

Assessing the Implications of Automated Merging Control in a Mixed and Heterogeneous Traffic Environment

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Abstract—Previous efforts to explore the implications of partial market penetration of connected and automated vehicles (CAVs) show a consensus on the benefits of higher market penetration rates (MPR) of vehicles enabled with connectivity and/or automation. There is, however, a level of uncertainty regarding the effects of lower market penetration rates and the consideration of heterogeneous vehicle fleets. Using VISSIM to perform microscopic traffic simulation and, vehicle simulation models, we assess the impacts of different CAVs market penetration rates on fuel consumption considering a heterogeneous traffic environment. The results show that the fuel efficiency and emissions reduction benefits of optimal coordination control are maximized in moderate congested scenarios when the CAVs MPR exceeds 40%.

Index Terms—Connected and Automated Vehicles, Optimal Merging Control, Automated Merging Control, Heterogeneous traffic, Market Penetration Effect, Cooperative Driving, Highway On-ramps, Merging Highways

I. INTRODUCTION

A. Motivation

Transportation is a key driver of social and economic development, enabling people to have access to jobs, education, health and goods. But the way how it has evolved resulted in many unintended consequences that are currently threatening our society's sustainable development. About 64% of global oil consumption and 23% of the worldwide CO_2 emissions are attributed to transportation [1] and, every year congestion accounts for billions of dollars due to wasted travel time and fuel consumption [2]. Connected and Automated Vehicles (CAVs) hold the potential to improve the current operational efficiency and safety of the transportation system by relieving drivers from some or all driving tasks. Several research efforts have revealed benefits of CAVs [3]. However, many challenges still remain while CAVs start being deployed on the roads and interacting with non-CAVs, adding a new level of complexity in the transportation system. Early efforts to explore the implications of these complex interactions are summarized in the next section.

B. Related Work

Talebpour and Mahmassani [4] presented a framework that uses different models and technology-related assumptions to simulate vehicles with different levels of communication and automation capabilities.

Ito et al. [6] proposed a coordination method for smoother traffic merging in mixed traffic environments. The approach was evaluated through microscopic simulations and its effectiveness assessed in terms of the smoothness of merging, the rate of failed merging, and the average vehicle merging velocity. Arvin et al. [7] developed a framework to explore the safety impacts of different market penetration rates (MPR) of vehicles with different levels of automation. The proposed framework builds upon modifications to the Wiedemann car following model to represent both manually driven vehicles and vehicles with different levels of automation.

Subraveti et al. [8] proposed a rule based advisory system for longitudinal control of main road vehicles with the aim to creating gaps and improving traffic performance. The proposed approach was evaluated through microscopic traffic simulation using a single traffic demand. The results showed a slight reduction in total travel time (1.9%) only at higher MPR with no improvement at lower MPRs. The hierarchical framework for cooperative merging under mixed traffic developed by Ding et al [9] was used to investigate the microscopic, i.e., vehicle trajectories, and macroscopic, i.e., throughput, delay and fuel consumption impact of different CAVs MPR. The authors evaluated three different cases: a baseline with only human drivers (0%), a full CAV penetration (100%) and a partial CAV MPR with different penetration rates from 10% to 90%. Simulations were performed for two different vehicle arrival rates.

Most of the previous research attempting to explore the implications of automated merging control in mixed traffic environments, have been mainly focused on assessing the impacts on safety and traffic flow. A few of them have explored the impacts on fuel consumption. However, to the best of our knowledge, there has not been attempts to assess the benefits of automated merging control considering a heterogeneous fleet of vehicles and using a diverse set of fuel consumption models, i.e., fuel consumption models for a diverse vehicle fleet. This paper aims to fill this gap by presenting a comprehensive assessment considering a heterogeneous fleet of vehicles and fuel estimation models.

C. Contribution of the paper

Early efforts to explore the implications of the interactions between CAVs and human-driven vehicles seem to show a con-



Fig. 1. Merging Scenario Illustrating the Control Zone.

sensus on the benefits of higher market penetration of vehicles enabled with connectivity and/or automation. However, there is still a level of uncertainty regarding the effects at lower market penetrations. Furthermore, to the best of our knowledge, there has not been attempts to explore these effects considering heterogeneous traffic, i.e., heavy duty and light duty vehicles and higher fidelity fuel consumption models for diverse powertrain types. In this paper we refer to higher fidelity vehicle models as the fuel consumption models that include the relevant vehicles subsystems for the total fuel consumption estimation. Through microscopic traffic simulation, this paper has two main objectives: (1) to enhance our previous merging control algorithm [5] by using an alternative calculation of the time to reach the merging zone that accounts for vehicle type, (2) to assess the fuel implications of partial penetration of CAVs in an heterogeneous traffic environment.

D. Organization of the paper

Section II includes an overview of the Optimal Merging Coordination problem. The simulation setup and an overview of the fuel models are included in sections III and IV. Finally, a discussion of results and concluding remarks are included in sections V and VI.

II. OPTIMAL MERGING COORDINATION PROBLEM

The aim of the optimal coordination system (OCS) is to coordinate the vehicles driving inside a predefined control zone while minimizing a cost function (figure 1). In this work, we aim at minimizing the acceleration for each vehicle on the control zone to reduce the overall fuel consumption.

The optimal coordination framework involves two phases: (1) determining the desired time to reach the merging zone (TTMZ) to avoid rear-end or lateral collisions between consecutive vehicles during merging, and (2) solving the optimal control problem to find the optimal control policy that minimizes the desired cost function for each vehicle driving inside the control zone.

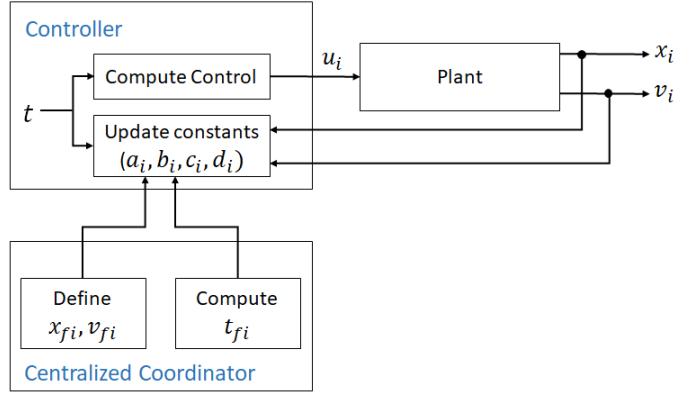


Fig. 2. closed-loop controller representation

To compute the TTMZ, we assume that once the vehicles enter the control zone, they can share their states, i.e., speed and position with a central coordinator that will assign a vehicle index. This index determines the sequence the vehicles will follow to reach the merging zone. In this work, the sequence is determined by a First-In-First-Out (FIFO) queue. Once the sequence is defined it allows to compute the TTMZ in a recursive way, i.e., the TTMZ for the host vehicle is computed as a function of the TTMZ of the lead vehicle in the FIFO sequence. Notably, this communication architecture could be adapted according to other desired criteria.

The host vehicle uses the TTMZ information of the lead vehicle in the FIFO queue to determine the time at which it should reach the merging zone while keeping a safe headway. The TTMZ is a key parameter to solve the optimal control problem that will compute the optimal trajectory to reach the merging zone. Once the vehicle reaches the merging zone, it will continue driving at the predefined speed limit.

The controller operates in a closed loop fashion, i.e., the vehicles' TTMZ and optimal control are updated at each sample time considering the current and the desired final states of the system (figure 2). For each vehicle i the centralized coordinator will communicate the desired final position x_{fi} and speed v_{fi} as well as the time to reach the control zone $TTMZ_i$ which is a function of the $TTMZ$ for vehicle $i - 1$. On the other hand, the controller uses information from the centralized coordinator to update the optimization constants that allow the computation of the optimal control input and states.

Implementing a closed loop control helps with collision prevention in case of unexpected events or malfunctions, enabling the vehicles to brake or come to a full stop if needed. For the VISSIM implementation we used the optimal coefficients to find the corresponding speed reference that the vehicle will follow. This speed is passed to the vehicles in the VISSIM environment using the COM interface.

There is an upper layer of control which will switch off the optimal controller if a predefined minimum safety gap is violated while the vehicles are driving in the control zone. If

this safety layer is activated, the vehicle will continue driving with the aim to keep a safe gap from the lead vehicle using the default VISSIM driver model.

A. Definition of the Time to Reach the Merging Zone (TTMZ)

The TTMZ is the time vehicle i will take to reach the merging zone.

If there is not a vehicle preceding the host vehicle or the preceding (lead) vehicle distance with respect to the host is $\rho_{th} > 100m$, the TTMZ is computed as:

$$TTMZ_i = t_{cz,i}^0 + \frac{L}{v_{f,i}}, \quad (1)$$

Where $t_{cz,i}^0$ is the time at which vehicle i reaches the control zone, L is the length of the control zone and $v_{f,i}$ is the speed the vehicle should follow at the time it reaches the merging zone.

if there is a preceding vehicle and its distance with respect to the host is $\rho_{th} \leq 100m$, the TTMZ is computed as:

$$TTMZ_i = TTMZ_{i-1} + \rho \quad (2)$$

Where $TTMZ_{i-1}$ is the time to reach the merging zone for the preceding vehicle vehicle $(i-1)$ and ρ is the desired headway that can be defined according to the vehicle type, ideally larger for an heavy duty vehicle (HDV) than for a light duty vehicle (LDV).

III. OPTIMAL CONTROL PROBLEM

We include a brief overview of the optimal control problem and its solution, for more details the readers can refer to [5]. Each vehicle is modeled by a second order dynamics as in (3).

$$\begin{aligned} \dot{p}_i &= v_i(t), \\ v_i &= u_i(t). \end{aligned} \quad (3)$$

Where p_i , v_i and u_i denote the position, speed and acceleration/deceleration (control input) of vehicle i . We aim to minimize the L_2 norm of the acceleration, subject to the vehicle longitudinal model in (3). The optimal control problem is mathematically stated as:

$$\begin{aligned} \text{minimize} \quad & \int_{t_{0,i}}^{t_{f,i}} u_i^2 dt \\ \text{subject to} \quad & \dot{p}_i = v_i, \quad \dot{v}_i = u_i, \\ & -u_{min} \leq u_i \leq u_{max} \end{aligned} \quad (4)$$

Where $t_{0,i}$ is the current time and $t_{f,i}$ is the TTMZ. The boundary conditions for this problem are defined by:

$$\begin{aligned} v_{t_{0,i}} &= v_i(t), \quad v_{t_{f,i}} = v_f, \\ p_{t_{0,i}} &= p_i(t), \quad p_{t_{f,i}} = p_f, \end{aligned} \quad (5)$$

Where $v_i(t)$, $p_i(t)$ are current speed and position and v_f , p_f are the speed and position at which the vehicle leaves the control zone.

To find the analytical solution and the online implementation of problem (4) we apply Hamiltonian analysis [10]. To simplify the analysis, we consider the unconstrained problem, meaning that the optimal solution would not provide limits for the state and control [5]. The Hamiltonian analysis allows finding the optimal control input and the speed and position for each vehicle as a function of time, namely:

$$u_i^*(t) = a_i t + b_i, \quad (6)$$

$$v_i^*(t) = \frac{1}{2} a_i t^2 + b_i t + c_i, \quad (7)$$

$$p_i^*(t) = \frac{1}{3} a_i t^3 + \frac{1}{2} b_i t^2 + c_i t + d_i, \quad (8)$$

Where a_i , b_i , c_i , d_i are constants of integration. To derive the optimal control for each vehicle in real time, the integration constants need to be updated at each sample time. Equations (7) and (8) along with the boundary conditions allow to form a system of equations of the form $\mathbf{T}_i \mathbf{b}_i = \mathbf{q}_i$, that can be solved in real time to update the integration constants.

IV. SIMULATION SETUP

A. Merging Scenario Overview

In this study, the optimal coordination system is implemented as a driver assistance feature, where the controller will take over control from the driver when reaching the control zone and will release control back to the drivers once they merge onto the main road or reach the end of the control zone. To represent the manual driving (or human-driven) portions of the simulation we used the Wiedemann car following model.

To perform a comprehensive evaluation of the optimal coordination control impacts and performance, we simulated traffic flow on the W I 94/N US 23 On-Ramp in Washtenaw county, Ann Arbor, Michigan (Figure 3) in VISSIM. A total of 1,086 m of the highway right-most lane and 560 m of the on-ramp are used for simulation.

We implemented the optimal coordination system using VISSIM's COM interface and defined a control zone of 400 m in both the main and ramp roads. To assess the implications of the optimal coordination control, we simulated a baseline scenario in which all the vehicles are assumed to be human-driven without coordination. The optimal coordination system was then simulated using the same traffic conditions as in the baseline and different CAV MPR were considered.

B. Main assumptions

- All the vehicles can share their speed and position, and are able to communicate via V2X with other vehicles and infrastructure;
- Perfect communication;
- Similar to [11], [12] only the rightmost lane of the highway is modeled;
- The vehicles are not allowed to perform lane changes while driving inside the control zone;
- The default lane change model in VISSIM is used to allow



Fig. 3. Traffic Simulation network in VISSIM based on the W I 94/N US 23 On-Ramp in the Washtenaw county in Ann Arbor, Michigan.

the vehicles to join the main road once they leave the control zone;

- vi) Traffic split: 60% - 40% between main and ramp roads;
- vii) 10% of the total number of vehicles on the road are HDVs.

C. Fuel Consumption Estimation

To estimate the fuel consumption, we used a compilation of Autonomie vehicle models [13] that were provided by the Center for Transportation Research - Argonne National Laboratory. These models were compiled to simulate a current fleet distribution scenario, i.e., different light duty and heavy duty vehicle makes and models [14]. The compiled models use the driving profiles generated by VISSIM as input and estimate fuel/energy consumption, emission measures, and driving distance as output. Each driving profile is randomly assigned to a vehicle make and model to estimate the respective outputs.

D. Simulated Traffic Flow Scenarios

Case Study 1: The fuel consumption and emissions implications were analyzed for a traffic flow of 2200 veh/h with a speed limit of 60 km/h and the MPRs listed in table I. A total of ten runs were considered to account for variability in traffic conditions.

Case Study 2: To get insights on how the traffic flow value influence the results, single simulations runs were conducted for different traffic flow values: 1600 veh/h, 1800 veh/h, 2000 veh/h, 2200 veh/h and 2400 veh/h considering the MPRs listed in table I.

V. SIMULATION RESULTS AND DISCUSSION

A. Case Study 1

The box plot in figure 4 shows that the implemented optimal control is effective at reducing average fuel consumption for all the simulated MPRs. Interestingly, higher variability is observed at lower MPRs which is due to the fact that CAVs have to react to the instantaneous maneuvers of a larger amount of human drivers. As a driver unexpectedly

TABLE I
MARKET PENETRATION RATES (MPRs) CONSIDERED FOR SIMULATION

Scenario	% Light Duty CAVs	% Heavy Duty CAVs
Baseline	0	0
2	0	100
3	5	100
4	20	100
5	40	100
6	60	100
7	80	100
8	100	100

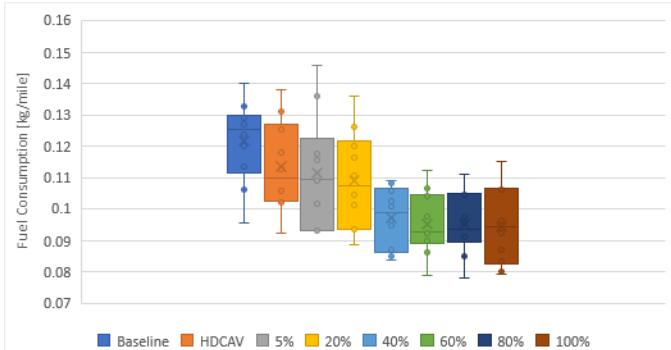


Fig. 4. Fuel consumption for different market penetration rates -traffic flow = 2200 veh/h, speed limit = 60 km/h.

reacts to merging vehicles, chained reactions of deceleration-acceleration maneuvers are required in the upstream traffic, thus increasing the variability and in some cases, resulting in higher fuel consumption values. This result suggests that, in the real world and under low MPR, there will likely be higher variability.

It is also evident that a substantial reduction in the average fuel consumption was achieved for MPRs greater than 40% and the reduction remains at an almost constant value after 60% MPR. The second box in figure 4 represents the scenario where all the HDVs on the road (10% of the total amount of vehicles) are connected and automated and are coordinated. In this case, the average fuel consumption reduction is about 6.5%, this means that, on a congested highway where the number of trucks correspond to about 10% of the traffic, enabling them to perform an automated coordinated merging can render fuel reduction on the overall vehicular network of more than 6% (considering only the fuel consumed around the merge point).

Overall, the average fuel reduction varied between 6.5% and 22.4%. Once the market penetration rate of LD-CAVs reaches 60% the average fuel reduction benefits remained above 20%.

Figure 5 summarizes the average fuel consumption difference with respect to baseline for the 10 simulated seeds under a traffic flow scenario of 2200 veh/h. Although in a few cases there was an increase in fuel consumption (ranging between 2% to 9%), once the MPR is greater than or equal to 40% the OCS consistently reduces fuel consumption. This confirms

	HDCAV	5%	20%	40%	60%	80%	100%
Scenario 1	6.79	10.37	8.00	16.46	18.58	19.76	26.71
Scenario 2	25.95	14.86	24.71	33.73	29.56	30.37	27.88
Scenario 3	-2.05	10.10	1.75	21.52	27.71	28.75	28.09
Scenario 4	-4.08	-9.79	-2.62	18.34	21.55	20.00	19.89
Scenario 5	12.74	15.12	21.49	25.17	24.38	24.38	25.43
Scenario 6	15.19	22.59	22.33	27.77	25.07	23.81	27.70
Scenario 7	3.42	2.13	2.04	10.78	9.73	10.96	16.89
Scenario 8	1.36	7.36	8.57	14.32	16.29	17.78	15.68
Scenario 9	13.40	11.81	10.55	17.06	26.37	22.84	21.79
Scenario 10	3.07	12.17	16.36	21.15	25.65	26.51	24.56

Fig. 5. Average fuel consumption % difference with respect to baseline for different market penetration rates and seed scenarios -traffic flow= 2200 veh/h, speed limit = 60 km/h.

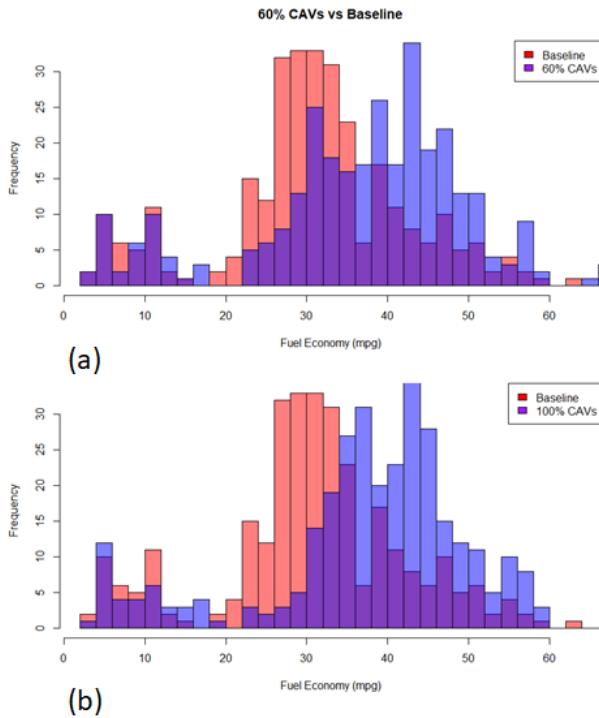


Fig. 6. Comparison of fuel economy histograms for the baseline and (a) 60% MPR, (b) 100% MPR -traffic flow = 2200 veh/h, speed limit = 60 km/h.

the potential of the OCS to reduce fuel consumption under moderate traffic conditions.

The histograms in figure 6 illustrate the increased fuel economy values that are achieved through coordination at 60% and full CAVs MPR. While the fuel economy values for the baseline case are mostly concentrated between 20 MPG and 40 MPG, when all the vehicles are optimally coordinated (MPR = 100%) the achieved fuel economy values are mostly concentrated between 30 MPG and 50 MPG.

Speed and acceleration patterns for 20 consecutive vehicles in the baseline, 20% MPR, 60% MPR and 100% MPR scenarios are shown in figure 7. These speed traces illustrate how the speed profiles smooth out once the MPR reaches 100%.

In the baseline scenario (a), once approaching the merging

point at 800 m, the modeled human drivers in the main road reacted to the incoming merging vehicles coming from the on-ramp. Similarly, some on-ramp vehicles will slow down, waiting for a gap, or accelerate to merge onto the main road. These reactions result in harsh deceleration/acceleration patterns that reduces the traffic efficiency performance. As the number of CAVs start increasing, more vehicles communicate and coordinate the merging maneuver, improving the performance in most cases. It is important to note that, however, at lower penetrations the CAVs are still constrained by the apparently "random" driving behavior of human drivers and the lack of accurate information about their decisions and intentions. Thus, in the presence of human-driven vehicles with no communication capabilities, CAVs have to rely on their sensors information to ensure collision free trajectories. Particularly in the case of low CAVs MPR, CAVs performance will be highly affected by the merging maneuvers of humans thus having to perform harder acceleration/deceleration and eventually stopping maneuvers to preserve safety. Those harsh maneuvers exacerbate the variability in fuel consumption trends. Since the control was applied in a receding horizon fashion, i.e., the optimal control input is updated at every sample time of the simulation, CAVs can react to the maneuvers of a driver who unexpectedly accelerates to merge onto the main road, ensuring collision-free trajectories. This erratic reaction comes at the price of additional acceleration patterns needed later by the CAV to arrive at the merging point at the required speed limit. These changes in acceleration for the sake of safety could affect the upstream traffic, lead to variable results and, in some cases, increase fuel. As the CAVs MPR increases, more vehicles will perform the coordinated merging, thus more homogeneous behaviors can be observed among human-driven vehicles and automated vehicles.

B. Case Study 2

To gain some insights into the sensitivity of the controller to changes in traffic flow, we performed simulations with a single seed for the following traffic flows: 1600 veh/h, 1800 veh/h, 2000 veh/h, 2200 veh/h and 2400 veh/h at a set speed limit of 60 km/h. Figure 8 shows that there is higher variability in the fuel consumption results and, in some cases increased fuel consumption, when the MPR is below 20%. Once the market penetration rate reached 40%, the fuel consumption was reduced in all the simulated cases when compared to the respective baseline. The results in figure 9 suggest that fuel consumption savings increase proportionally with the traffic flow when the MPR exceeds 40%. They also suggest that the benefits of optimal merging coordination are higher under congested scenarios. Conversely, when the MPR is lower than 20%, there is higher variability in the results, which can actually lead to increased fuel consumption since the CAVs performance is limited by a higher number of human drivers. In this case, CAVs will be "reacting" to the instantaneous maneuvers of drivers. These results make apparent that optimal merging coordination at lower traffic volumes will mainly render benefits when the MPR is higher than 40%.

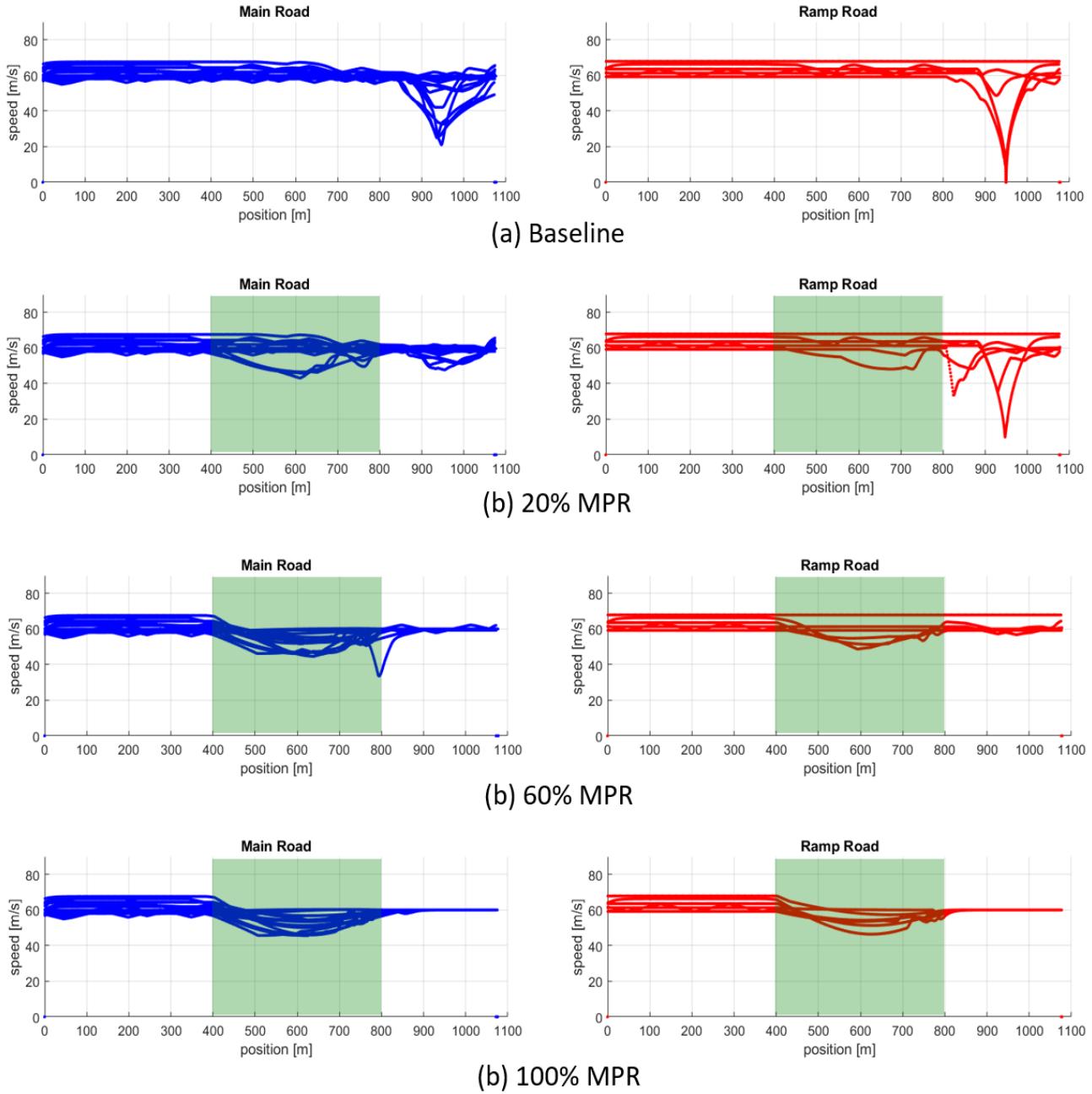


Fig. 7. Speed and acceleration patterns for 20 consecutive vehicles -traffic flow = 2200 veh/h, speed limit = 60 km/h. The shaded area illustrates the control zone.

The results show the importance of comprehensive analysis when assessing the benefits of different CAVs control strategies, which should consider a diversity of scenarios, traffic flow levels and speed limits.

Although the Autonomie fleets used for analysis included electrified powertrains, the sample was too small to ensure statistical significance and thus no conclusions are drawn in this work regarding the benefits for electrified vehicles. Future work will be devoted to estimate energy impacts considering

larger fleets of electrified vehicles.

VI. CONCLUSIONS

This paper presented an comprehensive assessment of the fuel and emissions implications of optimal merging coordination considering partial CAVs market penetration in a heterogeneous traffic environment. We modified the upper layer of the optimal merging coordination control to define the time to merge according to vehicle type. We used a database of vehicle models to estimate fuel consumption for fleets

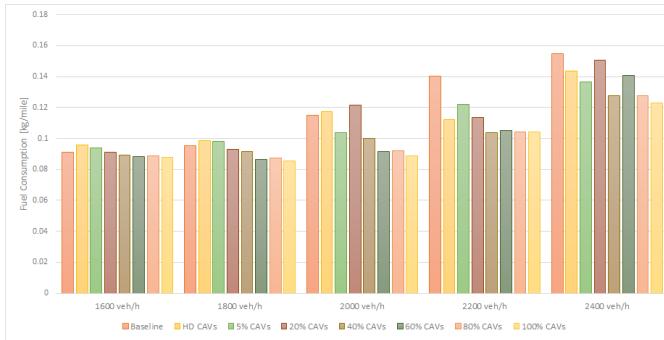


Fig. 8. Average fuel consumption for different CAVs market penetration rates and traffic flow values

	HD CAVs	5%	20%	40%	60%	80%	100%
1600 veh/h	-5.50	-3.40	-0.48	1.58	3.07	2.10	3.26
1800 veh/h	-3.04	-2.84	2.43	4.08	9.38	8.64	10.44
2000 veh/h	-2.09	9.76	-5.64	12.82	20.27	19.98	22.54
2200 veh/h	25.95	14.86	24.71	33.73	29.56	30.37	27.88
2400 veh/h	7.30	11.73	2.75	17.64	9.06	17.44	20.72

Fig. 9. Fuel consumption reduction with respect to baseline for different CAVs market penetration rates and traffic flow values

that are representative of fleets on the roads. The estimation models can simulate a fleet of diverse vehicle types, including conventional and electrified (electric and hybrid electric) light duty vehicles as well as a diverse set of heavy duty vehicles makes and models. Finally, we assessed the benefits of merging coordination on fuel consumption and emissions reduction using microscopic traffic simulations and considering different traffic flows.

The results show that the fuel efficiency and emissions reduction benefits of optimal coordination control are maximized in congested scenarios when the CAVs MPR exceeds 40%. Under moderate congestion, the fuel reduction with respect to the baseline case ranges from 8% to 24% depending on the CAVs MPR. Also, higher variability was observed at lower MPRs due to the limitations imposed over the CAVs maneuvers by the instantaneous decisions of a larger amount of human drivers. These results suggest that higher variability is likely to occur in real world as well.

The higher variations found at lower MPRs underline the importance of comprehensive analyses when assessing the benefits of CAVs control strategies. Such analyses should consider a diversity of scenarios, traffic flow levels and speed limits. Additional factors that could be considered to characterize the levels of variability include different ratios of HDV/LDV on the road, different traffic conditions throughout a day, week, or at highway corridors, etc.

The results of this study are highly tied to traffic demands and the merging road's geometry. The desired parameters and boundaries used to solve the optimal control problem should be properly adjusted to apply to other on-ramp merging types, e.g., a cloverleaf interchange. Note that the overall problem definition and solution of the optimal coordination control systems can be generalized for other conflict points

in a traffic network, i.e., intersections, roundabouts and speed harmonization applications.

Current work is undergoing to validate the real time capabilities and the fuel efficiency benefits of the proposed merging control using real vehicles in a test track environment.

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