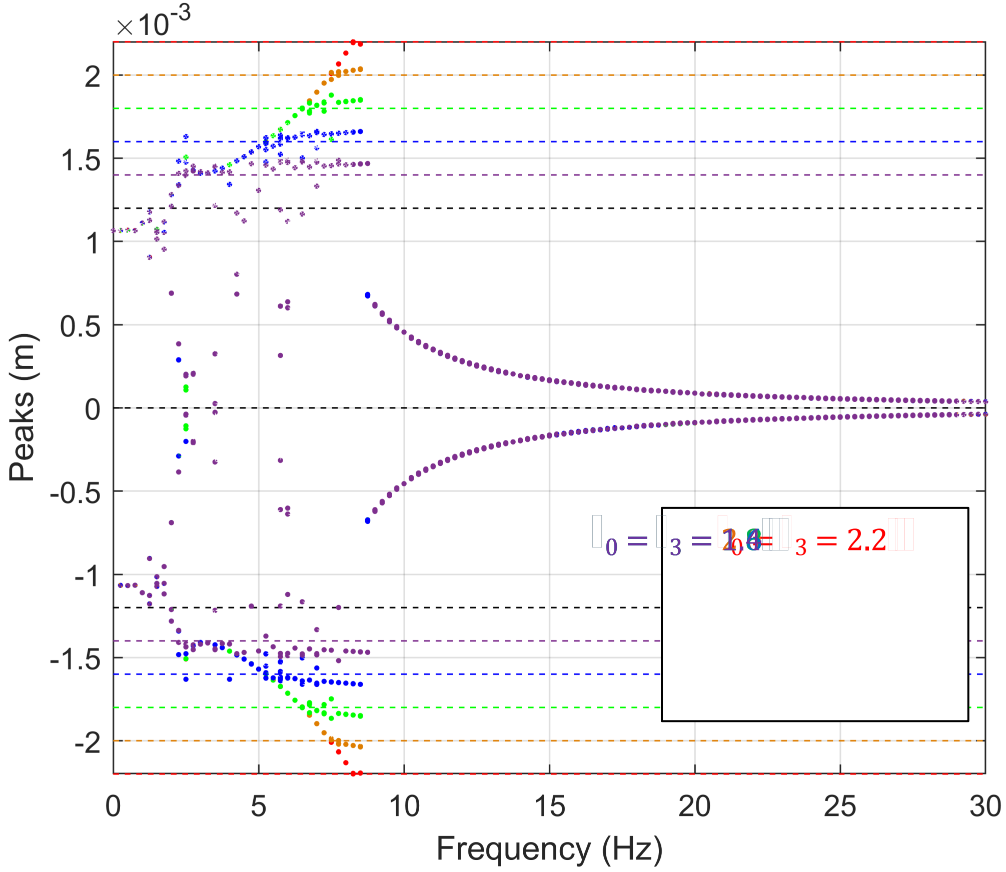
2D:



Changing outer gap size

Large inner gap,
$j_1 = j_2 = 1.2\ mm,$
$K_{c1} = 1.4 * 10^4 N/m$
$K_{c2} = 1.4 * 10^6 N/m$

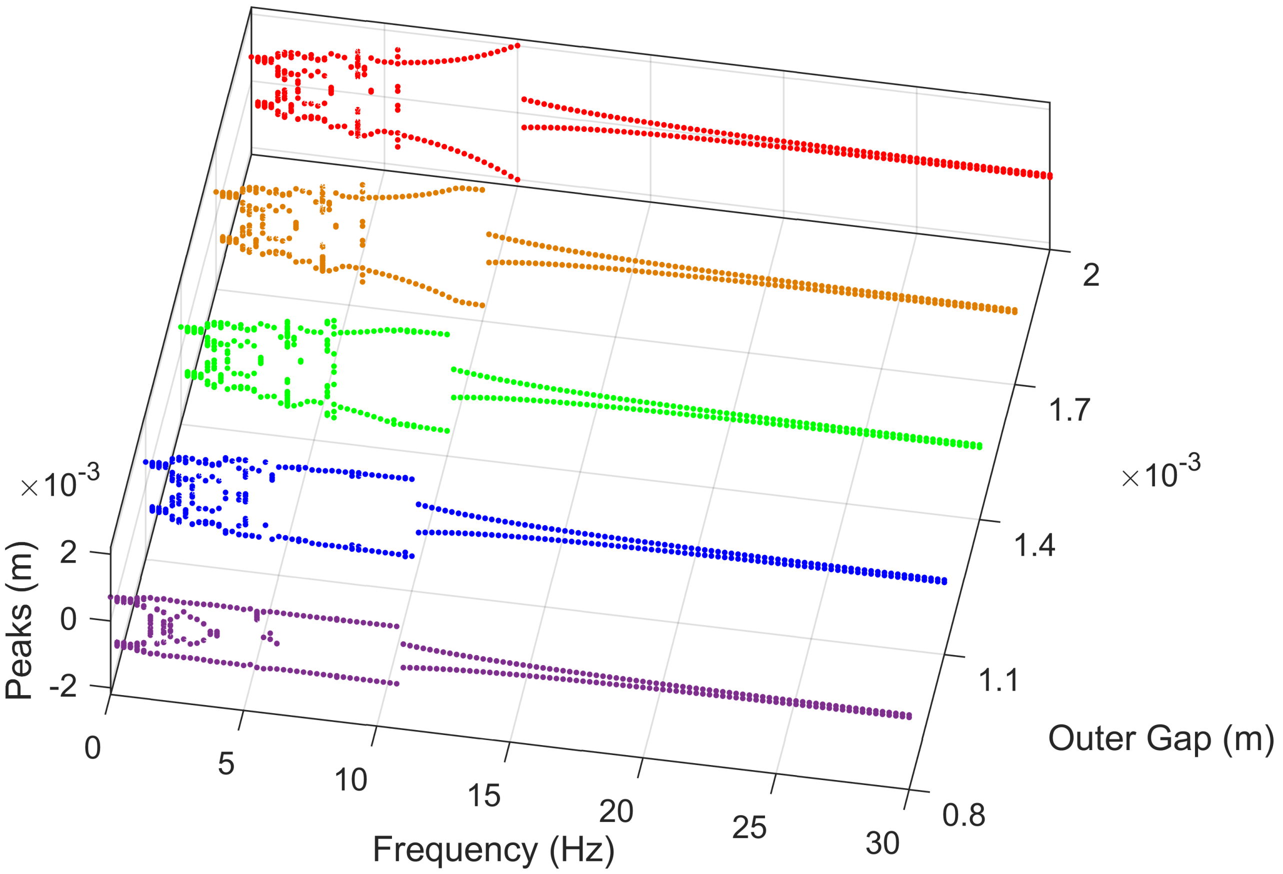


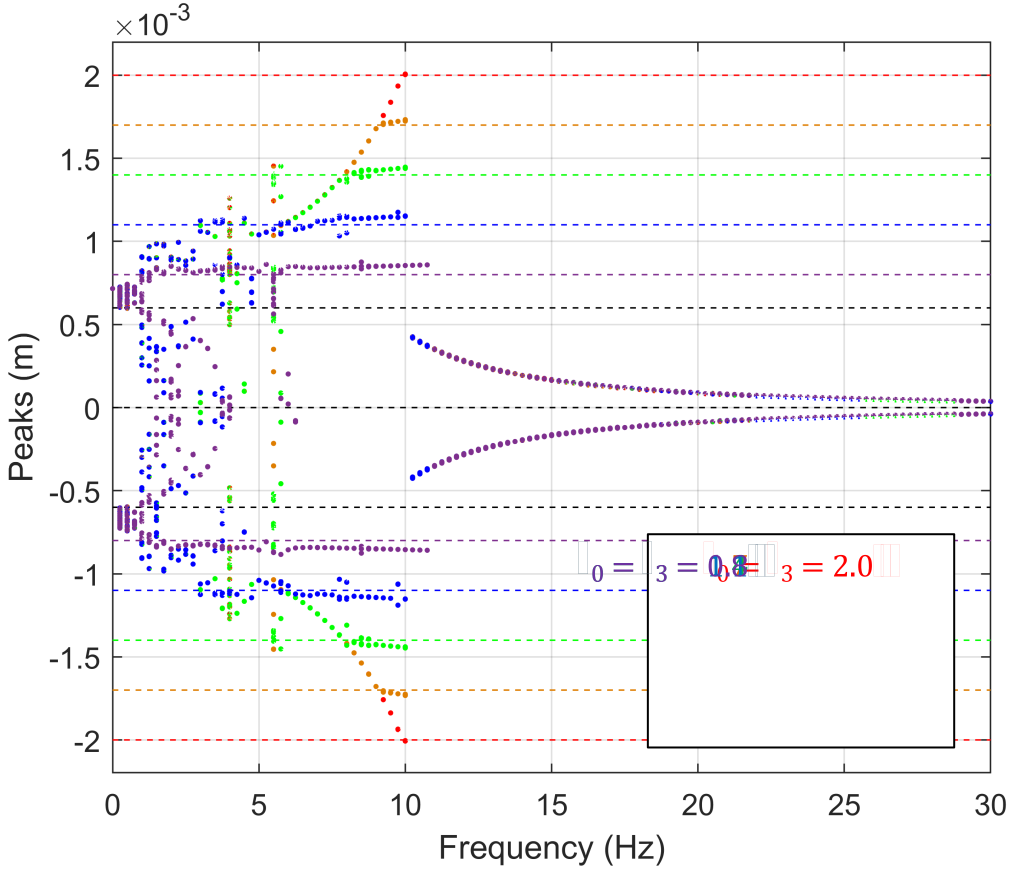


- Outer contact abruptly limits resonance peak and causes grazing contact
- Increase in superharmonic resonance behavior

Changing outer gap size

Small inner gap,
$j_1 = j_2 = 0.6\ mm$,
$K_{c1} = 1.4 * 10^4 N/m$
$K_{c2} = 1.4 * 10^6 N/m$

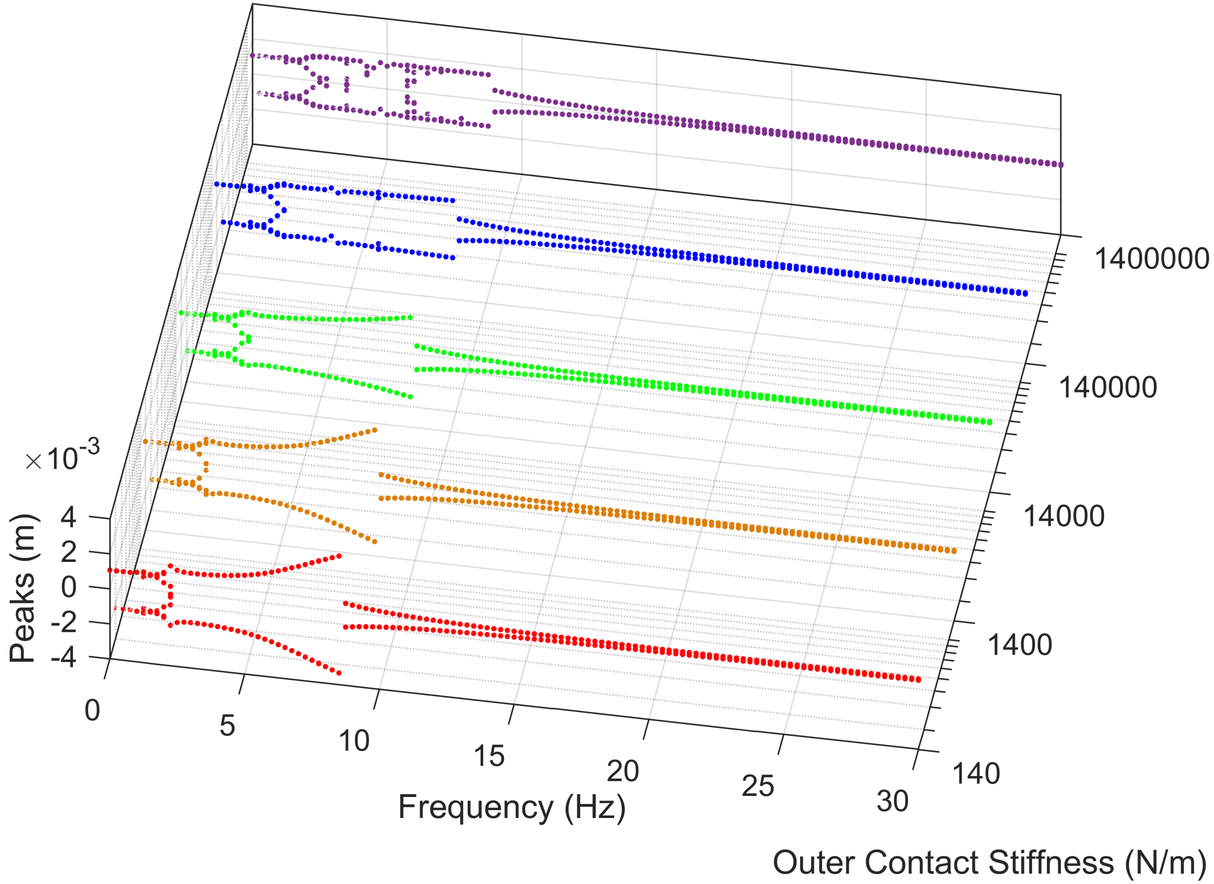


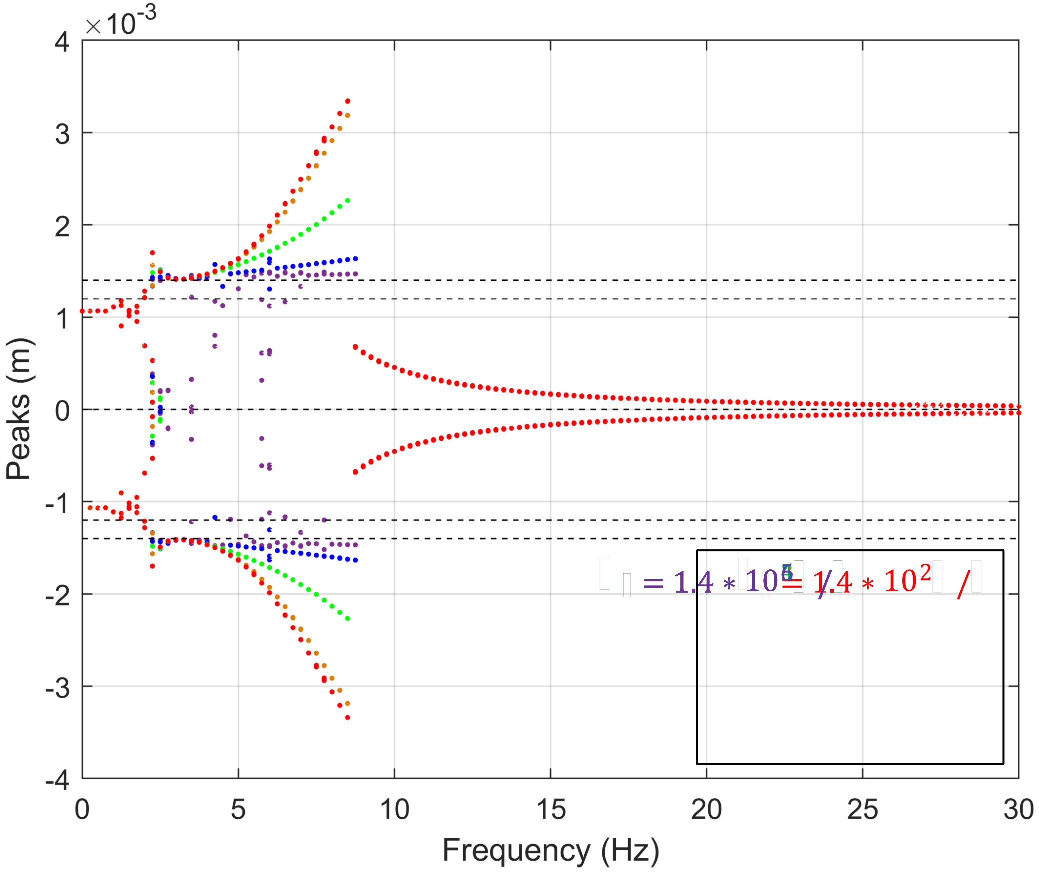


- Similar grazing contact as before
- Some bands of superharmonic resonance behavior disappear ($\sim 4 - 6Hz$)
- Outer segments have minor effect on the location of the jump phenomenon

Changing contact stiffness

Large inner gap,
$j_1 = j_2 = 1.2\ mm$,
$j_0 = j_3 = 1.4\ mm$

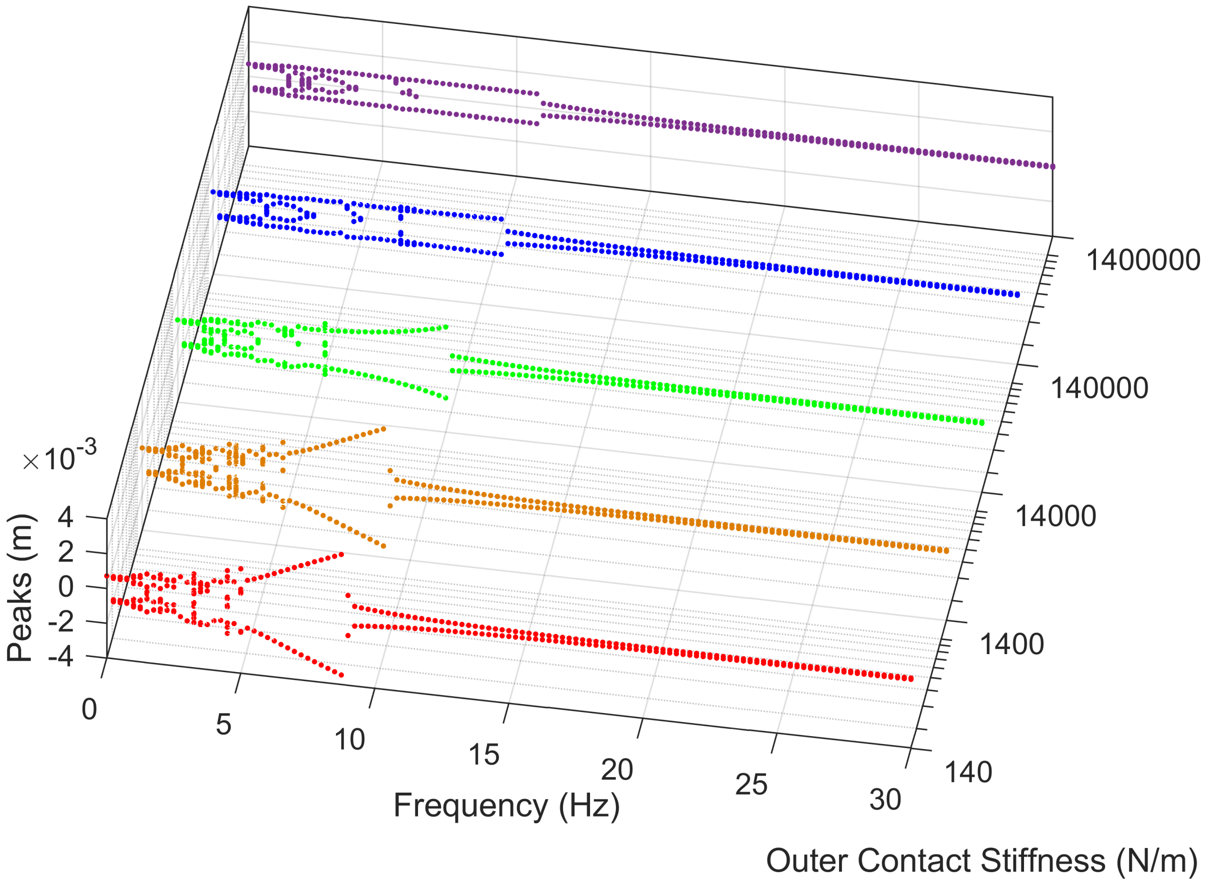


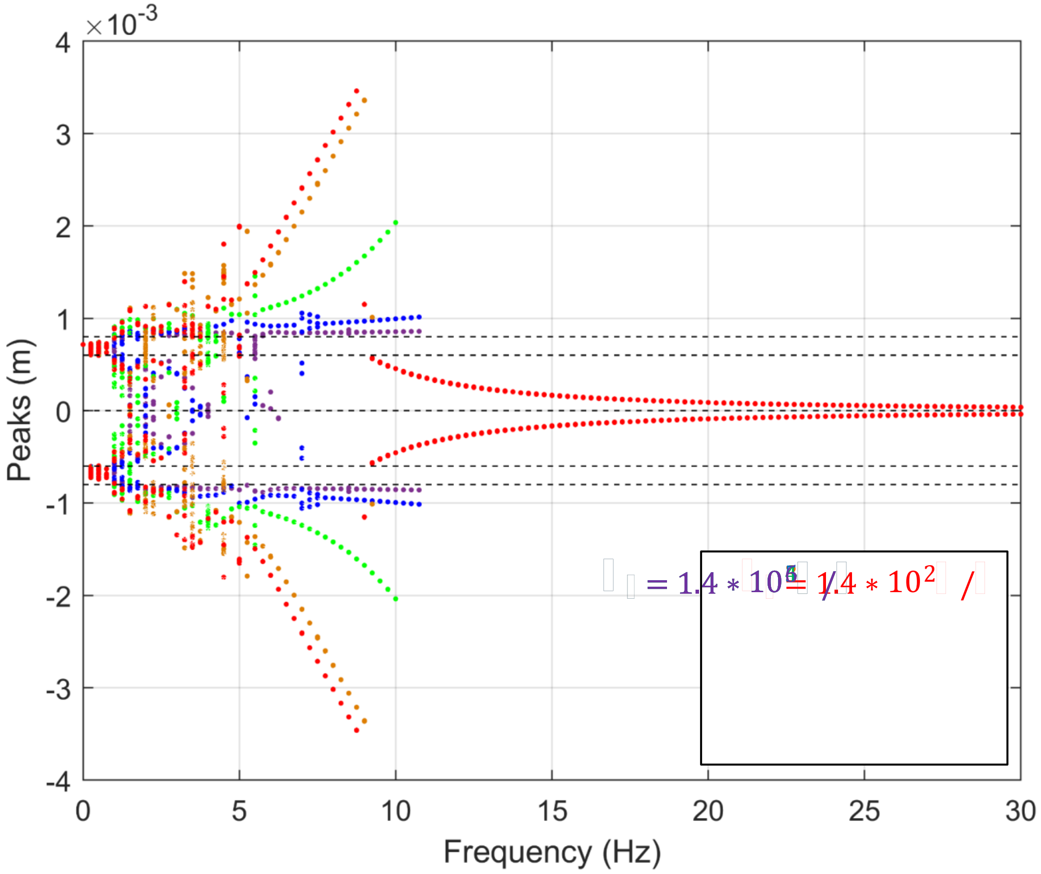


- No additional nonlinear behavior occurs for soft outer contact, but primary resonance peak increases
- The bands of superharmonic resonance behavior appear for hard outer contact

Changing contact stiffness

Small inner gap,
$j_1 = j_2 = 0.6\ mm$,
$j_0 = j_3 = 0.8\ mm$

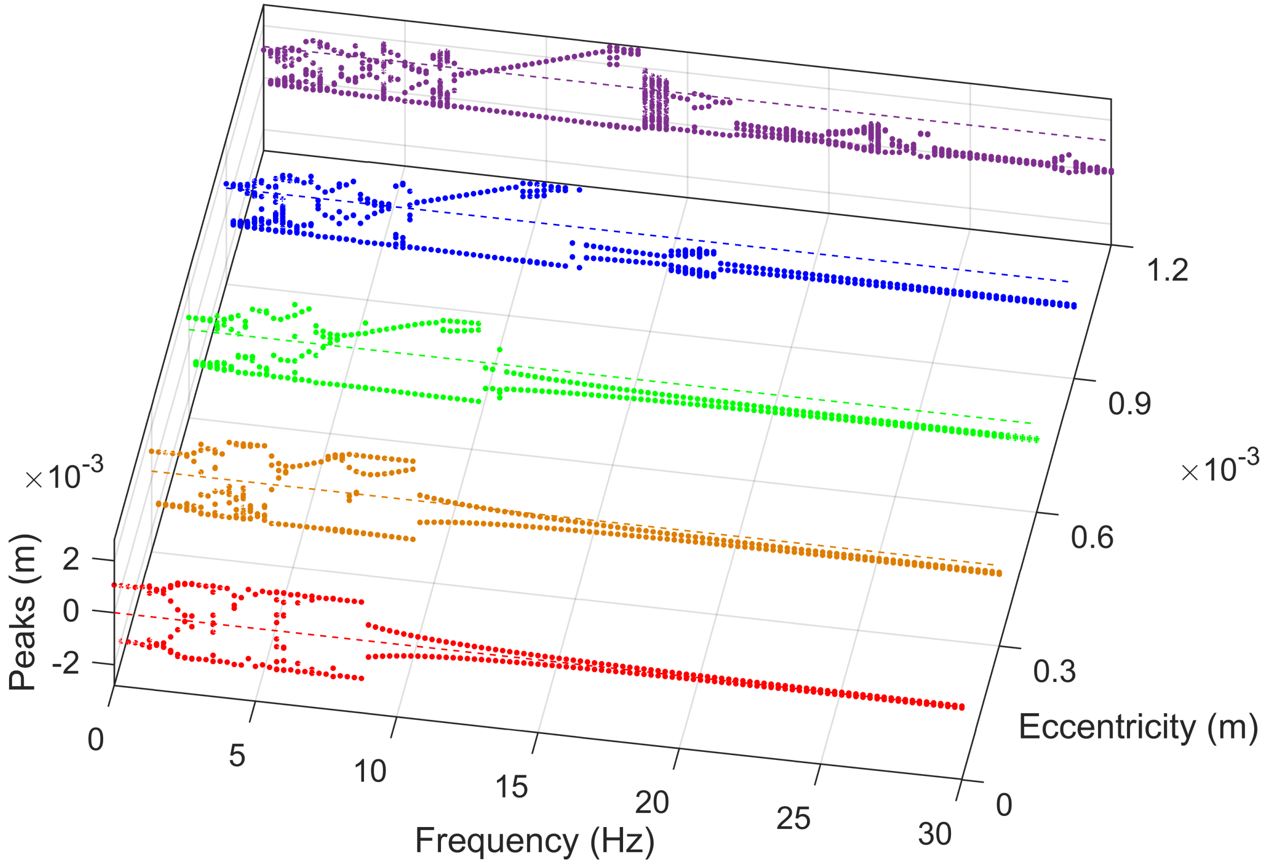


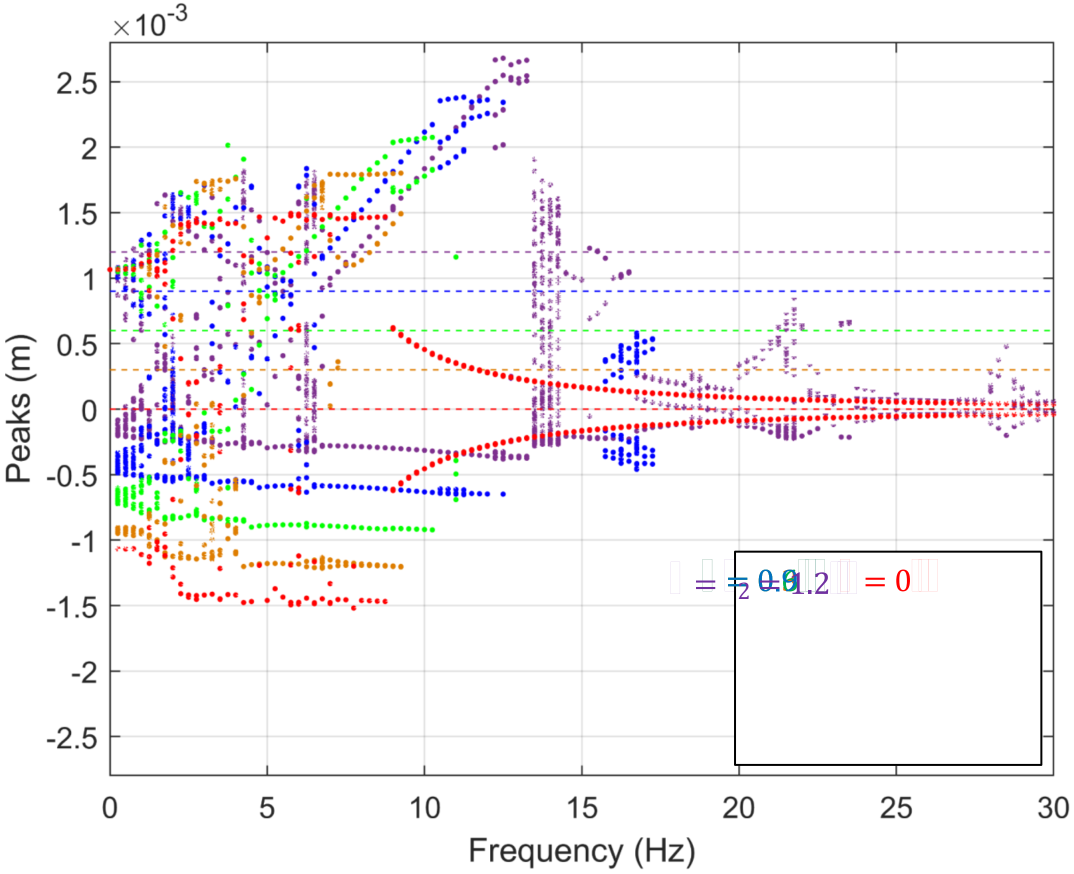


- Harder and softer outer contact stiffnesses activate different bands of superharmonic resonance behavior

Changing freeplay asymmetry/eccentricity

Large inner gap,
$j_1 = j_2 = 1.2\ mm,$
$j_0 = j_3 = 1.4\ mm$

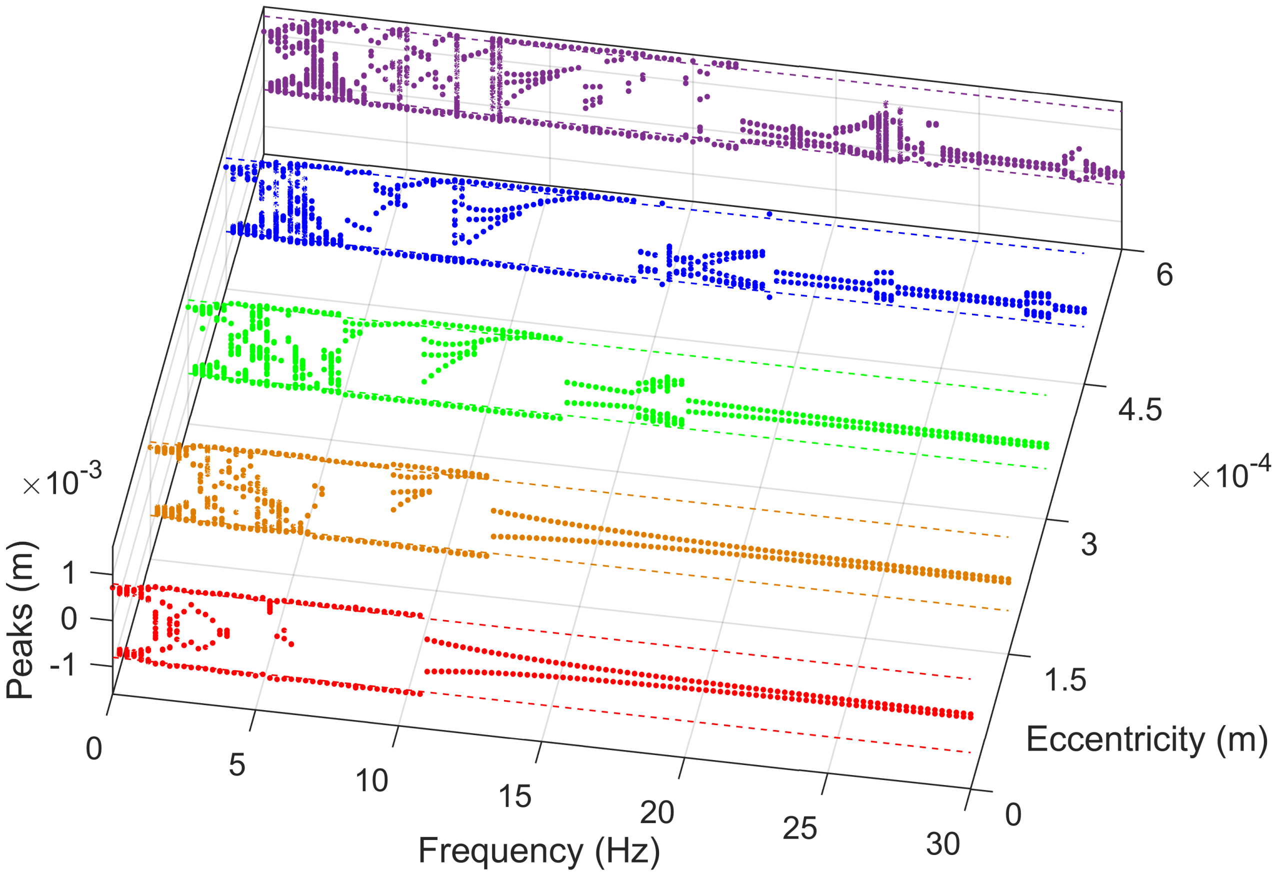


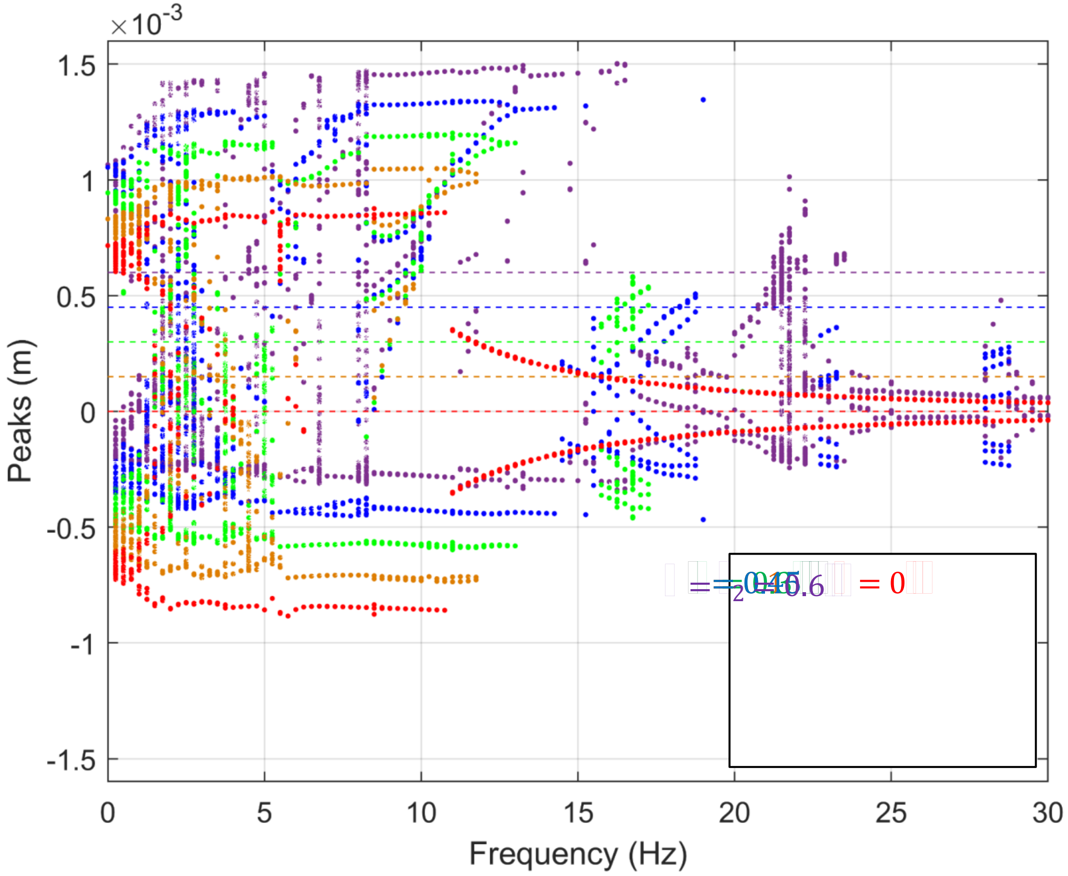


- Freeplay asymmetry has significant effect on the nonlinear behavior even for the large gap
- Multiple bands of subharmonic resonance behavior, including chaos, appear
- Nonlinear behavior on the primary resonance branch and at the peak

Changing freeplay asymmetry/eccentricity

Small inner gap,
$j_1 = j_2 = 0.6\ mm$,
$j_0 = j_3 = 0.8\ mm$

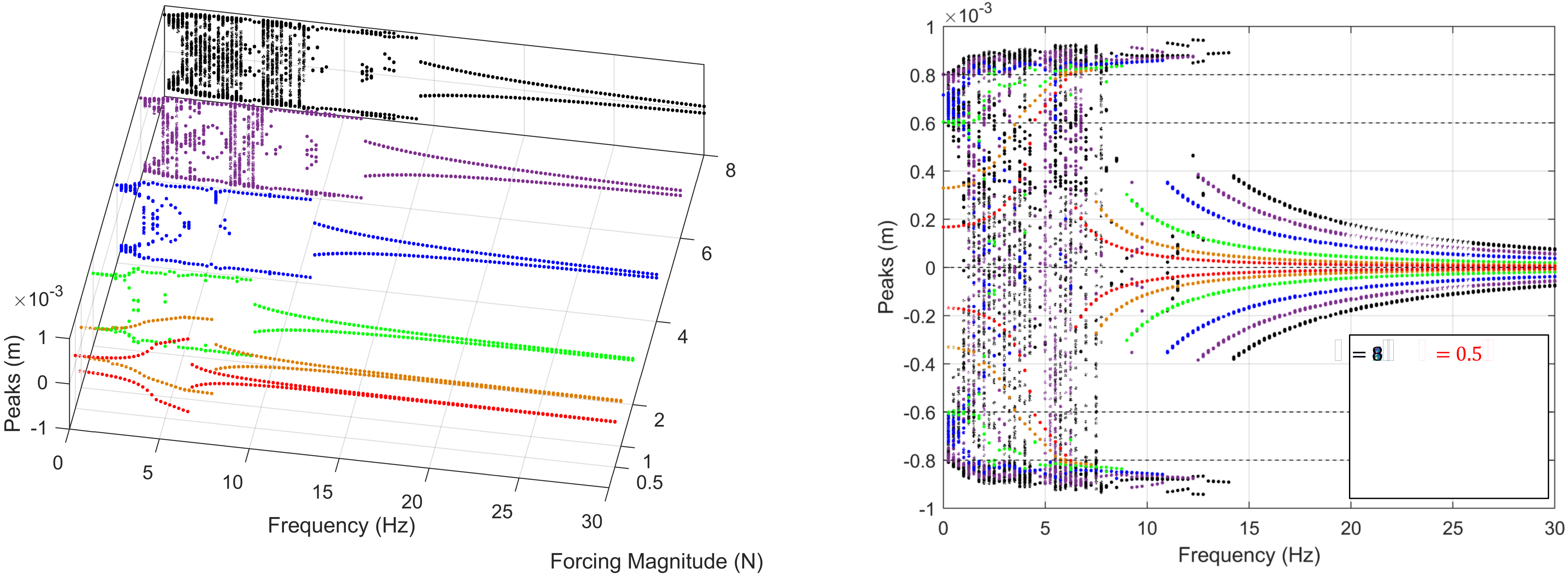




- Similar to before: increase in super- and sub-harmonic resonance behaviors
- More nonlinear behavior appears after the jump phenomenon appears, except for highest asymmetry
- Structures similar to period-adding bifurcations appear on the primary resonance branch

Changing forcing magnitude

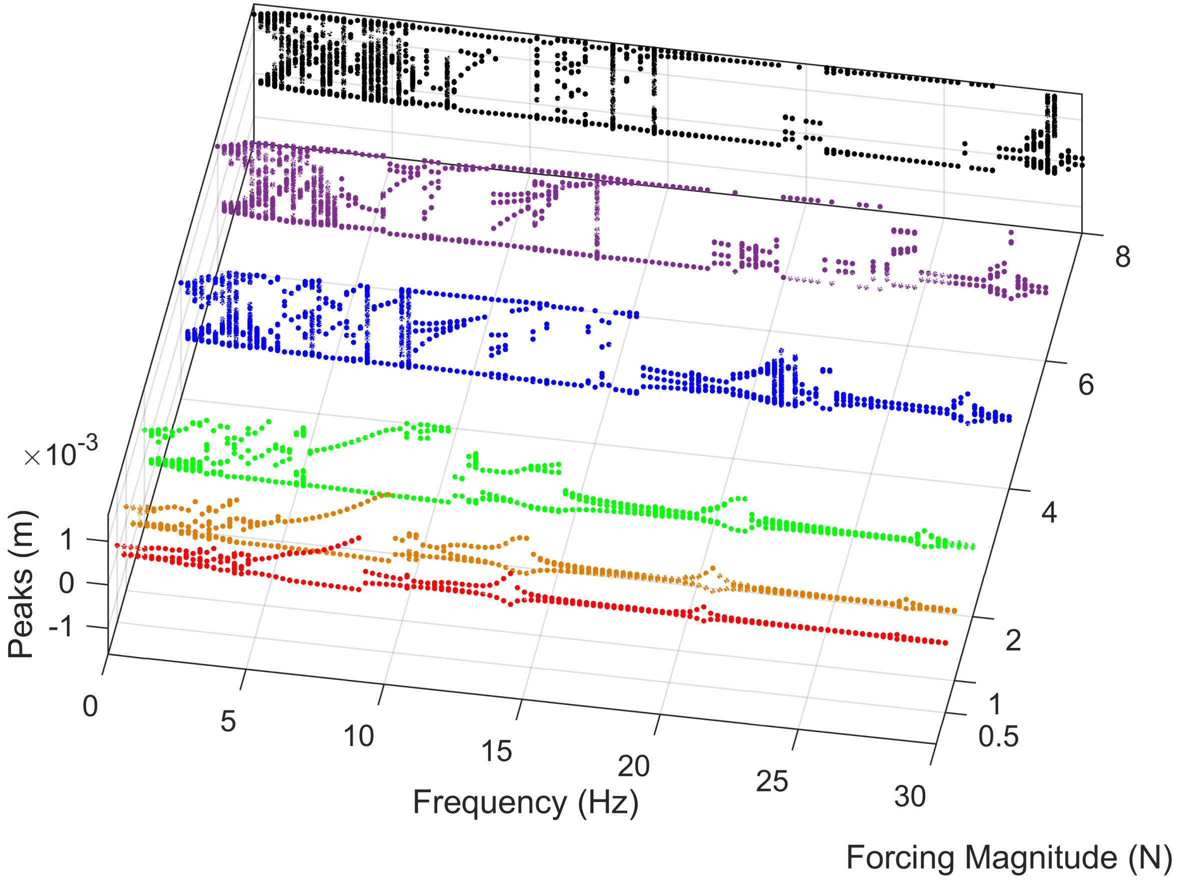
Small inner gap,
$j_1 = j_2 = 0.6\ mm,$
$j_0 = j_3 = 0.8\ mm,$
$e = 0\ mm$

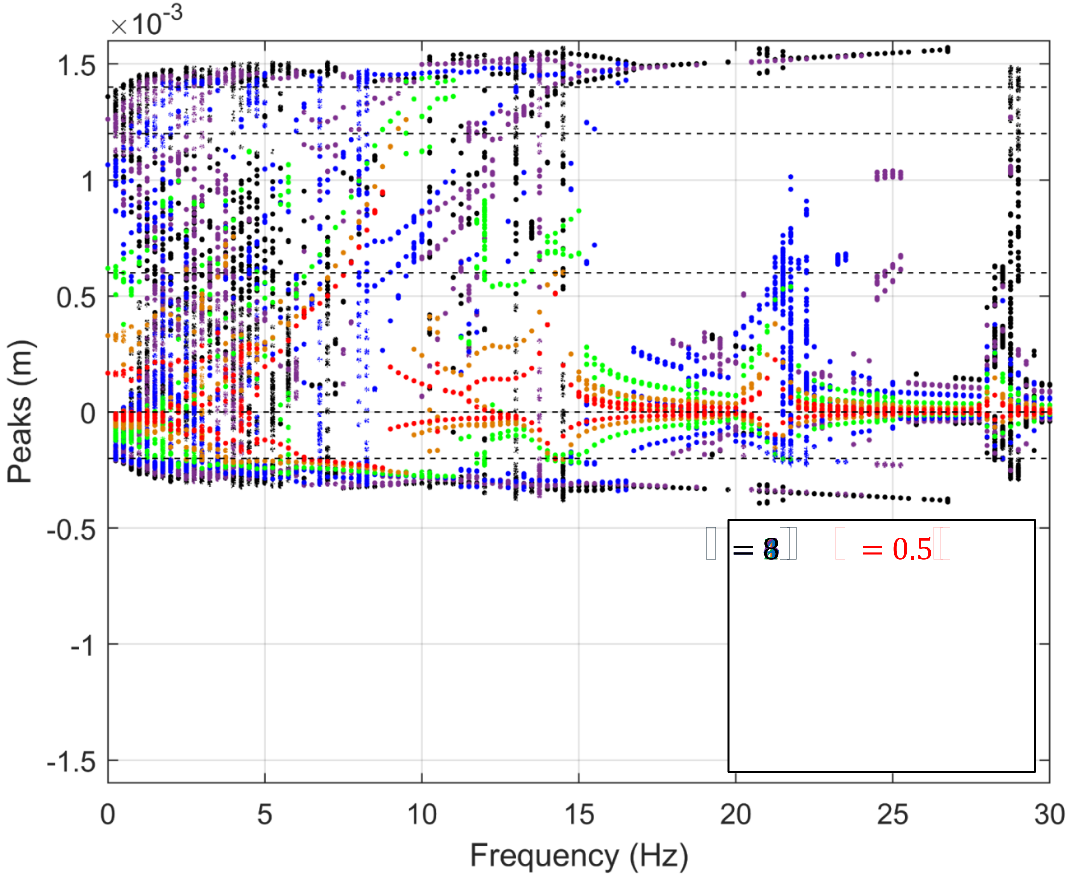


- Reduced forcing causes the superharmonic resonance behavior to disappear
- Stronger forcing primarily causes bands of superharmonic chaos to widen
- No subharmonic resonance behavior appears

Changing forcing magnitude

Small inner gap,
$j_1 = j_2 = 0.6\ mm$,
$j_0 = j_3 = 0.8\ mm$,
$e = 0.6\ mm$





- Reduced forcing causes bands of chaos to disappear and makes subharmonic resonance peaks well-defined
- Stronger forcing causes interspersed periodic and chaotic or nonlinear regions
- Harmonic regions grow significantly with forcing

Conclusions

- An oscillator system with a multi-segmented freeplay nonlinearity was studied to determine the effects on the physics of the system
- Bifurcation diagrams with respect to outer gap size, outer contact stiffness, freeplay asymmetry, and forcing magnitude were analyzed
- A decreasing outer gap increases the number of grazing contacts, but does not by itself activate much nonlinear behavior
  - Primary and superharmonic resonance amplitudes are more affected than lower-amplitude subharmonic resonance regions
- Increasing the outer contact stiffness (route to impact) or the forcing magnitude causes more superharmonic resonance and chaotic behavior to appear
- The asymmetry in the freeplay has the most significant effect on the system's behavior
  - Rich dynamics occur for the primary and secondary resonances, including bands of chaos

Acknowledgements

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Thank you for your attention!
Any questions?

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