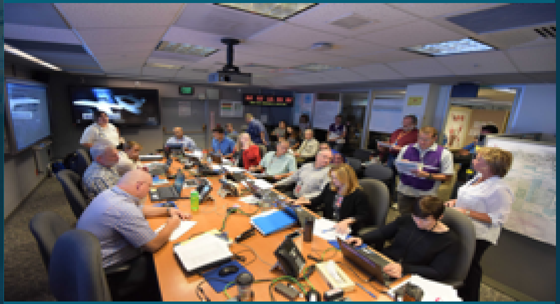
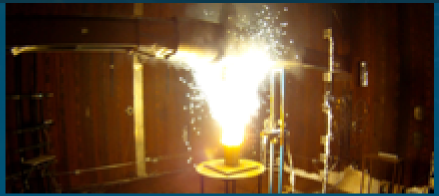
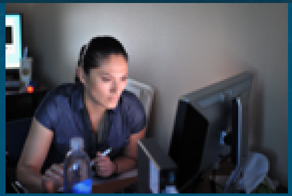
Sandia National Laboratories

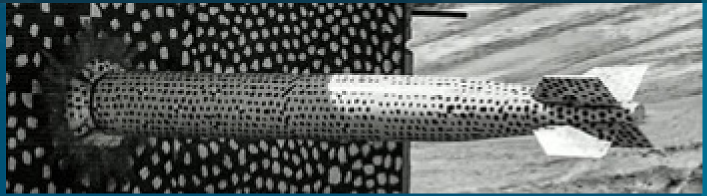
SAND2020-13210C

Adaptive Computational Plasticity with a Composite Tetrahedral Element









Eccomas Congress 2020 & 14th WWCM

Virtual Congress, January 11-15, 2021

Presented by

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U.S. DEPARTMENT OF ENERGY NNSA

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Introduction

- Background
  - Tetrahedral workflows: drastically ↓ time to solution vs. hexahedral (100x)
  - However: FEM + plasticity with tetrahedra → often non-standard (locking)
  - Composite tetrahedral element: demonstrated promise in this application space
  - Adaptivity: ability to ↓ time to solution while maintaining computational accuracy
  - Idea: Combine Composite tetrahedra + adaptivity for dramatic ↓ time to solution
  - However: adaptivity with higher order tetrahedra is currently a research topic
- Goals of this talk
  - Outline initial approach for higher order adaptive tetrahedral workflows

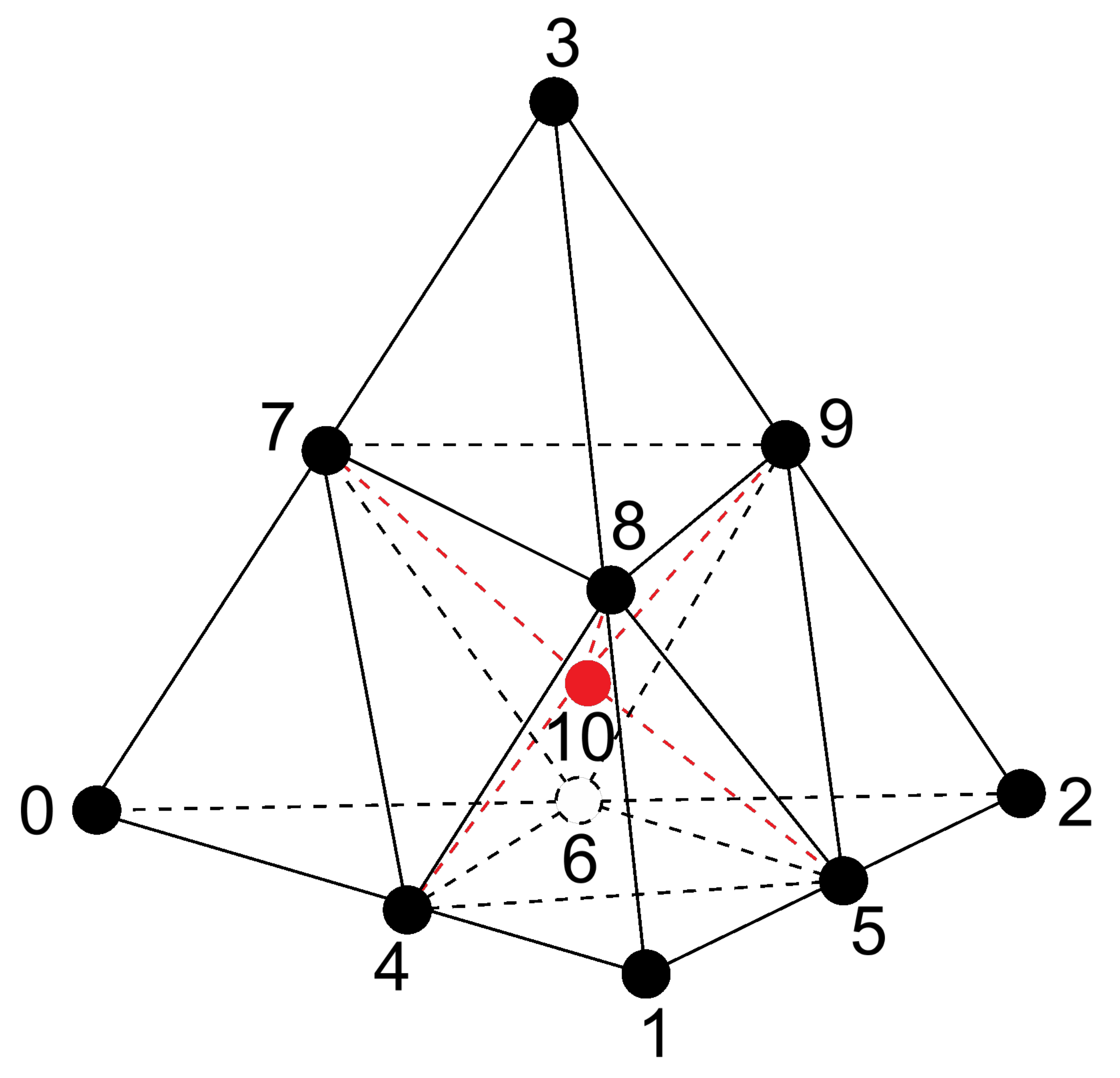
Composite Tetrahedron

Composite Tetrahedral Element

- Composite of twelve sub-tetrahedra
- Piecewise linear basis defined over sub-tetrahedra
- Assumed linear gradient space over entire element
- Analytic consistent mass matrix
- Four point numerical integration rule
- Volume averaged pressure and det(F) to circumvent l

Sub-tetrahedron	Nodes				Sub-tetrahedron	Nodes			
E_0	0	4	6	7	E_6	9	8	7	10
E_1	1	5	4	8	E_7	7	8	4	10
E_2	2	6	5	9	E_8	4	5	6	10
E_3	3	8	7	9	E_9	5	9	6	10
E_4	4	8	5	10	E_{10}	9	7	6	10
E_5	5	8	9	10	E_{11}	7	4	6	10

Table 4-1. Connectivity for the twelve sub-tetrahedra.



* Ostien, J. T., Foulk, J. W., Mota, A., & Veilleux, M. G. (2016). A 10 - node composite tetrahedral finite element for solid mechanics. *International Journal for Numerical Methods in Engineering*, *107*(13), 1145-1170.

Adaptive Strategy

Overview

1. Generate a mesh of composite tetrahedra that conforms to a CAD geometry.
2. Run a Lagrangian explicit dynamics simulation until adaptation criteria is met
3. Update the reference configuration for the Lagrangian simulation
4. Prescribe a “mesh size field” to drive adaptation
5. Perform conformal mesh adaptivity with local cavity operations on *linear* elements by disregarding edge-nodes positions.
6. Re-insert known edge-nodes at edges that were not modified by adaptivity
7. Insert edge-nodes at edge mid-point for any newly created edges during adaptivity
8. Straighten edges to prevent inverted composite tetrahedral elements
9. Perform solution mapping from previous mesh to new mesh
10. Repeat steps 2 through 7 until the final simulation time

All of these steps occur in-core (in-memory), i.e. no file-based transfer.

Main idea: mix robustness of mesh adaptivity on linear elements with enabling technology of higher order composite tetrahedral element.

Explicit Lagrangian Dynamics

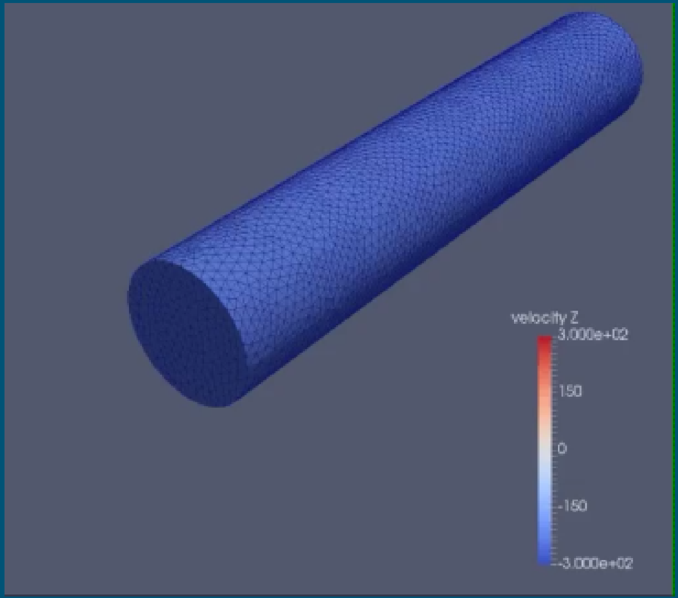
$$\boldsymbol{x} = \boldsymbol{\varphi}(\boldsymbol{X}, t) : \Omega \times [0, T] \to \mathbb{R}^3,$$

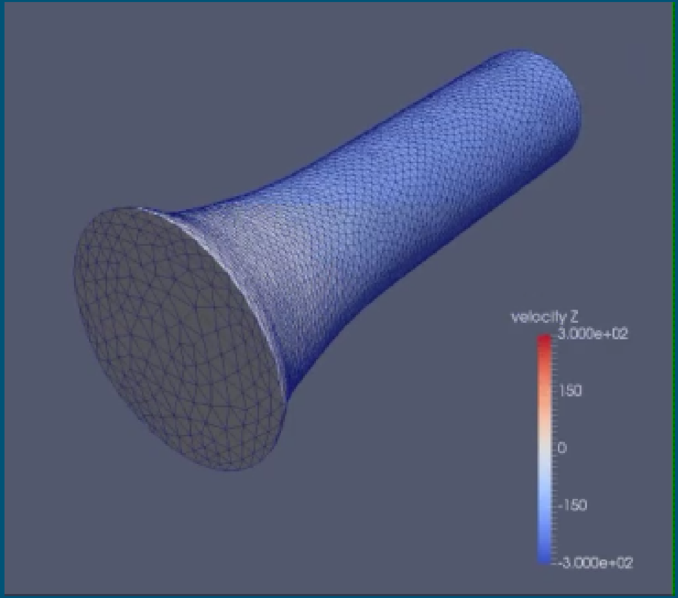
$$\boldsymbol{F} := \operatorname{Grad} \boldsymbol{\varphi}$$

$$\int_0^T \left[\int_\Omega (\operatorname{Div} \boldsymbol{P} + R\boldsymbol{B} - R\ddot{\boldsymbol{\varphi}}) \cdot \boldsymbol{\xi} \, \mathrm{d}V + \int_{\partial_{\boldsymbol{T}}\Omega} \boldsymbol{T} \cdot \boldsymbol{\xi} \, \mathrm{d}S \right] \mathrm{d}t = 0,$$

$$\mathcal{S} := \left\{ \boldsymbol{\varphi} \in W_2^1(\Omega) : \boldsymbol{\varphi} = \boldsymbol{\chi} \text{ on } \partial_{\boldsymbol{\varphi}}\Omega \times [0, T] \right\}$$

$$\mathcal{V} := \left\{ \boldsymbol{\xi} \in W_2^1(\Omega) : \boldsymbol{\xi} = \boldsymbol{0} \text{ on } \partial_{\boldsymbol{\varphi}}\Omega \times [0, T] \right\}$$





Triggering Mesh Adaptivity: Element Quality

$$\eta_k = \left(\frac{V_k}{\gamma_k (\bar{l}_k)^3} \right)^{\frac{2}{3}}.$$

Mean ratio element (k) quality metric

$$\bar{l}_k := \left(\frac{1}{6} \sum_{i=0}^{6} (l_{k,i})^2 \right)^{\frac{1}{2}},$$

Root mean-squared edge length of tetrahedron

$$\gamma_k := \frac{1}{\sqrt{72}}.$$

Volume of equilateral tetrahedron with unit edge length

$$\eta_{k,\min} := \min_k \eta_k,$$

Given these definitions, compute minimum quality element over all elements k

$$\eta_{k,\min} < \eta_{k,\min}^{\text{desired}} \quad \text{AND} \quad \begin{cases} \eta_{k,\min} \leq \eta_{k,\min}^{\text{old}} - 0.02 \quad \text{OR} \\ \eta_{k,\min} \leq \eta_{k,\min}^{\text{allowed}} + 0.02 \end{cases}$$

Adapt if quality is below a *desired* threshold AND is decreasing too rapidly (in a transient sense) OR is below a *required* threshold

Motivation: maintain high quality elements so characteristic element size (h) is well-controlled → well-controlled explicit time step size

Mesh Size Field

- Input to mesh adaptation software
- Defines desired edge lengths (h) at mesh vertices
- For this work: specified two mesh size fields

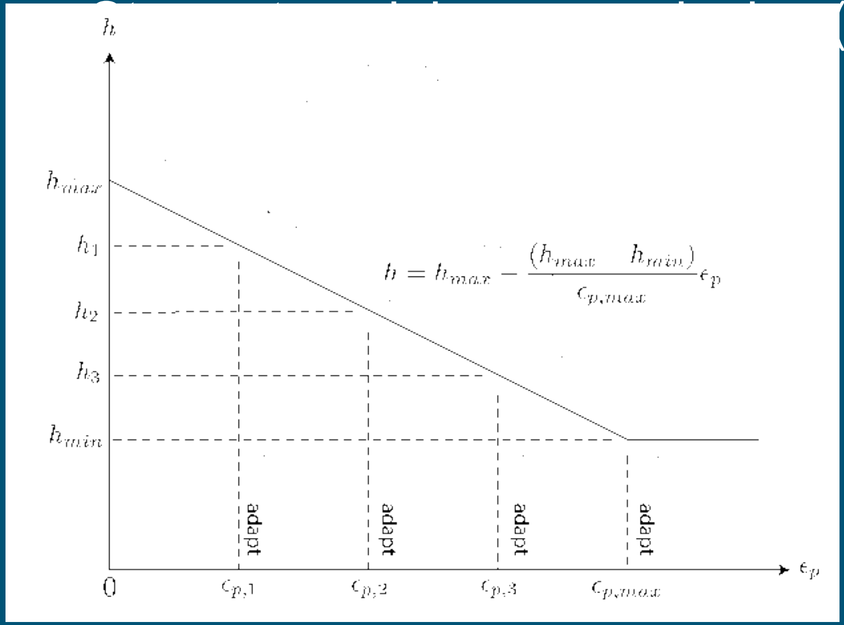
$$h_v^{\text{iso}} = \frac{\sum_{edge=1}^{n_{\text{adj}}} h_{\text{edge}}}{n_{\text{adj}}},$$

- Implied isotropic size field
  - Attempts to maintain element size throughout a simulation
  - Sets size at vertex as average of all adjacent edge lengths to the chosen vertex (v)
  - Useful in explicit dynamics where h → 0 implies dt → 0

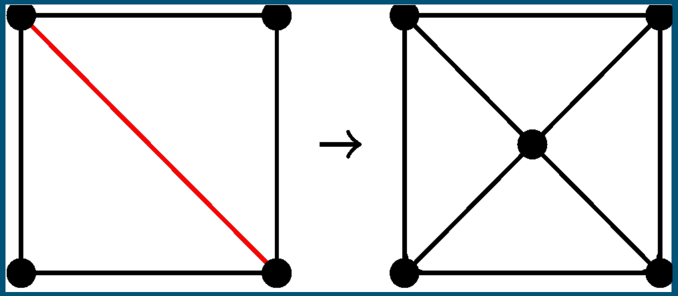
$$\bar{\epsilon}_{p,v} = \frac{\sum_{elem=1}^{n_{\text{adj}}} \sum_{qp=1}^{n_{\text{qps}}} \epsilon_{p,elem}\big|_{qp}}{n_{\text{adj}} n_{\text{qps}}}.$$

$$h_v^{\text{eqps}} = \begin{cases} h_{\max} - \dfrac{(h_{\max} - h_{\min})}{\epsilon_{p,\max}} \bar{\epsilon}_{p,v} & \text{if} \quad \bar{\epsilon}_{p,v} \leq \epsilon_{p,max} \\ h_{\min} & \text{otherwise} \end{cases}$$

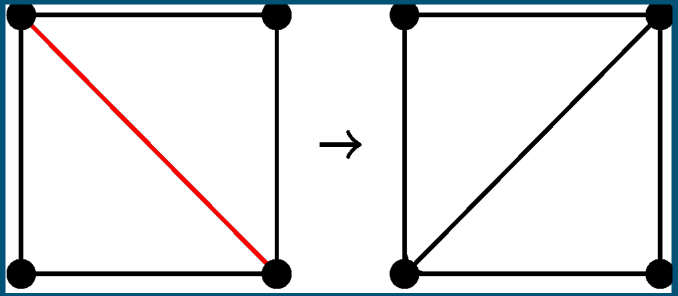
- Refinement with equivalent plastic strain
  - Mesh size (h_v)↓ linearly as eqps ↑
  - (h_min)



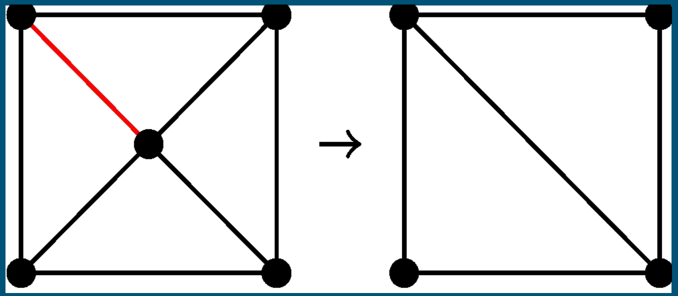
Cavity-Based Mesh Modification (Edge Operations)



- Split: A new vertex is placed mid-edge and new edges formed by connecting new vertex to cavity vertices.
  (2 elements → 4 elements, refinement)

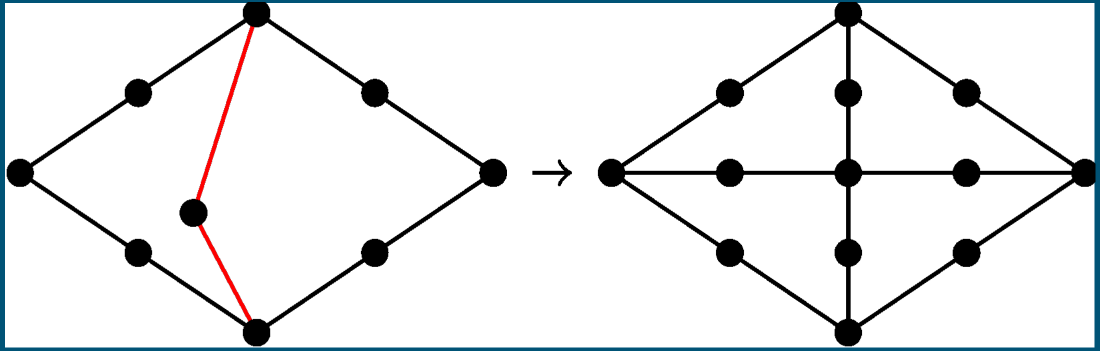


- Swap: A new edge is creating by `swapping' the vertices that define the edge.
  (2 elements → 2 elements, quality)



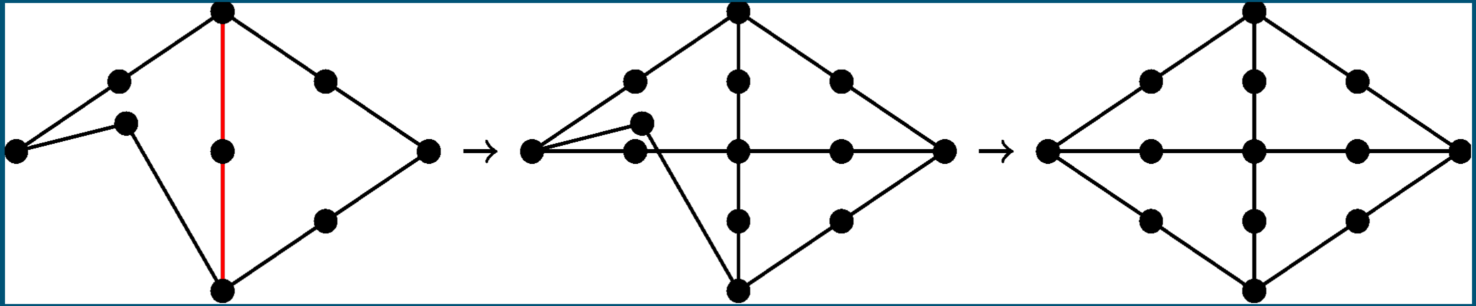
- Collapse: Two vertices that define an edge are `collapsed' into a single vertex.
  (4 elements → 2 elements, coarsening)

Edge Straightening



- In some scenarios, placing edge nodes at mid-point for new edges actually improves element quality
- e.g. Splitting a curved edge (red) results in four new straight edges and four new elements with good quality

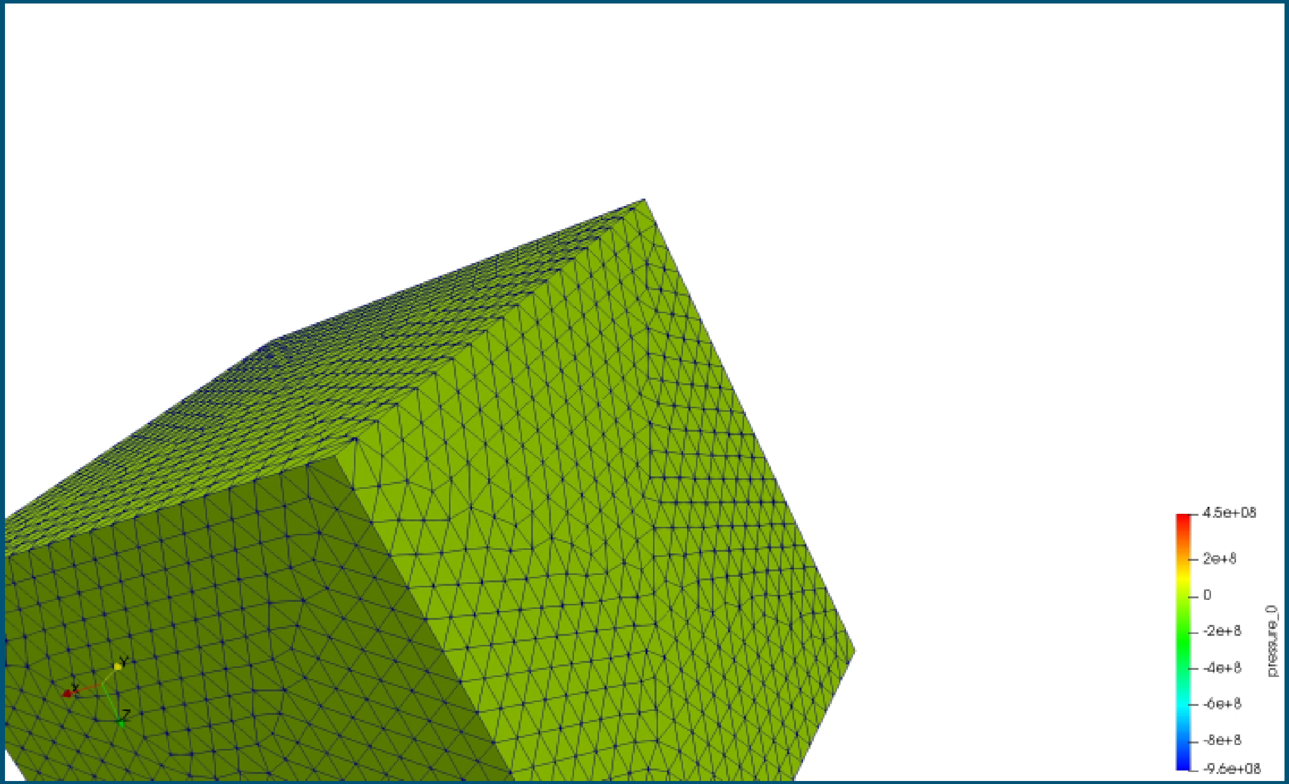
- In other scenarios, it can be catastrophic!
- e.g. Splitting an edge (red) adjacent to a curved element results in an inverted composite tetrahedral element.
- Solution: Straighten all edges adjacent to inverted elements
- Must be done iteratively

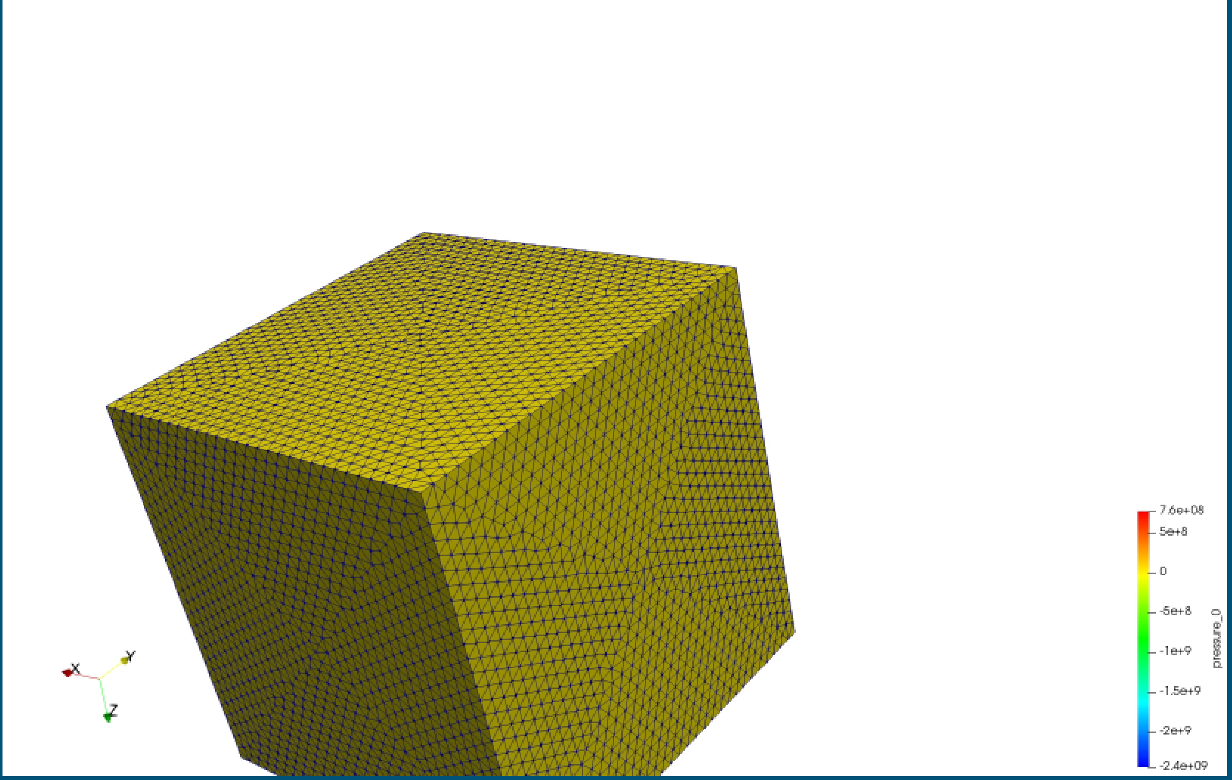


Results

Maintaining a constant time step

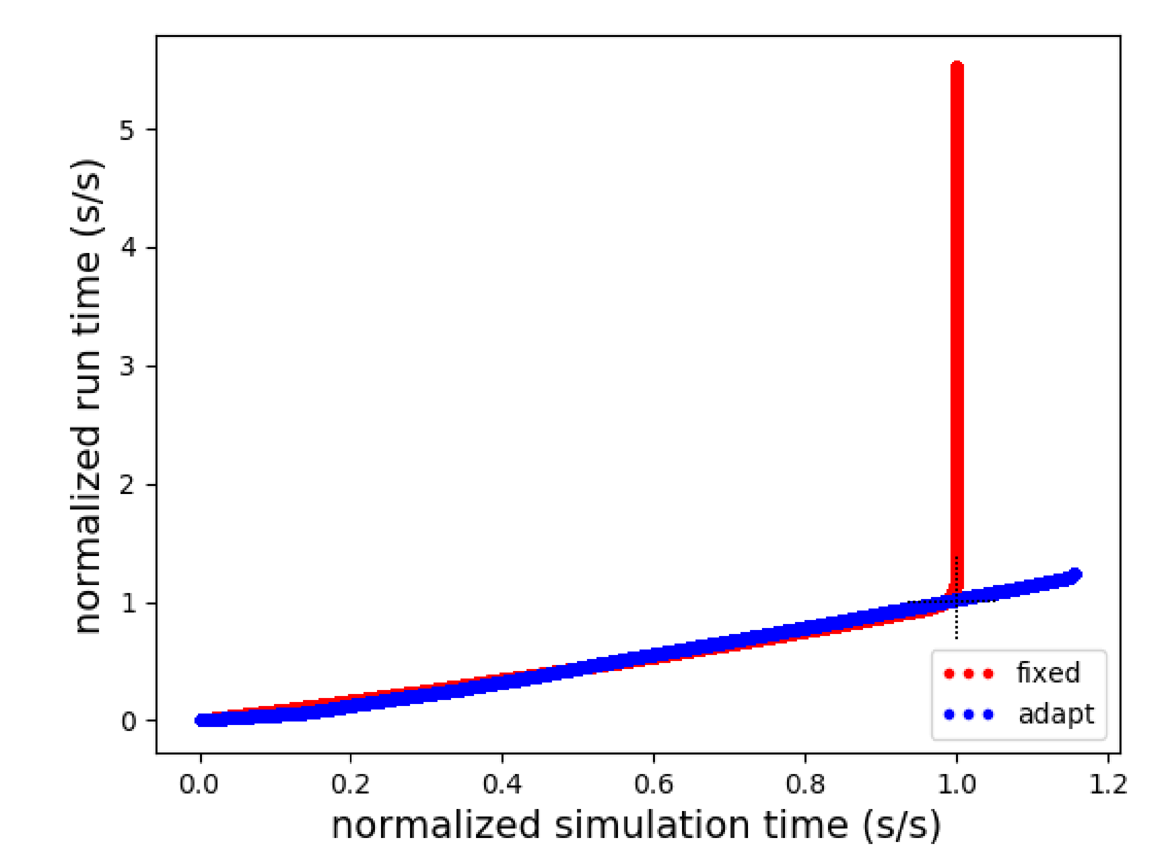
- Without adaptivity, the time step decreases by 10 orders of magnitude (initial: *4e-7*, final: *6e-17*)
- Status quo: want to keep time step constant? *Kill elements.*

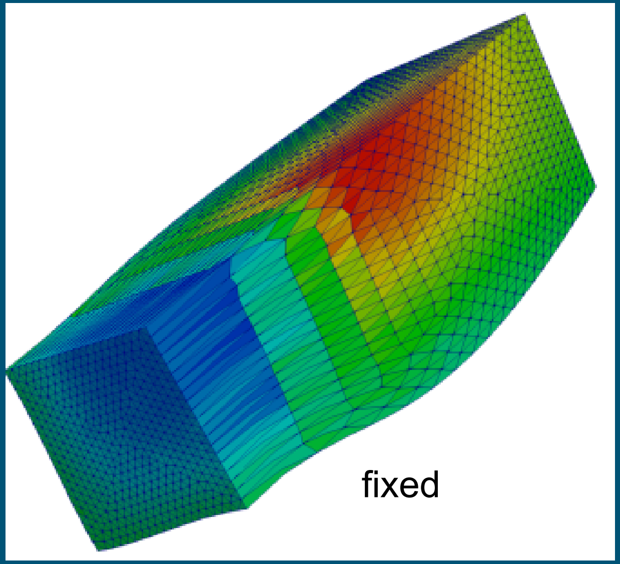


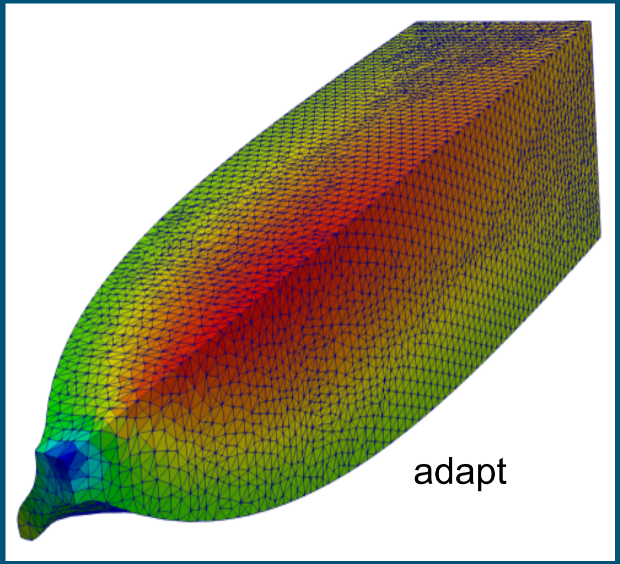


- With adaptivity, time step only decreases by a factor of 2: (initial *4e-7*, final: *1.8e-7*)
- Resource driven adaptivity: in my model, I can afford X, do your best with X

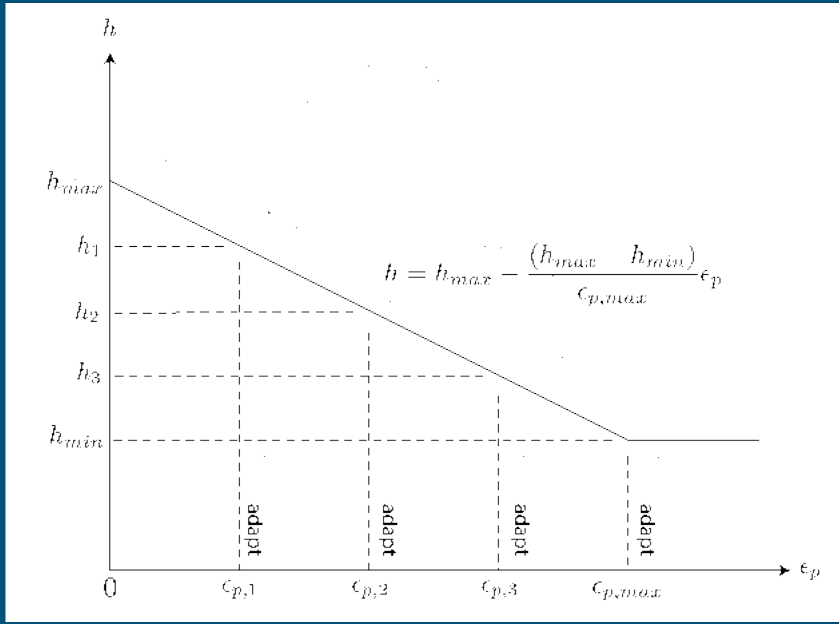
Maintaining a Constant Time Step





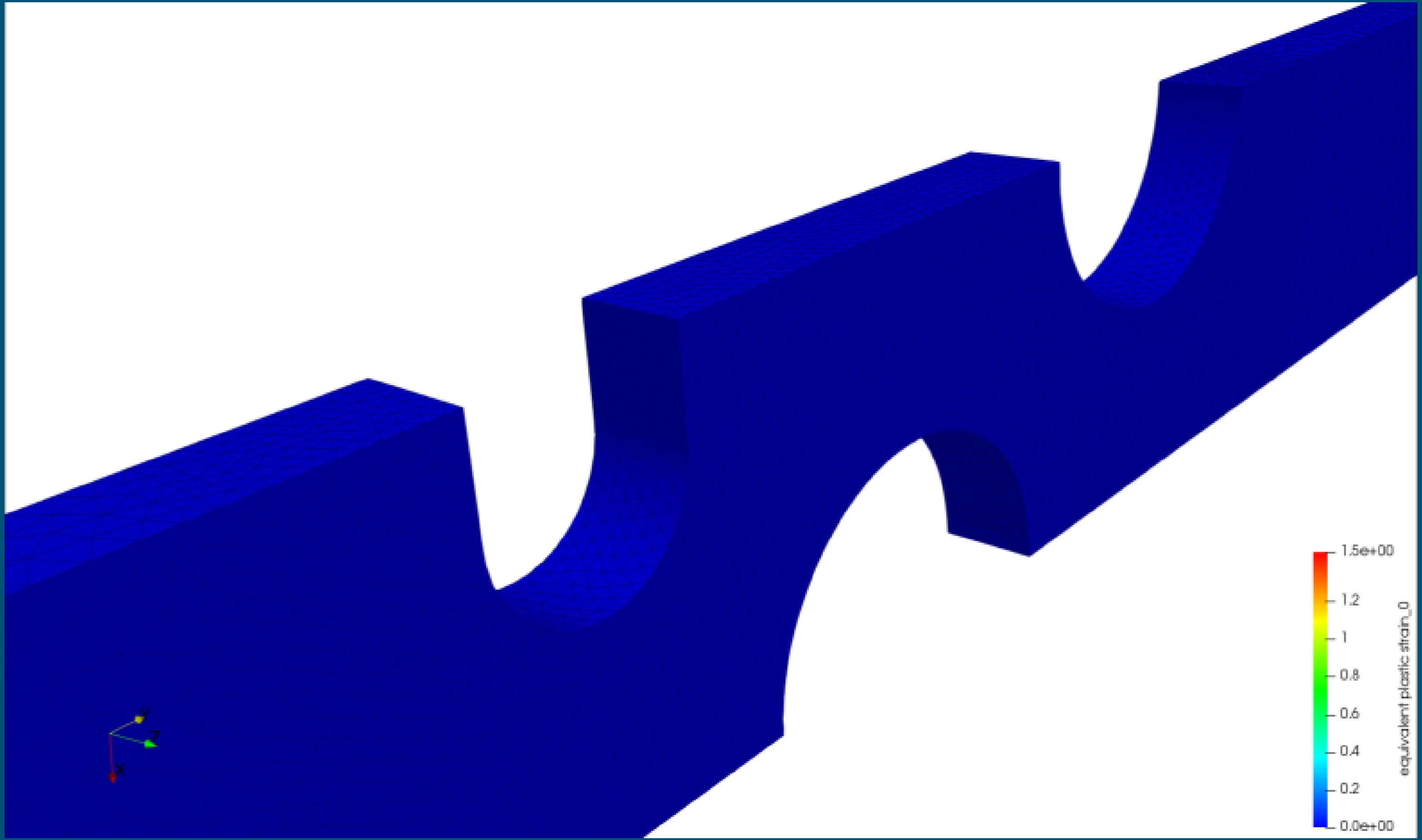


Resolution of Physics: Refinement to eqps

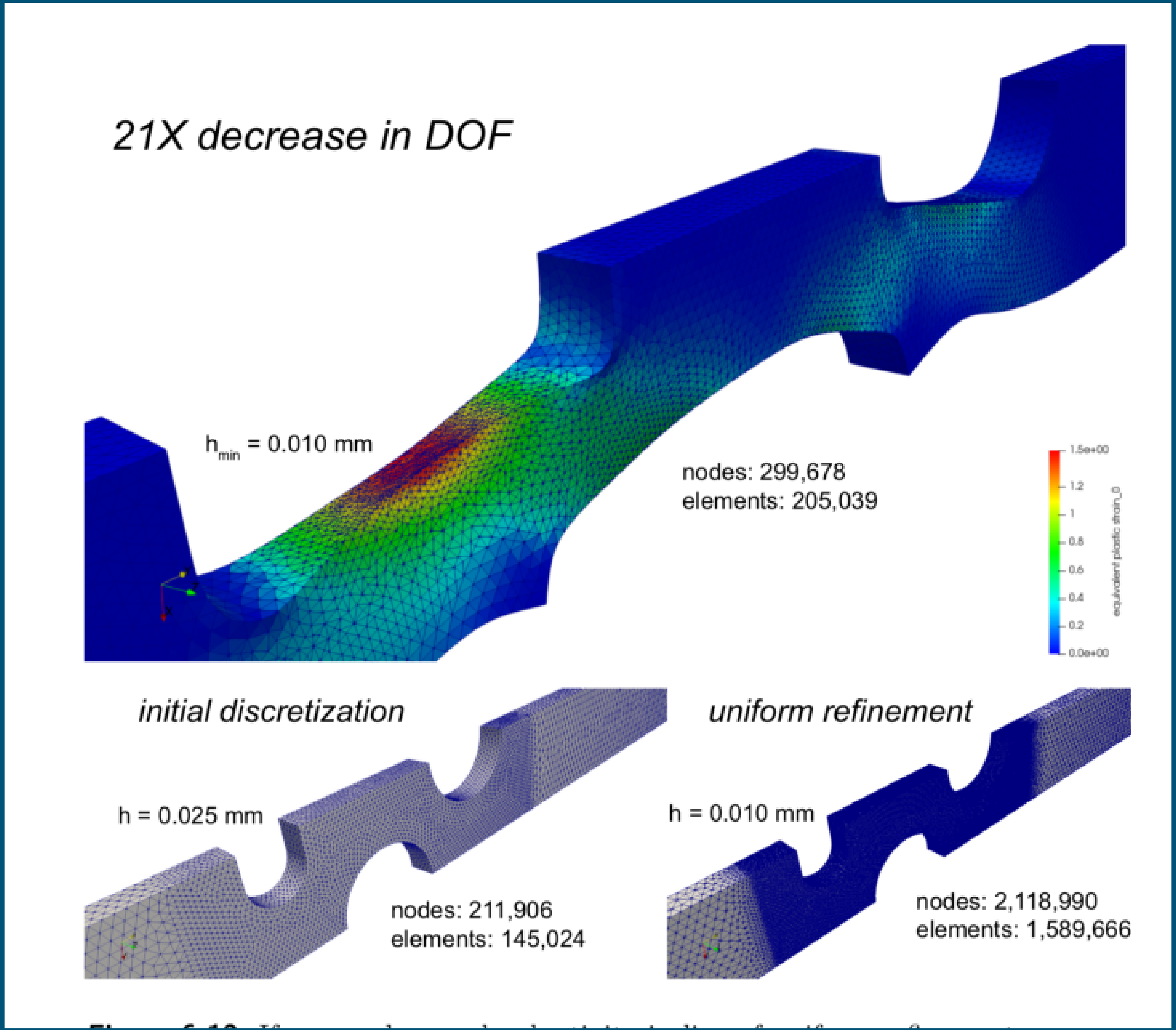


$$h_{max} = 0.25 \text{ mm}$$
$$h_{min} = 0.10 \text{ mm}$$
$$\epsilon_{p,max} = 1.0$$

mesh adapts when the linear tet mean ratio metric < 0.3



Resolution of Physics: Refinement to eqps



Conclusions and Future Outlook

Conclusions and Future Outlook

- Developed a very initial workflow for adaptive plasticity FEM simulations
- Main idea: combine robustness of linear tetrahedral adaptivity with enabling technology of a newly developed higher-order composite tetrahedral element
- To do: incorporate edge node information into mesh adaptivity operations for truly higher-order mesh adaptivity
- Not touched on: great work done in space of solution transfer for internal and primary solution variables, incorporate this into current adaptive plasticity workflows

THANK YOU!

QUESTIONS?