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# Insights on the bifurcation behavior of a freeplay system with piecewise and continuous representations

Brian Evan Saunders[1], Rui M.G. Vasconcellos[2], Robert J. Kuether[3], and Abdessattar Abdelkefi[1]

[1] Mechanical & Aerospace Engineering Department
New Mexico State University, Las Cruces, NM, USA

[2] Campus of São João da Boa Vista
São Paulo State University, São João da Boa Vista, Brazil

[3] Sandia National Laboratories, Albuquerque, NM, USA

## ABSTRACT

Dynamical systems containing contact/impact between parts can be modeled as piecewise-smooth reduced-order models. The most common example is freeplay, which can manifest as a loose support, worn hinges, or backlash. Freeplay causes very complex, nonlinear responses in a system that range from isolated resonances to grazing bifurcations to chaos. This can be an issue because classical solution methods, such as direct time-integration (e.g. Runge-Kutta) or harmonic balance methods, can fail to accurately detect some of the nonlinear behavior or fail to run altogether. To deal with this limitation, researchers often approximate piecewise freeplay terms in the equations of motion using continuous, fully smooth functions. While this strategy can be convenient, it may not always be appropriate for use. For example, past investigation on freeplay in an aeroelastic control surface showed that, compared to the exact piecewise representation, some approximations are not as effective at capturing freeplay behavior as other ones. Another potential issue is the effectiveness of continuous representations at capturing grazing contacts and grazing-type bifurcations. These can cause the system to transition to high-amplitude responses with frequent contact/impact and be particularly damaging. In this work, a bifurcation study is performed on a model of a forced Duffing oscillator with freeplay nonlinearity. Various representations are used to approximate the freeplay including polynomial, absolute-value, and hyperbolic-tangent representations. Bifurcation analysis results for each type are compared to results using the exact piecewise-smooth representation computed using Matlab® Event Location. The effectiveness of each representation is compared and ranked in terms of numerical accuracy, ability to capture multiple response types, ability to predict chaos, and computation time.

**Keywords:** Bifurcation analysis, nonlinear dynamics, freeplay, continuous representation

## INTRODUCTION

Vibro-contact and vibro-impact dynamical systems are very common across engineering fields, with examples ranging from large aeroelastic structures [1] to small energy harvesters [2]. Various numerical methods and models have been used and developed to represent contact/impact behavior. In reduced-order models, an important consideration is how to adequately represent the contact force(s), particularly because contact can induce very strong nonlinearities into a dynamical system. A realistic contact representation is a piecewise-smooth force curve; e.g. in a freeplay system, there is a gap between parts and there is no contact until the parts' displacement is larger than the gap size. The non-smooth behavior, however, can lead to numerical problems and roundoff error if the switching points from no-contact to contact are not captured. Contact points can be accurately captured if Henon's method [3] or another numerical scheme (e.g. Matlab® *Event Location*) is used, but this often increases computation time.

Other representations that are continuous and fully smooth have been developed, which may be based on combinations of absolute-value and polynomial functions [2, 4], the hyperbolic tangent function [1, 5], or similar functions/combinations. These continuous contact-force representations allow for low computational costs and remove the accumulating roundoff error, but there are tradeoffs with other sources of error. For example, the contact force within the freeplay gap may be nonzero or the contact stiffness beyond the gap may inappropriately harden or soften with increasing displacement. Vasconcellos et al. [1] found that, when used in the model of an aeroelastic system with control surface freeplay, some continuous representations are

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unable to capture all of the nonlinear behavior that may be present. This includes potentially dangerous responses, such as grazing contact and grazing bifurcations.

The goal of this work is to study how different contact-force representations can affect a more general nonlinear system, namely, a forced Duffing oscillator with freeplay [6]. Bifurcation analysis results for each representation are compared to results using the exact piecewise-smooth representation, computed using Matlab® ode45 with *Event Location*. The effectiveness of each representation is compared and ranked in terms of numerical accuracy, ability to capture multiple response types, ability to predict chaos, and computation time. This work intends to explore under what conditions a smooth contact-force representation may be used instead of the exact piecewise representation. Results for one representation (hyperbolic-tangent) are presented in this extended abstract for brevity; results for other representations (absolute-value, polynomial, etc.) are reserved for the final conference presentation.

## SYSTEM MODELING

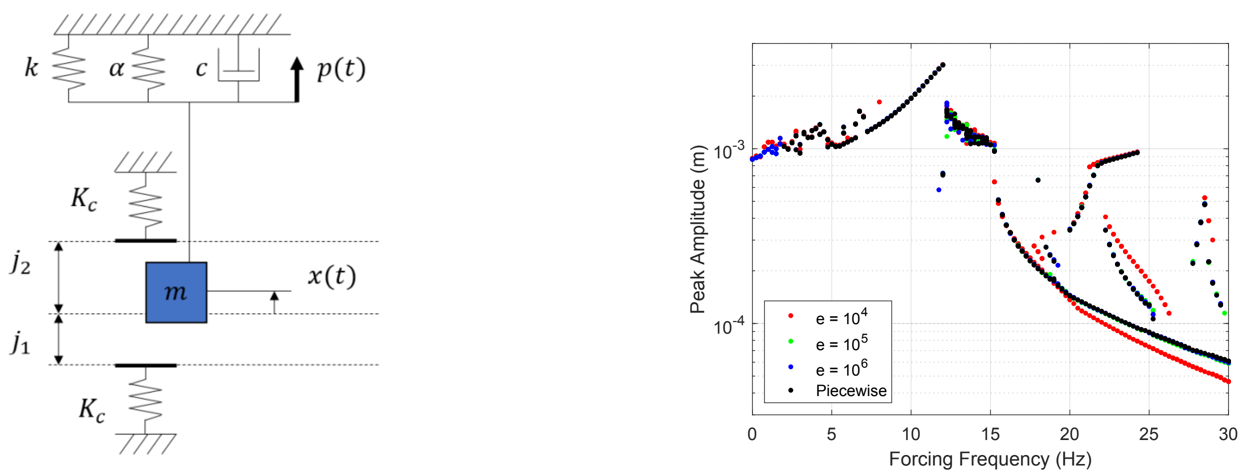
The equations of motion for the Duffing-freeplay system [6] are given by:

$$\ddot{x} + 2\omega_n\zeta\dot{x} + \omega_n^2 x + \frac{\alpha}{m}x^3 + \frac{F_c}{m} = \frac{p}{m}cos(\omega t), F_c = \begin{cases} K_c(x+j_1), & x < -j_1 \\ 0, & -j_1 < x < j_2 \\ K_c(x-j_2), & x < j_2 \end{cases} \#(1)$$

where $\alpha$ is the cubic stiffness, $K_c$ is the contact stiffness, and $j_1$,$j_2$ are the freeplay gap boundaries. To solve the system with piecewise-smooth representation, Matlab® ode45 with *Event Location* is used to accurately capture the switching points between freeplay regions. Figure 1(a) shows a schematic of the system. Figure 1(b) presents frequency response curves for the system using both the piecewise representation and the hyperbolic-tangent representation studied in [1], given by:

$$F_c = K_c\left(\frac{1}{2}[1 - tanh(e(x+j_1))](x+j_1) + \frac{1}{2}[1 + tanh(e(x-j_2))](x-j_2) + P\right)\#(2)$$

where $e$ is a "tolerance" parameter; in the limit $e \rightarrow \infty$, the representation approaches the piecewise representation. System parameters are $m = 5kg$,$\omega_n = 5Hz$,$\zeta = 0.03$,$\alpha = 7 * 10^8 N/m^3$,$p = 4N$,$j_1 = 0mm$, $j_2 = 0.8mm$,$K_c = 1.4 * 10^4 N/m$




(a) (b)

Figure 1: (a) schematic of the Duffing-freeplay system and (b) frequency response curves of the system with asymmetric gap, using both the piecewise and hyperbolic-tangent [1] representations

## EFFECTIVENSS OF CONTINUOUS REPRESENTATIONS FOR SYSTEM WITH FREEPLAY NONLINEARITY

Figure 1(b) indicates that results can significantly diverge as forcing frequency increases past the primary resonance peak if a large enough value of $e$ is not used, meaning regions of subharmonic resonance may be inaccurately predicted. A graph of the contact force versus displacement for even the coarsest value of $e = 10^4$ used in Figure 1(b) appears acceptable (omitted from this extended abstract for brevity), though, and does not indicate that frequency-response results will diverge. Thus, a convergence analysis is necessary. The low-frequency superharmonic resonances and chaotic behavior seem to be relatively unaffected, in addition to the primary resonance peak, for all values of $e$. However, a good agreement in frequency-response results is not always a good indicator that results agree globally and that system physics are not lost [7]. Nonlinear characterization (omitted for brevity) is also performed to determine how well the continuous representations can capture the overall physics of the system response.

## CONCLUSIONS

In this work, bifurcation analysis was carried out on a forced Duffing oscillator system with freeplay nonlinearity for different mathematical representations of the freeplay contact force. Results using a hyperbolic-tangent representation indicated good frequency-response agreement after a parameter convergence analysis was performed. This convergence was required because the contact force-displacement may look acceptable for an unconverged model, but the frequency response significantly diverges as forcing frequency increases past the primary resonance peak. This is particularly dangerous because subharmonic resonances often lead to high-amplitude responses which can be damaging. Results for other representations (absolute-value, polynomial, etc.) are reserved for the final conference presentation.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