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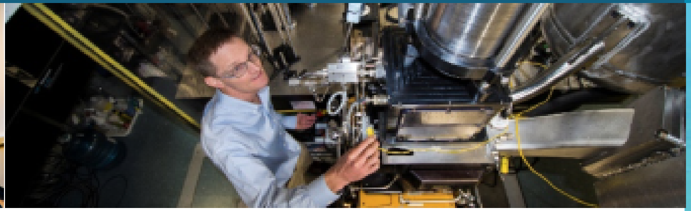
Sensitivity analysis of fiber reinforced polymer composite damage criteria: cooling induced residual stress



PRESENTED BY

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U.S. DEPARTMENT OF ENERGY NNSA National Nuclear Security Administration

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Introduction

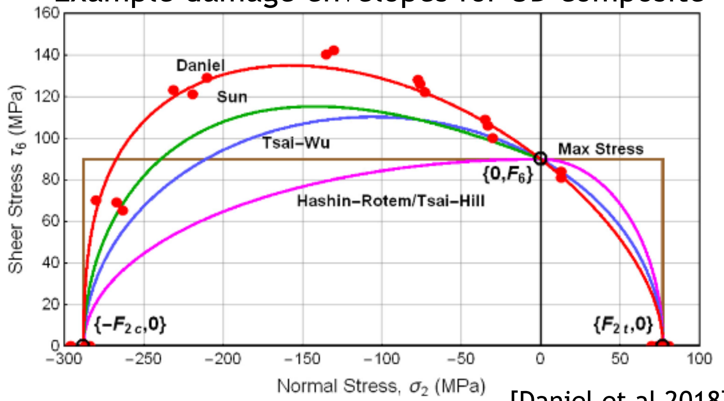
Motivation:

- Fiber reinforced composite materials have many desirable qualities including high strength-to-weight ratios, low manufacturing costs, and corrosion resistance
- Composite damage may involve various mechanisms including fiber fracture, fiber buckling, matrix cracking, matrix crushing, debonding of fibers and matrix, and delamination
- Numerous models with varying levels of complexity have been proposed to define damage envelopes for composite materials
- In order to predict and mitigate damage, it is advantageous to explore how material parameters influence the development of stress components and damage metrics.

Project goals:

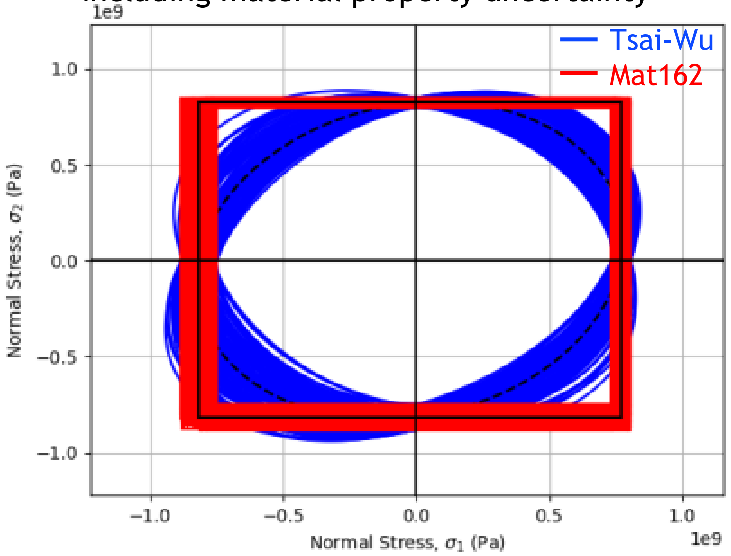
- Perform sensitivity analyses of damage criteria on elastic and strength properties of woven carbon fiber reinforced polymer for a residual stress scenario induced from isothermal cooling
- Determine the most critical parameters for each damage criterion
- Identify key differences and similarities among the damage criteria

Example damage envelopes for UD composite



[Daniel et al 2018]

Analytic damage envelopes for woven composite including material property uncertainty



Problem set up: isothermal cooling of composite cylinder

Geometry: open aluminum cylinder with 3 outer layers of carbon fiber reinforced polymer (CFRP)

- overall: 127 mm length by approx. 100mm inner diameter
- Ply thickness = 0.5mm (1.5mm total)
- Aluminum thickness =1.5mm

Mesh: 41,984 8-noded hex elements with uniform gradient formulation

Boundary conditions: isothermal cooling 120°C to -50°C (Δ170°C)

Aluminum material properties		
E (GPa)	Nu (-)	CTE (ppm/˚C)
68.9	0.33	23.4

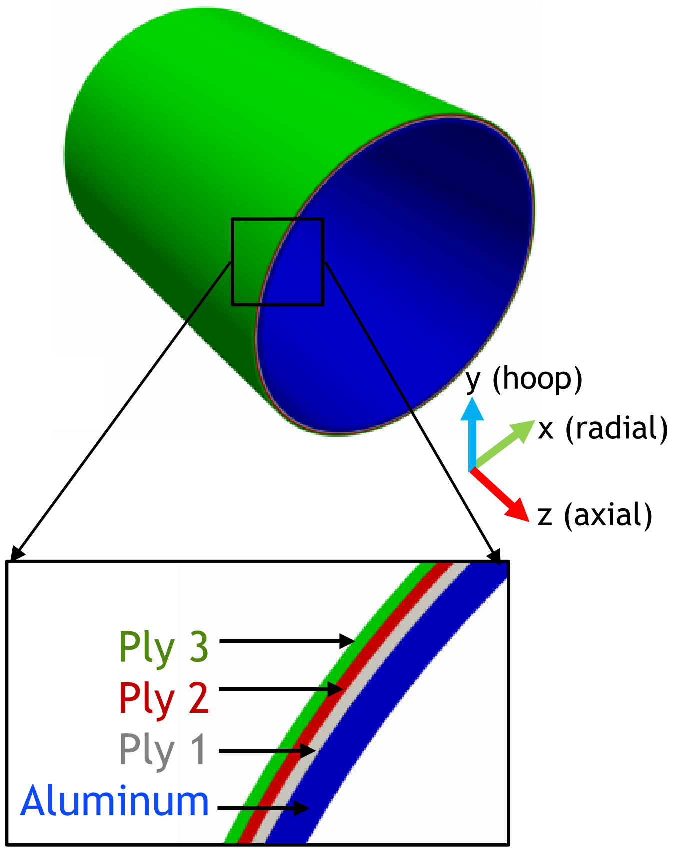
Aluminum

- Elastic model

CFRP	Glassy	Rubbery
CTE11 (ppm/˚C)	3.399	0.948
CTE22 (ppm/˚C)	3.359	1.358
CTE33 (ppm/˚C)	71.99	283.3

Carbon fiber reinforced polymer

- Elastic orthotropic model
- Damage criteria
  - Absolute max principal stress - common, simple one term metric
  - Tsai-Wu metric - single equation of fully interactive terms including all stress components
  - Mat162 metric - maximum value of a set of mechanism-based damage criteria



Sensitivity analyses

- Incremental Latin hypercube sampling (LHS) – identify most correlated parameters for down-select
- Variance-based decomposition Sobol analysis – determine main and total effects indices

Problem set up: damage metrics

Tsai-Wu criterion

- General expression

$$F_i\sigma_i + F_{ij}\sigma_i\sigma_j = 1 \quad i,j = 1, \ldots, 6$$

- Woven composites
  - Assume orthotropic symmetry conditions
  - Assume no normal-shear and shear-shear coupling
  - Coefficients may be written in terms of 12 material strength parameters determined by experiments

$$F_1 = \frac{1}{S_{1T}} - \frac{1}{S_{1C}}, \quad F_2 = \frac{1}{S_{2T}} - \frac{1}{S_{2C}}, \quad F_3 = \frac{1}{S_{3T}} - \frac{1}{S_{3C}}$$

$$F_4 = F_5 = F_6 = 0$$

$$F_{11} = \frac{1}{S_{1T}S_{1C}}, \quad F_{22} = \frac{1}{S_{2T}S_{2C}}, \quad F_{33} = \frac{1}{S_{3T}S_{3C}}$$

$$F_{44} = \frac{1}{S_{23}^2}, \quad F_{55} = \frac{1}{S_{13}^2}, \quad F_{66} = \frac{1}{S_{12}^2}$$

$$F_{12} = \frac{1}{2S_{biax12}^2}\{1 - S_{biax12}(F_1 + F_2) - S_{biax12}^2(F_{11} + F_{22})\}$$

$$F_{13} = \frac{1}{2S_{biax13}^2}\{1 - S_{biax13}(F_1 + F_3) - S_{biax13}^2(F_{11} + F_{33})\}$$

$$F_{23} = \frac{1}{2S_{biax23}^2}\{1 - S_{biax23}(F_2 + F_3) - S_{biax23}^2(F_{22} + F_{33})\}$$

Mat162 criteria

- Maximum value of a set of mechanism-based damage criteria
- Tensile/shear fiber mode

f1 $$\left(\frac{\langle\sigma_{11}\rangle}{S_{1T}}\right)^2 + \left(\frac{\sigma_{13}}{S_{1FS}}\right)^2 = 1$$

f2 $$\left(\frac{\langle\sigma_{22}\rangle}{S_{2T}}\right)^2 + \left(\frac{\sigma_{23}}{S_{2FS}}\right)^2 = 1$$

Macaulay bracket

$$\langle x\rangle = \begin{cases} 0 & x < 0 \\ x & x \geq 0 \end{cases}$$

- Compressive fiber mode

f3 $$\left(\frac{-\langle\sigma_{11}\rangle_- + \langle\sigma_{33}\rangle_-}{S_{1c}}\right)^2 = 1$$

f4 $$\left(\frac{-\langle\sigma_{22}\rangle_- + \langle\sigma_{33}\rangle_-}{S_{2c}}\right)^2 = 1$$

Inverse Macaulay bracket

$$\langle x\rangle_- = \begin{cases} x & x < 0 \\ 0 & x \geq 0 \end{cases}$$

- Crush (matrix) mode

f5 $$\left(\frac{\langle\sigma_{33}\rangle_-}{S_{3C}}\right)^2 = 1$$

- In-plane shear matrix mode

f6 $$\left(\frac{\sigma_{12}}{S_{12}}\right)^2 = 1$$

- Delamination mode

f7 $$S^2\left\{\left(\frac{\langle\sigma_{33}\rangle}{S_{3T}}\right)^2 + \left(\frac{\sigma_{23}}{S_{23} + S_{SR}}\right)^2 + \left(\frac{\sigma_{13}}{S_{13} + S_{SR}}\right)^2\right\} = 1$$

$$S_{SR} = -\langle\sigma_{33}\rangle_- \tan(\varphi)$$

Absolute max principal stress criterion

- For these cases: minimum principle stresses are compressive, and have greatest magnitudes

(Tsai & Wu, 1971 ; Xiao, et al., 2007; Haque, 2017; LSDYNA manual)

Problem set up: sensitivity variables

Uniform uncertain CFRP material input parameters

Elastic parameters	nominal	min	max
E11 (GPa)	63.86	61.46	66.26
E22 (GPa)	62.74	60.34	65.14
E33 (GPa)	8.585	8.285	8.885
Nu12 (-)	0.0480	0.0415	0.0545
Nu13 (-)	0.4080	0.4015	0.4145
Nu23 (-)	0.4075	0.401	0.414
G12 (GPa)	3.43	3.33	3.53
G13 (GPa)	3.265	3.165	3.365
G23 (GPa)	3.25	3.15	3.35

Damage parameters	nominal	min	max
F1t (MPa)	769	732	806
F1c (MPa)	816	747	885
F2t (MPa)	823	797	849
F2c (MPa)	816	747	885
F3t (MPa)	56.2	43.2	69.2
F3c (MPa)	56.2	43.2	69.2
S12 (MPa)	48.4	47.56	49.24
S13 (MPa)	32.4	25	39.8
S23 (MPa)	32.4	25	39.8

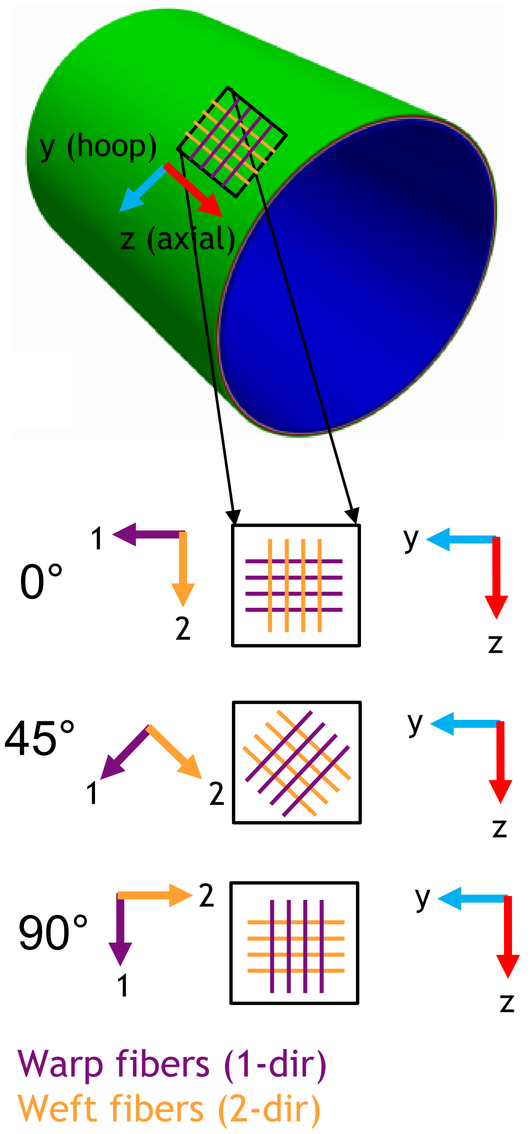
Tsai-Wu parameters	nominal	min	max
Biax12 (MPa) *est	600	510	690
Biax13*	55	46.75	63.25
Biax23*	55	46.75	63.25

Mat162 parameters	nominal	min	max
Safs (MPa) *	385	327.25	442.75
Sbfs (MPa) *	412	350.2	473.8

Ply stack sequences explored: (inner ply/middle ply/outer ply)

- Orientations indicate **warp fibers** direction relative to global **hoop direction (y-dir)**

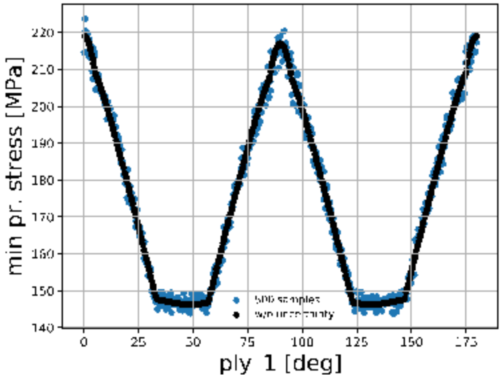
- (0°/90°/0°) : inner and outer plies have warp fiber oriented in the global hoop direction (weft fibers in axial direction), middle ply has warp fiber oriented in global axial direction (weft fibers in hoop direction)
- (45°/-45°/45°)
- (90°/0°/90°)

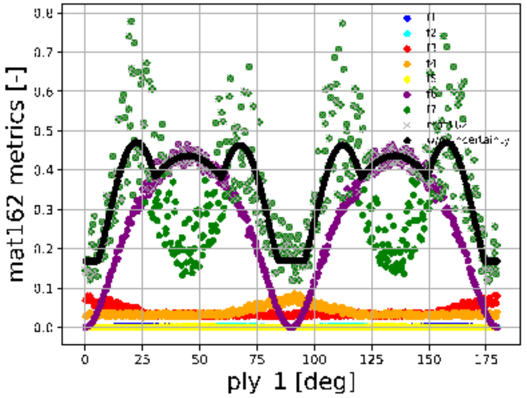


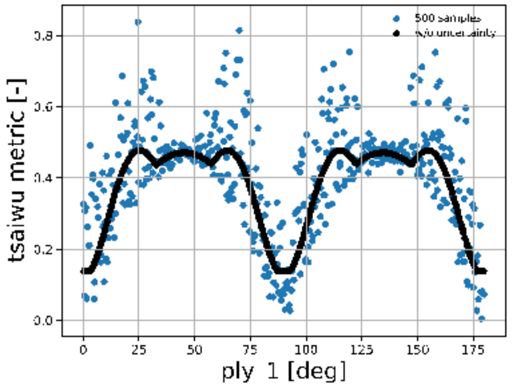
Preliminary problem bounding: orientation effects

Single orientation

- *with* and *without* (black) material property uncertainty
- Min pr stress: ~0.18-0.27 (-)
  - Norm by f1c=f2c=816mpa:
- Tsaiwu: ~0.13-0.475 (-)
- Mat162: ~0.17 – 0.47 (-)

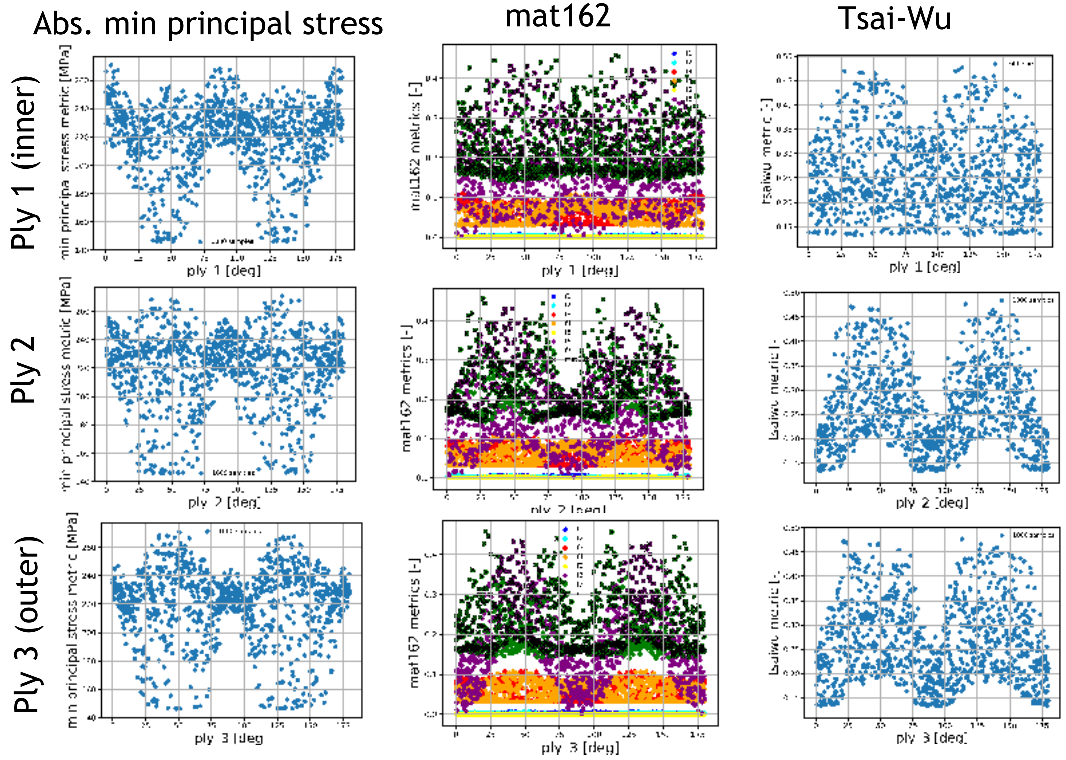






Three orientations ranges of metrics *without* material property uncertainty

- Min pr stress: ~0.18-0.33 (-)
- Tsai-wu: 0.13-0.48
- Mat162: 0.13-0.48



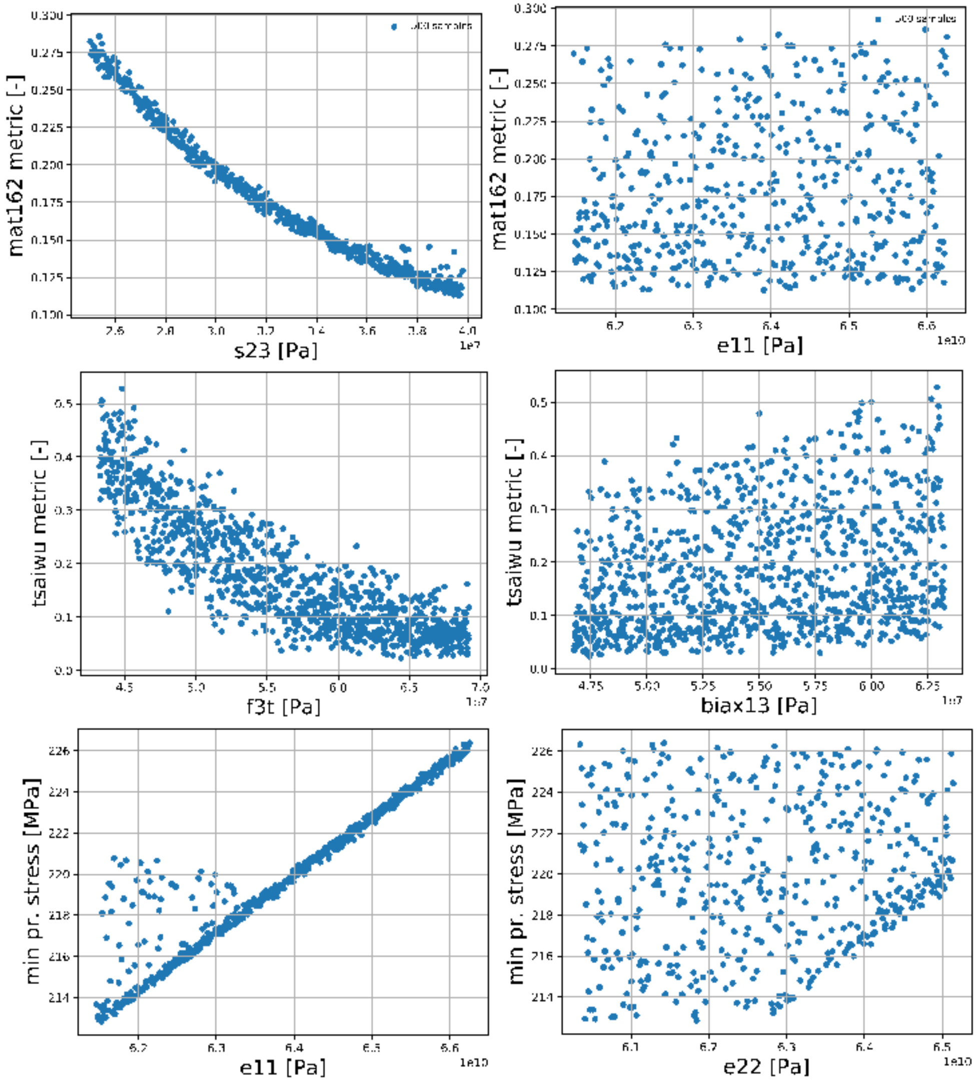
- Tsai-Wu and mat162 plots show similar behavior
  - Avoid orientations of 20°-70° (110°-160°)
  - lowest damage metrics near 0°, 90°, 180°
- Abs. min principal stress metric indicated opposite behavior

(0°/90°/0°) correlation statistics

Correlations for 1000 Latin hypercube samples (LHS)

- Bold variables chosen for variance-based decomposition
- The Tsai-Wu metric shows a significantly higher range than the mat162 and min pr. stress metrics
- The mat 162 and min. principal stress metric show a similar high-end range of approx. 0.28 when min pr. stress is normalized by f1c=f2c=816MPa

	mat162 metric			Tsai-Wu metric		min pr. stress	
variable	partial rank	simple rank	variable	partial rank	simple rank	partial rank	simple rank
e11	**5.25E-01**	6.78E-02	e11	**1.54E-01**	4.52E-02	**9.58E-01**	9.52E-01
e22	**4.97E-01**	5.06E-02	e22	**1.31E-01**	2.95E-02	**3.30E-01**	1.01E-01
e33	-1.15E-02	2.15E-03	e33	7.30E-02	-1.55E-03	-1.14E-03	-6.48E-03
v12	**1.75E-01**	1.46E-02	v12	3.05E-02	6.93E-03	**1.35E-01**	5.27E-02
v13	-8.37E-03	1.02E-02	v13	5.20E-03	4.42E-03	-1.01E-02	-3.47E-03
v23	-5.19E-02	-2.45E-02	v23	-7.22E-03	-5.63E-03	-4.43E-03	-1.41E-02
g12	-1.72E-03	-1.43E-02	g12	2.30E-02	-1.09E-02	2.96E-02	1.06E-02
g13	-1.56E-02	-2.81E-03	g13	1.09E-02	2.41E-02	3.31E-02	1.78E-02
g23	5.18E-02	1.09E-02	g23	2.53E-02	1.46E-02	-5.66E-02	-1.77E-02
f1t	9.20E-03	2.73E-03	f1t	**2.21E-01**	5.24E-02		
f1c	-1.22E-02	1.39E-02	f1c	**-3.99E-01**	-9.26E-02		
f2t	-4.27E-02	-1.22E-02	f2t	4.52E-02	2.86E-02		
f2c	2.31E-02	-6.23E-03	f2c	**-3.39E-01**	-7.22E-02		
f3t	**-3.45E-01**	-3.39E-02	f3t	**-9.67E-01**	-8.63E-01		
f3c	-2.50E-02	-1.15E-02	f3c	**3.00E-01**	4.77E-02		
s12	5.21E-02	1.20E-02	s12	6.21E-03	1.43E-03		
s13	**-2.49E-01**	-2.30E-02	s13	-6.91E-02	-2.10E-02		
s23	**-9.97E-01**	-9.93E-01	s23	**-6.99E-01**	-2.33E-01		
safs	-6.09E-02	-2.11E-03	biax12	**-2.53E-01**	-6.48E-02		
sbfs	-3.75E-02	-3.65E-03	biax13	**7.85E-01**	2.79E-01		
			biax23	**6.13E-01**	1.64E-01		

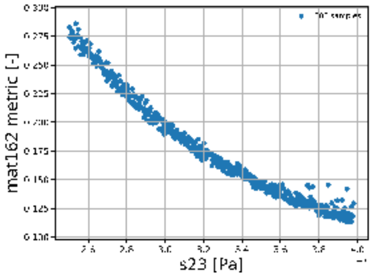


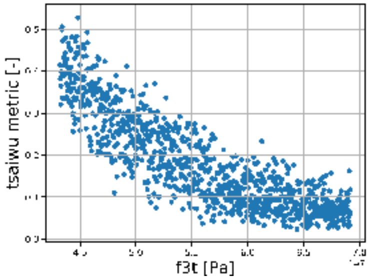
Scatter plots of three damage metrics with two most correlated variables for (0°/90°/0°) ply layup

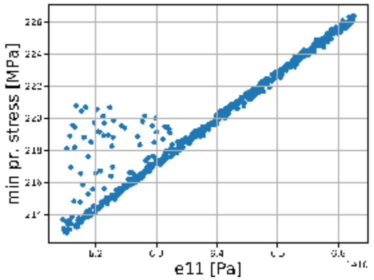
(0°/90°/0°) Variance-based decomposition Sobol sensitivity indices

- **Main effects index**: contribution of each individual variable *alone* to the variance in the model quantity of interest (QOI)
- **Total effects index**: contribution of each individual variable *in combination with all other variables* to the variance in the QOI

	mat162 metric		Tsai-Wu		min pr. stress	
variable	main effects	total effects	main effects	total effects	main effects	total effects
e11	0.003	0.003	0	0.001	0.883	0.952
e22	0	0.002	0	0	0.02	0.071
v12	0	0	0	0	0.002	0.001
f1t	-	-	0	0.002		
f1c	-	-	0.008	0.011		
f2c	-	-	0.006	0.004		
f3t	0.003	0.001	0.776	0.83		
f3c	-	-	0.005	0.005		
s13	0.002	0.002	0.026	0.042		
s23	0.992	0.995	0.001	0.003		
biax12			0.114	0.112		
biax13			0.039	0.025		
biax23			0.024	0.026		







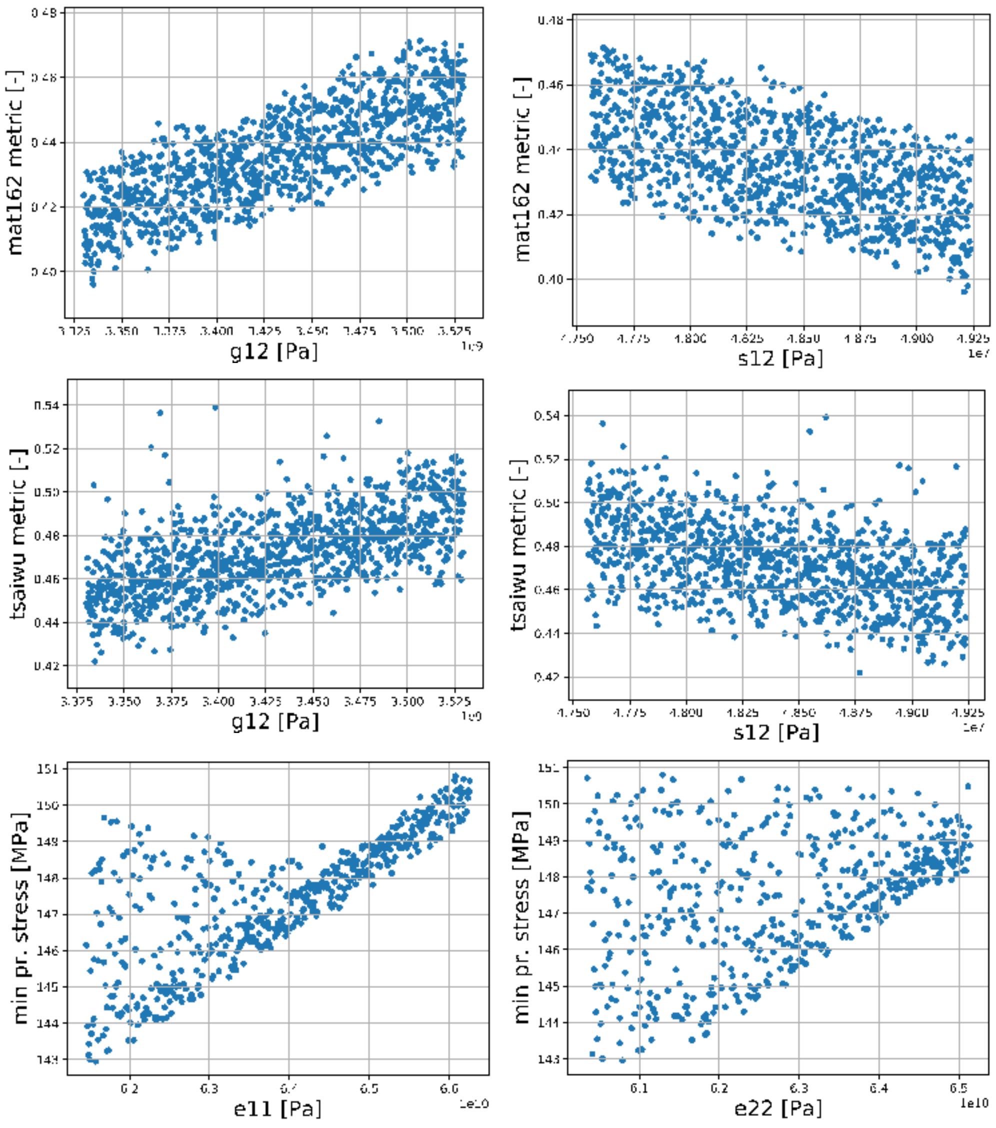
- Variance in **S23** *alone* is responsible for **99%** of variance in the **mat162** metric
- Variance in **f3t** *alone* is responsible for **78%** of variance in the **Tsai-Wu** metric
  - Variance in **f3t** *in combination with other variables* accounts for **83%** of variance in the **Tsai-Wu** metric
- Variance in **e11** alone is responsible for **88%** of variance in the **abs. minimum principal stress** metric
  - Variance in **e11** *in combination with other variables* accounts for **95%** of variance in the **abs. minimum principal stress** metric

(45°/-45°/45°) correlation statistics

Correlations for 1000 LHS

- Bold variables chosen for variance-based decomposition
- The Tsai-Wu metric shows a slightly higher range than the mat162 metric
- The min. principal stress metric shows a much smaller range of 0.17-0.19 when normalized by f1c=f2c=816MPa

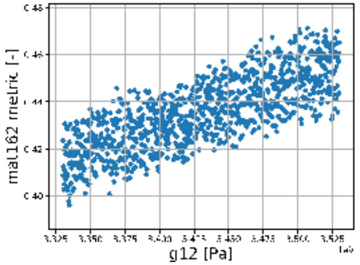
	mat162 metric			Tsai-Wu metric		min pr. stress	
variable	partial rank	simple rank	variable	partial rank	simple rank	partial rank	simple rank
e11	**7.22E-01**	1.24E-01	e11	**5.65E-01**	1.45E-01	**8.45E-01**	7.83E-01
e22	**7.41E-01**	1.44E-01	e22	**6.34E-01**	1.74E-01	**5.39E-01**	3.19E-01
e33	-2.61E-02	-4.05E-04	e33	2.62E-02	4.07E-03	-1.93E-02	-1.55E-02
v12	**3.80E-01**	5.86E-02	v12	**2.85E-01**	6.66E-02	**3.64E-01**	2.08E-01
v13	7.11E-03	4.73E-03	v13	-4.60E-02	-9.69E-03	-1.66E-02	-1.11E-02
v23	6.22E-03	8.92E-03	v23	-3.11E-02	-6.59E-03	-2.33E-02	-2.16E-02
g12	**9.84E-01**	7.84E-01	g12	**9.50E-01**	6.32E-01	2.20E-02	1.48E-02
g13	4.45E-02	1.03E-02	g13	1.35E-01	4.90E-02	-1.51E-03	7.90E-03
g23	4.89E-02	1.11E-02	g23	1.01E-01	2.55E-02	-4.05E-02	-2.03E-02
f1t	2.35E-02	-1.12E-02	f1t	2.57E-01	4.96E-02		
f1c	-2.01E-02	-1.67E-02	f1c	**-3.84E-01**	-9.87E-02		
f2t	-1.16E-02	2.54E-03	f2t	1.79E-01	4.05E-02		
f2c	1.01E-02	4.52E-03	f2c	**-3.86E-01**	-6.49E-02		
f3t	-4.70E-02	-1.71E-03	f3t	**-8.30E-01**	-3.07E-01		
f3c	-3.02E-03	1.01E-02	f3c	**4.32E-01**	9.88E-02		
s12	**-9.70E-01**	-5.65E-01	s12	**-9.11E-01**	-4.67E-01		
s13	-1.51E-02	-1.29E-04	s13	**-7.89E-01**	-2.77E-01		
s23	-5.42E-02	2.68E-03	s23	**-7.86E-01**	-2.54E-01		
safs	-1.83E-03	1.45E-02	biax12	-1.17E-01	-2.58E-02		
sbfs	-1.75E-02	-6.78E-03	biax13	**3.39E-01**	7.39E-02		
			biax23	**3.05E-01**	5.00E-02		

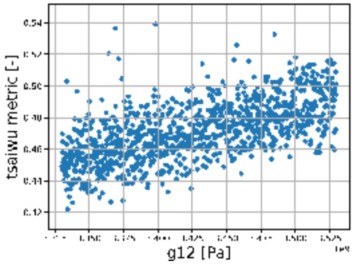


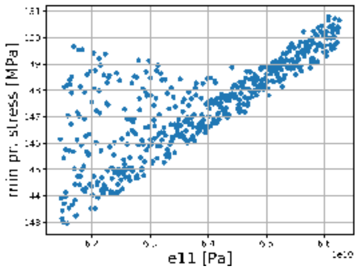
Scatter plots of three damage metrics with two most correlated variables for (45°/-45°/45°) ply layup

(45°/-45°/45°) Variance-based decomposition Sobol sensitivity indices

	mat162 metric		Tsai-Wu		min pr. stress	
variable	main effects	total effects	main effects	total effects	main effects	total effects
e11	0.021	0.024	0.02	0.026	0.725	0.905
e22	0.038	0.031	0.039	0.029	0.129	0.315
v12	0.003	0.003	0.001	0.004	0.04	0.041
g12	0.583	0.571	0.403	0.391	0	0
f1c	-	-	0.019	0.014		
f2c	-	-	0.016	0.017		
f3t	-	-	0.129	0.138		
f3c	-	-	0.01	0.01		
s12	0.32	0.322	0.217	0.219		
s13	-	-	0.085	0.087		
s23	-	-	0.077	0.103		
biax13			0.001	0.014		
biax23			0.005	0.013		







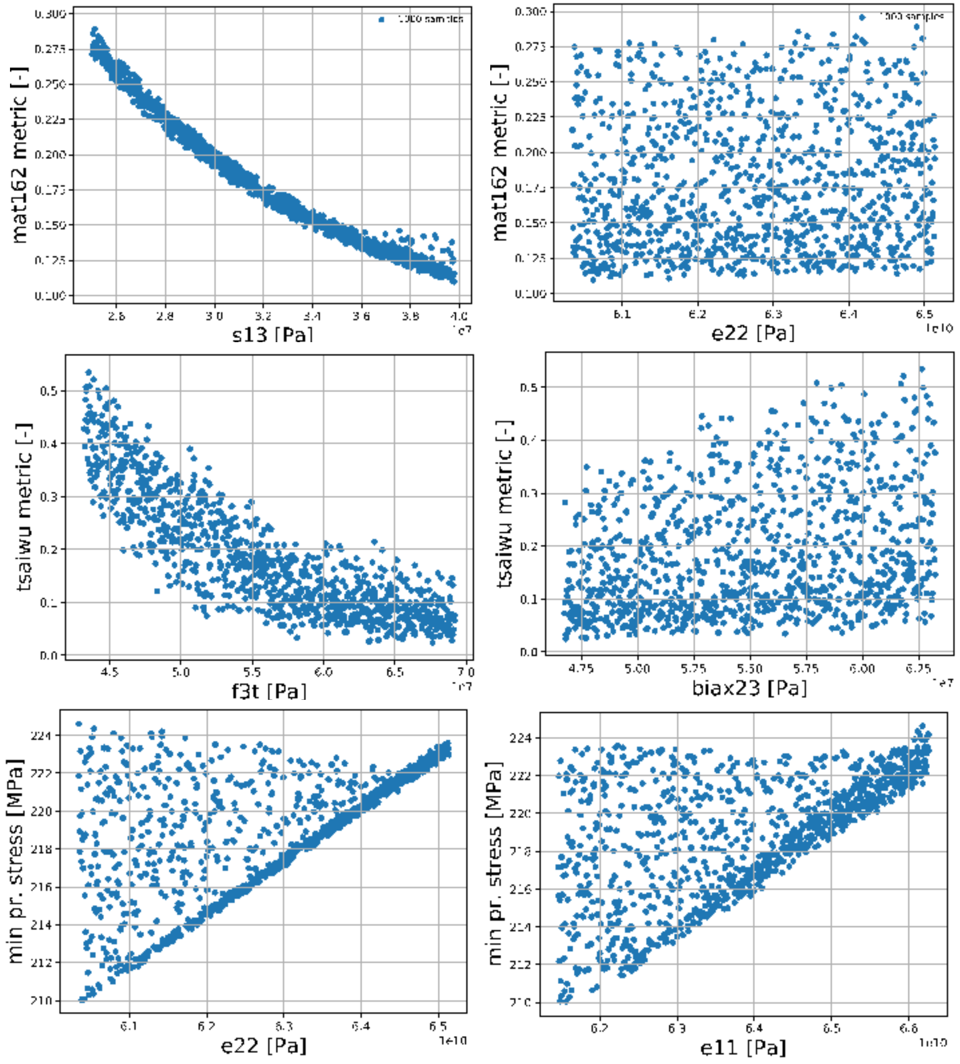
- Variance in **g12** including interactions with other variables is responsible for **57%** of variance in the **mat162** metric
- Variance in **g12** including interactions with other variables is responsible for **39%** of variance in the **Tsai-Wu** metric
- Variance in **e11** including interactions with other variables is responsible for **91%** of variance in the **abs. minimum principal stress** metric
  - ***Significant coupling for e11 and e22*** indicated by increase in total effects indices (91% and 32%) over main effects indices (73% and 12.9%) for the abs. min principal stress metric

(90°/0°/90°) correlation statistics

Correlations for 1000 Latin hypercube samples (LHS)

- Bold variables chosen for variance-based decomposition
- The Tsai-Wu metric shows a significantly higher range than the mat162 and min pr. stress metrics
- The mat 162 and min. principal stress metric show a similar high-end range of approx. 0.28 when min pr. stress is normalized by f1c=f2c=816MPa

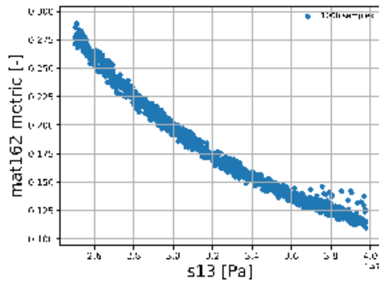
	mat162 metric			Tsai-Wu metric		min pr. stress	
variable	partial rank	simple rank	variable	partial rank	simple rank	partial rank	simple rank
e11	**4.38E-01**	5.53E-02	e11	**8.03E-02**	3.34E-02	**6.66E-01**	5.54E-01
e22	**5.48E-01**	6.85E-02	e22	**1.26E-01**	2.94E-02	**6.65E-01**	5.52E-01
e33	3.90E-03	6.29E-03	e33	4.82E-02	-7.53E-03	-3.40E-02	-2.55E-02
v12	**1.32E-01**	1.20E-02	v12	**2.94E-02**	6.78E-03	**7.15E-02**	5.90E-02
v13	4.33E-02	-4.94E-04	v13	2.29E-02	1.06E-02	-7.08E-03	-6.47E-03
v23	-4.38E-02	-1.68E-02	v23	-3.18E-02	-7.33E-03	-2.53E-02	-2.38E-02
g12	-7.94E-03	-9.41E-03	g12	-2.42E-02	-2.23E-02	9.75E-03	9.35E-03
g13	7.28E-02	1.98E-02	g13	6.45E-03	1.79E-02	-1.52E-02	-3.35E-03
g23	-5.29E-03	-9.72E-03	g23	3.74E-02	1.58E-02	-3.95E-02	-2.24E-02
f1t	1.58E-02	7.75E-03	f1t	1.72E-01	4.15E-02		
f1c	-8.37E-02	-1.85E-02	f1c	-2.09E-01	-4.13E-02		
f2t	-8.88E-03	-9.98E-03	f2t	1.05E-01	4.26E-02		
f2c	-3.02E-02	2.83E-03	f2c	**-5.12E-01**	-1.29E-01		
f3t	**-3.20E-01**	-1.34E-02	f3t	**-9.66E-01**	-8.59E-01		
f3c	4.13E-02	7.12E-03	f3c	2.70E-01	3.60E-02		
s12	6.28E-02	7.97E-03	s12	4.22E-03	-2.17E-03		
s13	**-9.97E-01**	-9.94E-01	s13	**-6.88E-01**	-2.19E-01		
s23	**-2.30E-01**	-2.12E-02	s23	-9.61E-02	-3.03E-02		
safs	9.03E-03	1.72E-02	biax12	-2.41E-01	-5.80E-02		
sbfs	1.92E-02	1.07E-02	biax13	**5.76E-01**	1.51E-01		
			biax23	**8.06E-01**	3.02E-01		

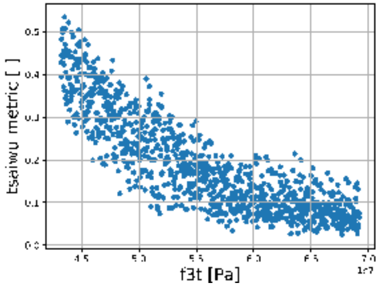


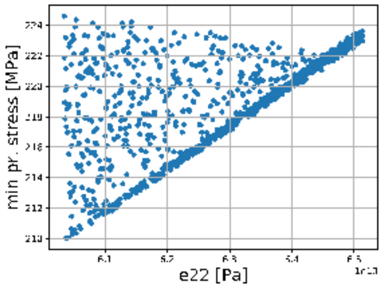
Scatter plots of three damage metrics with two most correlated variables for (90°/0°/90°) ply layup

(90°/0°/90°) Variance-based decomposition Sobol sensitivity indices

	mat162 metric		Tsai-Wu		min pr. stress	
variable	main effects	total effects	main effects	total effects	main effects	total effects
e11	0.003	0.002	0	0	0.344	0.603
e22	0.002	0.004	0.002	0.002	0.381	0.629
v12	0.001	0	0	0	0.001	0.002
f2c	-	-	0.016	0.013		
f3t	0.001	0.001	0.801	0.822		
s13	0.978	0.984	0.041	0.044		
s23	0	0.002	-	-		
biax13			0.021	0.023		
biax23			0.109	0.114		







- Variance in **s13** including interactions with other variables is responsible for **98%** of variance in the **mat162** metric
- Variance in **f3t** including interactions with other variables is responsible for **82%** of variance in the **Tsai-Wu** metric
- Variance in **e22** including interactions with other variables is responsible for **63%** of variance in the **abs. minimum principal stress** metric
- Variance in **e11** including interactions with other variables is responsible for **60%** of variance in the **abs. minimum principal stress** metric
- ***Significant coupling for e11 and e22*** indicated by increase in total effects indices (60% and 63%) over main effects indices (34% and 38%) for the abs. min principal stress metric

Summary: isothermal cooling induced residual stress

Most critical parameters: important to have rigorous experimental data

- e11, e22, g12 (elastic properties); f3t, s13, s23 (strength properties)

The Tsai-Wu metric consistently showed a larger spread and peak damage metric than the other two metrics for the three ply stack cases considered

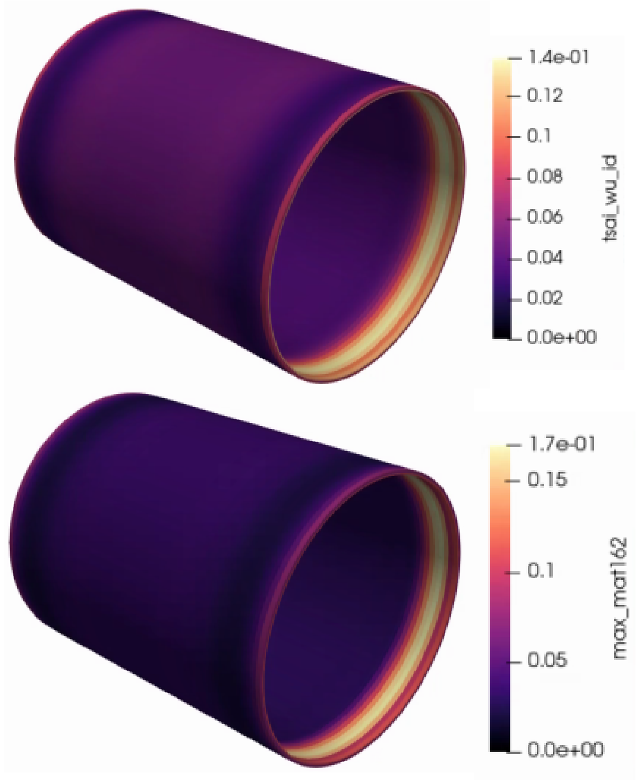
- Tsai-Wu metric is the most conservative option
- Differs from results for single orientation, which showed very similar behavior with mat162

Peak damage metrics always occurred at/near the edges

- Except for min principal stress metric for (45°/-45°/45°), peak in center of cylinder

Future work:

- Explore other ply sequences, separate metrics by ply
- Investigate the use of and comparison with surrogate methods and composite homogenization techniques



Fields of mat162 and Tsai-Wu metric values for (0°/90°/0°) shows peak near ends of inner ply adjacent to aluminum

	(0°/90°/0°)			(45°/-45°/45°)			(90°/0°/90°)		
Damage metric	Relative std dev [%]	Greatest Sobol index param	Total effects [%]	Relative std dev [%]	Greatest Sobol index param	Total effects [%]	Relative std dev [%]	Greatest Sobol index param	Total effects [%]
Mat162	26.2	S23	99.5	3.45	G12	57.1	26.1	s13	98.4
Tsai-Wu	61.5	f3t	83.0	3.91	G12	39.1	61.6	f3t	82.2
Min Pr. Stress	1.64	e11	95.2	1.19	e11	90.5	1.48	e22	62.9

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Thank you!