Collaborative Decision Approach for Electricity Pricing-demand Response Stackelberg Game

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***Abstract*—Demand response programs are considered as a valuable resource in smart grids that provide several advantages of load shifting, peak load reduction, mediating intermittency of renewable energy integration, etc. Flexible price-based incentives have been recognized as a critical strategy in motivating and compensating consumers' load adjustment actions for a successful implementation of demand response. Game theoretical approaches, especially Stackelberg games are popularly adopted to model the relationship between electricity price and customers' demand response and solved by the classical centralized backward induction (BI) method. However, the BI method generally requires convexity of the follower's model for necessary optimality conditions, and the computational time of any centralized approach increases sharply with larger problem instances. In this paper, the Stackelberg game of electricity pricing-demand response between a distribution system operator (DSO) and load aggregators (LAs) is decomposed based on a collaborative optimization (CO) framework, where each LA is treated as a discipline with its own domain constraints (e.g. building temperature control), while the DSO at the system level tries to reduce the solution discrepancy and guide the searching towards optimality. Several groups of comparison experiments have demonstrated the effectiveness of the proposed collaborative decision approach in solving the demand response game.**

***Index Terms*—Stackelberg Game, Bilevel Optimization, Decomposition Based Approach, Collaborative Optimization, Demand Response**

NOMENCLATURE

Indices

T, t, n	Decision period, index for hours, LAs

Parameters

$C_t, \overline{P}$	Cost of elec. generation, price upperbound
$\underline{L}_{n,t}, \overline{L}_{n,t}$	Lower, upperbound of demand limit for LAs
$Dh_{n,t}, Dd_{n,t}$	Nominal thermal, non-thermal demand of LAs
$\alpha_{n,t}, \theta$	Satisfaction preferences of LAs, penalty coef.
ϵ	Dissipation rate of virtual battery
$\underline{B}_{n,t}, \overline{B}_{n,t}$	Lower, upperbound of SOC for virtual battery

Variables

$p_t, dl_{n,t}$	Electricity price, total demand of each LA
$hr_{n,t}, dr_{n,t}$	Thermal, non-thermal load of each LA
$m, b_{n,t}$	Peak load, level of charge in virtual battery

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I. INTRODUCTION

With the high penetration of renewables, electricity grid systems face more challenges in improving grid reliability, enhancing operational flexibility for regulations, providing more contingency reserves, etc. An efficient strategy for dealing with these challenges is demand side management programs, which include every action on the demand side in order to improve the grid characteristics [1], [2]. Generally, two broad sub-categories of demand side management are energy efficiency programs and demand response programs. Demand response programs could be further classified as 1) incentive-based demand response programs in which consumers are awarded for consumption adjustment as per the desire of the supply-side (e.g., direct load control, load curtailment, emergency demand reduction), and 2) price-based demand response programs where electricity tariffs (e.g., time of use pricing, critical peak pricing, real time pricing) are utilized to motivate the consumption pattern change [3], [4].

Extensive research has been conducted on demand response programs, which can be categorized into 1) Aggregated demand response flexibility and potential quantification. A demand response estimation framework for residential and commercial buildings is presented in [5] based on the combination of EnergyPlus and two-state models for thermostatically controlled loads. Regression models are then fit to predict the demand response potential based on key inputs like hour of day, set point change, outside temperature, etc. The demand response flexibility of residential smart appliances is quantified based on the measurements from a large scale pilot project [6], and the maximum amount of time a certain increase or decrease of power that can be realized within the comfort requirement is calculated. A bottom-up approach for the quantification of building flexibility services is proposed in [7], where cost curves from the solution of optimal control problems with low-order models is computed to show both the amount and cost of the flexibility at a given time. 2) Interactive modeling between electricity pricing and demand response. Stackelberg game models are proposed for a distribution system operator (DSO) leader and load aggregators (LAs) followers, where the leader has the privilege of setting prices and the followers change their demand patterns in response to the price signals [4], [8]. A tri-level demand response model is developed in [9] for a grid operator, multiple service providers, and corresponding customers, where a two-loop Stackelberg game is proposed to capture the interactions between the different actors. In local energy transaction, the electricity pricing and trading behavior with demand response are modeled in [10] by both cooperative and non-cooperative games. The leader's attitude (selfish or benevolent) and price structure (uniform or discriminatory) are explored in the profit distribution of non-cooperative bilevel optimization. It is shown in [11] that plain time-of-use pricing is not a promising demand response policy if residential customers are equipped with home energy management systems. Then two new approaches are proposed to uncover the residential demand response: optimal load aggregation under augmented time-of-use pricing and active demand response participation in unit commitment under rewards. 3) Demand response with temperature control. The demand response potential from a large number of thermostatically controlled loads that have to maintain their temperatures within a fixed range through ON/OFF control is studied in [12]. In the proposed hierarchical framework, virtual battery constraints are developed based on building thermal properties to guarantee the thermal comfort range for end users [13] and aggregated for LAs, which are then incorporated into the electricity pricing-demand response game between the DSO and LAs at the upper layer. At the lower layer, a model-free control strategy is used to track and allocate the resulted demand profile.

Specifically, for the Stackelberg model of pricing-demand response game, the classical backward induction (BI) or Karush-Kuhn-Tucker (KKT) conditions has/have been applied in a number of studies [8] [4] [14]. The general two steps in the BI method are that given the electricity price from the leader, the equivalent optimality conditions of the followers' model are first derived and then substituted back into the upper level to transform the original bilevel model into a final single level model. Convexity is required to derive equivalent global optimal conditions of the original model. Since BI is a centralized solution approach, all the information of load aggregators needs to be collected by a central unit to make decisions, and the computational cost increases sharply with larger problem scale. Different distributed solution approaches [4] [15] [16] have been investigated for the purpose of reducing the solving time with less information sharing. In this work, we make a first attempt to apply a decomposition based distributed decision approach — collaborative optimization (CO) to the Stackelberg demand response game. CO origins from multi-disciplinary design problem, and it has been utilized in different domains, e.g., low energy building design [17], aircraft design [18], etc. The detailed procedure of collaborative optimization is elaborated in later sections.

The organization of this research is as follows. The electricity pricing and demand response game between a DSO and LAs is presented in Section II. Then, the Stackelberg game model is decomposed in collaborative optimization decision framework in Section III. Several groups of case studies are conducted to evaluate and compare the performance of the proposed approach in Section IV. The conclusions are drawn in Section V.

II. Demand Response Game

In the Stackelberg game here, the objective of the DSO is to maximize the profit in Eq.(1), which includes the electricity selling profit (first two terms), the overall satisfaction from load aggreagators (the third term in Eq.(1)) and the peak load penalty (the fourth term in Eq.(1)). As a leader, the DSO has the privilege of setting the price signal. The electricity price is limited by its marginal generation cost C_t and by an upper bound. Eq.(3) defines the peak load as the maximum total load for the whole duration.

$$\begin{aligned} DSO: \max U_o = \textstyle\sum_{n,t} p_t \cdot dl_{n,t} - \sum_{n,t} C_t \cdot dl_{n,t} \\ + \textstyle\sum_{n,t} S(Dl, dl) - \theta \cdot T \cdot m \end{aligned} \tag{1}$$

$$C_t \leq p_t \leq \overline{P},\ \forall t \tag{2}$$

$$m \geq \textstyle\sum_n dl_{n,t},\ \forall t \tag{3}$$

$$LAs: \max U_n = \textstyle\sum_t S(Dl, dl) - \sum_t p_t \cdot dl_{n,t} \tag{4}$$

$$dl_{n,t} = hr_{n,t} + dr_{n,t},\ \forall n, t \tag{5}$$

$$\underline{L}_{n,t} \leq dr_{n,t} \leq \overline{L}_{n,t},\ \forall n, t \tag{6}$$

$$0.9 \cdot \textstyle\sum_t Dd_{n,t} \leq \sum_t dr_{n,t} \leq 1.1 \cdot \sum_t Dd_{n,t},\ \forall n, t \tag{7}$$

$$\underline{B}_{n,t} \leq b_{n,t} \leq \overline{B}_{n,t},\ \forall n, t \tag{8}$$

$$b_{n,t} = \epsilon \cdot (b_{n,t-1} + hr_{n,t} - Dh_{n,t}),\ \forall n, t \tag{9}$$

As followers, LAs respond to the price signal from the leader and try to maximize their own utility functions in Eq.(4), including the satisfaction value of electricity consumption (first term) and bill payment (second term). The function $S(Dl, dl)$ in Eq.(10) is used to represent the monetary value of customers' satisfaction. It's value is 1 when the actual consumption after demand response is equal to the nominal demand.

$$S(Dl, dl) = (Dl_{n,t}) \cdot w_{n,t} \cdot (\frac{dl_{n,t}}{Dl_{n,t}})^{\alpha_{n,t}} \tag{10}$$

Eq.(10) is revised based on [8] to maintain the convex property. Note that $Dl_{n,t} = Dh_{n,t} + Dd_{n,t}$ is the total nominal load; w is a user defined parameter; α represents the sensitivity of demand shifting. Eq.(5) defines the total consumption as the summation of the thermal load hr and the non-thermal load dr. Eq.(6) limits the hourly non-thermal demand and Eq.(7) uses the range $\pm 10\%$ to ensure that the daily non-thermal load does not fluctuate too much for the required daily work. The stricter equality constraint $\sum_t dr_{n,t} = \sum_t Dd_{n,t}$ could be an alternative to Eq.(7), which means only demand shifting is allowed for non-thermal consumption and curtailment is not allowed.

Eq.(8)-(9) constrain the level of charge of the virtual battery for the aggregated buildings that is derived based on the building thermal characteristics. The difference between the actual power consumed hr by a thermal appliance and the nominal thermal load Dh determines whether the virtual battery is being charged or discharged. ϵ is the virtual battery dissipation rate, which depends on the properties of the thermal load (e.g. insulation characteristics) and can be determined empirically. For further details on developing the virtual battery constraints, readers are referred to our previous work [13].

III. Collaborative Optimization

CO is a decomposition based decision making strategy for distributed multi-disciplinary optimization architecture, and the key in its actual implementation is how to partition a complex system into several disciplines (or subsystem agents) that can make independent decisions with the consideration of the domain constraints [17] [19]. The motivation of developing a collaborative decision approach is to reduce the computational cost and communication requirements, and to provide a distributed design authority and modularized flexibility by disciplines. The system partition process depends on the characteristics of the target problem, which could be based on disciplines or sequential subcomponents [20]. Take the building design problem as an example, it could be decomposed by disciplines as architecture design, envelope design, energy system design, etc. Each of these disciplines needs perspective domain knowledge, and meantime, these subsystems are also coupled by some design variables.

In the studied Stackelberg problem here, the bilevel problem structure is well-defined, and thus we decompose the pricing-demand response problem into a system level coordination model for the DSO and several models for LAs at the lower or discipline level. The system level optimizer coordinates the activities of all discipline level problems, guiding the system towards optimality and consistency. The system consistency is ensured via the auxiliary constraints in Eq.(11), where $dl''_{n,t}$ is the decisions from each LA at the discipline level. Thus, the system level optimization now becomes the objective function Eq.(1), constraints Eq.(2)-(10) and Eq.(11).

$$\sum_t (dl_{n,t} - dl''_{n,t})^2 \leq 0.001 \tag{11}$$

The discrepancy between the system level and discipline level is represented by Eq.(12). The discipline level optimizer receives the decision $dl_{n,t}$ from the system level and then seeks to minimize this discrepancy while subjected to local constraints. The discipline level for each LA now have the optimization with objective function Eq.(12) and constraints Eq.(5)-(9).

$$LAs : \max U_n = \sum_t (dl'_{n,t} - dl_{n,t})^2 \tag{12}$$

The overall collaborative decision procedure for the electricity pricing-demand response game is as follows:

- Step 1: Using the total nominal demand of LAs as initial solution $dl''_{n,t} = Dl_{n,t}$. Solve the system level optimization to obtain $dl_{n,t}$ and pass it down to the discipline level.
- Step 2: For discipline level, solve the optimization model for each LA parallelly to obtain the solution $dl'_{n,t}$. Then pass this solution up to system level and let $dl''_{n,t} = dl'_{n,t}$.
- Step 3: Repeat steps 1-2, stop the process if the objective improvement of system level in two successive iterations is less than a small tolerance, and the objective value of each load aggregator at the discipline level is also less than the defined tolerance.

Note that, in Step 1, the nominal demand is used for the initial solution of $dl''_{n,t}$ for the sake of simplicity, other predefined solutions from heuristic algorithms could also be used as a warm start to speed up the convergence.

IV. Experimental Results

In this section, the results from the classical BI method are used as a baseline for comparison with the CO approach. The detailed BI solution process for solving the proposed Stackelberg game in Section II can be found in our previous study [4]. The solution from the BI method is a global optimal.

A. Data preparation

For the case studies here, ten LAs are considered with thermal demand (e.g. residential HVAC, commercial HVAC, water heaters) and non-thermal demand. The hourly nominal thermal demand profiles of ten LAs are simulated based on different weather conditions and building thermal characteristics, see Figure 1. The hourly nominal non-thermal demand of LAs are generated based on a combination of commercial building reference loads (e.g. large office, hospital, supermarket, etc.) from the U.S. Department of Energy, see Figure 2, and the marginal cost data is obtained from [8]. The α parameters in the satisfaction function of all LAs are assumed to be the same, shown in Figure 3. α represents the sensitivity of demand shifting, which is usually higher during peak hours as the electricity price is high, therefore, the overall peak reduction in demand response greatly depends on the trade-off potential between customers' comfort preference and monetary saving.

B. Case studies

To illustrate the proposed solution approach, several groups of case studies are designed based on whether a flat price (FIX) or time-of-use (TOU) price structure is adopted, and whether the penalty coefficient of peak load θ in Eq.(1) equals 10 or 20. Note that an additional constraint $p_t = p_{t-1}, \forall t \geq 2$ needs to be added into the system level when flat price

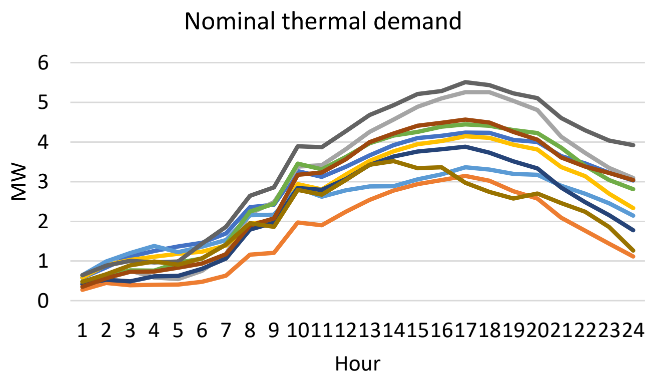

Fig. 1: Nominal thermal demand of load aggregators

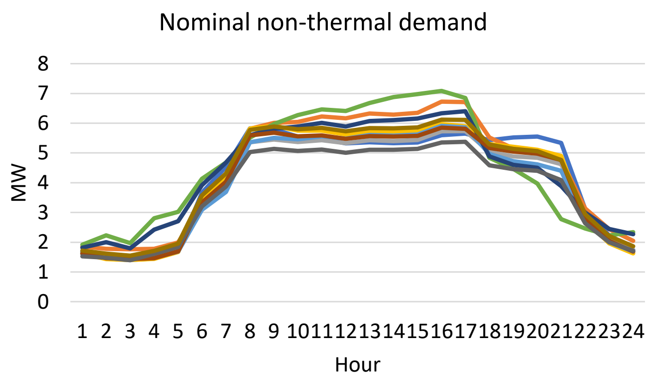

Fig. 2: Nominal non-thermal demand of load aggregators

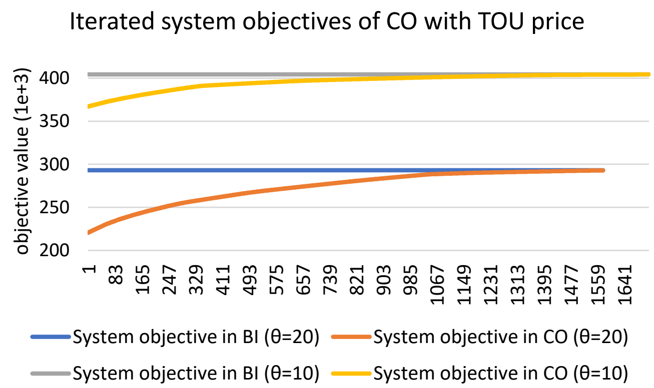

Fig. 4: Iterated system objective in CO under TOU price structure

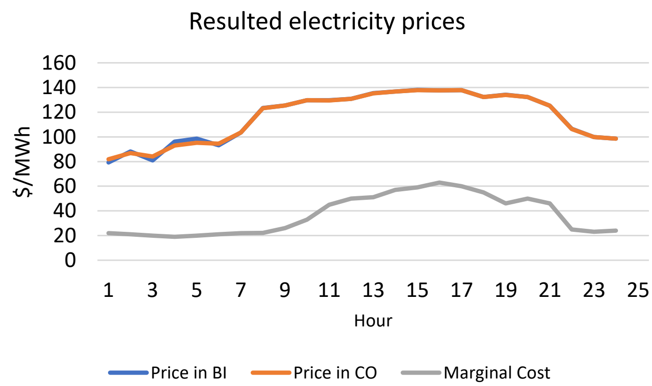

Fig. 5: Resulted electricity prices in BI & CO ($\theta = 20$)

is selected. The stopping criteria of the CO is when the system level objective value stops improving, and the objectives (solution discrepancy) of all LAs are zero. For ten LAs, the iterated system level objectives of CO are plotted in Figure 4 under TOU price structure, where the optimal system objectives from the BI are used as their upperbounds. Eventually, the solution of the CO converges to the optimal solution of BI.

In the CO approach, the electricity price p_t is the local decision variable in the system level and doesn't appear in the LA's model, while in the BI the price variable from the leader

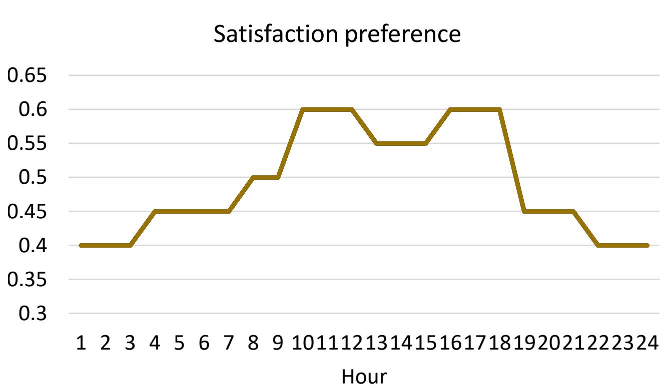

Fig. 3: α parameter in satisfaction function

model will be treated as given parameter in the follower's model. The marginal cost of electricity generation C_t and the resulted electricity prices from both CO and BI are shown in Figure 5. As shown, the two resulted prices are overlapped for most of the time.

As observed from Figure 6 and Figure 1, the peak load at about hour 17 are shifted to hour 22-24. Since the indoor temperature is guaranteed by virtual battery constraints, this shifting will flatten the total demand of all aggregators combined with the resulted non-thermal demand shown in Figure 7, which benefits the DSO. Also, the electricity price in hour 21-24 is lower than the one in hour 17 which leads to more cost saving. The peak reduction is demonstrated in Figure 8 which indicates that the resulted total demand profiles are almost the same in both BI and CO methods and the total demand is reduced from about 100 MW to about 72 MW.

For the cooling case here, a fully charged virtual battery means that the temperatures of thermal loads are less than the setpoint temperatures (e.g. 23.0 °C) as much as the comfort band limits (1.5 °C). It can be observed from the virtual battery level of all LAs in Figure 9 that the optimal strategy of thermal control with the premise of maintaining preferred temperature band is to charge the virtual battery (precooling) at non-peak hour 7, and then the thermal load keeps at a relatively lower level and building temperatures start to increase towards the upper limit during the long discharging

TABLE I: Detailed profit and cost result comparison for CO and BI

System Level	FIX ($\theta = 10$)		FIX ($\theta = 20$)		TOU ($\theta = 10$)		TOU ($\theta = 20$)	
	BI	CO	BI	CO	BI	CO	BI	CO
Peak Load	108.20	108.23	95.68	95.65	82.51	82.75	71.91	71.98
Average Load	71.48	71.49	64.40	64.38	71.34	71.42	64.99	64.99
PAR value	1.51	1.51	1.48	1.48	1.15	1.15	1.10	1.10
DSO revenue	200993	201021	190219	190190	196620	196721	185945	186022
DSO cost	72622	72642.4	65186	65166	66995	67108	59858	59876
DSO profit	128370	128379	125033	125023	129624	129613	126087	126145
Discipline Level	FIX ($\theta = 10$)		FIX ($\theta = 20$)		TOU ($\theta = 10$)		TOU ($\theta = 20$)	
	BI	CO	BI	CO	BI	CO	BI	CO
bill payment of LA1	20785	20788	19686	19683	20428	20426	19336	19343
bill payment of LA5	18689	18692	17692	17690	18353	18366	17377	17384
bill payment of LA10	18943	18946	17925	17922	18551	18558	17539	17546
satisfaction of LA1	41335	41341	39241	39235	41187	41194	39164	39173
satisfaction of LA5	37002	37007	35111	35106	36849	36878	35057	35066
satisfaction of LA10	37313	37318	35388	35383	37046	37060	35188	35197

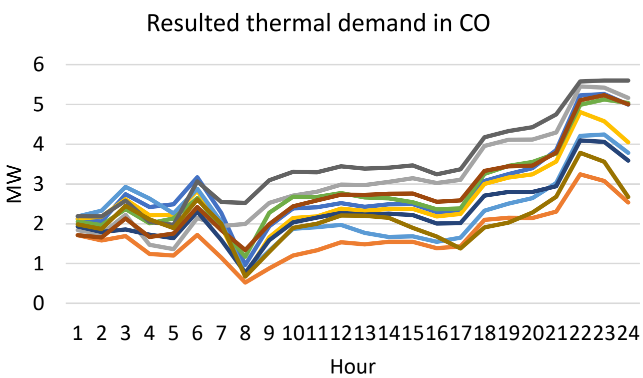

Fig. 6: Resulted thermal demand of load aggregators ($\theta = 20$)

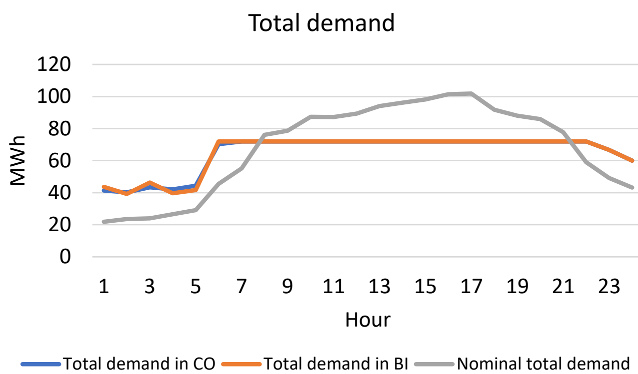

Fig. 8: Total demand of all load aggregators ($\theta = 20$)

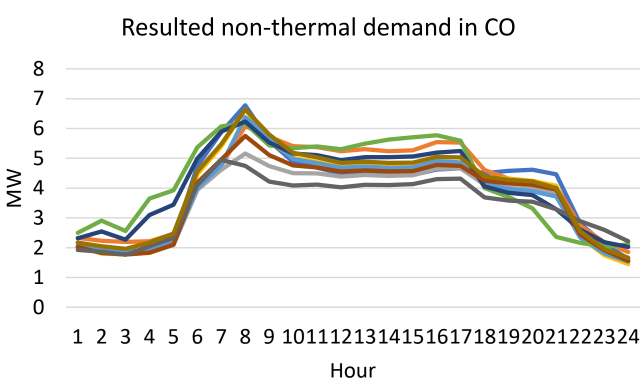

Fig. 7: Resulted non-thermal demand of load aggregators ($\theta = 20$)

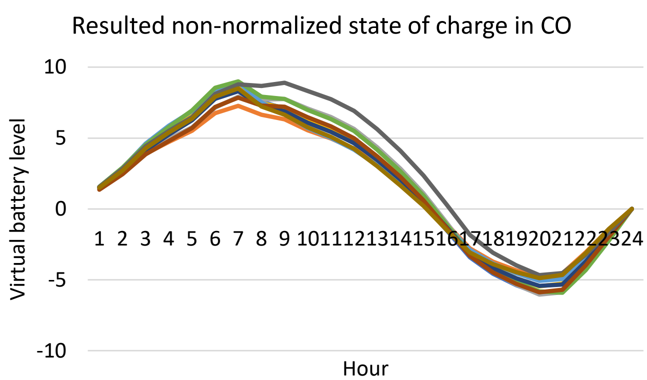

Fig. 9: Virtual battery level of load aggregators ($\theta = 20$)

of virtual battery from hour 7-20. At the end hours, the virtual battery starts to charge again that brings down the building indoor temperatures.

The detailed experimental results of the several case studies have been summarized in Table I for comparison. Load aggregators LA1, LA5 and LA10 are used as an illustration, the results for the other LAs have a similar pattern. Under the same price structure, larger penalty coefficient θ results in more peak reduction and thus smaller PAR (peak-to-average) values. Meantime, more peak reduction means that more load shifting is required for LAs, therefore, the satisfaction is getting lower with more monetary compensation (cost savings in the bill payment). Under the same θ value, more load shifting potential could be exploited in TOU price structure than FIX price. As shown in the table, results from CO and BI are very close for the DSO at the system level and each of the LAs at the lower level. This indicted the effectiveness of applying the CO approach to solve one leader multiple followers games in a distributed way.

V. Conclusion

In this research, CO, a system decomposition based distributed solution approach, is studied and applied to a popular Stackelberg game of electricity pricing-demand response. DSO serves as system coordinator and guides the solution towards its benefit maximization with the consideration of system consistency. Given the demand profile decisions from the DSO, each LA testifies the feasibility of the demand profile on its domain constraints and tries to minimize the solution discrepency between its decision and system decision. The effectiveness of CO in solving the presented Stackelberg game is demonstrated in simulation on ten LAs in comparison to the BI centralized method. For future study, the scalability of collaborative decision approach will be tested using a larger scale Stackelberg model, and different initial solution generation methods will be explored to accelerate the convergence speed.

Acknowledgment

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