

Real-time Parameter Estimation of Lithium-Ion Battery Models Using a Novel Tanks-in-Series Approach

Modeling, Analysis, and Process-control Laboratory for Electrochemical systems

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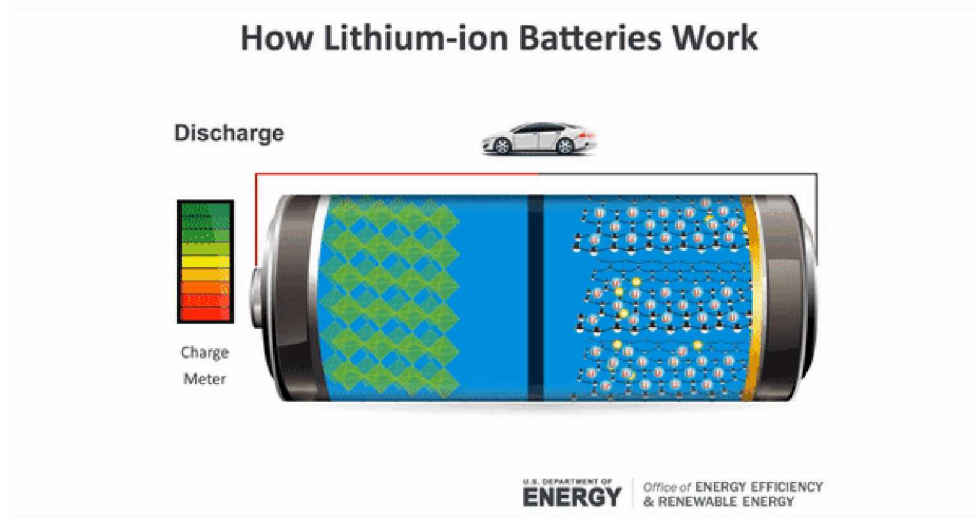
Overview

- **Introduction**
- **Background**
- **Novel Tanks-in-Series model**
- **Problem Formulation**
- **Proposed Methodology**
- **Results and Discussions**
- **Conclusion**



Introduction

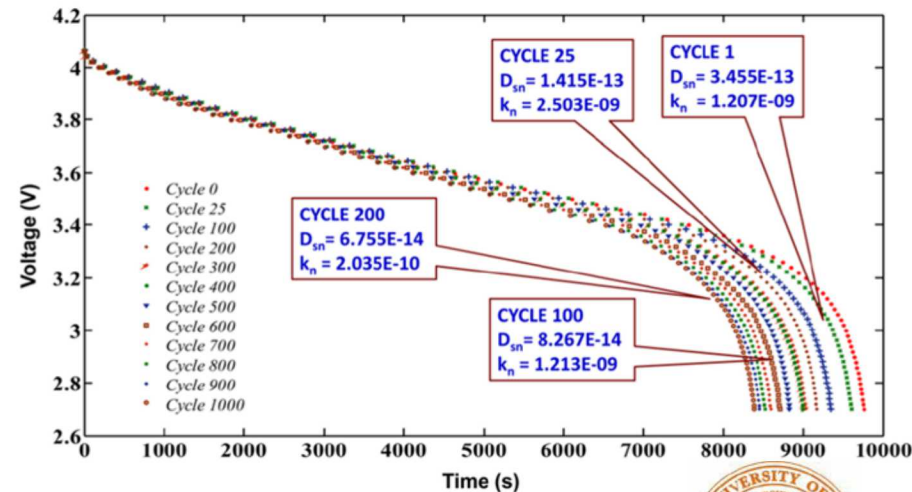
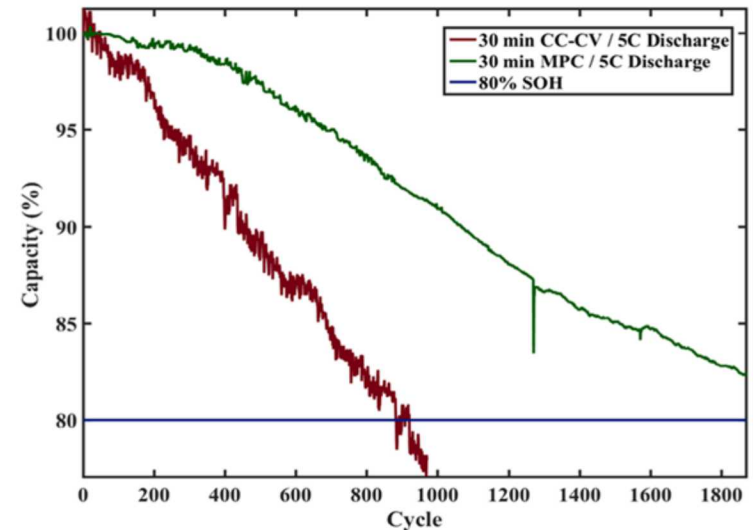
- **Lithium-Ion batteries are becoming increasingly popular.**



- **To monitor, control, and predict the performance of Li-ion batteries, advanced electrochemical model-based battery management systems (BMS) play a vital role.**

Physics Based Battery Management System

- Model-based BMS can double the cycle life.
- Physics-based battery models are complex, and parameters change over cycles.
- Accuracy of the model parameters determines model's predictability of internal states.



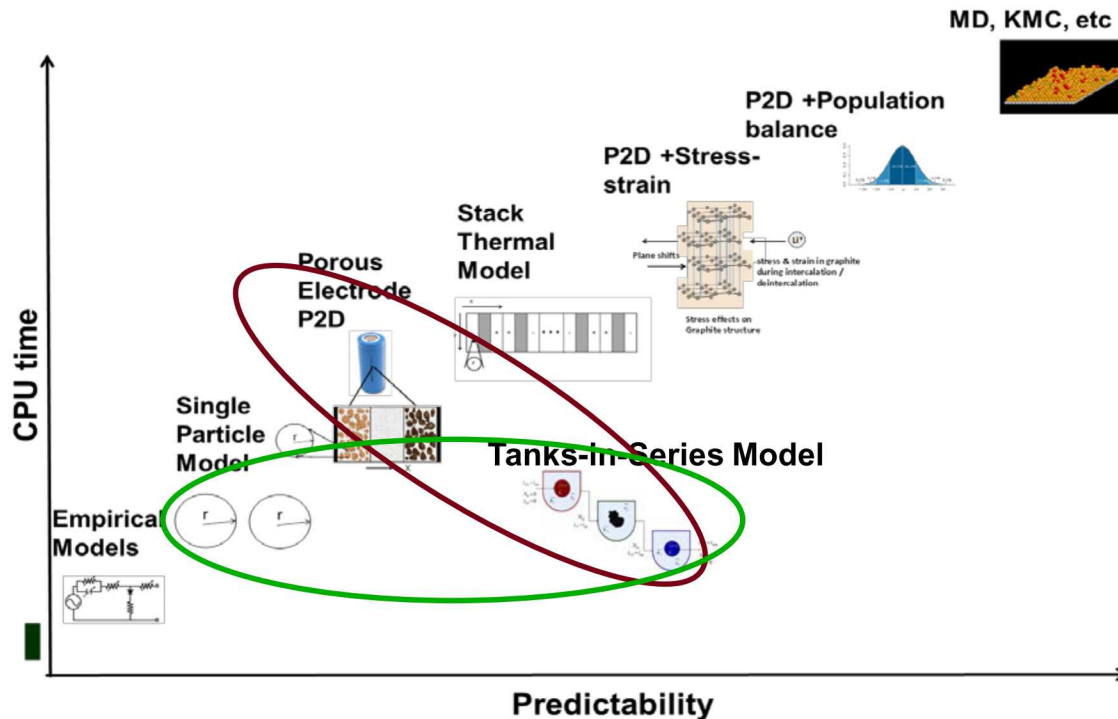
Ref: Pathak, M. *et. al.* (2017). ECS transactions, 75(23), 51.

Ramadesigan, V. *et. al.* (2011). JES, 158(9), 1048-1054.



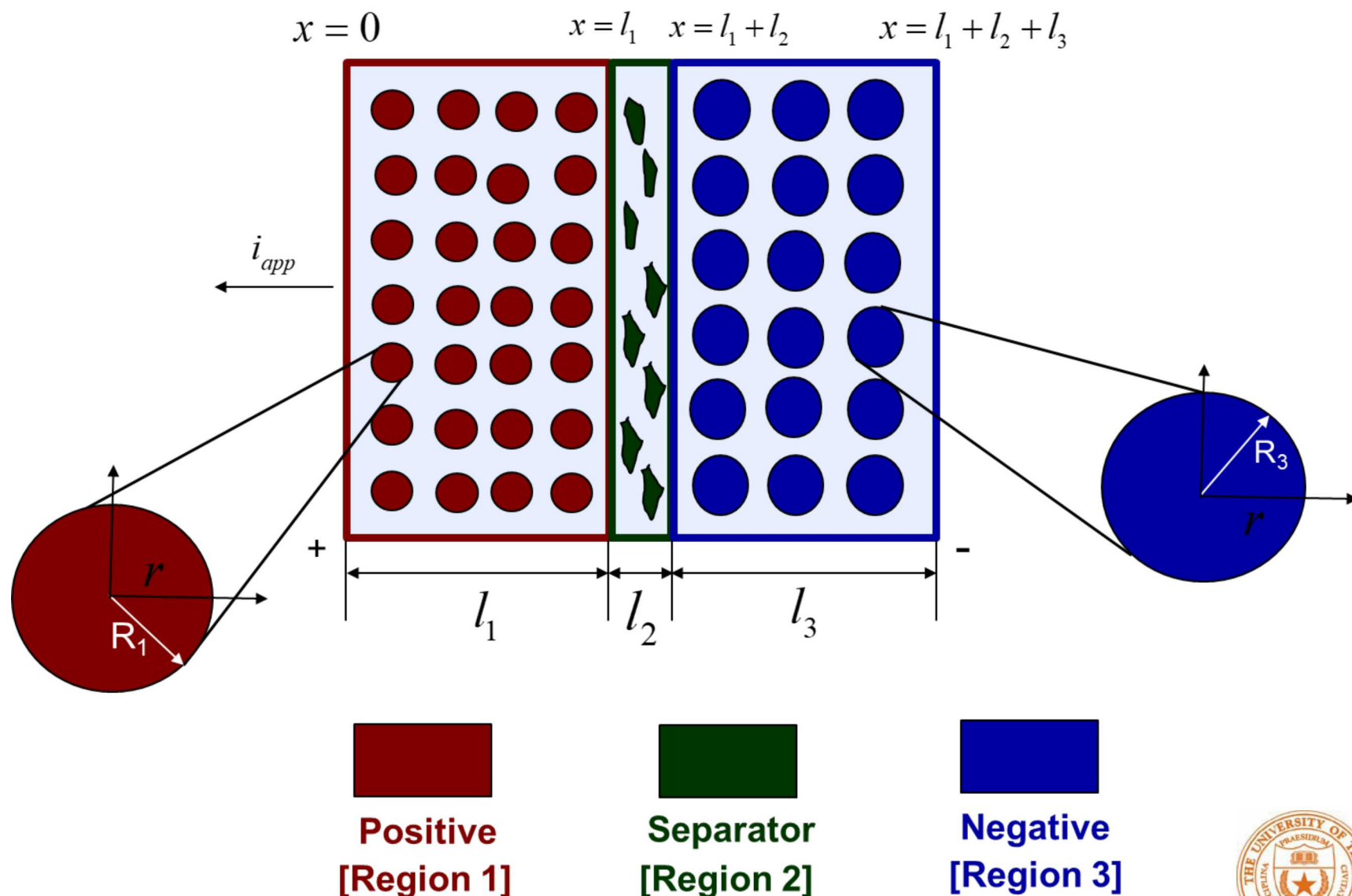
Speed Vs Accuracy

- Trade-off between CPU time vs. predictability.



- Develop a robust approach for estimating the parameters of physics-based lithium-ion battery model.

The p2D Model For Lithium-ion Batteries



The p2D Li-ion Battery Model

• Coupled Conservation Laws :

Electronic Charge :

$$\sigma_{\text{eff,p}} \frac{\partial^2 \Phi_1}{\partial x^2} = a_p F j_p; \Phi_1 \text{ is the solid-phase potential}$$

Overall Charge :

$$-\sigma_{\text{eff,p}} \frac{\partial \Phi_1}{\partial x} - \kappa_{\text{eff,p}} \frac{\partial \Phi_2}{\partial x} + \frac{2\kappa_{\text{eff,p}} RT}{F} (1 - t_+) \frac{\partial \ln c}{\partial x} = I;$$

Φ_2 is the liquid phase potential

Electrolyte Mass:

$$\varepsilon_p \frac{\partial c}{\partial t} = D_{\text{eff,p}} \frac{\partial^2 c}{\partial x^2} + a_p (1 - t_+) j_p;$$

c is the electrolyte concentration

Solid Phase Mass:

$$\frac{\partial c_p^s}{\partial t} = D_{s,p} \left(\frac{\partial^2 c_p^s}{\partial r^2} + \frac{2}{r} \frac{\partial c_p^s}{\partial r} \right); c_p^s \text{ is the solid phase concentration}$$

Parameters and Constitutive Equations

$$\kappa_{\text{eff,p}} = \varepsilon_p^{\text{bruggp}} \kappa = 0.01775 \times \left(4.1253 \times 10^{-2} + 5.007 \times 10^{-1} c - 4.7212 \times 10^{-1} c^2 \right. \\ \left. + 1.5094 \times 10^{-1} c^3 - 1.6018 \times 10^{-2} c^4 \right)$$

$$j_p = 2k_p \left(c_{s,\text{max,p}} - c_p^s \right)^{0.5} c_p^{s0.5} c^{0.5} \sinh \left[\frac{0.5F}{RT} (\Phi_1 - \Phi_2 - U_p) \right]$$

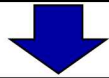
$$U_p = f(\text{SOC})$$

Computational Complexity has spurred active investigation of different simplification and reformulation techniques

The Single Particle Model

Neglect Electrolyte Effects:

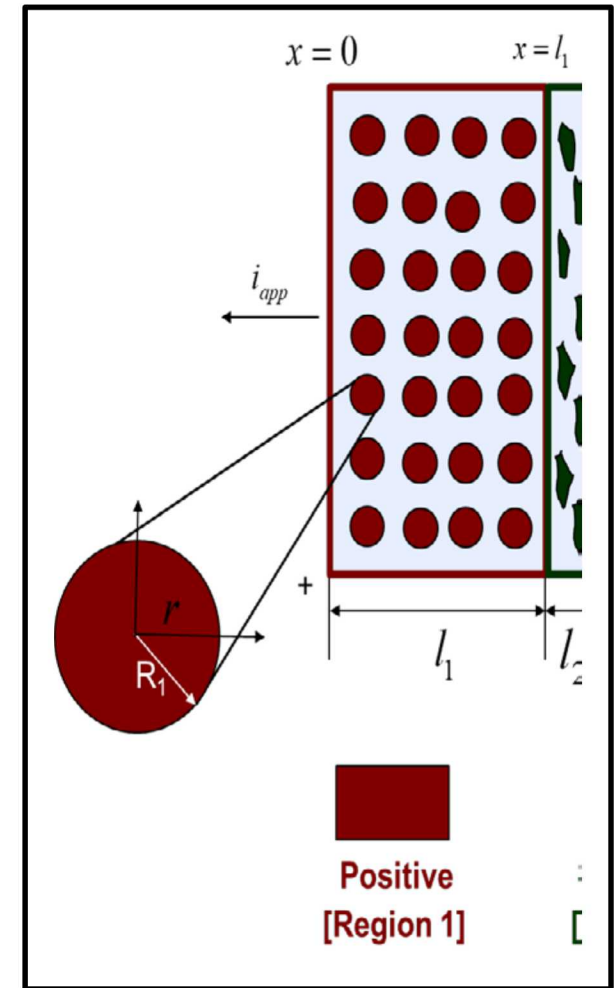
$$\frac{\partial c_1^s}{\partial t} = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 D_1^s \frac{\partial c_1^s}{\partial r} \right)$$



$$\frac{\partial \int_{x=0}^{x=l_1} c_1^s dx}{\partial t} = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 D_1^s \frac{\partial \int_{x=0}^{x=l_1} c_1^s dx}{\partial r} \right)$$

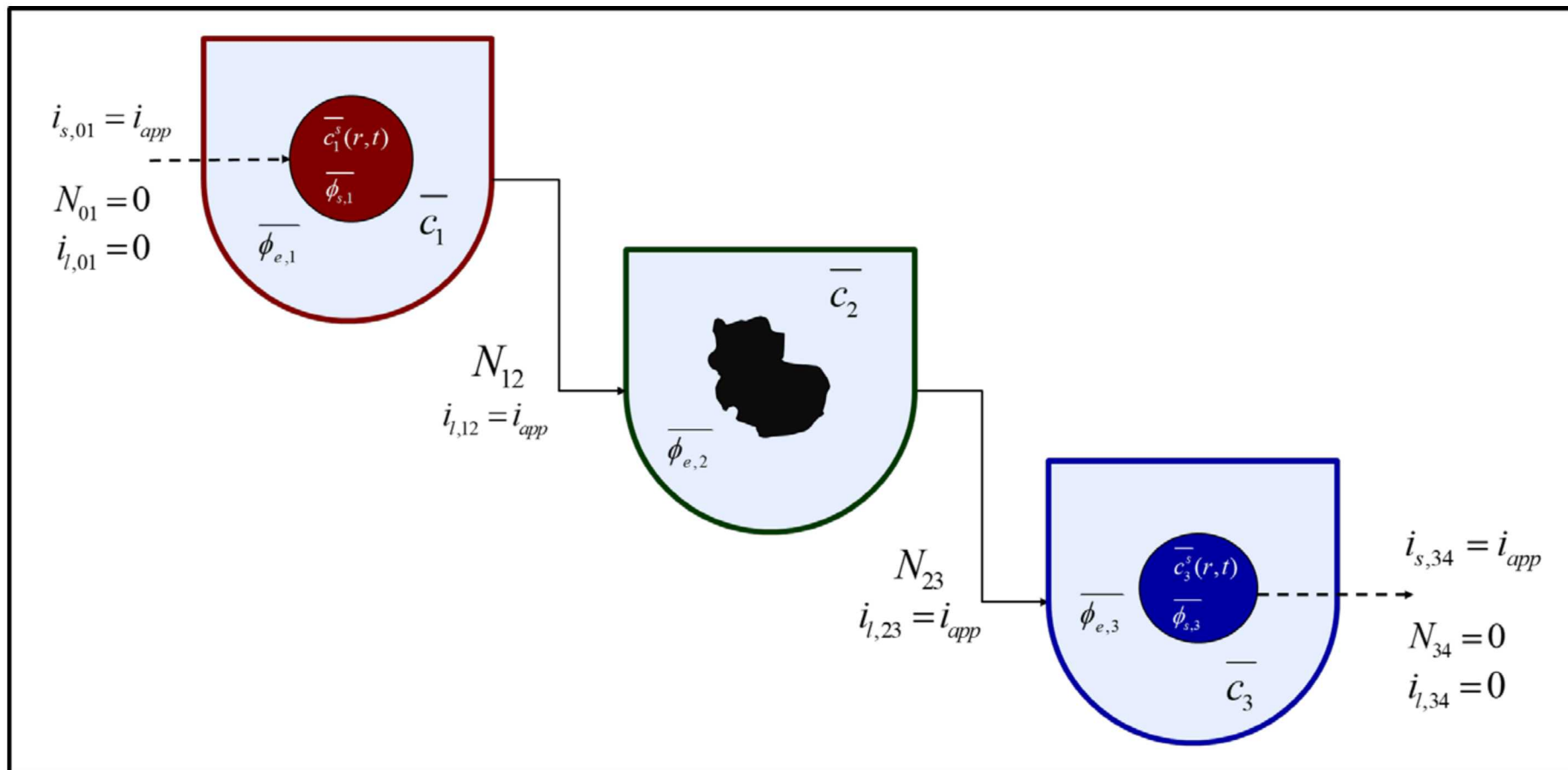


$$\frac{\partial \overline{c_1^s}}{\partial t} = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 D_1^s \frac{\partial \overline{c_1^s}}{\partial r} \right)$$



Restricted to moderate current scenarios, where liquid phase polarizations aren't significant

Lithium-ion Battery as Tanks in Series

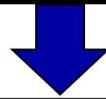


Can we incorporate average electrolyte dynamics by a similar volume-averaging approach?

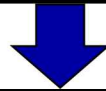
Generating the Tank Model

Example Conservation Equation:

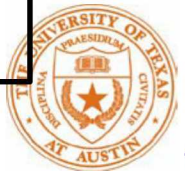
$$\varepsilon_1 \frac{\partial c_1}{\partial t} = -\frac{\partial N_1}{\partial x} + a_1 (1 - t_+^0) j_1$$



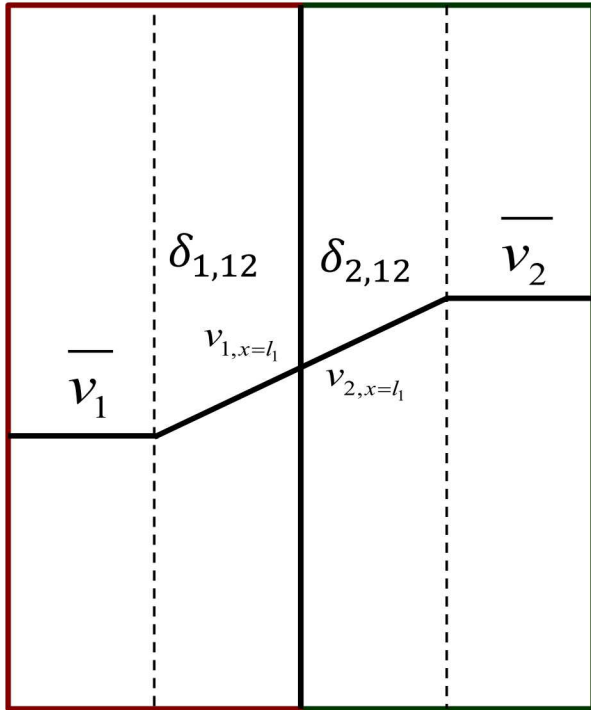
$$\frac{d \int_{V_1} \varepsilon_1 c_1 dV}{dt} = - \int_{V_1} \frac{\partial N_1}{\partial x} dV + \int_{V_1} a_1 (1 - t_+^0) j_1 dV$$



$$\varepsilon_1 \frac{d\bar{c}_1}{dt} = - \frac{\int_{x=0}^{x=l_1} \frac{\partial N_1}{\partial x} dx}{l_1} + a_1 (1 - t_+) \bar{j}_1 \left(\frac{N_{1,x=0} - N_{1,x=l_1}}{l_1} \right) + a_1 (1 - t_+) \bar{j}_1$$



Approximating Interfacial Fluxes



$$N_{1,x=l_1} = -D(c_{1,x=l_1})\varepsilon_1^{b_1} \frac{\partial c_1}{\partial x} \bigg|_{x=l_1} \approx D(c_{1,x=l_1})\varepsilon_1^{b_1} \left(\frac{\Delta c_1}{\delta_{1,12}} \right) = D(c_{1,x=l_1})\varepsilon_1^{b_1} \left(\frac{\bar{c}_1 - c_{1,x=l_1}}{\frac{l_1}{2}} \right)$$

$$N_{2,x=l_1} = -D(c_{2,x=l_1})\varepsilon_2^{b_2} \frac{\partial c_2}{\partial x} \bigg|_{x=l_2} \approx D(c_{2,x=l_1})\varepsilon_2^{b_2} \left(\frac{\Delta c_2}{\delta_{2,12}} \right) = D(c_{2,x=l_1})\varepsilon_2^{b_2} \left(\frac{-\bar{c}_2 + c_{2,x=l_1}}{\frac{l_2}{2}} \right)$$

$$D(c_{1,x=l_1})\varepsilon_1^{b_1} \left(\frac{\bar{c}_1 - c_{1,x=l_1}}{\frac{l_1}{2}} \right) = D(c_{2,x=l_1})\varepsilon_2^{b_2} \left(\frac{-\bar{c}_2 + c_{2,x=l_1}}{\frac{l_2}{2}} \right)$$

Naive assumptions of 'film thickness', can be regarded an adjustable parameter



Tank Model DAE System

Electronic Charge :

$$\bar{j}_1 = \frac{i_{app}}{a_1 F l_1} \quad \bar{j}_3 = -\frac{i_{app}}{a_3 F l_3}$$

Electrolyte Mass:

$$\begin{aligned} \varepsilon_1 \frac{d\bar{c}_1}{dt} &= \frac{\frac{2D(c_{12})(\bar{c}_2 - \bar{c}_1)}{\frac{l_1}{\varepsilon_1^{b_1}} + \frac{l_2}{\varepsilon_2^{b_2}}}}{l_1} + a_1(1-t_+) \bar{j}_1 \\ \varepsilon_2 \frac{d\bar{c}_2}{dt} &= \frac{\frac{-2D(c_{12})(\bar{c}_2 - \bar{c}_1)}{\frac{l_1}{\varepsilon_1^{b_1}} + \frac{l_2}{\varepsilon_2^{b_2}}} + \frac{2D(c_{23})(\bar{c}_3 - \bar{c}_2)}{\frac{l_2}{\varepsilon_2^{b_2}} + \frac{l_3}{\varepsilon_3^{b_3}}}}{l_2} \\ \varepsilon_3 \frac{d\bar{c}_3}{dt} &= \frac{\frac{-2D(c_{23})(\bar{c}_3 - \bar{c}_2)}{\frac{l_2}{\varepsilon_2^{b_2}} + \frac{l_3}{\varepsilon_3^{b_3}}}}{l_3} + a_3(1-t_+) \bar{j}_3 \end{aligned}$$

Liquid Charge (plus reference) :

$$\begin{aligned} i_{l,1,x=l_1} &= -2\kappa(c_{12}) \left(\frac{\frac{\bar{\phi}_{l,2} - \bar{\phi}_{l,1}}{\frac{l_1}{\varepsilon_1^{b_1}} + \frac{l_2}{\varepsilon_2^{b_2}}}}{\frac{l_1}{\varepsilon_1^{b_1}} + \frac{l_2}{\varepsilon_2^{b_2}}} \right) + \frac{4RT(1-t_+^0)}{F} \kappa(c_{12}) \frac{1}{c_{12}} \left(\frac{\bar{c}_2 - \bar{c}_1}{\frac{l_1}{\varepsilon_1^{b_1}} + \frac{l_2}{\varepsilon_2^{b_2}}} \right) = i_{app} \\ i_{l,3,x=l_1+l_2} &= -2\kappa(c_{23}) \left(\frac{\frac{\bar{\phi}_{l,3} - \bar{\phi}_{l,2}}{\frac{l_3}{\varepsilon_3^{b_3}} + \frac{l_2}{\varepsilon_2^{b_2}}}}{\frac{l_3}{\varepsilon_3^{b_3}} + \frac{l_2}{\varepsilon_2^{b_2}}} \right) + \frac{4RT(1-t_+^0)}{F} \kappa(c_{23}) \frac{1}{c_{23}} \left(\frac{\bar{c}_3 - \bar{c}_2}{\frac{l_2}{\varepsilon_2^{b_2}} + \frac{l_3}{\varepsilon_3^{b_3}}} \right) = i_{app} \\ \phi_{l,12} &= \left(\frac{\frac{\varepsilon_1^{b_1}}{l_1} \bar{\phi}_{l,1} + \frac{\varepsilon_2^{b_2}}{l_2} \bar{\phi}_{l,2}}{\frac{\varepsilon_1^{b_1}}{l_1} + \frac{\varepsilon_2^{b_2}}{l_2}} \right) = 0 \end{aligned}$$

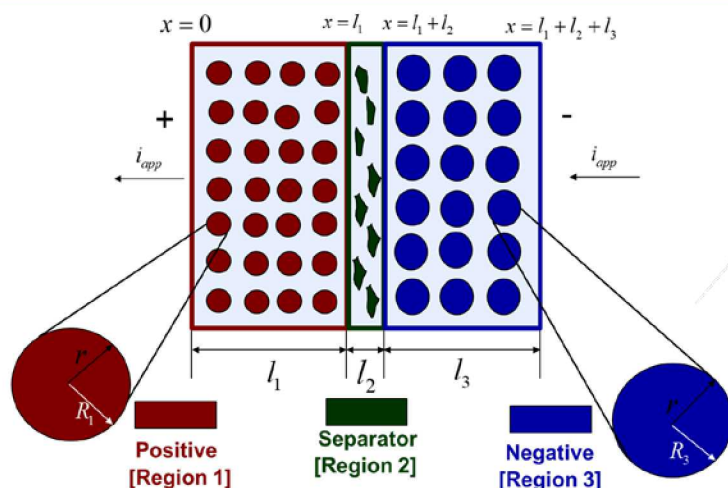
Solid Phase Mass (SPM):

$$\frac{\partial \bar{c}_1^s}{\partial t} = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 D_1^s \frac{\partial \bar{c}_1^s}{\partial r} \right)$$

PDE Model is thus reduced to simplified DAE system for liquid phase.

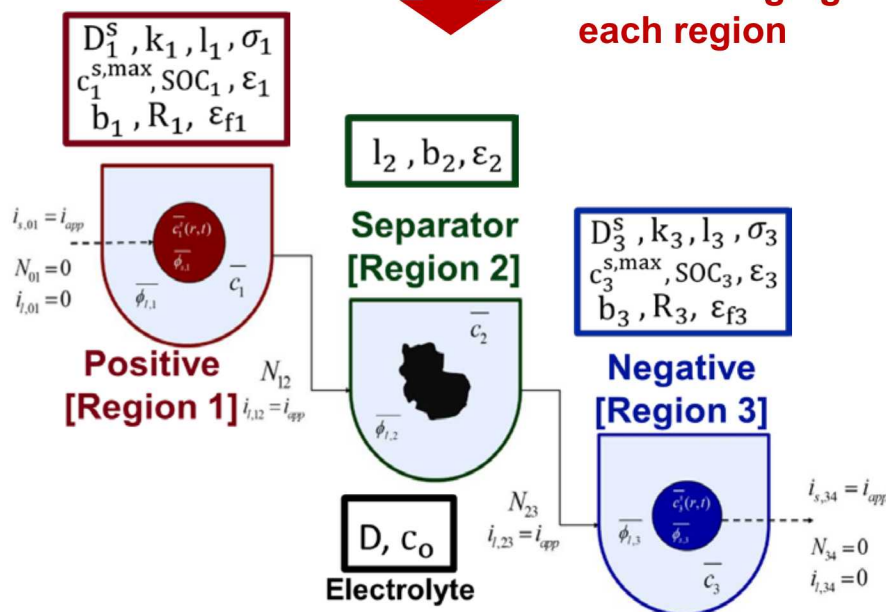


Novel Tanks-in-Series Model



p2D model > 1000 DAEs (complex)
(Computation time, C++: 1493ms)

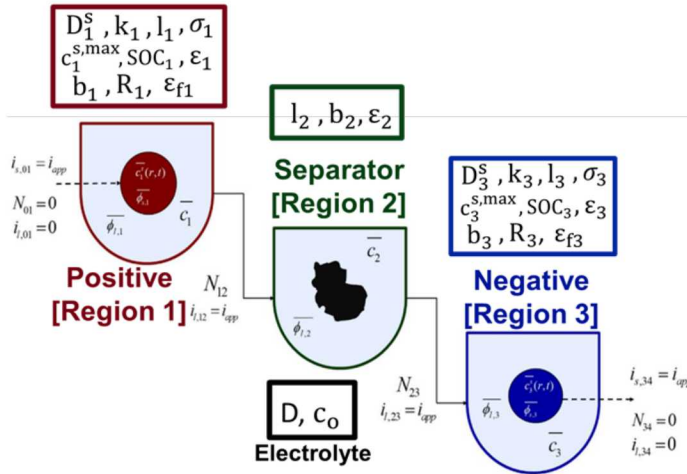
**Volume averaging
each region**



Tanks-in-Series model ~10 DAEs
(Computation time, C++: 2.1ms)

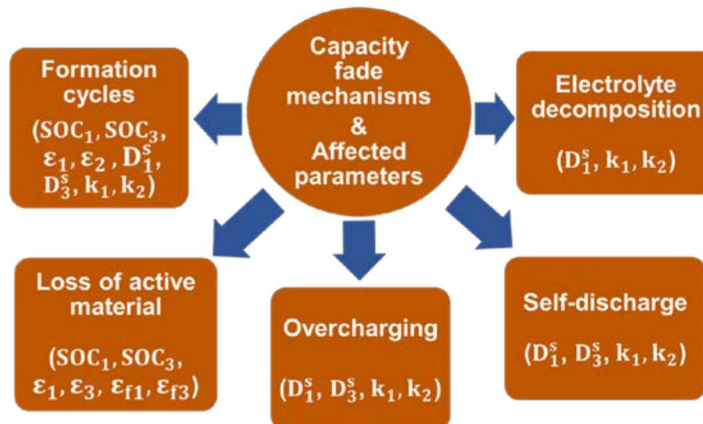
<1% error observed as compared
to existing physics-based battery
models (SPM, P2D, etc.)

Novel Tanks-in-Series Model



List of Tanks-in-Series model parameters	
Parameter	Symbol
Electrolyte Diffusivity	D
Solid Phase Diffusivity	D_1^s, D_3^s
Reaction Rate Constant	k_1, k_3
Solid Phase Conductivity	σ_1, σ_3
Electrolyte Conductivity	κ
Electrolyte Concentration	c_0
Maximum Solid Phase Concentration	$c_1^{s,max}, c_3^{s,max}$
Initial SOC	SOC_1, SOC_3
Transference Number	t_+^0
Porosity	ϵ_1, ϵ_2
Filler Fraction	$\epsilon_{f1}, \epsilon_{f3}$
Bruggeman Constant	b_1, b_2, b_3
Thickness	l_1, l_2, l_3
Particle Size	R_1, R_3
index: 1 (+ve electrode), 2 (separator), 3 (-ve electrode)	

Parameters to be estimated to model Battery aging



Optimization Formulation

Objective:
$$\min \sqrt{\frac{\left[\sum_{t_0=0}^{t_n=t_f} \left(V_{\text{exp}}(t_i) - V_{\text{model}}(t_i) \right)^2 \right]}{N_{\text{exp}}}}$$

where, V_{exp} : experimental voltage profile

V_{model} : Tanks – in – Series model voltage profile

Subject to:

$$\left. \begin{array}{ll} \frac{\partial}{\partial \bar{x}} g_j(\bar{x}) = 0, & j = 1, \dots, J \\ h_k(\bar{x}) = 0, & k = 1, \dots, K \end{array} \right\} \begin{array}{l} \text{Ordinary differential} \\ \text{equations (ODEs)} \\ \\ \text{Algebraic} \\ \text{equations (AEs)} \end{array} \quad \left. \vphantom{\begin{array}{l} \text{Ordinary differential} \\ \text{equations (ODEs)} \\ \\ \text{Algebraic} \\ \text{equations (AEs)} \end{array}} \right\} \begin{array}{l} \text{Differential} \\ \text{algebraic} \\ \text{equations} \\ \text{(DAEs)} \end{array}$$

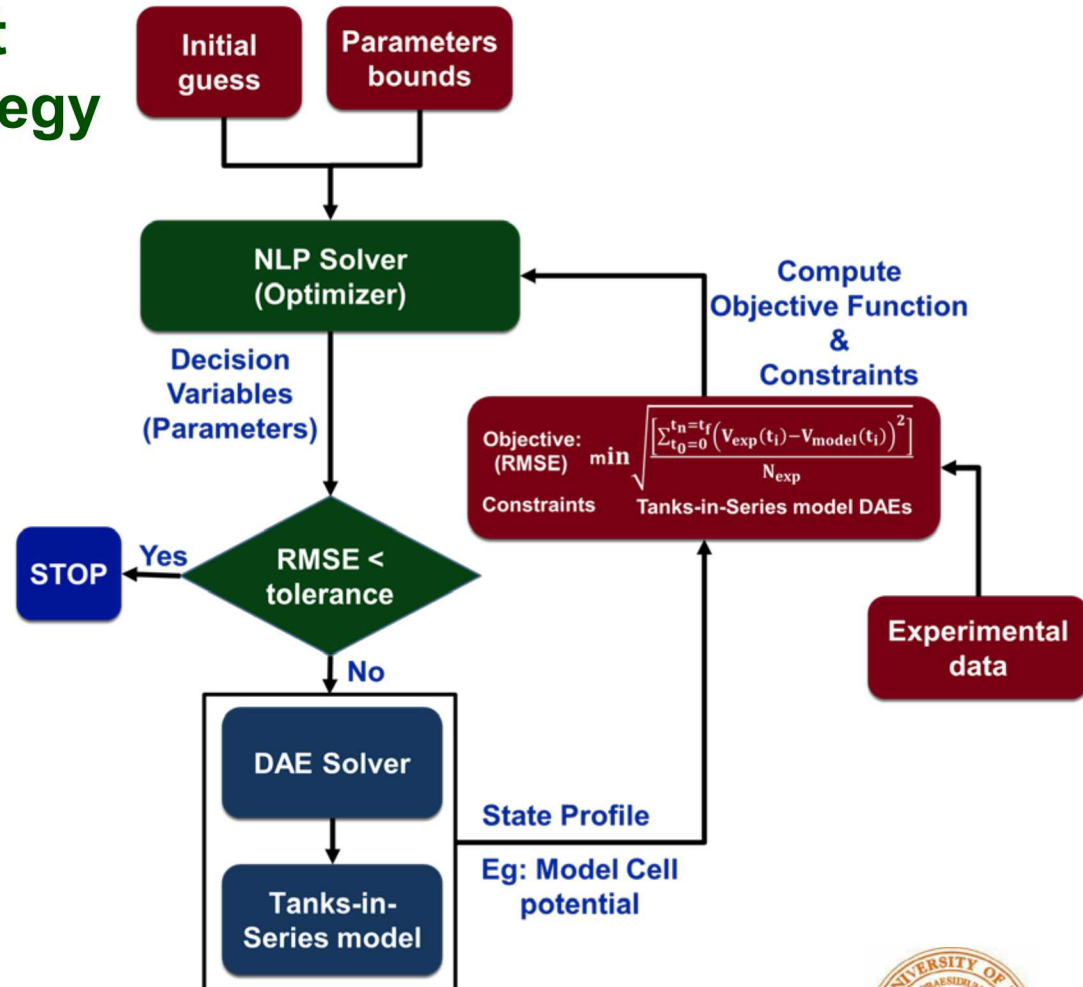
Variables: $\bar{x} \in \Omega$

$lb \leq \bar{x} \leq ub$

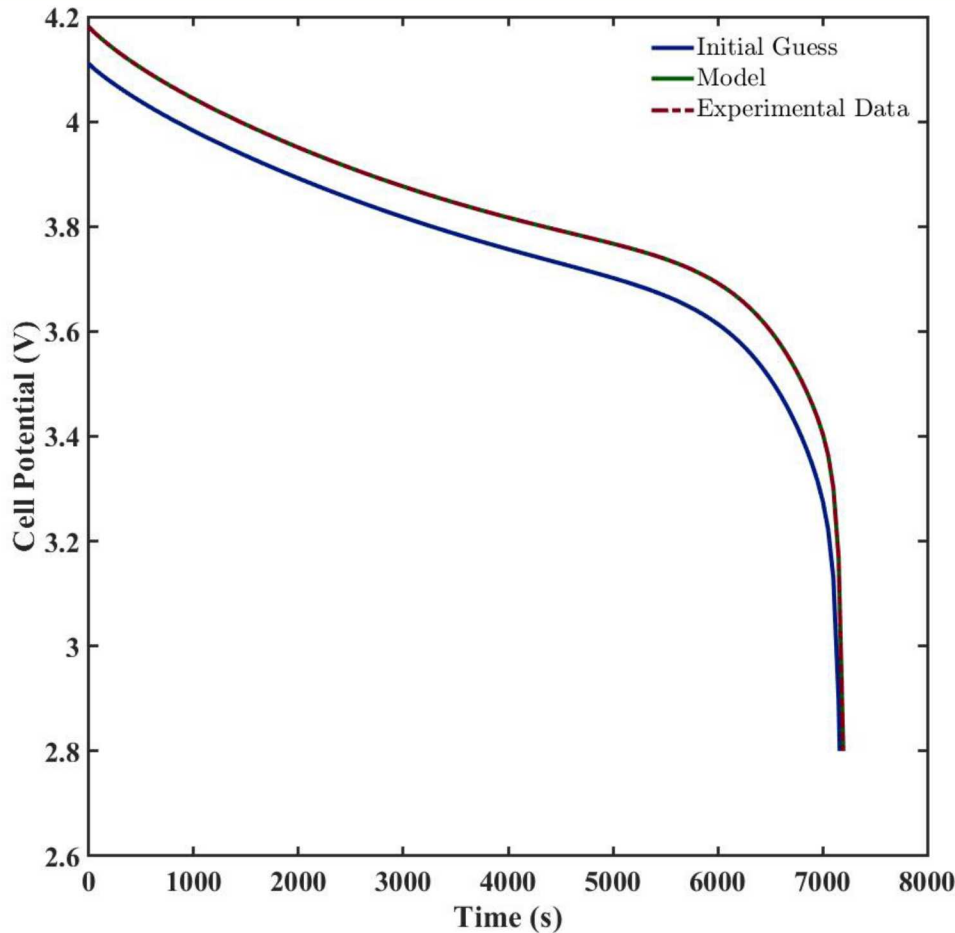


Proposed Methodology

Sequential parameter estimation using robust dynamic optimization strategy



Fast Computation



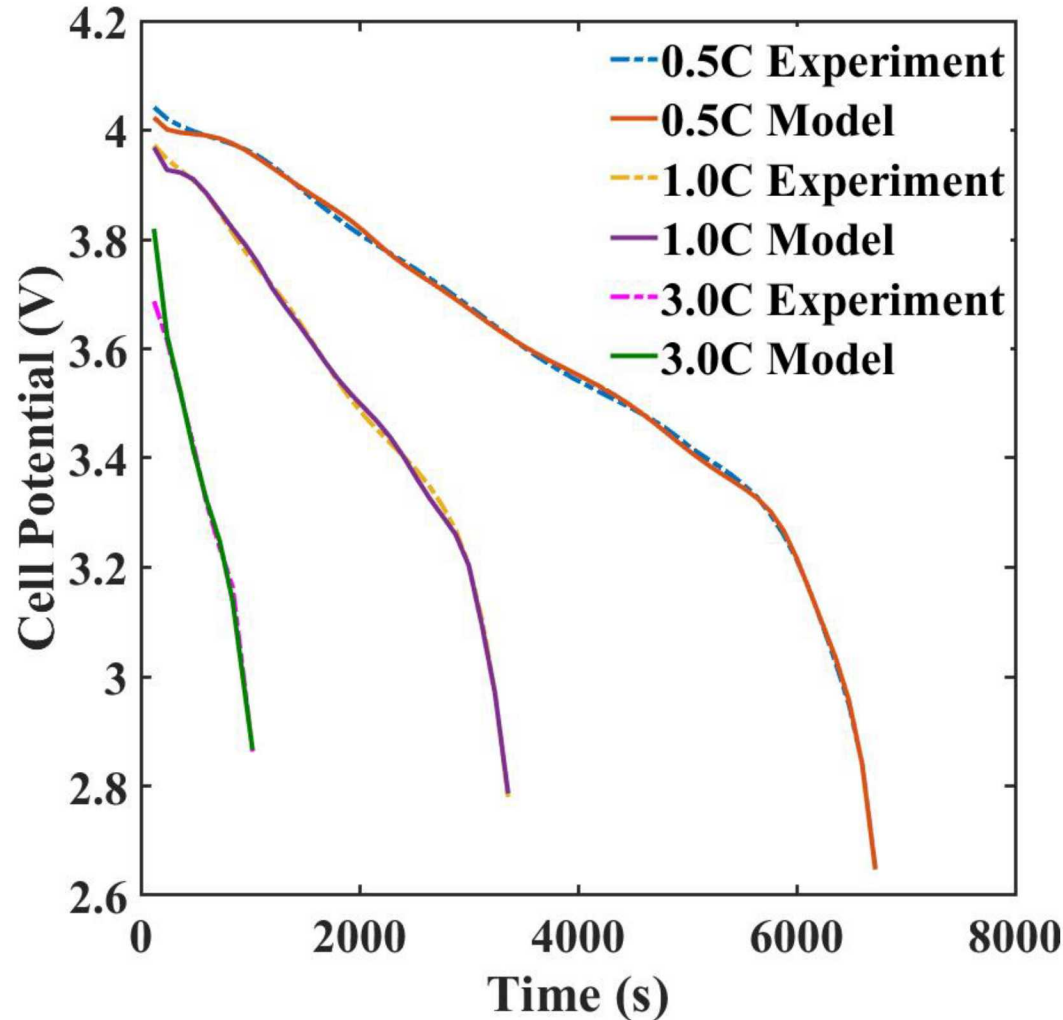
► **Tanks-in-Series model enables Fast Computation in MATLAB**

Battery Model	Total Estimation Time (s)
SPM	~40
Tanks-in-Series	~45
p2D	~900

Comparison plot between Initial Guess and Optimal parameters of Tanks-in-Series model for 0.5C synthetic discharge data.



Results and Discussions



RMSE error values (in mV)

Crate	RMSE (mV)
0.5	7.22
1.0	10.13
3.0	11.64

Note: Experiments are performed on 3Ah 18650 cylindrical cell (Cathode: NMC811 and Anode: Si-C) at different discharge rates (C/2,1C and 3C) at Sandia National Laboratories.



Conclusions

- **Novel Tanks-in-Series approach is adopted to model the cell behavior at different discharge rates (0.5C, 1C, and 3C).**
- **The adopted model competes with SPM (in terms of simulation time) and complies with the synthetic experimental data within 15 mV error.**
- **A robust sequential optimization approach is proposed to estimate the parameters of the battery model with <12 mV error.**



Acknowledgements

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THANK YOU!

Any Questions?

