

Computational simulation of coupled poromechanical processes in deformable fractured and porous media

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Introduction

The formulation for poromechanical problems can be fully characterized with four equations: three geomechanics equations and one fluid mechanics equation. In a fully-coupled scheme, this set of equations is solved simultaneously with computational methods that model both flow and mechanics. However, the fully coupled approach requires significant computational resources and a unified flow-mechanics scheme. Alternatively, an identical solution can be obtained with much lower computational cost through sequentially coupled methods, where separate solvers for mechanics and flow are used. A fixed stress scheme is one feasible solution method. In a

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Verification with Benchmark Problems

The fixed stress scheme implementation was verified with poromechanical benchmark problems. In all three problems, both the solid and liquid phases were considered compressible. The analytical solution and fully coupled numerical solution were compared with the fixed stress results. The solutions are compared on the basis of pore pressure time history at a specified point in the mesh.

Terzaghi Problem

The problem is a one-dimensional representation of a confined compression test. The conditions posed here are slightly modified from Terzaghi's original, as both the pore fluid and the solid particles to be compressible [3]. A comparison of the pore pressure at mid height is shown:

A comparison of the root mean square error and the computational time are shown below:

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Verification with Experimental Data

Conclusions

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INTRODUCTION

Motivation

Prediction of flow, transport, and deformation in fractured porous media is critical to improving our scientific understanding of coupled thermal-hydrological-mechanical-chemical (THMC) processes across subsurface energy activities. As an example, poroelastic coupling can account for the elastic deformation of a solid material as a reaction to pore pressure driven by mechanical stressing. Biot's formulation for poroelastic problems can be solved via a fully-coupled or iteratively-coupled schemes.

Objectives

- Implementation of poromechanics coupling via a fixed stress scheme into an existing Sandia Sierra multiphysics software
- Development of validation problems against well-controlled laboratory data

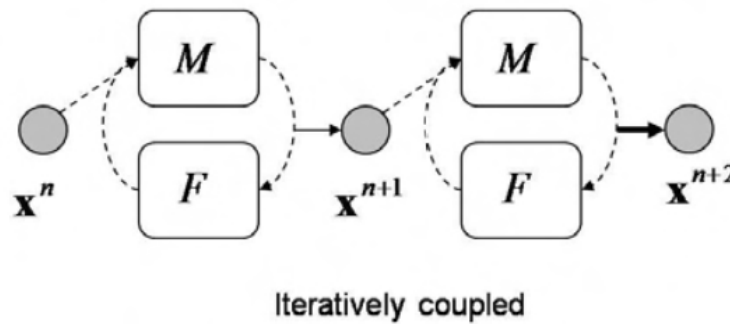
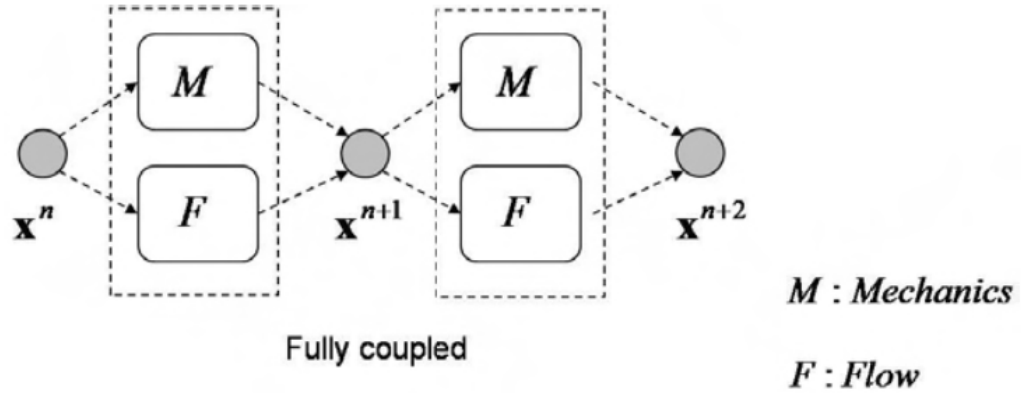
Approach

- Verification of the fixed stress scheme for poroelasticity using three benchmark problems (one dimensional (1D) Terzaghi, 2D Mandel, and 3D Cryer sphere problems)
- Comparison of the fully coupled and fixed stress solutions against analytical solutions
- Evaluate the convergence, accuracy, and computational efficiency over a range of permeability and elastic properties
- Construction of validation case based on hydro-mechanical testing in fractured media

MODELING DETAILS

Poro-mechanics Coupling Schemes

Both fully-coupled and fixed-stress schemes are investigated. Within a fixed-stress scheme, the flow problem (i.e, pore pressure) is solved first while setting the time derivative of total mean stress to zero. Then, the mechanics problem (i.e., displacement) is solved as below (from Kim et al. [1]):



$$\begin{bmatrix} \mathbf{u}^n \\ \mathbf{p}^n \end{bmatrix} \xrightarrow{\mathcal{A}_{fc}} \begin{bmatrix} \mathbf{u}^{n+1} \\ \mathbf{p}^{n+1} \end{bmatrix} \text{ where } \mathcal{A}_{fc}: \begin{cases} \nabla \cdot \boldsymbol{\sigma} + \rho_b \mathbf{g} = \mathbf{0} \\ \dot{m} + \nabla \cdot \mathbf{w} = \rho_{f,0} f \end{cases}$$

$$\begin{bmatrix} \mathbf{u}^n \\ \mathbf{p}^n \end{bmatrix} \xrightarrow{\mathcal{A}_{ss}^p} \begin{bmatrix} \mathbf{u}^* \\ \mathbf{p}^{n+1} \end{bmatrix} \xrightarrow{\mathcal{A}_{ss}^u} \begin{bmatrix} \mathbf{u}^{n+1} \\ \mathbf{p}^{n+1} \end{bmatrix} \text{ where } \begin{cases} \mathcal{A}_{ss}^u: \nabla \cdot \boldsymbol{\sigma} + \rho_b \mathbf{g} = \mathbf{0}, p: \text{prescribed} \\ \mathcal{A}_{ss}^p: \dot{m} + \nabla \cdot \mathbf{w} = \rho_{f,0} f, \delta \dot{\boldsymbol{\sigma}} = 0 \end{cases}$$

Sandia Sierra Multiphysics Toolkit

The fixed-stress scheme is implemented in the Sandia Sierra multiphysics toolkit, composed of a suite of highly parallelized finite element analysis code modules including a thermal/fluid mechanics module (Aria) and a solid mechanics module (SM). The module coupling approach is solved using Arpeggio [2-4].

- Fully-coupled approach
 - Sierra/Aria for poroelasticity
- Fixed-stress split scheme
 - Implementation in Sierra/Aria
 - Expansion into Arpeggio - coupling Sierra/Aria with Sierra/SM
- Shell elements for fractured media

In Aria, the mass and fluid flow of the porous medium are obtained according to Darcy's Law. The material uses a Coussy porosity model and a compressible Boussinesq model for density. Linear hex elements were used in the benchmark problems, unless noted otherwise.

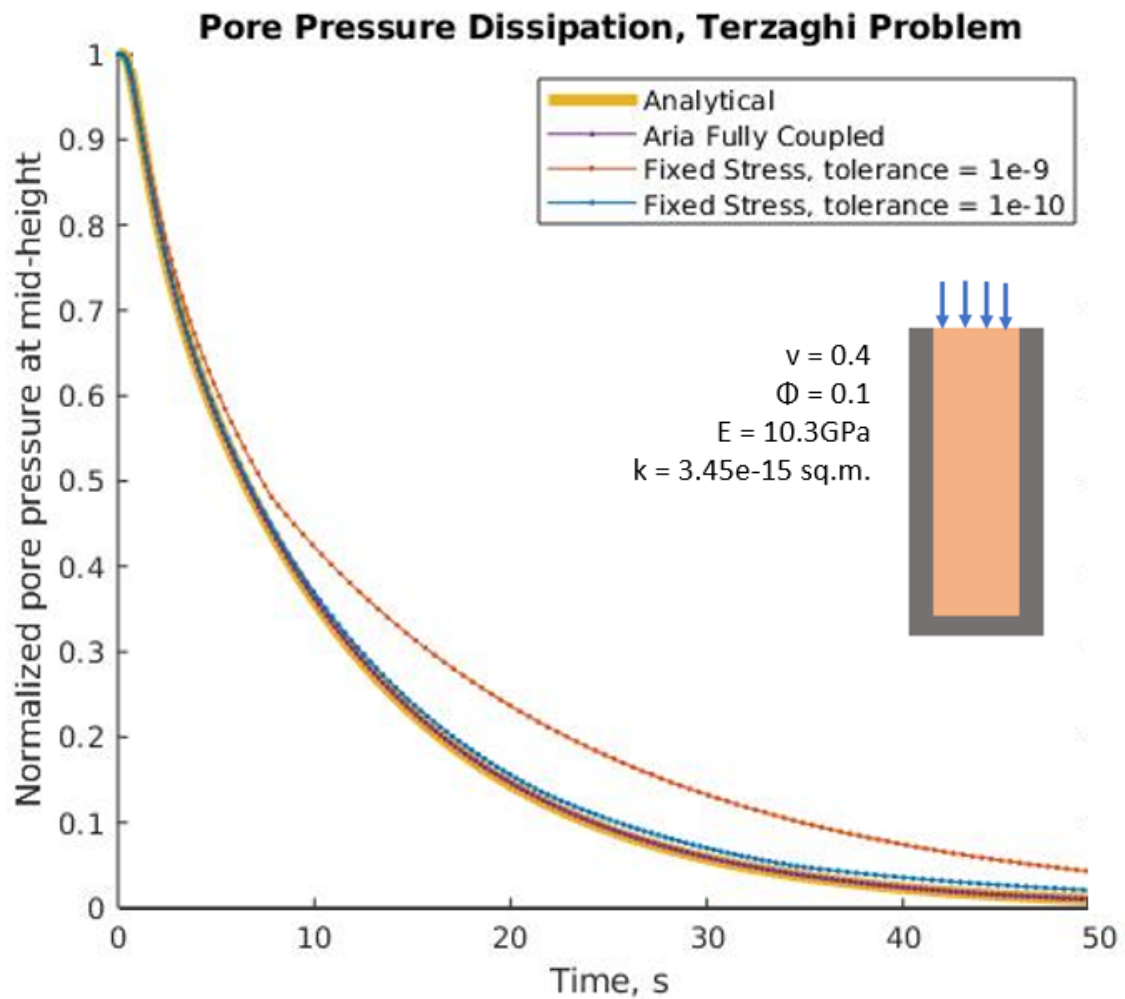
VERIFICATION WITH BENCHMARK PROBLEMS

For three poroelastic benchmark problems, both the solid and liquid phases were considered compressible. To verify the fixed-stress scheme, results of pore pressure time history at a specified point were compared with both analytical solution from Verruijt [5] and the fully-coupled numerical solution.

Terzaghi Problem

- 1D representation of a confined compression test with drainage at top
- 1m tall soil column with properties of Bandera sandstone from Mann and Fatt [6]

Comparison of pore pressure at mid-height:

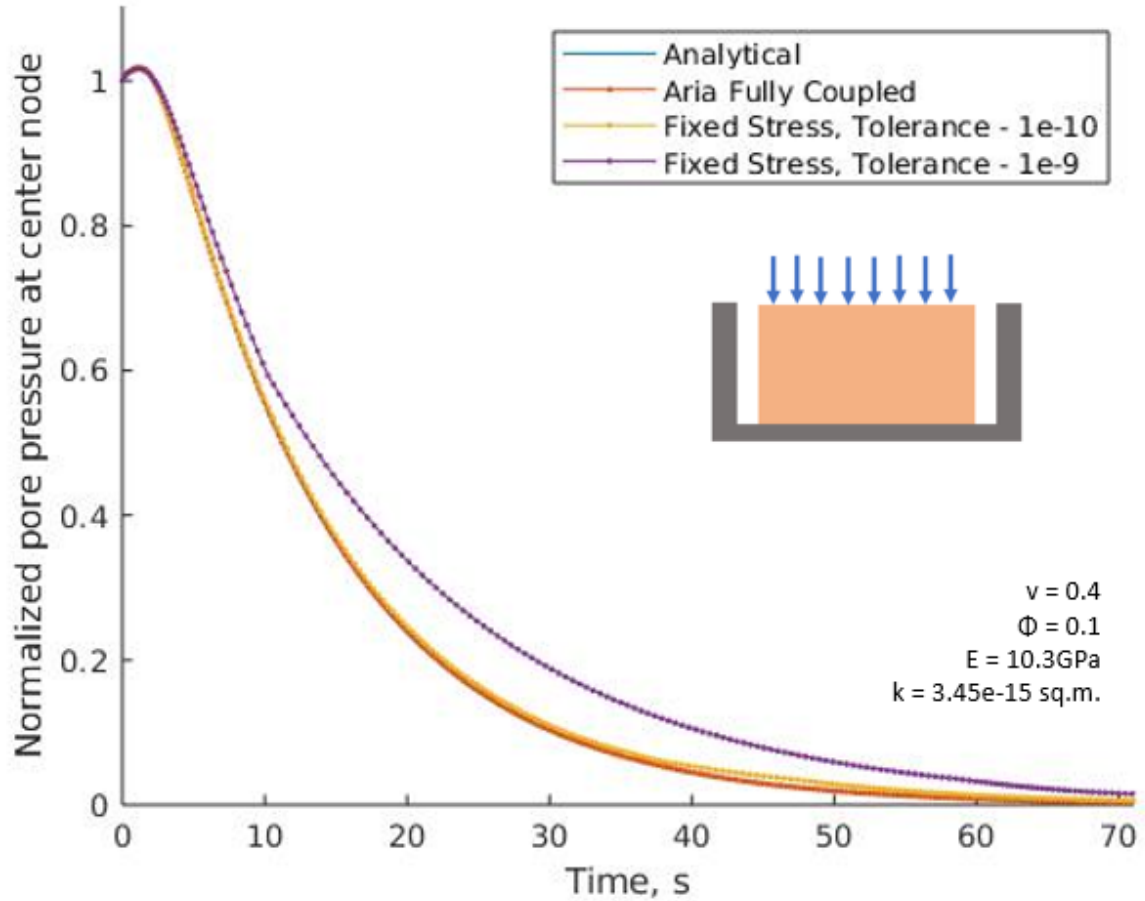


	Tolerance	RMS Error	Solution Time (s)
Fully Coupled	-	0.0257	11.8
Fixed Stress (Aria)	1e-10	0.127	9.03
Fixed Stress (Aria)	1e-9	0.680	6.31

Mandel Problem

- 2D plane strain problem, vertically compressed soil with lateral drainage
- Surface traction of unity, model length $a=1\text{m}$ and height $b = 0.2\text{m}$, and properties of Bandera Sandstone from Mann and Fatt [6]
- Prescribed displacement on top surface according to analytical solution, since problem requires uniform vertical deformation at the top plane

Compare pore pressure at the center point of the problem:



	Tolerance	RMS Error	Solution Time (s)
Fully Coupled	-	.0299	40.67
Fixed Stress (Aria)	$1e-10$.111	59.45
Fixed Stress (Aria)	$1e-9$.765	35.01

- For both the Terzaghi and Mandel problems, mesh refinement causes the fixed-stress solution to converge to the analytical solution at larger tolerances

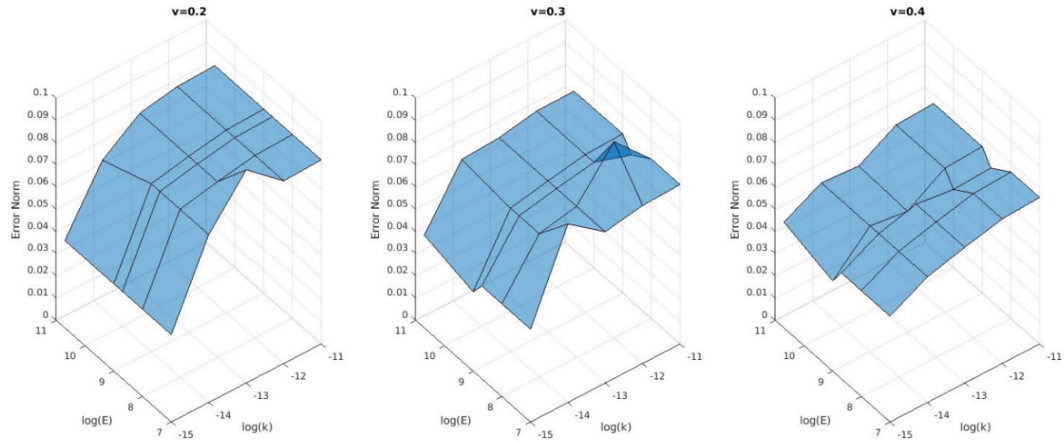
Cryer Sphere Problem

- 3D poroelasticity problem (1/8 of a sphere)
- Sensitivity study for material parameters:

- Elastic modulus (10MPa to 50GPa)
- Permeability (1e-11 to 1e-15 m²)
- Poisson's ratio (0.2 to 0.4).

Sensitivity study

A total of 75 cases were tested. The RMS error of these cases are plotted below.

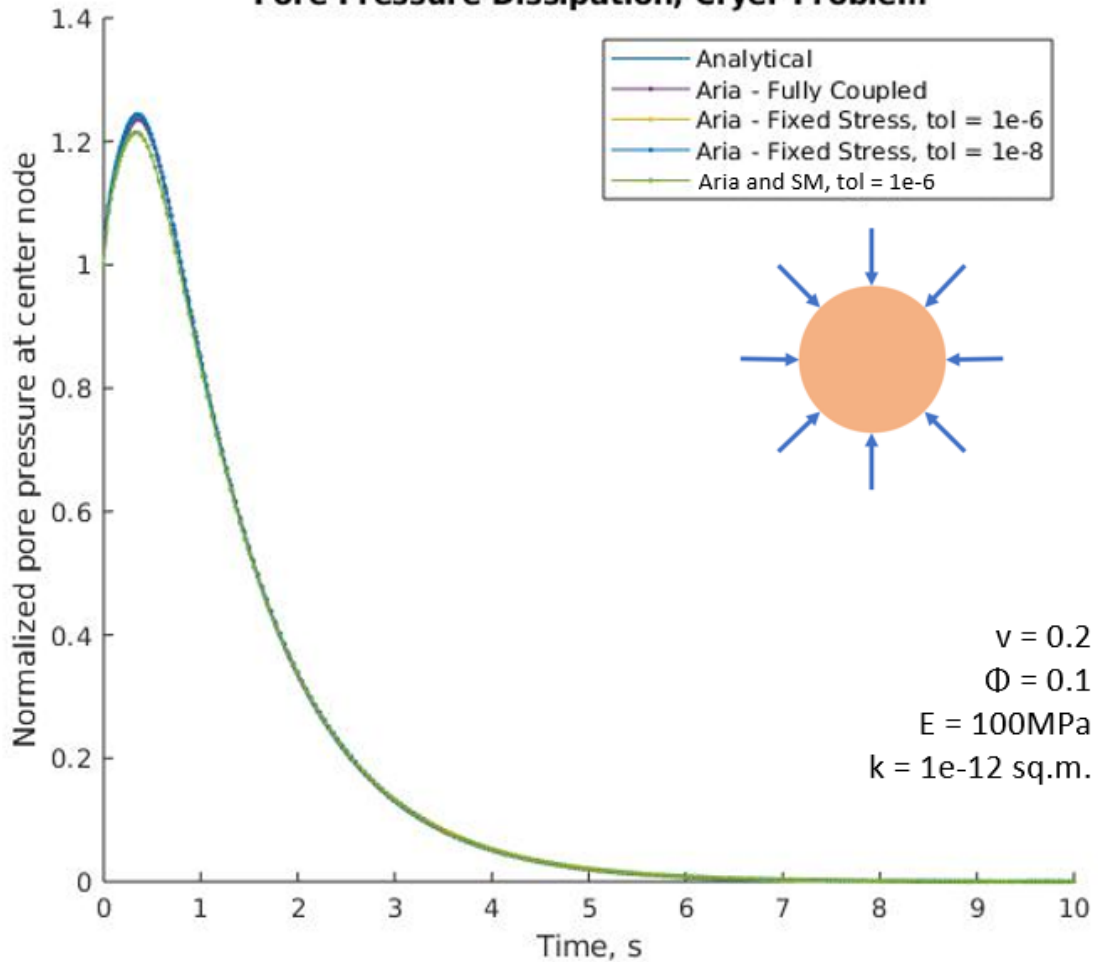


- The error tends to be higher in magnitude for stiffer, more permeable porous media.
- Some parameter combinations with errors fell outside of the general trends
 - Peak-like irregularities in above plot
 - These cases exhibited comparatively steep pressure increases (i.e., Mandel-Cryer Effect) before pressure began to dissipate [5].

Accuracy and computational time comparison

To further evaluate the accuracy of these cases, a porous medium with permeability of 1e-12 m², elastic modulus of 100MPa, and Poisson ratio of 0.2 was investigated. The pore pressure profile over time is shown below:

Pore Pressure Dissipation, Cryer Problem



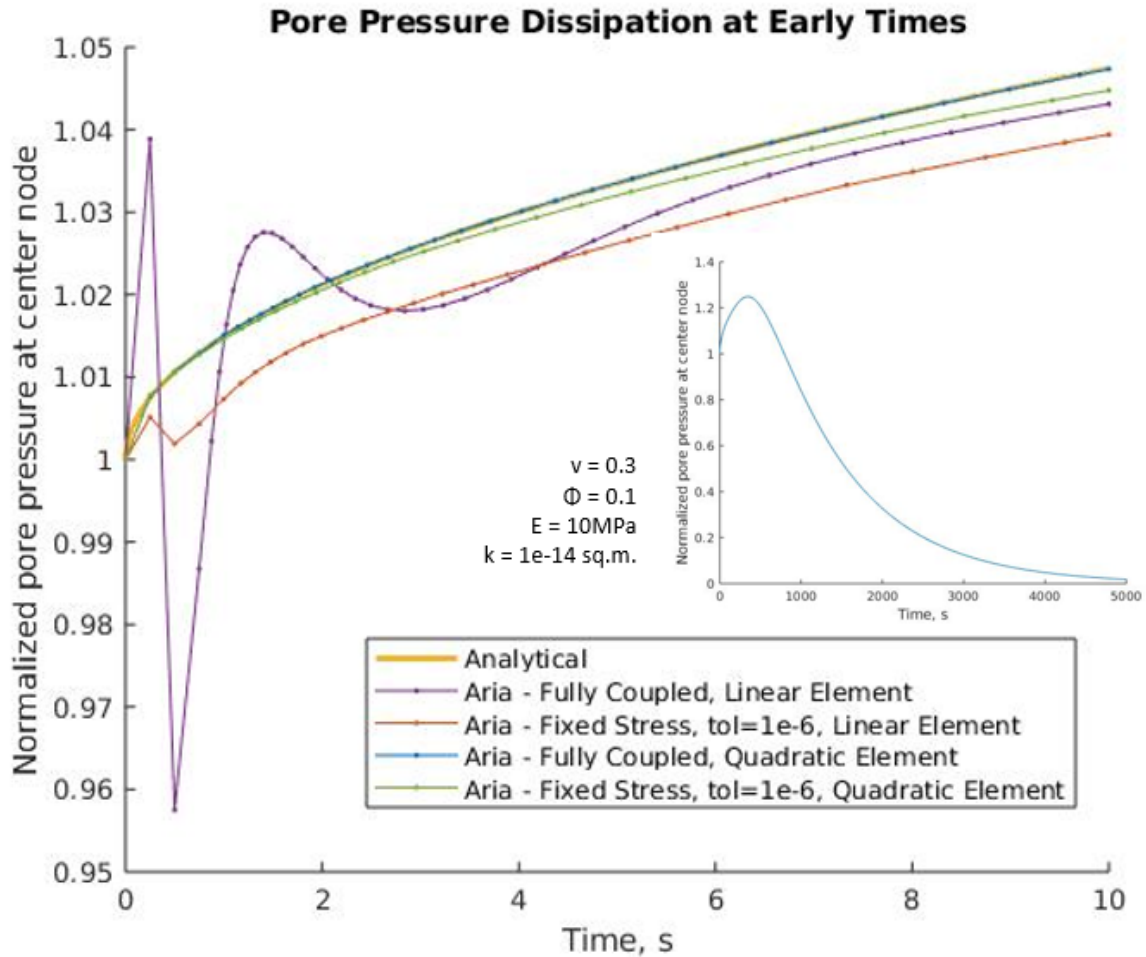
- Most of cases show acceptable accuracy
- The fixed stress scheme with data passed between Aria and SM was the least accurate primarily due to different formula implemented in different fluid mechanics and solid mechanics modules.
- Mesh was refined to have a lower error, but significantly higher computational time

	Number of Elements	Tolerance	Initial Timestep (s)	RMS Error	Solution Time (s)	
Fully Coupled	1900	-	0.05	.0690	986	
			0.025	.0407	805	
Fixed Stress (Aria)		1e-6	0.05	.0855	329	
			0.025	.0644	362	
		1e-8	0.05	.0747	518	
			0.025	.0527	579	
Fixed Stress (Aria and Sierra SM)		36250	1e-6	0.05	.3926	155
				0.025	.3951	182
	0.05			.1624	5368	
	0.025			.1449	6153	

- The accuracy of all three solution methods are improved by reducing the initial timestep.
 - However, excessively lowering initial timestep causes significant fluctuations of the pore pressure at very early times. This is especially apparent in very compressible geomaterials with comparatively low porosity.

Stability study at early time

To further investigate stability at very early times, the Cryer sphere model was evaluated with permeability of $1e-14 \text{ m}^2$, elastic modulus of 10MPa, and Poisson ratio of 0.3.



	Tolerance	Element Formulation	RMS Error	Solution Time (s)
Fully Coupled	-	Linear	.0776	204.3
		Quadratic	.000438	4912
Fixed Stress (Aria)	1e-6	Linear	.0373	164.5
		Quadratic	.00660	1812.8

- Increasing the timestep of the analysis allows the result output to "skip" over the early fluctuations, but the matrix of systems of equations is increasingly ill-conditioned.
- The fixed stress scheme was more stable than the fully coupled scheme
 - Condition number of matrix for the fluid problem with the fixed stress scheme was an order of magnitude smaller than that of the fully coupled method.
- Quadratic elements decreased the initial instabilities, but increased the computational time significantly.

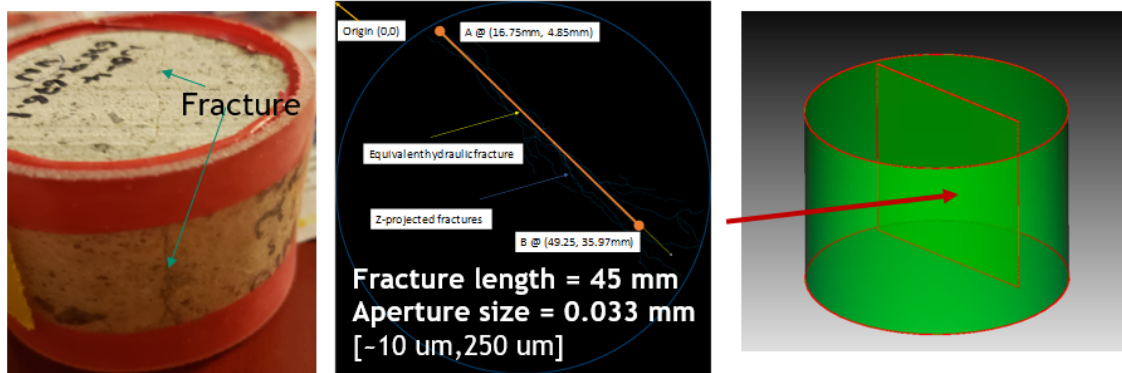
VALIDATION WITH EXPERIMENTAL DATA

Coupled Poro-elastic Problem in Fracture Media

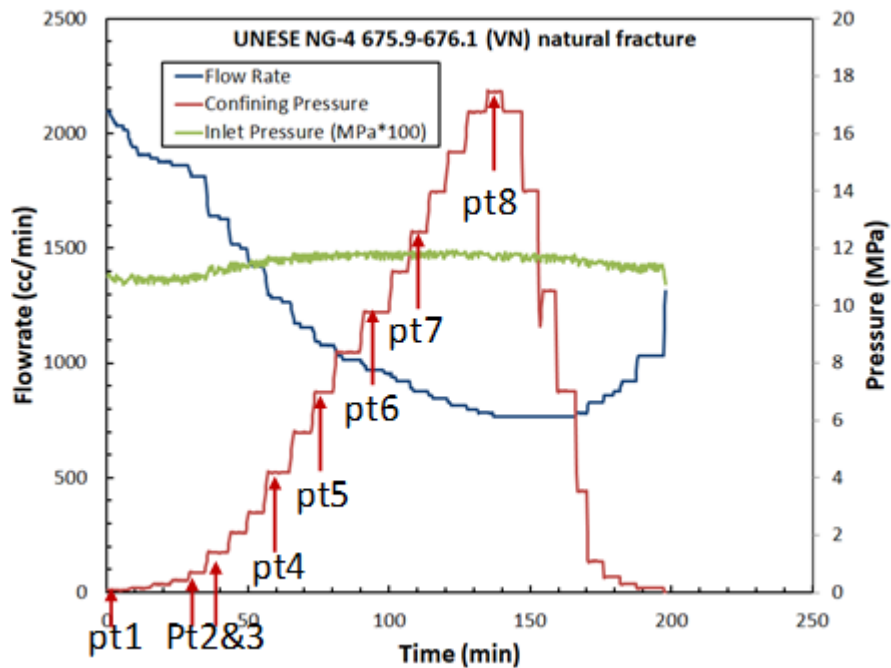
Experimental study

- Single core sample of vitric nonwelded tuff was tested to evaluate permeability change under confinement (Miller et al., 2017 [7])

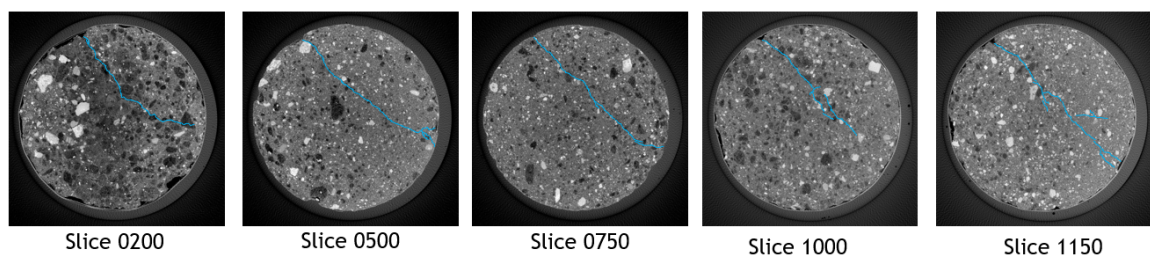
Testing sample of fracture media and simplified single fracture model domain



Experimental results of permeability and flowrate changes at different confined pressures



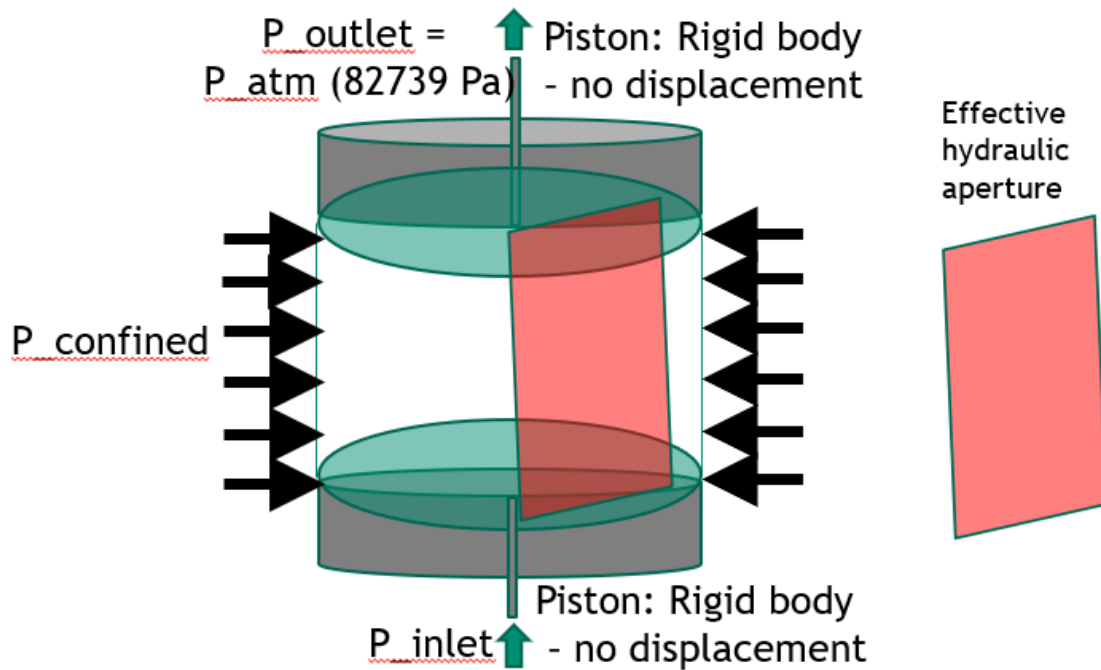
3D microCT images and thin section analysis



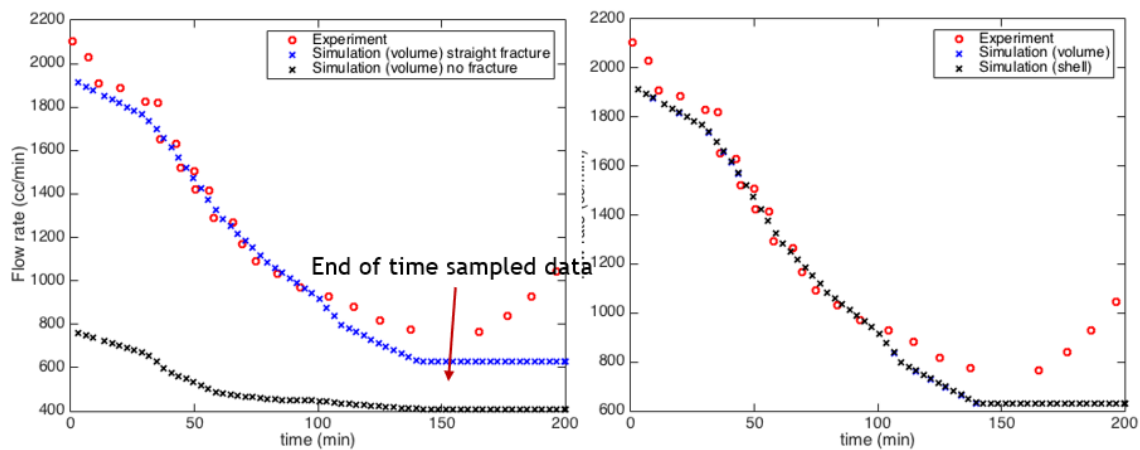
- Variations in primary fracture, microfractures, matrix porosity, and compositions affect
 - Aperture and connectivity of fracture network
 - Interaction between fracture and matrix
 - Permeability response as a function of stress
 - Flow and transport as a function of stress
 - Mechanical properties

Numerical simulation

- Investigate flow rate prediction with both volume and shell modeling approaches
- Conceptual model of single, uniformly plane fractured media



- Comparison of flow rates of simulations with experimental results

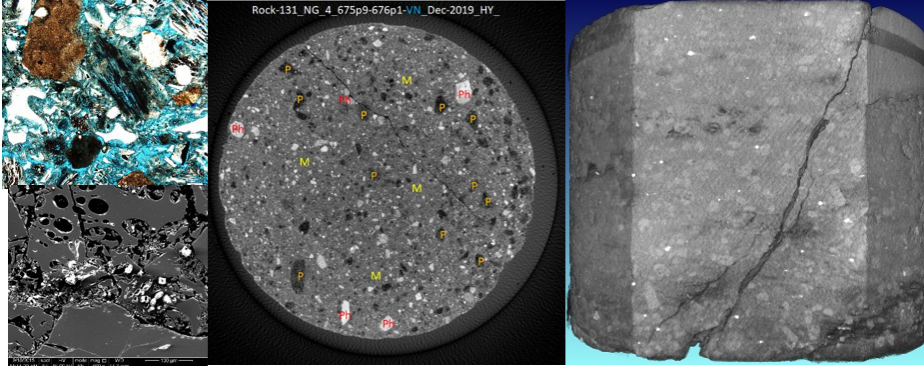


- Comparison shows simulation results match experimental data relatively well, indicating flow is dominant through fracture
- After a certain point, simulation results predict lower flow rates, indicating that real fracture apertures may not be reduced below a certain point and there may exist with fracture toughness under a certain confinement
- Fully coupled poromechanics in fractured media is under development

CONCLUSIONS

- In this work we implemented the fixed-stress scheme into Sandia Sierra Multiphysics software modules (Aria and Solid Mechanics)
- The fixed-stress scheme lowered computational times while maintaining error commensurate to fully-coupled methods
- The fixed-stress scheme can be implemented with different modules with a reasonable accuracy
- A sensitivity study was conducted to demonstrate the accuracy of models with parameter combinations
- For challenging problems such as more compressible materials with low permeability or stiffer material with high permeability, a significant Mandel-Cryer effect can be resolved using a reduced initial timestep
- A significantly reduced initial timestep can introduce numerical instabilities of pore pressure at early times in the Cryer problem. Along with decreasing the computational demand, the use of a fixed stress scheme has been shown to reduce instabilities in pore pressure at early simulation times

FUTURE WORK



- Investigate fracture modeling techniques
 - Inclined uniform fracture with a regular mesh and shell element
 - Effect of varying length and aperture size based on CT image
- Machine Learning Applications (upscaling, reduced order modeling)
- Validation through applications for experimental data (from a part of a U.S. DOE NNSA non-proliferation campaign)

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ABSTRACT

Prediction of flow, reactive transport, and deformation in fractured porous media is critical to improving our scientific understanding of coupled thermal-hydrological-mechanical-chemical (THMC) processes related to many subsurface energy activities and storage. With the existing numerical simulators an iterative coupling method of different physical principles such as fluid and solid mechanics is a viable approach to improve our simulation capabilities. It is now well-known that a certain class of iteration schemes such as the fixed stress scheme can provide an accurate approach compared to the fully coupled monolithic approach. In this work we implemented the fixed stress scheme into an existing Sandia Sierra Multiphysics toolkit where thermal/fluid mechanics module is coupled with the solid mechanics module. First, we rigorously compared the fixed stress scheme with the fully coupled method implemented in the Sierra fluid mechanics module for poroelastic problems such as one-dimensional (Terzaghi), two-dimensional (Mandel), and three-dimensional (Cryer Sphere) problems. For these three benchmark problems, both methods are also compared with analytical solutions. The effect of convergence criteria is examined to investigate the trade-off between accuracy and computational demand. Further, the sensitivity of the benchmark problems to material parameters of the porous media is evaluated. After model verification with the fixed stress scheme, the model is validated against well-controlled laboratory experimental data where flow and transport processes as a function of confined pressure have been investigated with different morphological geomaterials. We will evaluate the dependency of permeability and diffusion processes on the response of fracture aperture and pore structures given the confined pressure. This comparison will reveal the importance of coupled poromechanics on permeability evolution and diffusion processes in deformable porous media. SNL is managed and operated by NTESS under DOE NNSA contract DE-NA0003525.

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