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Using Modal Projection Error to Evaluate SEREP Modal Expansion

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Modal Expansion with the System Equivalent Reduction Expansion Process (SEREP)

Goals of this presentation

- Expand our understanding of the SEREP expansion process
- Determine if the Modal Projection Error can be used to aid the execution of the SEREP expansion

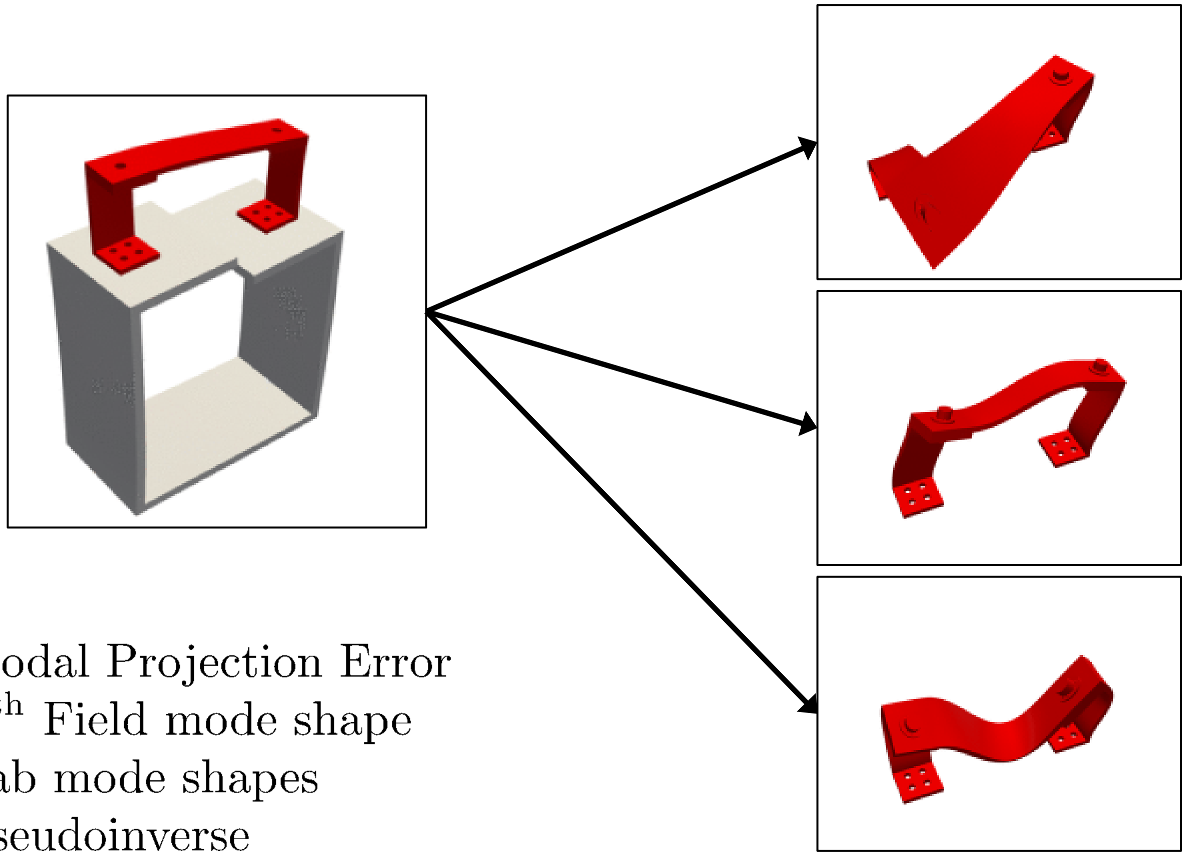
SEREP(1) $$\ddot{x}_{(n)} = \phi_{FEM(nxm)}\phi^{+}_{FEM(axm)}\ddot{x}_{f(a)}$$

From a limited set of field degrees of freedom (f) of count a, use the m mode shapes of the finite element model as a set of basis vectors to fit to the measured data. Then use those fitted shapes to interpolate and extrapolate to the (n) dofs.

[1] John C O'Callhan. System equivalent reduction expansion process. In *Proc. of the 7th Inter. Modal Analysis Conf., 1989.*

Explanation of the Modal Projection Error

$$MPE = \Psi_n^2 = 1 - \bar{\phi}_{Fn}^{+} \phi_L \phi_L{}^{+} \bar{\phi}_{Fn}$$



Ψ_n^2 = Modal Projection Error
$\bar{\phi}_{Fn}$ = n$^{\text{th}}$ Field mode shape
ϕ_L = Lab mode shapes
$^{+}$ = Pseudoinverse

T. Schoenherr, J. Rouse, and J. Harvie. Characterizing Dynamic Test Fixtures Through the Modal Projection Error. Technical Report SAND2020-1119, Sandia National Laboratories, January 2020.

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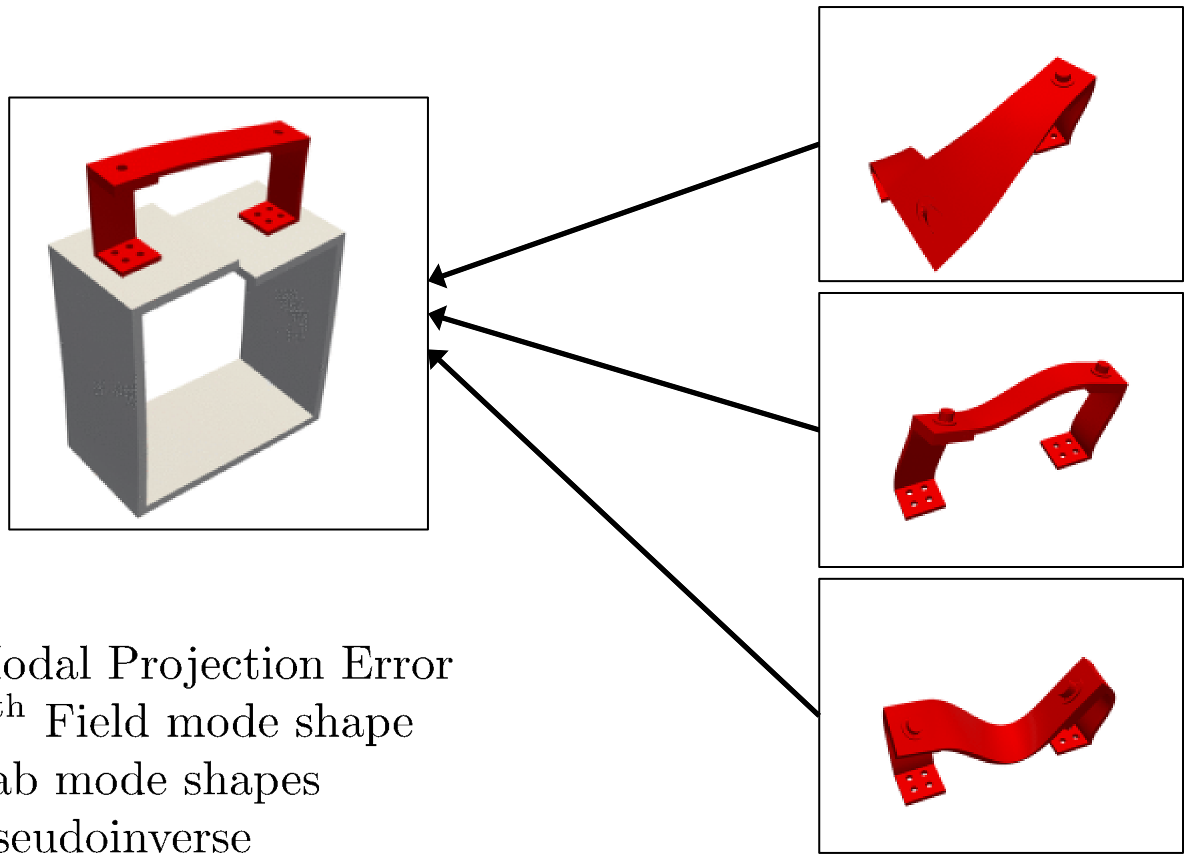

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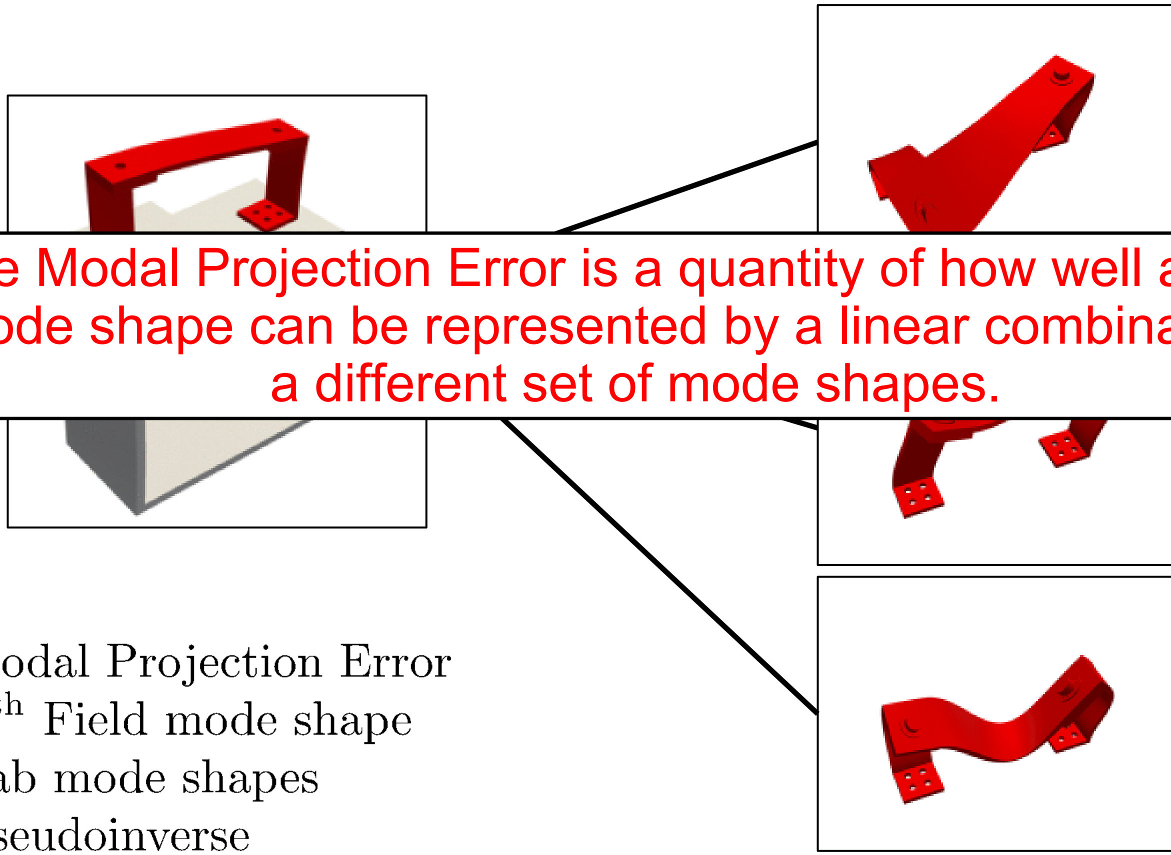

The Modal Projection Error is a quantity of how well a single mode shape can be represented by a linear combination of a different set of mode shapes.

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Modal Projection Error and SEREP

To help gain an understanding of SEREP, I created a fictious example using polynomial curve fitting in EXCEL®.

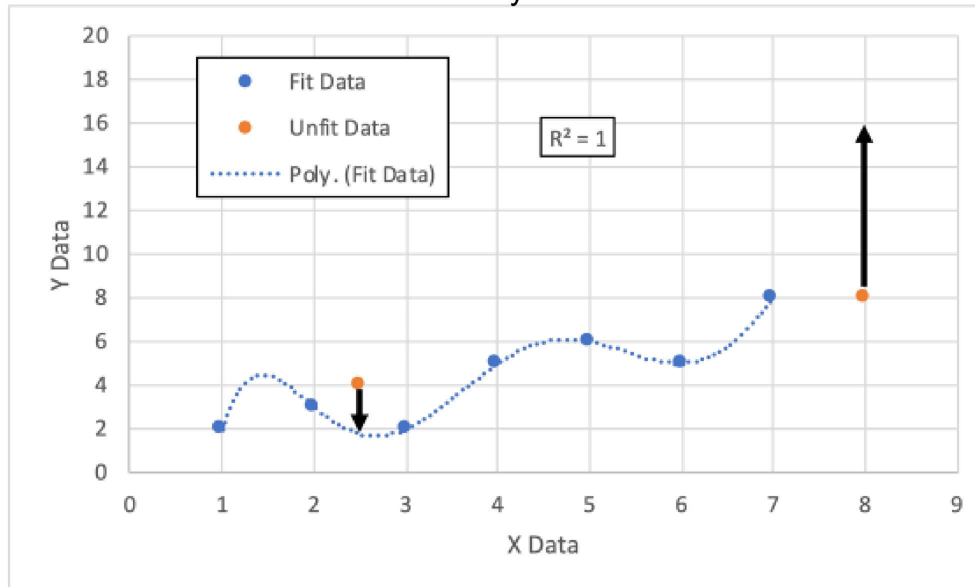
For a given set of measured experimental data (blue dots), a polynomial was fit in a least squared sense.

A 6th order polynomial was fit to the 7 data points and a perfect fit was acquired (Coefficient of Determination, R = 1). However, the unfit data (interpolation and extrapolation) had large error.

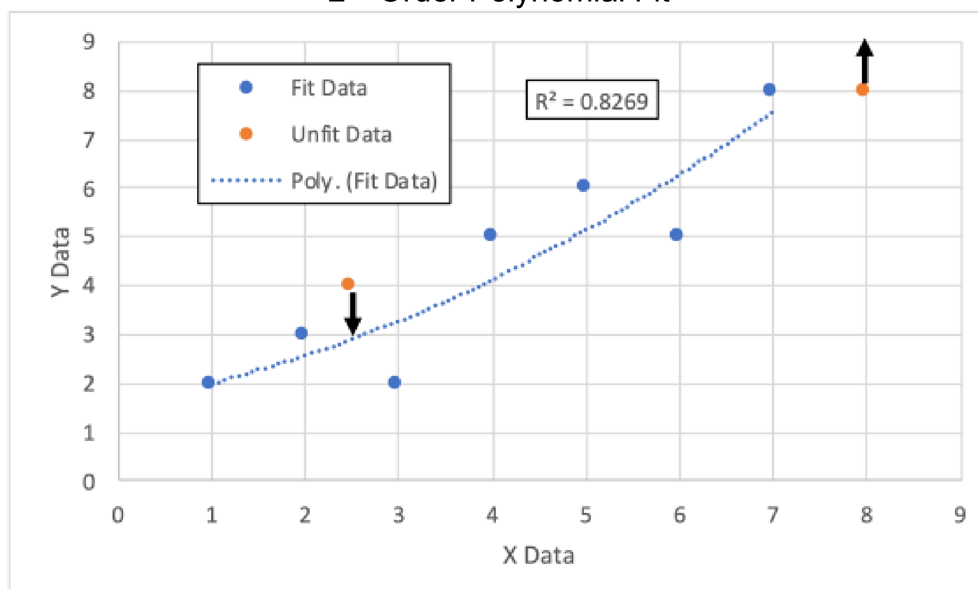
The same data was fit with a 2nd order polynomial and we can qualitatively conclude that this polynomial is a better fit to the data as the error to the unfit data is lower.

The R value is no longer 1, but provides more information on the fit to the data. The modal projection error is analogous to R.

6th Order Polynomial Fit



2nd Order Polynomial Fit



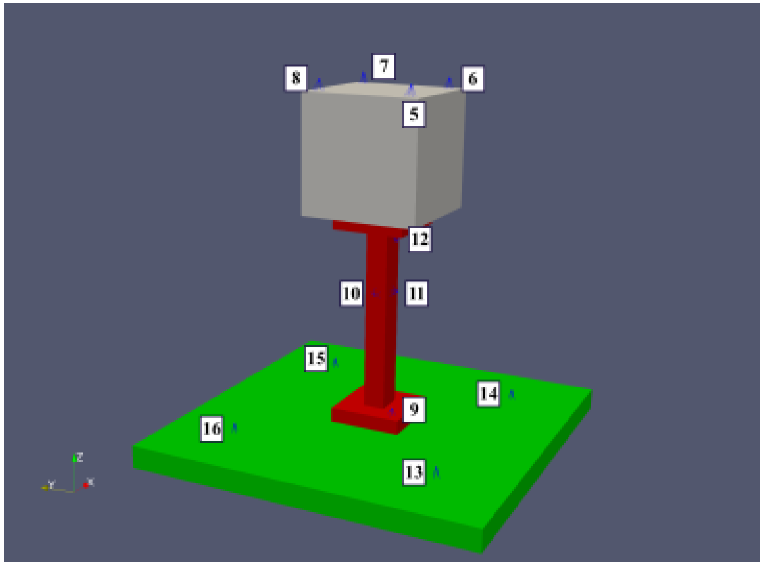
Experimental Data

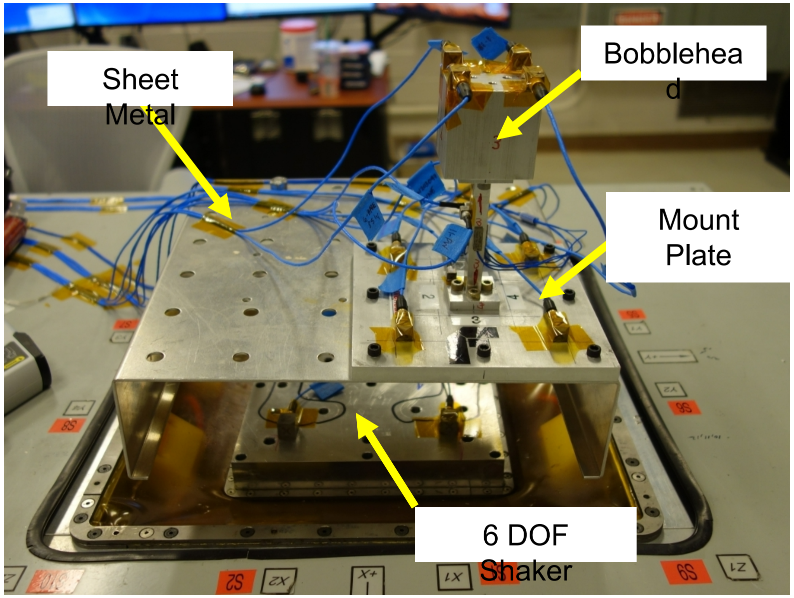
A FEM was created of the part of interest. The FEM included the "bobblehead" and its mounting plate.

Two versions of the model were created. One with free boundary conditions, one with boundary conditions fixed at the bolted locations

The bobblehead and mounting plate were attached to a thin piece of bent sheet metal and the sheet metal was attached to the 6 DOF shaker via a single threaded rod.

An environment was run on a 6 DOF shaker.





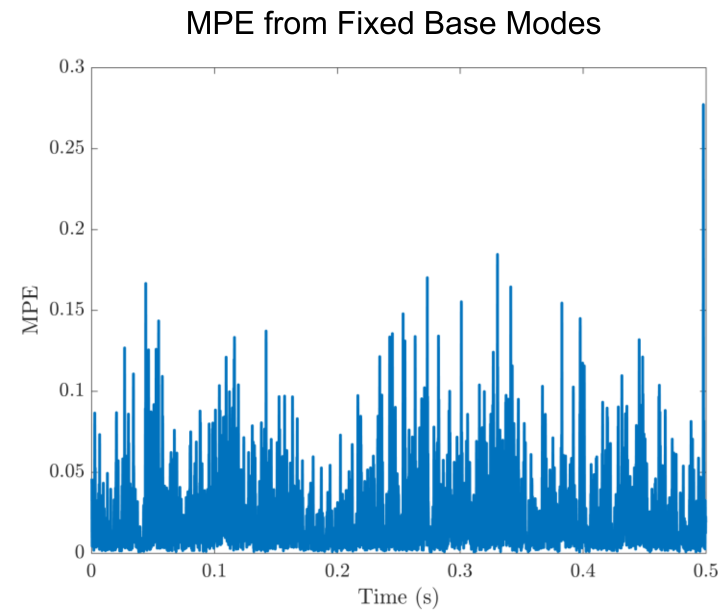
SEREP Expansion

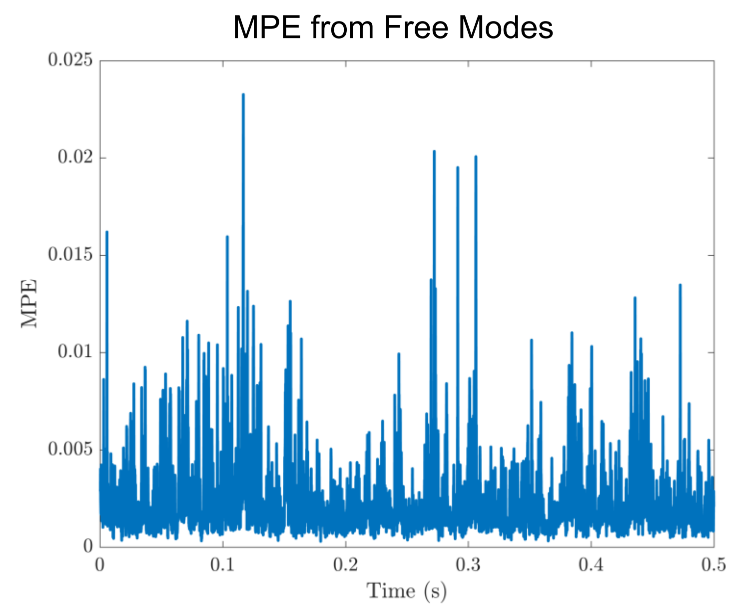
A 0.5 second snapshot of time was analyzed and expanded through the fixed and free FEM mode shapes (RBM included for both).

The MPE was calculated at each timestep. Using the free modes as the basis vectors provided a MPE that was about an order of magnitude lower.

The amount of modes used as the set of basis vectors was less than half of the DOFs measured. This improves the least squares fitting and gives the MPE more validity.

The TRAC and visual comparison between the measured and expanded data gave the same conclusion that the free shapes were a better set of basis vectors.





Using MPE to identify the most important modes

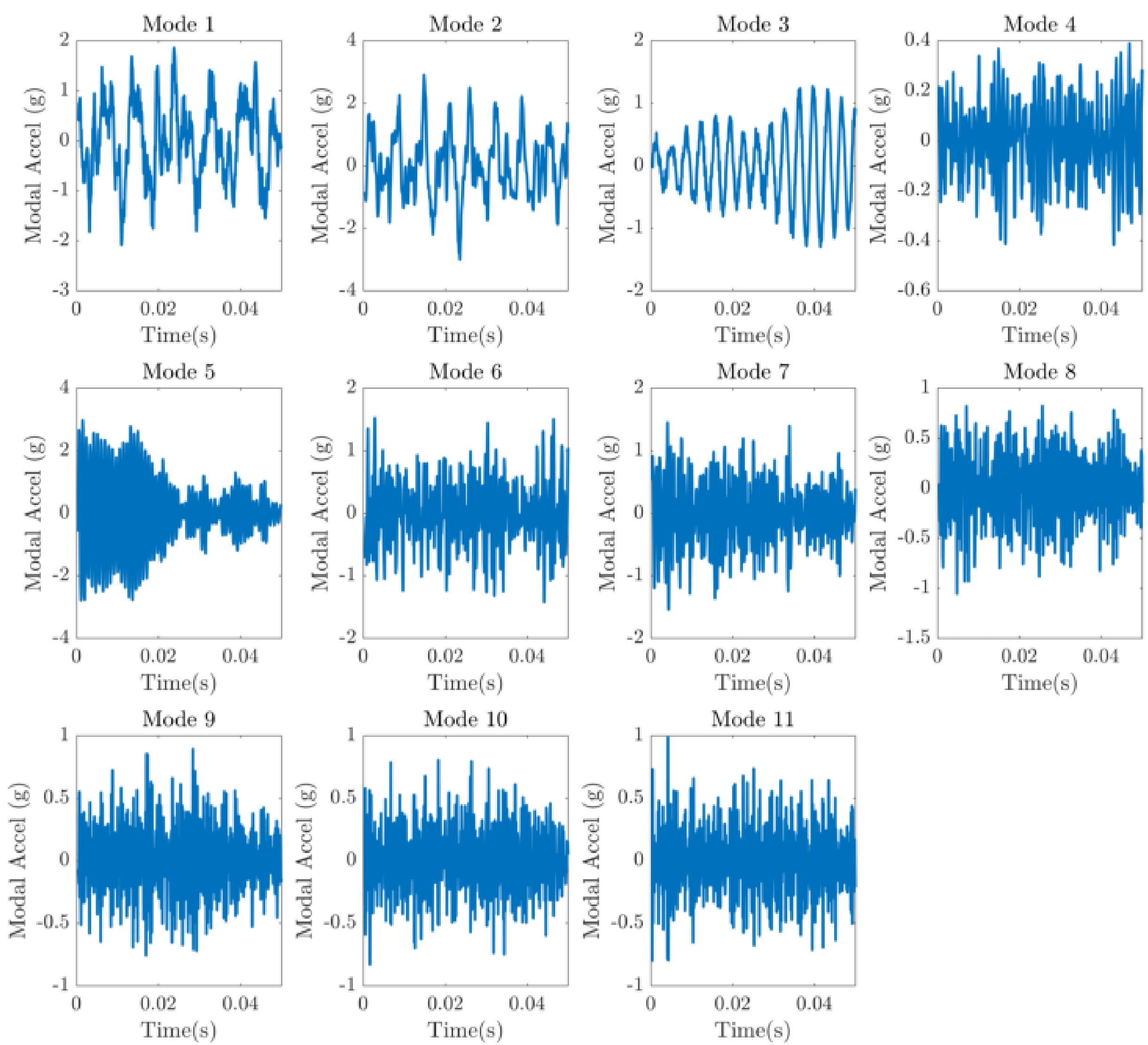
The modal coefficients were calculated to determine if we could determine which modes were less important in the expansion.

Although the frequency and amplitude of the modal coefficients varied, there is no justification to determine which of these modes is the least important.

$$\ddot{x}(t) = \phi\, q(t)$$

Modal Coefficient

Modal Coefficients of the First 11 Elastic Modes



Using MPE to identify the most important modes

Removing a single mode from the basis set and recalculating the MPE each time was done.

If the mode did not increase the error much when removed, then it was determined that the mode is not prevalent in the data and is not needed for the expansion.

Mode 9 and 10 are the least important modes in this basis set. Not mode 4 based on modal coefficient amplitude.

This process can be done iteratively to determine the most important modes.

Mode Removed	MPE ave	MPE max
none	2.20e-3	2.33e-2
1	4.38e-2	5.12e-1
2	7.42e-2	6.15e-1
3	11.4e-2	7.54e-1
4	1.11e-2	2.23e-1
5	13.6e-2	7.94e-1
6	2.82e-2	5.45e-1
7	3.49e-2	4.90e-1
8	5.10e-3	1.01e-1
9	4.80e-3	7.86e-2
10	4.90e-3	7.50e-2
11	8.50e-3	1.86e-1

Summary

The modal projection error can be used to determine which modes are most important in reconstructing your environment.

Having a fairly rectangular shape matrix for projection is critical when projecting experimental motion onto finite element shapes. This is where the least squares fitting happens which leads to more confidence in interpolation and extrapolation.

Bonus Information:

Performing expansion on strain responses can be beneficial because that eliminates the need for DC or rigid body response because rigid body motion produces no strain.