

# Nonlinear Variability due to Mode Coupling in a Bolted Benchmark Structure

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This paper presents a set of tests on a bolted benchmark structure called the S4 beam with a focus on evaluating coupling between the first two modes due to nonlinearity. Bolted joints are of interest in dynamically loaded structures because frictional slipping at the contact interface can introduce amplitude dependent nonlinearities into the system, where the frequency of the structure decreases, and the damping increases. The challenge to model this phenomenon is even more difficult if the modes of the structure become coupled, violating a common assumption of mode orthogonality. This work presents a detailed set of measurements in which the nonlinearities of a bolted structure are highly coupled for the first two modes. Two nominally identical bolted structures are excited using an impact hammer test. The nonlinear damping curves for each beam are calculated using the Hilbert Transform. Although the two structures have different frequency and damping characteristics, the mode coupling relationship between the first two modes of the structures is shown to be consistent and significant. The data is intended as a challenge problem for interested researchers; all data from these tests are available upon request.

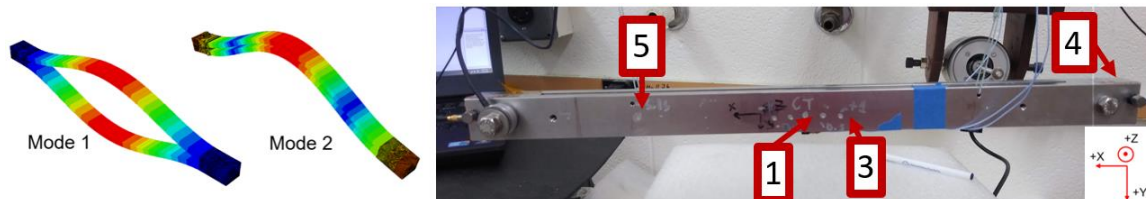
## Introduction:

Bolted joints present a challenge for characterizing and modeling the dynamics of a structure. As a structure is loaded, frictional slip at the joint contact interface results in a decrease in stiffness and an increase in damping. Although this is typically considered a weak nonlinearity, bolted joints can account for up to 90% of the damping in some structures [1]. This type of nonlinearity is referred to as a microslip nonlinearity because only part of the contact region in the joint begins to slip while some part of it remains fully stuck. This work is concerned with experimentally quantifying the nonlinearity in a bolted structure. Only the damping nonlinearity is discussed here, but similar results are seen for the frequency nonlinearity as well, although the frequency shifts are smaller than the changes in damping. The Hilbert transform is used to calculate the damping of the mode as a function of amplitude, which was originally proposed by [2] and has been developed further by [3].

When possible, it is very convenient to treat the modes of a jointed structure as uncoupled so that the linearized eigenvectors can be used to decompose the model into a set of uncoupled, nonlinear equations of motion. Previous studies have demonstrated that this assumption holds for some systems [4], [5]. This work presents a set of test results where mode coupling has a significant effect on the damping of the structure. Multiple drive points are used on two sets of nominally identical beams to excite different combinations of modes to different amplitudes; In each case the modal damping is found and mode coupling is observed when the damping of a mode changes due to the amplitude of another mode. The two structures used in this work will be referred to as the 2017 beam, which was previously used in [6], [7], the 2020 beam which was previously used in [8].

## Experimental Setup and Procedure:

The experimental setup for the S4 beam is shown in Figure 1. The 2017 beam is shown but the same setup was used for the 2020 beam. The beams were suspended by bungee cords at two points. The bolt preload on the left bolt was measured by a load washer which uses a strain gauge to measure the tension in the bolt. The torque to reach the desired preload was recorded and then the same torque was used to tighten the other bolt. Five uniaxial accelerometers were used to instrument the beam: one at each end in the Z direction near the bolts, one at the center in the Z direction between the two beams, and one on each beam in the center on the bottom facing side.



**Figure 1: Left: Mode 1 and Mode 2 of the S4 beam from a FEM. Experimental setup for the S4 beam with indexes shown for each drive point and the reference coordinate system.**

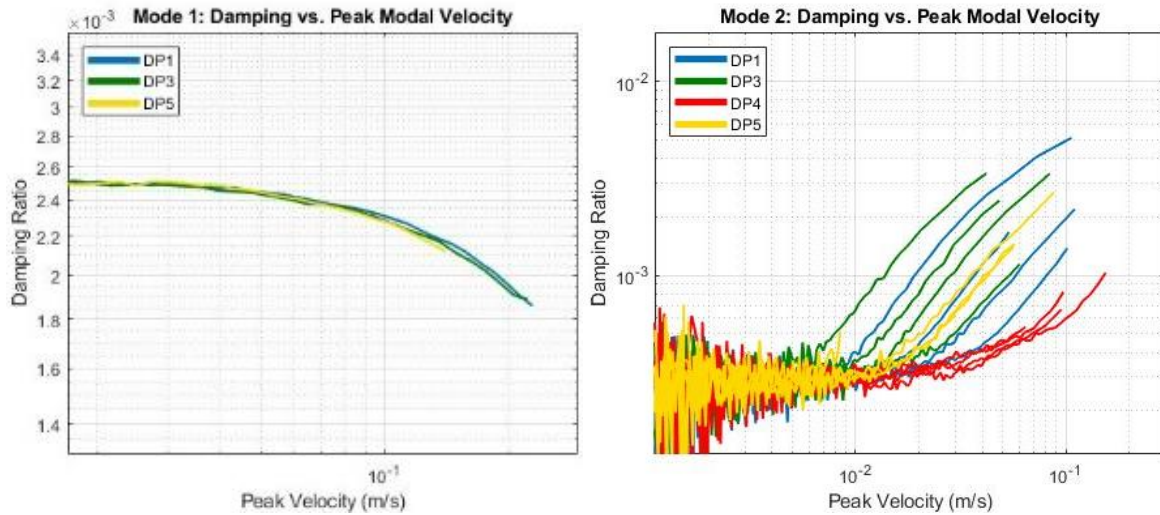
Five drive points were used in the tests, each selected to excite the modes of the structure to different relative amplitudes. The two modes of interest for the S4 beam are Mode 1 and Mode 2, shown from a finite element model (FEM) of the S4 beam from [6] in Figure 1. The first mode is the first out-of-phase bending mode and the second mode is the first in-phase bending mode. The drive points are named DP1, DP3, DP4, and DP5. DP1 is in the Z-direction in the middle of the beam and will excite Mode 1 and 2. DP3 is 25.4 mm (1 in.) right of DP1, which will excite Mode 1 more than Mode 2. DP4 is in the Z-direction near the bolt, which will not excite Mode 1 but will excite Mode 2. Finally, DP5 is in the Z-direction about 5 inches left from DP1, which is close to a node for Mode 2 based on the FEM, so the Mode 1 amplitude should be greater than Mode 2.

The first step of the test procedure was to measure the linear mode shapes of the system. This was done by using small impacts with an amplitude of less than 1 N to excite the structure. The second step of the testing used a large impact hammer with a soft rubber tip, which ranged in impact amplitudes from roughly 100 N – 600 N. The ringdown of the nonlinear impact was recorded, the time data was modally filtered with the linear mode shapes, bandpass filtered, and the Hilbert transform was used to calculate the damping ratio of the mode of interest versus the peak modal velocity of the mode of interest.

### Results:

Results for Mode 1 and Mode 2 of the 2017 beam are shown in Figure 2. For Mode 1, across all drive points, the damping is very consistent. DP4 is omitted since it did not produce a high enough Mode 1 amplitude. The damping for Mode 1 decreases with increasing amplitude, which is the opposite of what one would expect for a microslip joint nonlinearity [9], suggesting the nonlinearity in this mode is dominated by something other than microslip.

Mode 2 shows quite different characteristics. The damping for a given impact can vary by almost an order of magnitude depending on what drive point is used. There is also some correlation between higher damping tests and force amplitude although those details are omitted for brevity. The lowest damping for Mode 2 is achieved at DP4, where there is almost no Mode 1 excitation. This may be considered the baseline damping for Mode 2 when it is isolated from Mode 1. In DP5 Mode 1 is excited more than Mode 2, and the results for this drive point show a relatively repeatable but higher amount of damping. The large variability in damping only occurs at DP1 and DP3, where both Mode 1 and Mode 2 would be excited to relatively high amplitudes. This drive point dependence is the basis for the assertion that mode coupling is a strong contributor to the damping in some modes. Interestingly, this appears to be a one-way coupling relationship, the damping in Mode 1 is independent of the drive point (and hence the amplitude of Mode 2), but Mode 1 greatly affects the damping in Mode 2. Clearly mode coupling must be accounted for to model or predict the nonlinear damping of Mode 2.



**Figure 2: Damping vs. peak modal amplitude for Mode 1 (left) and 2 (right) of the 2017 beam.**

The results for Mode 1 and 2 of the 2020 beam are shown in Figure 3. The damping curves for each mode are distinct from the result for the 2017 beam. The damping increase at lower amplitudes for both modes, in a manner consistent with a microslip joint nonlinearity. However, at higher amplitudes, both modes exhibit a decrease in damping and Mode 2 shows a clear increase at even higher amplitudes.

For Mode 1, though the trends for damping are quite different, the repeatability observations for the 2017 beam still hold. At lower amplitudes before passing the peak damping, the curves are quite repeatable between DP1 and DP5. DP3 was not used in this test because it was shown to give nearly identical results to DP1 from the last set of tests.

Mode 2 again shows a larger spread in possible damping values depending on the drive point. As with the 2017 beam, when excited at DP4, the results show a repeatable damping curve, at least if some of the results at low amplitudes can be disregarded. This again reinforces the assertion that when Mode 2 is isolated from Mode 1 the damping will be the most repeatable. DP1, with a significant Mode 1 and Mode 2 excitation, again shows a larger spread in damping over a range of amplitudes. Although the damping curves are quite different for the two beams in general, the results show that the mode coupling relationship between the two beams is similar.

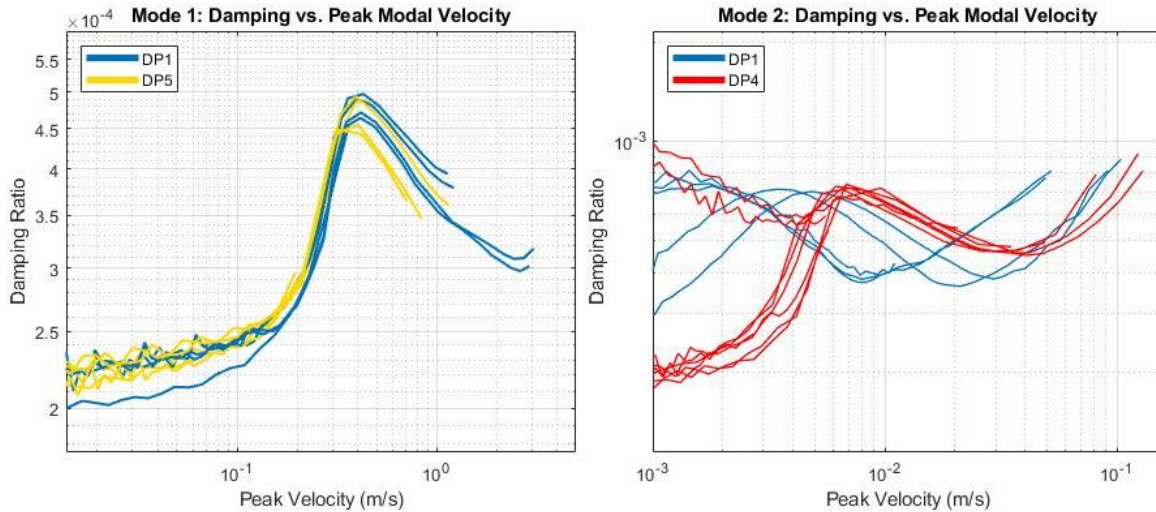


Figure 3: Damping vs. peak modal amplitude for Mode 1 (left) and 2 (right) of the 2017 beam.

### Conclusion:

The experimental results here present a case where the common assumption of uncoupled modes is not accurate. While the damping of Mode 1 was independent of the amplitude of Mode 2, the damping of Mode 2 changed by an order of magnitude depending on the drive point used, so mode coupling was very important for that mode. The authors are not aware of any models that can capture this behavior, so this presents an interesting challenge for the community. Interestingly, even though the damping curves for the two structures were very different, the mode coupling was qualitatively similar. This suggests that the mode coupling depends more on the structure of the system, i.e. the mode shapes and natural frequencies, which are similar between the beam sets, rather than on the details of stick and slip at the interfaces, which were very different between the beams.

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