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LLNL-JRNL-825945

Kinetic Friction _ The Physics Teacher

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August 19, 2021

The Physics Teacher

Exploring Newton's Second Law and Kinetic Friction using the Accelerometer Sensor in Smartphones

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Introduction

Decades of improvements in microelectromechanical systems (MEMS) have enabled high-performance compact sensors to become routinely integrated into smartphones. When combined with incredible touch screen displays, high-performance microprocessors for data analysis, and high-speed data transfer rates using Wi-Fi and Bluetooth, smartphones provide an unprecedented technical capability for conducting scientific investigations. The remarkable capability of smartphones to sense the world around us combined with their nearly universal availability to high school and college students has the potential to revolutionize inquiry-based learning in physics education. In recent years, there has been a growing awareness of this underutilized potential. Physics experiments enabled by the sensors embedded in smartphones have recently been reviewed by O'Brien[1] and a growing number of resources are available online[2,3,4]. This paper describes a novel approach for determining the coefficient of kinetic friction, which simultaneously incorporates the opportunity for students to explore many of the foundational disciplinary core ideas in mechanics using smartphones.

There are several traditional approaches to determine the coefficient of friction in student laboratories[5,6]. The first involves sliding a block down an inclined plane at a constant velocity where the force acting on the block due to gravity can be calculated from the incline angle. The second approach involves pulling a block on a horizontal surface with a pulley system or a spring scale. In both experiments, a known force opposing the force resulting from kinetic friction allows the determination of the coefficient of kinetic friction between the two surfaces. With the availability of MEMS accelerometers in smartphones, these approaches have been adapted to directly measure acceleration using both inclines[7] and pulley systems[8]. Recently, Leblond and Hicks have demonstrated the use of a commercially available platform (iOLab[9]), which includes an accelerometer sensor to measure the kinetic friction of the sliding device[10]. This paper presents an alternative experimental design where the phone is initially accelerated by an elastic force (rubber band). The coefficient of friction is then determined by measuring the acceleration of the phone as it slides along a horizontal surface during the time when the only force acting on the phone is the result of kinetic friction. This approach takes full advantage of precise and high-speed measurements recorded with the MEMS accelerometer and enables the full characterization of the velocity and displacement of the phone during the experiment. This approach can then be easily implemented to measure the coefficient of kinetic friction for a wide range of surface combinations. While this experiment has a goal of measuring the coefficient of kinetic friction, students engage in a rich journey of mechanics as they examine, interpret, and analyze the experimental data.

Experimental Design

An example experimental design is shown in Fig. 1. The experiment should be conducted on a uniform horizontal surface (e.g., lab bench, kitchen counter, wood floor, or table). The acceleration of a phone is measured for the short duration of the phone's motion (~1 second) while it experiences several different time varying forces. The phone should be in a case and/or secured to a simple sled (e.g., a piece of cardboard, felt, wood, or plastic) to provide a smooth uniform surface to slide along the horizontal surface. Note that different combinations of surfaces will result in different values for the coefficient of friction. The phone or sled should be connected to a chain of rubber bands used to provide an initial force to accelerate the phone. Students can explore many potential ways to attach the rubber bands to the phone. The phone is initially held in place while extending the rubber band until it is taut, and the desired initial elastic potential energy is achieved. The "desired" initial elastic energy will vary depending on the mass of the phone and the coefficient of friction. Upon release, the phone will slide across the uniform horizontal surface for tens of centimeters before coming to rest again. The extent of the phone's acceleration and resulting displacement can be adjusted by incrementally increasing the initial elastic force. The experiment is designed so that after the rubber band chain returns to its equilibrium length, the phone should continue to slide due to the phone's momentum for >50 ms (ideally a few hundred ms) before coming to a stop. These approximate times can be achieved through experimentation by observing the real-time data as the initial elastic potential energy is adjusted. The goal is to accelerate the phone along a single axis (y-axis will be the most stable), which can be achieved through small adjustments of the rubber band connection to minimize any off-axis forces.

Measuring Acceleration

The acceleration data can be collected using any of several free applications (e.g., phyphox or Physics Toolbox) that allow access to the 3-axis accelerometer data and enable exporting of the data for analysis. The data collection rates for most modern smartphones are in the range of 100-500 Hz, which is sufficient to resolve the changes in acceleration required in this experiment. The data that follows was collected using phyphox and an iPhone collecting data at 100 Hz.

Graphs of the acceleration versus time measured along all three axes for a typical experiment are shown in Fig. 2a. While the change in acceleration is relatively small for the x and z axes, there is a large and well-defined change in the acceleration along the y-axis, which is anticipated due to the displacement directed along the y-axis of the phone. An expanded view of the y-axis acceleration data is shown in Fig. 3, where 1.25 seconds of data is displayed. Regions where specific changes in the acceleration are observed have been labeled to facilitate investigation of the rich physics associated with the movement of the phone. Table 1 provides a summary of the forces for each region and the associated motion. The initial large acceleration results when the phone is released, and the elastic force accelerates the phone along the y-axis. As the rubber band begins to return toward its equilibrium length and converts the elastic potential energy into kinetic energy, the elastic force is reduced. The phone continues to gain linear momentum during the entire time the acceleration is positive. While the phone is sliding along the horizontal surface, a force resulting from kinetic friction is acting on the phone in the opposite direction of the velocity. This data provides a great opportunity for students to use free-body diagrams to describe the relative magnitude of the forces acting on the phone at various times as they observe the experimental acceleration.

Region 5 in Fig. 3 is of particular interest. According to Newton's Second Law, the phone should experience constant acceleration while a constant force is applied. The graph illustrates that the phone experiences constant acceleration for a period of >350 ms where we expect the only force acting on the phone is kinetic friction. We can also conclude that the force from kinetic friction is independent of velocity since the acceleration remains constant over a large change in velocity.

Calculating the Coefficient of Kinetic Friction

The experimental geometry used in this experiment leads to a very simple determination of the coefficient of kinetic friction, μ_k, during the time where the acceleration, a, is solely determined by the force of kinetic friction, F_k. The relationship between F_k and μ_k is shown in Eq. (1). Substitution of Eq. (1) into the equation for Newton's Second Law results a simple relationship for μ_k, where it can be determined by the ratio of the measured acceleration and the acceleration due to gravity, g, as shown in Eq. (2).

$$F_k = \mu_k F_{Normal} = \mu_k mg \quad (1)$$

$$F_k = ma \implies \mu_k mg = ma \implies \mu_k = \frac{a}{g} \quad (2)$$

Fig. 4 illustrates the determination of the average value of the acceleration from the graph, as well as the calculation of μ_k for the specific surfaces (i.e., Otterbox case and wooden table).

Calculating the Velocity and Displacement

In addition to determining the coefficient of kinetic friction in this experiment, it is possible to calculate both the velocity and the displacement from the acceleration data. The analysis starts with the definition of acceleration:

$$a = \frac{\Delta v}{\Delta t} \quad (3)$$

The change in velocity for each increment in time can then be estimated from the experimental data using:

$$\Delta v = a\Delta t \quad (4)$$

Starting with a velocity of 0.00 m/s, the change in velocity can be calculated for each increment of time in the data set. The velocity at any moment in time will be the sum up the changes in velocity which is easily calculated using a spreadsheet. In a similar fashion, the change in displacement can also be calculated:

$$\Delta d = v\Delta t \quad (5)$$

The total displacement can be determined by summing up the displacement for each time increment. The calculated velocity and displacement are shown in Fig. 5, along with the experimental acceleration. This figure illustrates that the maximum value of the velocity occurs when the acceleration is changing signs, when the elastic force and the force from kinetic friction are equal. The precision of the acceleration and the robustness of the calculation of the velocity is demonstrated by the fact that the velocity returns to zero at the end of the experiment. The determination of the displacement is also very accurate. In the experiment shown in Fig. 5, the displacement measured on the table from the starting location to the ending position was 60.0 cm. The value calculated from the acceleration data was 59.5 cm, which is in good agreement. This analysis provides a great opportunity to introduce students to integration and reinforces the concept of integration to those that have already taken calculus.

Investigating Kinetic Friction for Different Surfaces

Now that a simple method of determining the coefficient of friction has been established, it is straightforward to vary the surfaces producing the frictional force. The experimental data for four different surface combinations are shown in Fig. 6. The coefficients of friction determined from the acceleration data for the different surfaces are captured in Table 2. Examining the magnitude of the coefficient of kinetic friction will enable students to discuss its physical origins. Students will be able to investigate the contributions from macroscopic surface roughness, microscopic interactions that are often enhanced for smooth surfaces, and the impact of lubrication. This simple approach to measurement allows students to immediately visualize and quantify the coefficient of kinetic friction.

Discussion

This experimental design can also be used to extend the investigation of other variables associated with friction. Following the methods outlined above, students can further investigate the dependence on surface area and mass. If teachers are interested in adding additional complications to the analysis, they can ask students to demonstrate this approach on an incline plane where students will use trigonometry to include additional forces in the analysis.

In summary, this simple laboratory experiment requires limited resources beyond a student's smartphone. The method produces high-quality data and provides an opportunity to reinforce many disciplinary core ideas of mechanics. The activity also has many opportunities for student-initiated inquiries to extend the investigation of the very important phenomena of friction.

Acknowledgements

Support for this work was provided in part by the NNSA and Minority Serving Institutions Partnership Program grant number DE-NA0003866 (Consortium for High Energy Density Science). Additional support was provided through the 2021 STEM Teacher and Researcher Program by Chevron (www.chevron.com), the National Science Foundation through the Robert Noyce Program under Grant #1836335, the California State University Office of the Chancellor, and California Polytechnic State University. This work is performed under the auspices of the U. S. Department of Energy by Lawrence Livermore National Laboratory under Contract DE-AC52-07NA27344.

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[2] Experiments designed to use smartphone sensors using the Physics Toolbox application from Vieyra Software: https://www.vieyrasoftware.net/browse-lessons

[3] Experiments designed to use smartphone sensors using the phyphox application from RWTH Aachen University: https://phyphox.org/experiments/

[4] Experiments designed to use smartphone sensors from Lawrence Livermore National Laboratory: Physics with Phones | Science and Technology (llnl.gov)

[5] Raymond A. Serway and John W. Jewett, Jr, *Physics for Scientists and Engineers*, 10th ed. (Cengage, Boston, 2019), pp. 116-119.

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[10] Louis Leblond and Melissa Hicks, "Designing laboratories for online instruction using the iOLab device," *Phys. Teach.* **59**, pp. 351-355 (April 2021).

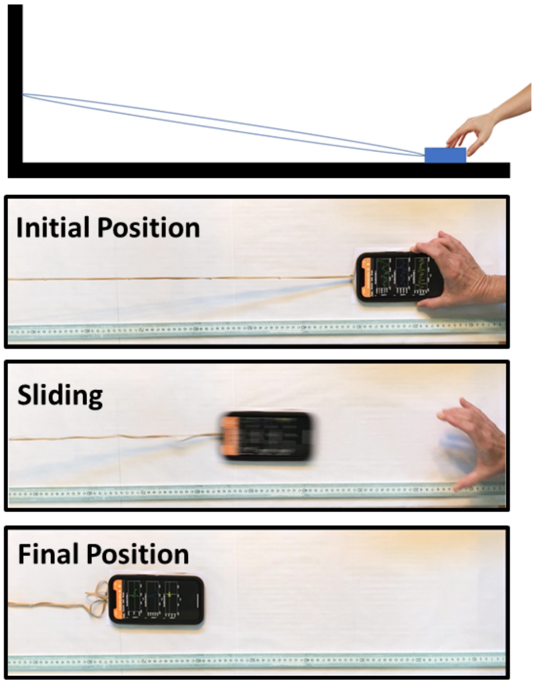

Fig. 1. Experimental design used to measure coefficient of kinetic friction. The top frame shows a schematic diagram of the experiment before the phone is released. The next three frames are images captured from a video at the beginning, middle, and end of the experiment.

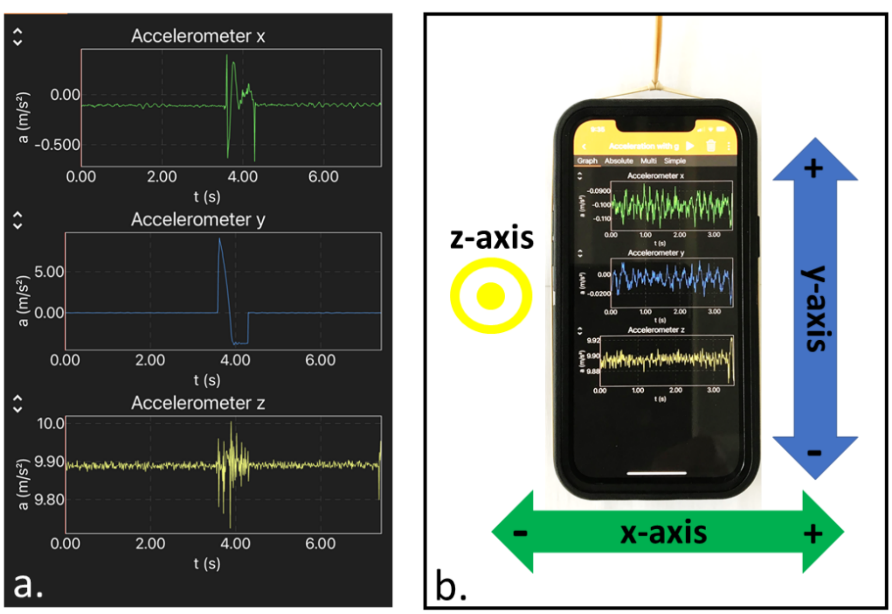

Fig. 2. Fig. 2a. is a screen shot of the acceleration measured along all three axes of the phone using the experimental design illustrated in Fig. 1. Fig 2b. shows the sensor axes relative to the phone orientation, where the positive z-axis is coming out of the plane. The rubber band is attached at the top of the phone resulting in an initial acceleration along the positive y-axis.

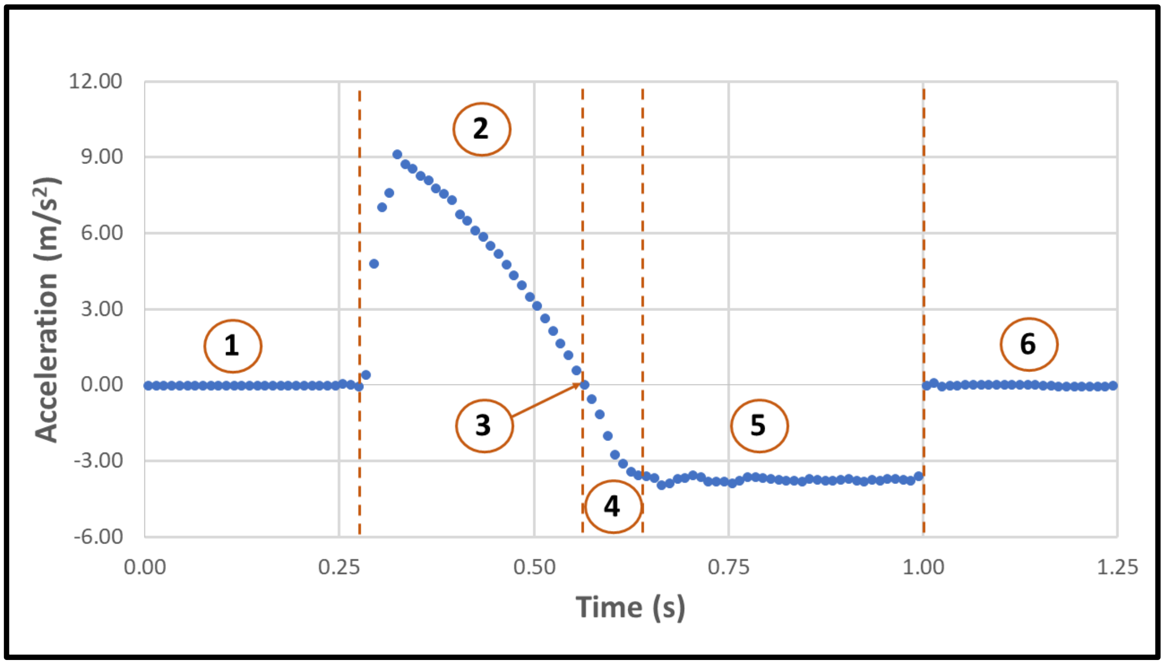

Fig. 3. Graph of the y-acceleration vs time for an iPhone 12 in an Otterbox Defender Series case, sliding on a wooden table using the experimental design illustrated in Fig. 1. Numbers and dotted lines are used to indicate periods of time describing the mechanics of motion in Table 1.

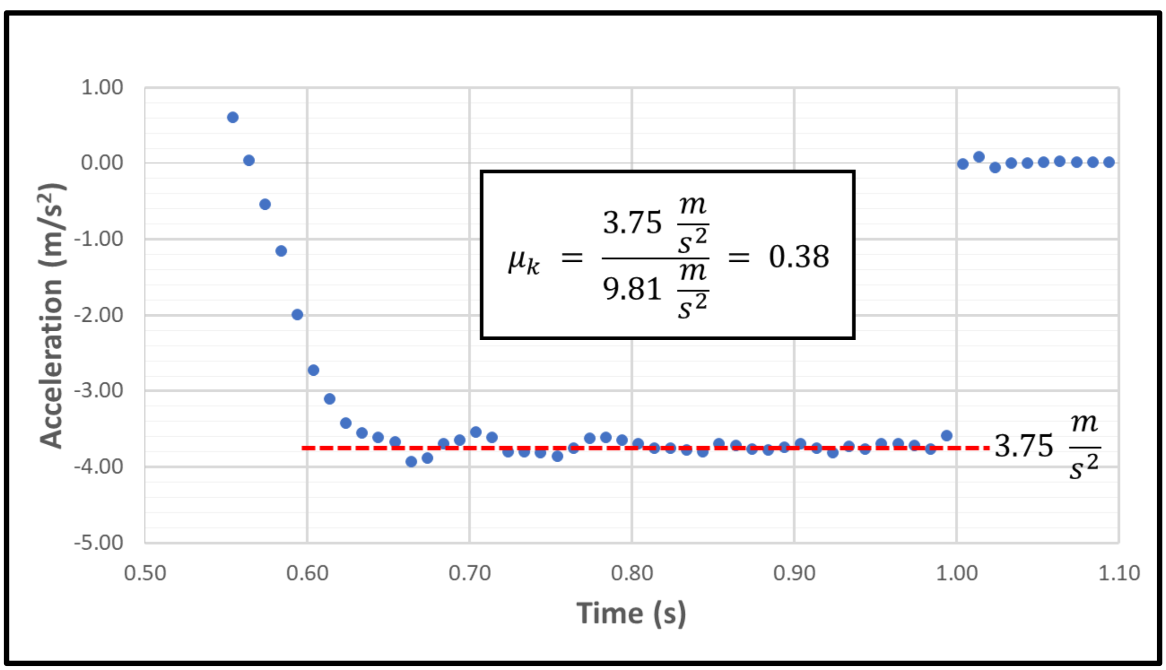

Fig. 4. Acceleration vs time data from Fig. 3 is graphed for a 600 ms period allowing closer examination of the time where only the force of friction is acting on the phone. An estimate of the average acceleration was determined graphically to be 3.75 m/s^2 during this time. The coefficeint of kinetic friction, μ_k, is determined to be 0.38, using Eq. 2 and the average value of the acceleration.

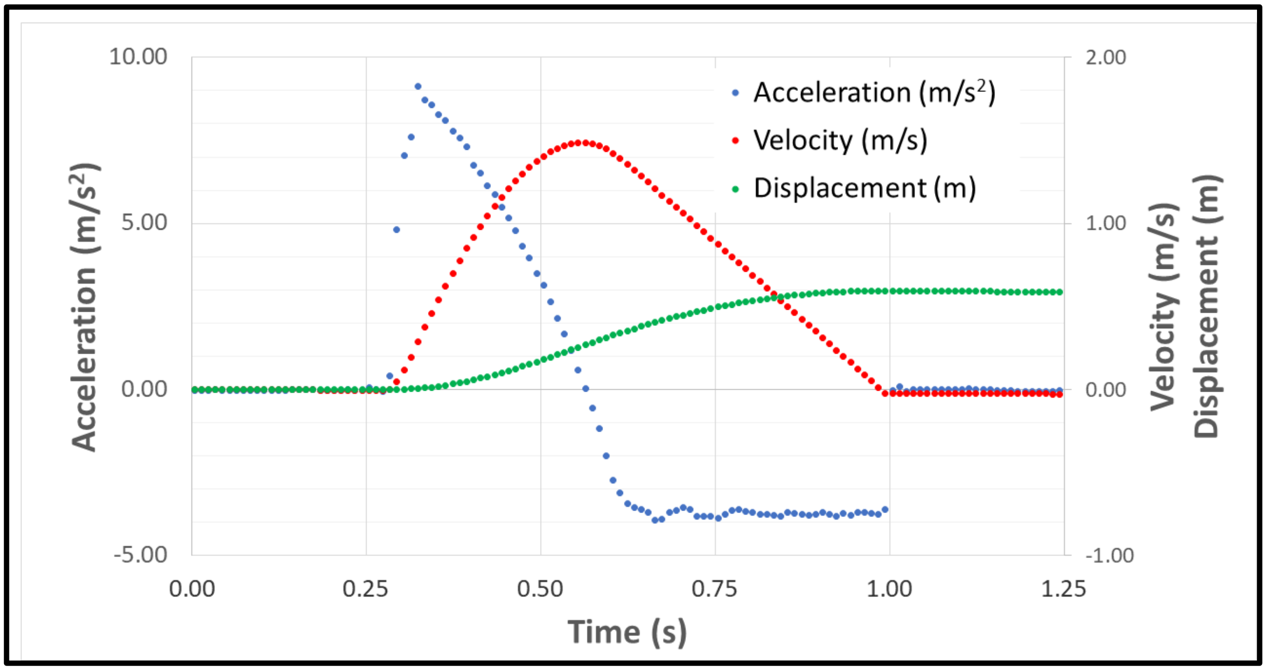

Fig. 5. The calculated velocity and displacement are graphed along with the experimental acceleration. The graph shows that the final velocity reaches a maximum of 1.49 m/s and then returns to zero (calculated value of -0.02 m/s) as expected at the end of the experiment. The calculated total displacement was 59.5 cm (experimental displacement was 60 cm).

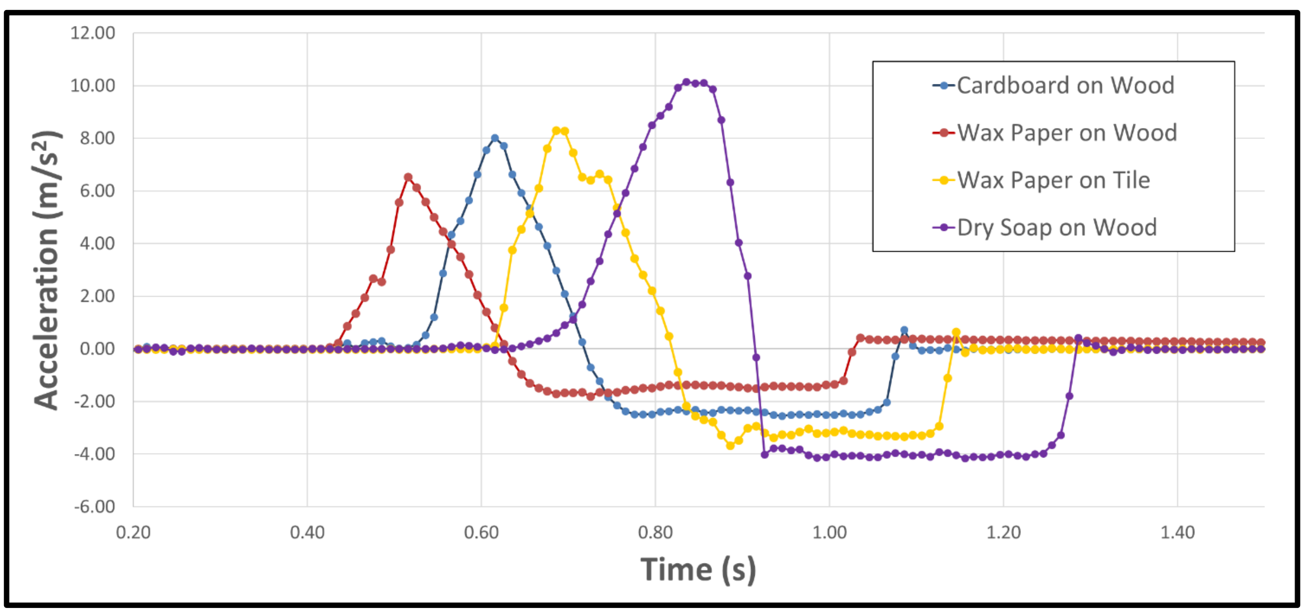

Fig. 6. Example data for four different surface combinations demonstrating the variation in observed acceleration due to the force of friction. The coefficient of friction determined from these data sets are included in Table 2.

Region from Fig. 2	Time (s)	Forces Acting in Horizontal Plane ; Resulting Motion
1	0 - 0.26	$\sum F = 0$; Phone at rest.
2	0.27 - 0.55	$F_{elastic} >> F_k$; Positive acceleration along y-axis.
3	0.56	$F_{elastic} = F_k$; $a=0$; Maximum velocity
4	0.57 - 0.61	$F_{elastic} < F_k$; Negative acceleration along y-axis
5	0.62 - 1.00	$F_{elastic} = 0$; $F_k = \text{constant}$; Constant negative acceleration
6	1.01 - 1.50	$\sum F = 0$; Phone at rest.

Table 1: Description of forces and the resulting motion for the regions indicated in Fig. 2.

Horizontal Surface	Phone "Sled" Surface	Average Acceleration due to Friction (m/s²)	Coefficient of Friction μ_k
Wood	Cardboard	2.29	0.234
	Wax Paper	1.47	0.150
	Dry Soap	3.98	0.406
	Wet Soap	2.16	0.220
Tile	Cardboard	4.49	0.459
	Wax Paper	3.07	0.313
	Dry Soap	5.20	0.531
	Wet Soap	2.54	0.259

Table 2: The coefficient of friction for eight different surface combinations are tabulated with the experimental values of the average acceleration. Graphs of the experimental acceleration vs time for four of the surface combinations are shown in Fig. 6.