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Bifurcation analysis of a piecewise-smooth freeplay system

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ABSTRACT

Physical systems that are subject to intermittent contact/impact are often studied using piecewise-smooth models. Freeplay is a common type of piecewise-smooth system and has been studied extensively for gear systems (backlash) and aeroelastic systems (control surfaces like ailerons and rudders). These systems can experience complex nonlinear behavior including isolated resonance, chaos, and discontinuity-induced bifurcations. This behavior can lead to undesired damaging responses in the system. In this work, bifurcation analysis is performed for a forced Duffing oscillator with freeplay. The freeplay nonlinearity in this system is dependent on the contact stiffness, the size of the freeplay region, and the symmetry/asymmetry of the freeplay region with respect to the system's equilibrium. Past work on this system has shown that a rich variety of nonlinear behaviors is present. Modern methods of nonlinear dynamics are used to characterize the transitions in system response including phase portraits, frequency spectra, and Poincaré maps. Different freeplay contact stiffnesses are studied including soft, medium, and hard in order to determine how the system response changes as the freeplay transitions from soft contact to near-impact. Particular focus is given to the effects of different initial conditions on the activation of secondary- and isolated-resonance responses. Preliminary results show isolated resonances to occur only for softer-contact cases, regions of superharmonic resonances are more prevalent for harder-contact cases, and more nonlinear behavior occurs for higher initial conditions.

Keywords: Bifurcation analysis, nonlinear dynamics, contact, freeplay, piecewise-smooth

INTRODUCTION

Dynamical systems subject to some form of intermittent contact or impact are very common across engineering fields. In many vibro-contact systems, the contact is predictable and can easily be expressed mathematically based on the critical displacement for contact to occur. A common type of easily expressible contact is freeplay, which occurs when there is a clearance between parts in a system. When parts sufficiently displace enough to cause contact, the resulting contact force can be modeled as a piecewise-smooth force-displacement curve. Examples of freeplay include backlash between gears and control surfaces in aeroelastic systems [1, 2]. Freeplay and other contact behaviors cause a dynamical system to behave nonlinearly; some notable behaviors that can occur includes isolated or subharmonic resonances, chaos, and discontinuity-induced bifurcations. When undesired, these can lead to unexpected aircraft flutter instability or premature wear and failure of parts.

Past researchers have studied relatively simple vibro-contact systems to gain a fundamental understanding of how contact can affect the nonlinear response of a system. Shaw [3] studied a spring-mass oscillator with freeplay in which the contact was rigid-body hard impact with variable coefficient of restitution. deLangre et al. [4] developed an experimental setup of the nonlinear Duffing oscillator but used a rigid stopper and contact springs of relatively soft stiffness. Researchers have also extensively studied the contact problem for aeroelasticity applications [5, 6], often including bifurcation analyses to determine how the contact can cause transitions in the systems' responses. Literature review on more general, variable-stiffness contact systems has turned up little focus on bifurcation analysis. This would be useful to fundamentally determine how a variable contact nonlinearity can affect a vibrating system and even how it can interact and couple with other present nonlinearities. Thus, in this work, bifurcation analysis is carried out on a forced Duffing oscillator with freeplay. Modern methods of nonlinear

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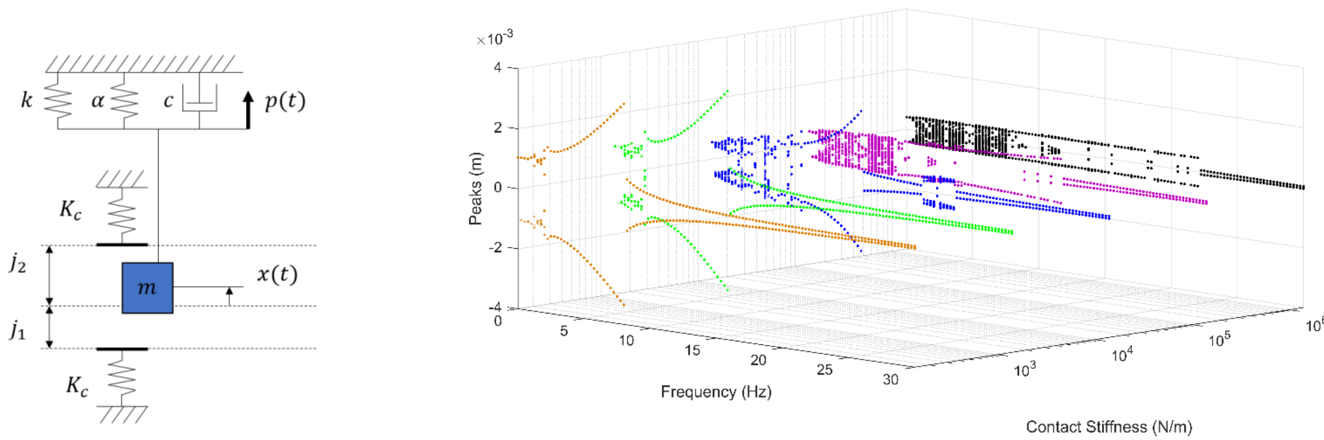
dynamics including phase portraits, frequency spectra, and Poincaré maps are used to characterize the transitions in the system response. Particular attention is paid to how the freeplay transitions from soft contact to hard impact and to the effects of different initial conditions on the activation of secondary resonances.

SYSTEM'S MODELING

The equations of motion for the Duffing-freeplay system as used by deLangre et al. [4] are given by:

$$\ddot{x} + 2\omega_n \zeta \dot{x} + \omega_n^2 x + \frac{\alpha}{m} x^3 + \frac{F_c}{m} = \frac{p}{m} cos(\omega t), F_c = \begin{cases} K_c(x + j_1), & x < -j_1 \\ 0, & -j_1 < x < j_2 \\ K_c(x - j_2), & x < j_2 \end{cases} \#(1)$$

where α is the cubic stiffness nonlinearity, K_c is the contact stiffness, and j_1,j_2 are the freeplay gap boundaries. Matlab® ode45 with *Event Location* is used to perform simulations to accurately capture the switching points between freeplay regions. Figure 1(a) shows a schematic of the system. The freeplay gap is kept symmetric about the origin, so $j_1 = j_2$.



(a) (b)

Figure 1: (a) Schematic of the Duffing-freeplay system and (b) 3D bifurcation diagrams showing the effects of contact stiffness on the Duffing-freeplay system for $m = 5kg$, $\omega_n = 5Hz$, $\zeta = 0.03$, $\alpha = 7 * 10^8 N/m^3$, $p = 4N$, $j_1 = j_2 = 0.4mm$, $x_0 = 0.32mm$.

NONLINEAR CHARACTERIZATION

Figure 1(b) presents 3D bifurcation diagrams showing all peaks in the steady-state time history at a given forcing frequency, for different values of contact stiffness. Results show that frequency bands of superharmonic resonance and chaos widen as contact stiffness hardens, but regions of subharmonic resonance will first appear and then disappear as contact stiffness hardens. More chaos occurs for hard contact, and intuitively hard contact strongly limits the maximum amplitude of the system to the span of the freeplay gap. This also implies the cubic stiffness is less influential for hard contact than for soft contact. Further, initial displacements x_0 near the origin do not always impart enough energy for the system to activate super- or sub-harmonic resonances compared to initial displacements closer to the freeplay boundaries.

CONCLUSIONS

In this work, bifurcation analysis was performed on a forced Duffing oscillator with freeplay nonlinearity. The effects of contact stiffness on the transitions in the system were studied. Results indicated that the system's secondary resonances are highly affected by harder contact stiffnesses and that the contact nonlinearity is generally more dominant over the system response than the cubic nonlinearity. The contact also causes a strong dependence on initial conditions.

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