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Insights on stoppers effects on the effectiveness of energy harvesting absorbers for dynamical systems

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Abstract

A major concern while designing structures, systems, and components, is the diminution of unwanted harmful vibrations. A common source of these vibrations is base excitation. To combat this in exterior structures, researchers have turned to increase the stiffness of the structures or the inclusion of tuned-mass-dampers. In this study, model derivations are carried out of a system under base excitation with a piezoelectric energy harvesting absorber with the aim to convert the wasted mechanical energy of a tuned-mass-damper into usable electrical energy to power sensors in hard to reach locations. Additionally, amplitude stoppers will be included to the energy harvesting absorber to generate a broadband response, increasing the window of operational use for energy harvesting and system's control. A nonlinear reduced-order model is developed using spring-mass-damper system for the primary structure and Euler-Bernoulli beam's theory for the energy harvesting absorber. The Galerkin discretization is employed to approximate the system's response. Due to the nonlinearities due to the contact-impact between the energy harvester and stoppers, a converge analysis is carried out to determine the required modes in the Galerkin method. Some preliminary results show promising control of the primary system, as well as adequate power generation from the energy harvesting absorber.

I. Introduction

Controlling the undesirable oscillations in structures and buildings due to environmental forces is a large focus for researchers and engineers. These researchers aim to avoid the possibility of critical failure of the system due to the high amplitudes of oscillation by employing different tactics to reduce or eliminate the unwanted vibrations. One method used is to increase the stiffness of the structure to increase the structure's natural frequency. The environmental forces are most dangerous when they can match the system's natural frequencies, causing extreme oscillations of the system. By increasing the structure's stiffness, engineers aim to move the system's natural frequencies out of the range of possible environmental forces strength, eliminating the threat. A possible disadvantage of this approach is the stiffness might be needed to be increased by an excessive amount, that would detrimentally

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increase the weight of the system out of the required specifications. Another well-known method is the inclusion of tune-mass-dampers [1]. A tuned-mass-damper is a coupled subsystem designed to absorb the primary system's oscillations by vibrating at its coupled natural frequency. Tuned mass dampers These tuned-mass-dampers may be a good candidate to dissipate the energy of the primary structure and hence control it.

One common source of detrimental oscillations in exterior structures is base excitations. Because the resulting resonance of base excitation takes place when the forcing frequency matches one of the system's natural frequencies, the resonant oscillations are found in a narrowband region centered around one of the natural frequencies. Because of this narrowband of oscillations, tuned-mass-dampers are a great candidate to eliminate the induced vibrations. Tuned-mass-dampers come in many shapes and sizes. Some variations include the common spring mass system from researchers like Pellicer [2], and Bakre and Jangid [3], pendulum tuned mass dampers presented by Setareh et al. [4], and particle tuned-mass-dampers which take advantage of flowing fluids or marbles to generate damping forces [5]. Since tuned-mass-dampers disperse the vibrational energy as unusable mechanical energy, it is desired to convert this energy into usable electrical energy. A piezoelectric energy harvester is an excellent option to achieve minimal oscillations in the primary structure as well as collecting usable electrical energy, as shown in a case of base excitation by Abdelmoula et al. [6]. They focused their research on the optimization of the piezoelectric layers and electrical circuitry and determined the most influential parameters in order to obtain broadband regions for energy harvesting and controlling the primary structure in an efficient way. McNeil [7] also implemented a piezoelectric energy harvester to control a system under base excitation. He focused his research on optimizing the system's parameters to widen the operable range of harvesting energy by studying the impacts of the tip mass, dimensions of the substrate and piezoelectric layers on the primary structure amplitudes and levels of the harvested power of the energy harvesting absorber.

To further improve the energy harvesting absorber efficiency, amplitude stoppers will be included to introduce additional nonlinear forces to the system and hence the possibility of widening the broadband resonance region and increasing the levels of the harvested power. Previously, Zhou et al. [8] performed experiments on the effects the stopper's material on the response of a piezoelectric energy harvester under harmonic base excitation. They showed that the chosen stopper material can greatly affect the system's response and improve the broadband resonance region through a hardening effect, as shown in Figure 1. These nonlinear forces can be optimized to design ultra-wide bandwidth energy harvesting absorbers. This study aims to optimize a piezoelectric energy harvesting absorber to adequately damp a primary structure's oscillations due to base excitations effects, while creating a broadband response to increase the energy harvesting absorber's capabilities.

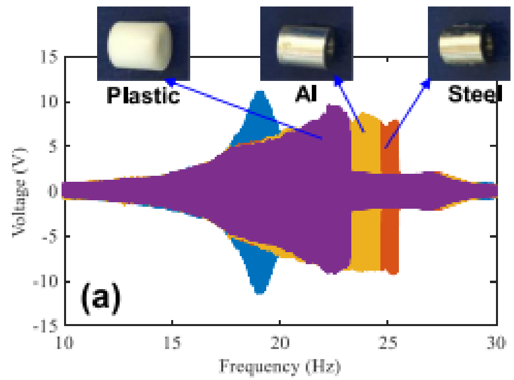


Figure 1. Voltage generated by the energy harvester with respect to the exciting frequency when using different materials for the stoppers [8].

II. System's dynamics and reduced-order modeling

The coupled system under investigation is modeled as a reduced-order model to avoid the computational complexity due to the high nonlinearity of stopper forces. Having a reduced-order model is beneficial because it reduces the amount of time and resources needed to solve the problem accurately. Instead of a high-fidelity model, the primary system is modeled as a spring-mass-damper system, with the stiffness and damping of the system being represented by the spring constant k and damping coefficient c, respectively. The coupled system is depicted in Figure 2, where w_1 represents the primary structure's displacement, w_2 denotes the energy harvesting absorber's displacement, and the green hashed blocks represent the locations of the stoppers.

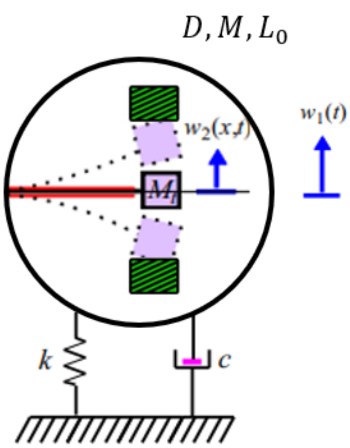


Figure 2. Schematic of the dynamical system subjected to base excitations.

The piezoelectric energy harvester consists of steel substrate partially covered with bimorph piezoelectric layers with a tunable tip mass. The coupled system's governing equations using the Euler-Lagrange principle are derived. The Euler-Lagrange equations require the kinetic and potential energies, which are given by:

$$T = \frac{1}{2} M \dot{w}_1^2 + \frac{1}{2} M_t \dot{r}_2{}^2 + \frac{1}{2} I_t w_2(L,t)'^2 + \frac{1}{2} \int_0^L M_b (w_1 + w_2(x,t))^2 dx \tag{1}$$

$$\Pi = \frac{1}{2} \int_{v_s} \sigma_x^s \varepsilon_x^s dv + \frac{1}{2} \int_{v_s} (\sigma_x^s \varepsilon_x^s - E_3 D_3) dv + \frac{1}{2} k w_1^2 \tag{2}$$

where $\dot{r}_2 = w_1 + w_2(L,t) + L_c w_2{}'(L,t)$, the second moment of inertia, $I_t = \frac{1}{3} M_t L_c^2$, and the electric displacement, $D_3 = d_3 E_p \varepsilon_x^p - \varepsilon_{33} E_3$.

The Galerkin method is used to approximate the displacement of the energy harvesting absorber. Hence, the following discretization is employed:

$$w_2(x,t) = \sum_{i=1}^{\infty} \varphi_i(x) r_i(t) \tag{3}$$

where $\varphi_i(x)$ represents the i^{th} mode shape of the energy harvesting absorber.

After substituting equation (3) back into equations (1) and (2), the kinetic and potential energies of the system can be rewritten as:

$$T = \frac{1}{2} M w_1{}^2 + \frac{1}{2} M_t \dot{r}_2{}^2 + \frac{1}{2} I_t \left(\sum_{i=1}^{\infty} \varphi_i'(x) r_i(t) \right)^2 + \frac{1}{2} M_{b1} \int_0^{L_1} \left(w_1 + \sum_{i=1}^{\infty} \varphi_i(x) \dot{r}_i(t) \right)^2 dx + \frac{1}{2} M_{b2} \int_{L_1}^{L} \left(w_1 + \sum_{i=1}^{\infty} \varphi_i(x) \dot{r}_i(t) \right)^2 dx \tag{4}$$

$$\Pi = EI_1 \int_0^{L_1} \left(\sum_i^{\infty} \varphi_i''(x) r_i(t) \right)^2 dx + EI_2 \int_{L_1}^{L} \left(\sum_i^{\infty} \varphi_i''(x) r_i(t) \right)^2 dx + b e_{31} \left(h_s + h_p \right) \int_0^{L_1} \left(\sum_i^{\infty} \varphi_i''(x) r_i(t) \right) V(t) dx - \frac{e_{31} b L_1}{h_p} V(t)^2 + \frac{1}{2} k w_1^2 \tag{5}$$

where the energy harvesting absorber's stiffnesses, $EI_1 = b E_s \frac{h_s^3}{12} + \frac{1}{2} b E_p h_s^2 h_p + b E_p h_s h_p^2 + b E_p \frac{2}{3} h_p^3$ and $EI_2 = b E_s \frac{h_s^3}{12}$, the mass of the beam, $M_{b1} = b \rho_s h_s + 2 b \rho_p h_p$ and $M_{b2} = b \rho_s h_s$, and $\dot{r}_2 = w_1 + \sum_{i=1}^{\infty} \varphi_i(L) \dot{r}_i(t) + L_c \sum_{i=1}^{\infty} \varphi_i'(L) r_i(t)$.

The last expressions needed for the Euler-Lagrange principle is the non-conservative forces of the system. A smoothened trilinear spring model has been shown to sufficiently depict this contact force by Paidoussis et al. [9]. This impact force is expressed in equation (6). The effect of varying the stoppers stiffness when considering three different configurations (soft, medium, and hard) can be seen in Figure 3.

$$F_{stopper} = k \left[\sum_{i=1}^{\infty} \varphi_i(L_f) r_i(t) - \frac{1}{2} \left(\left| \sum_{i=1}^{\infty} \varphi_i(L_f) r_i(t) + d \right| - \left| \sum_{i=1}^{\infty} \varphi_i(L_f) r_i(t) - d \right| \right) \right]^3 \tag{6}$$

where L_f denotes the distance along the beam where the stoppers are placed, and d is the initial distance between the beam and the stoppers.

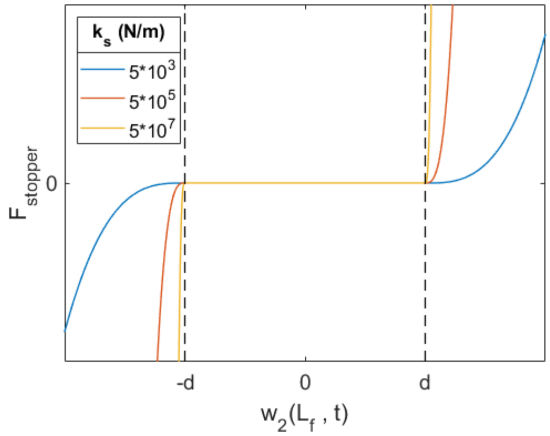


Figure 3. Impact force versus energy harvesting absorber's displacement at the location of the stoppers for varying stopper stiffness.

Considering the expressions of the kinetic energy, potential energy, and non-conservative forces and employing the Euler-Lagrange principle, the nonlinear reduced-order model can be written as:

$$(M + M_t + M_{b1}L_1 + M_{b2}(L - L_1))w_1 + cw_1 + kw_1 + \sum_{i=1}^{\infty}\left(M_t\varphi_i(L) + M_tL_c\varphi_i'(L) + M_b\int_0^L \varphi_i(x)\, dx\right)\ddot{r}_i = Fcos(\omega_f t + \phi) \qquad (7)$$

$$\ddot{r}_i + 2\xi_i\omega_i\dot{r}_i + \omega_i r_i + \left(M_t\varphi_i(L) + M_tL_c\varphi_i'(L) + M_b\int_0^L \varphi_i(x)dx\right)w_1 - \theta_i V = \varphi_i(L_f)F_{stopper} \qquad (8)$$

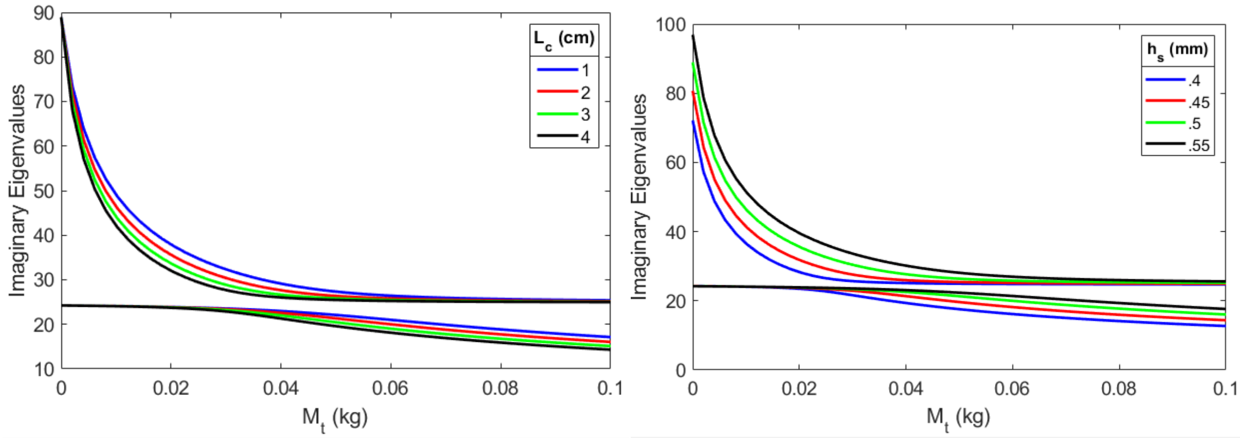
$$C_p\dot{V} + \frac{1}{R} + \sum_{i=1}^{\infty}\theta_i\dot{r}_i = 0 \qquad (9)$$

where the capacitance of the harvester is $C_p = 2\frac{\varepsilon_{33}bL_1}{h_p}$, and the piezoelectric coupling term is $\theta = E_p d_{31} b(h_p + h_s)\varphi_i'(L_1)$.

III. Selection of the energy harvesting absorber's dimensions for optimal control

First, a linear analysis is performed to optimize the energy harvesting absorber's parameters to force a strong coupling, and therefore efficiently control the primary system. To this end, an eigenvalue problem analysis is carried for the coupled electromechanical system under investigation. When looking at the imaginary eigenvalues, which represent the system's coupled natural frequencies, it is important to select the values of the parameters where the two couple frequencies are close together. By graphing the imaginary eigenvalues versus the tip mass with a variance in a parameter, as depicted in Figure 4, it is possible to select the optimal value for that parameter and the tip mass.

As the critical length of the tip mass increases, as presented in Figure 4(a), its second moment of inertia also increases which has more control over the system. With a higher critical length, the tip mass can have a lower mass. This shows that the critical length is a great parameter to tune the system. Additionally, varying the height of the substrate, shown in Figure 4(b), has an impact on the optimal tip mass. This is because the stiffness of the energy harvesting absorber is proportional to the cube of the height of the substrate. This means a large tip mass is needed to counter act the high stiffness that results from a big substrate height. After evaluating the energy harvesting absorber's parameters effects on the coupled frequencies of the system, 2 centimeters is chosen for the critical length, 0.5 millimeters is chosen for the height of the substrate, and 0.048 kilograms is chosen as the tip mass.



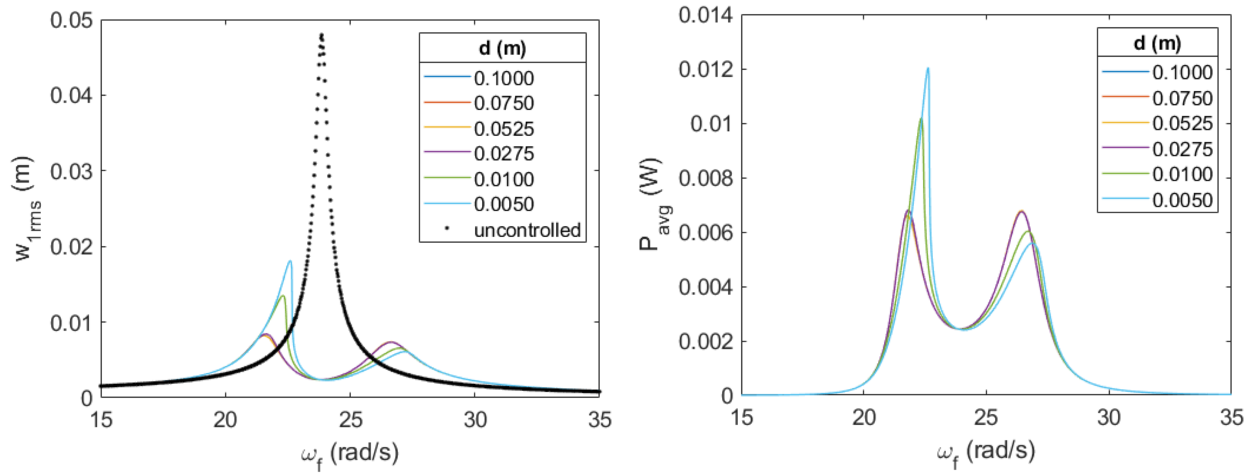
(a) (b)

Figure 4. Linear analysis of the imaginary eigenvalues versus the tip mass for varying (a) the critical length of the tip mass, and (b) the height of the substrate.

IV. Stoppers effects on the system's responses and harvester's efficiency

There are a few main considerations when looking at the effects of the stoppers. First, the control of the primary structure is evaluated. This is important because the main aspect of the energy harvesting absorber is to control the primary structure. Next, the effects of the stoppers on the peak power are investigated. Then, the bandwidth of the operable range of harvesting energy is evaluated. The latter two conditions are the important factors in generating consistent energy to provide enough power to operate various sensors. In this study, some preliminary results are shown. Indeed, two stoppers' stiffnesses are considered, namely, $5 * 10^3$ N/m and $5 * 10^7$ N/m. Multiple gap distances are considered to get a good understanding of the possible nonlinear effects of the two kinds of stoppers and the gap between the energy harvester and the stoppers.

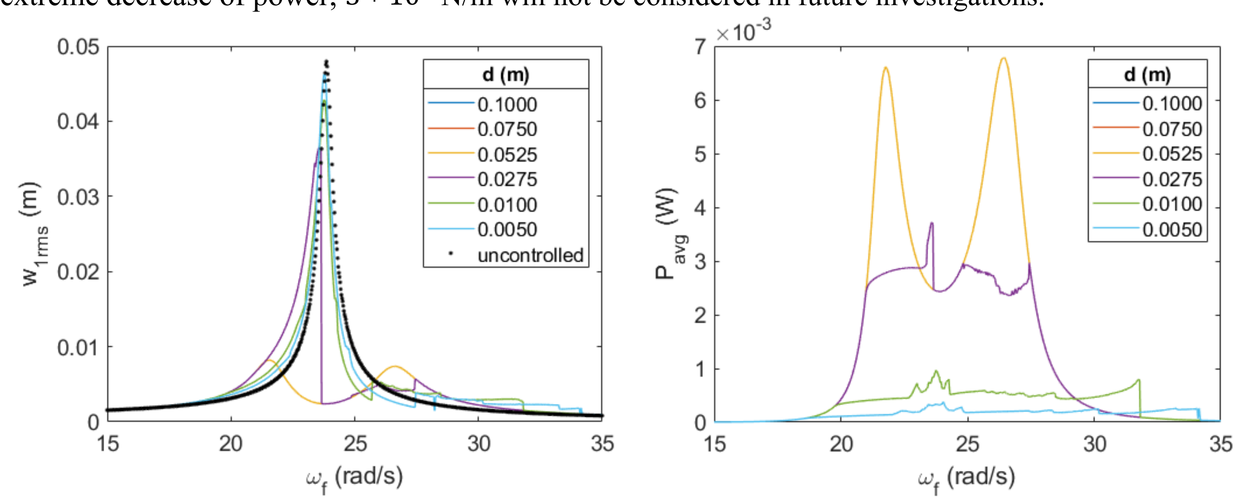
Figure 5 shows the effects of the stopper on the primary structure and the power generated by the energy harvesting absorber when considering a stiffness of $5 * 10^3$ N/m. It can be seen that even with small gaps, the energy harvesting absorber is beneficial for controlling the primary structure. All gap distances studied reduce the primary structures peak amplitude by 62% or greater. The impacts of the soft stopper on the generated power is also beneficial in terms of increasing the levels of the harvested for the resonant frequency and having broadband resonance region. In fact, while the gap decreases, the peak power increases. Additionally, the first peak shifts to the right, increasing the average power generated of the operable range. A broadband region does not develop, but the operable region does not narrow so these results are still desirable.



(a) (b)

Figure 5. Stopper stiffness of $5 * 10^3$ effects on (a) primary structural amplitude and (b) power harvested.

Considering the hard stopper, it can be seen from Figure 6(a) that once there is contact with the stoppers, there is practically a total loss of control of the primary structure. Additionally, the second peak, which corresponds to the energy harvesting absorber's coupled frequency, becomes distorted. This is the root mean square representation of an aperiodic region. This aperiodic region can be generated by multiple sources including grazing bifurcation and warrants more investigation. In Figure 6(b), it is seen that once there is contact with the amplitude stoppers, there is an immediate loss of power generated. Small gaps of 1 centimeter and 0.5 centimeters show the generation of a broadband region. As this is desired, the extreme decrease in the harvested power compared to the cases with no contact with the amplitude stoppers is a major concern. When looking at the loss of control of the primary structure and the extreme decrease of power, $5 * 10^7$ N/m will not be considered in future investigations.



(a) (b)

Figure 6. Stopper stiffness of $5 * 10^7$ effects on (a) primary structural amplitude and (b) power harvested.

Matching the energy harvesting absorber's fundamental natural frequency to the hardened frequency has shown an additional increase of the structure's response by an additional two percent. Further increasing the absorber's stiffness slightly decreases the structure's amplitude while widening the responses band, but it also greatly decreases the power harvested. Because of this, it was concluded that the broadband response of the structure was not large enough to overcome the loss of power from the decreased amplitude of the response. Furthermore, it is desirable to seek a solution to further widen the broadband response while maintaining a low structural amplitude and an adequate amount of power harvested. This target will be deeply investigated and discussed by studying the impacts of the stoppers on the dynamics and energy harvesting of the energy harvesting absorber and the suppression of the primary structure's amplitudes over a wide range of wind speeds.

IV. Conclusions

Base excitation can cause catastrophic failures in exterior structures, requiring the aid of tuned-mass-dampers or other implementations to limit the structures oscillations. To further the benefits of a tuned-mass-damper, the use of a piezoelectric energy harvester as a vibration absorber can adequately dampen the primary structures oscillations, as well as generate energy to power small and hard to reach sensors. The optimization of the energy harvesting absorber's characteristics was performed to minimize the primary structure's oscillations. The inclusion of amplitudes stoppers showed varying results dependent on the strength of a stiffness. With lower stiffnesses, great control of the primary structure and a promising increase in generated power was seen. With high stiffnesses, a total loss of control of the primary system and an immediate loss of power is seen with initial contact of the amplitude stoppers regardless of the stoppers gap was seen.

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