

Real-time Parameter Estimation of Lithium-Ion Battery Models Using a Novel Tanks-in-Series Approach

**Modeling, Analysis, and Process-control Laboratory for
Electrochemical systems**

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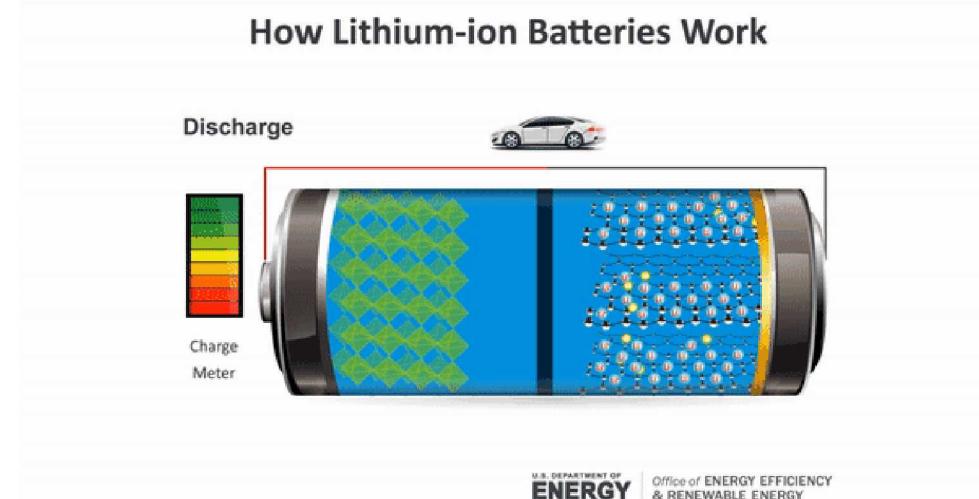
Overview

- ▶ **Introduction**
- ▶ **Background**
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- ▶ **Problem Formulation**
- ▶ **Proposed Methodology**
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Introduction

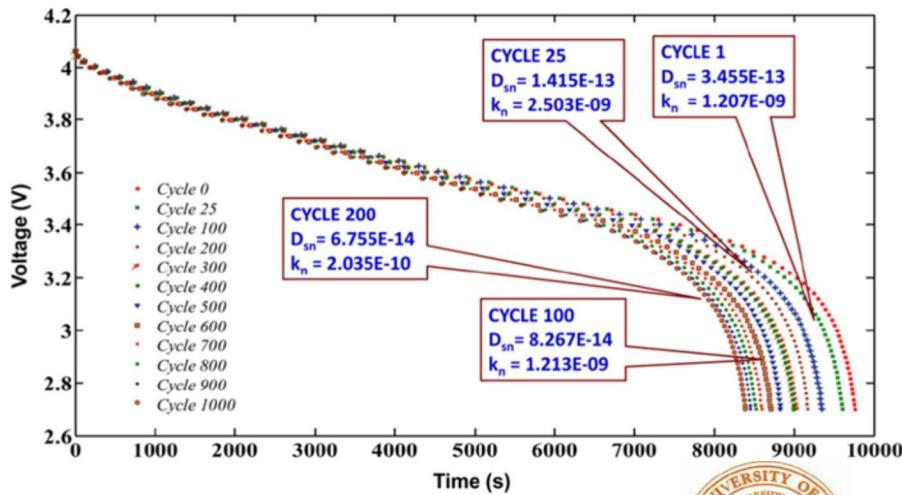
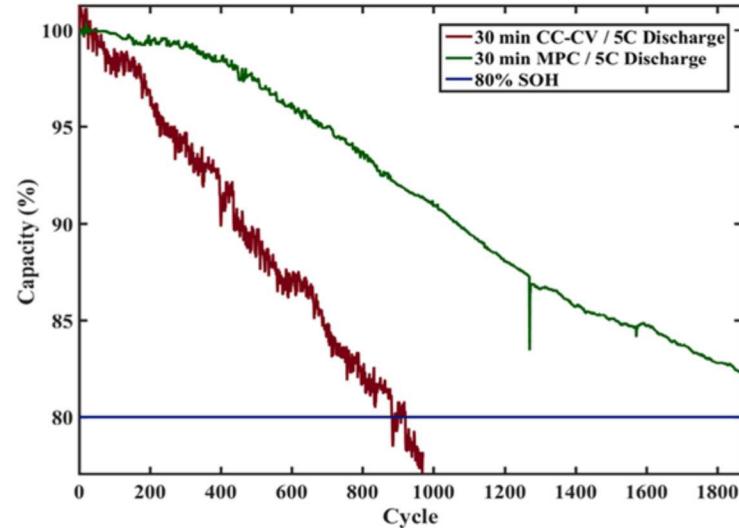
- **Lithium-Ion batteries are becoming increasingly popular.**



- **To monitor, control, and predict the performance of Li-ion batteries, advanced electrochemical model-based battery management systems (BMS) play a vital role.**

Physics Based Battery Management System

- Model-based BMS can double the cycle life.
- Physics-based battery models are complex, and parameters change over cycles.
- Accuracy of the model parameters determines model's predictability of internal states.



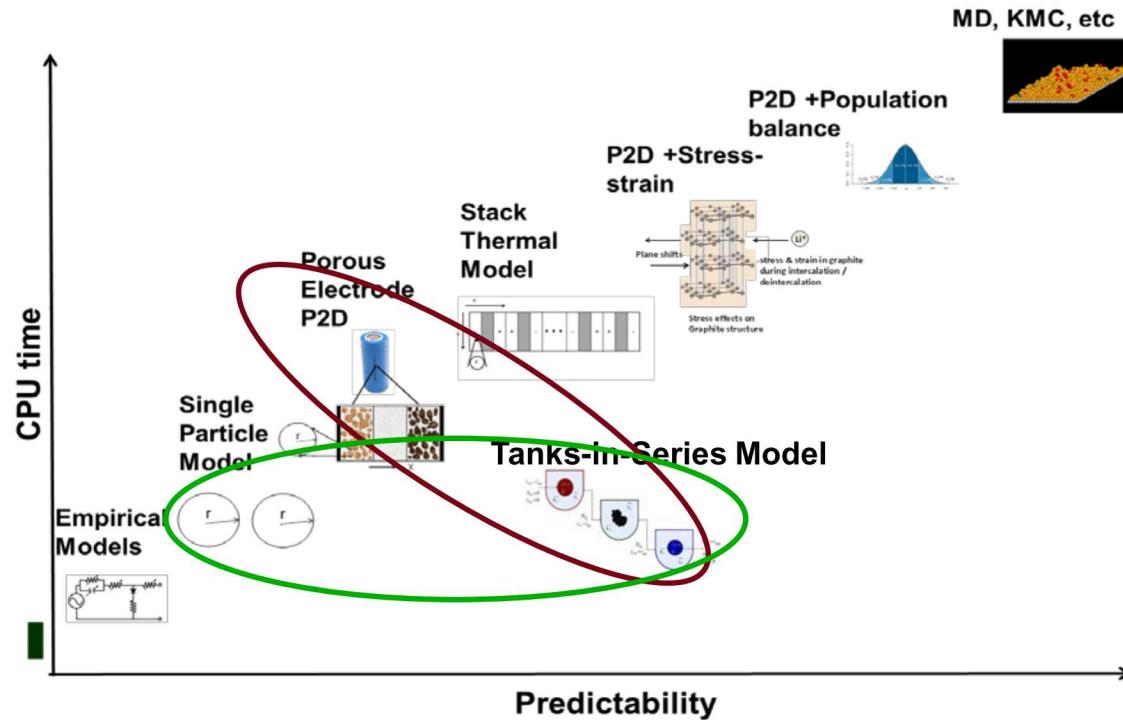
Ref: Pathak, M. et. al. (2017). ECS transactions, 75(23), 51.

Ramadesigan, V. et. al. (2011). JES, 158(9), 1048-1054.



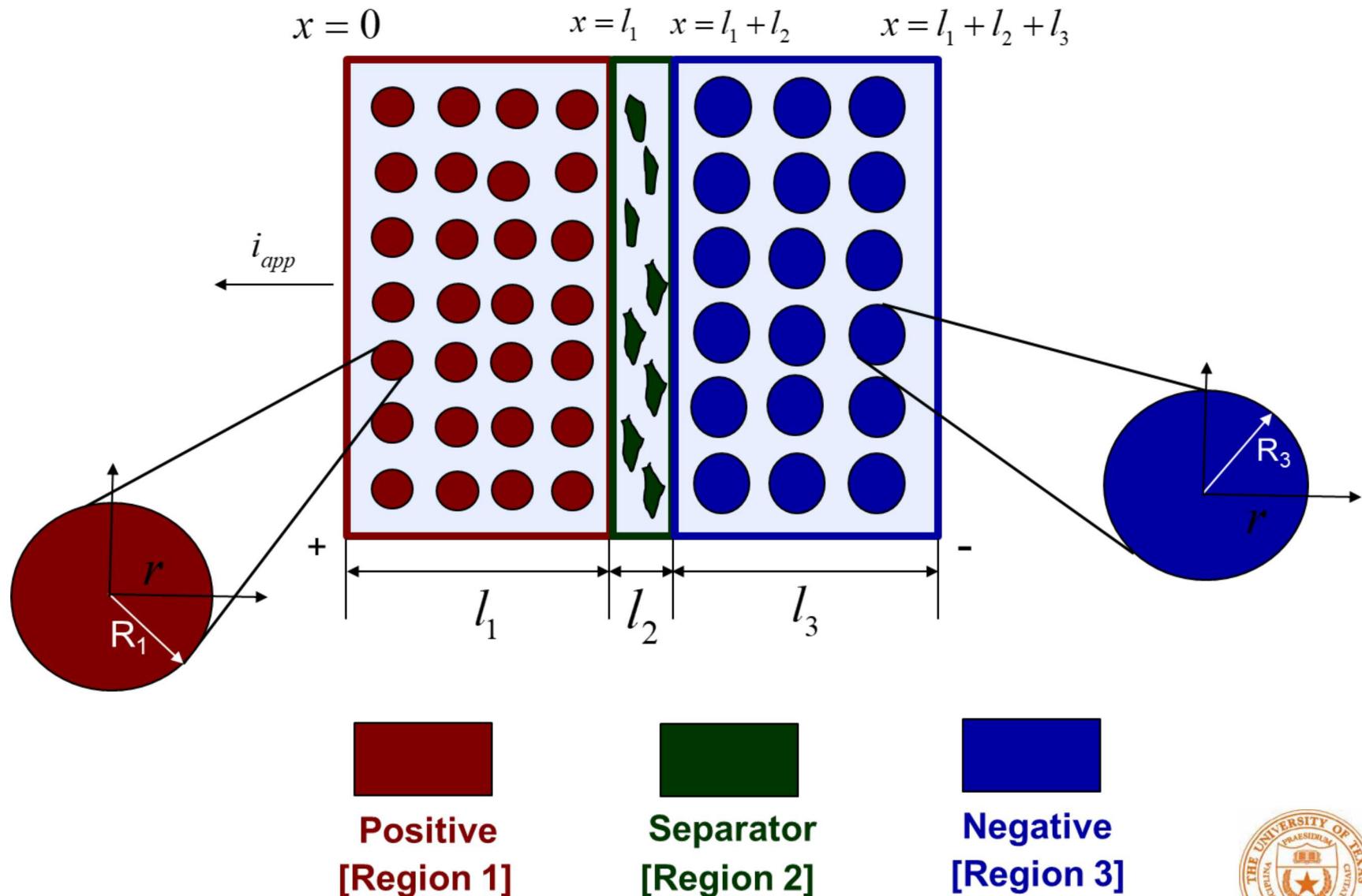
Speed Vs Accuracy

► Trade-off between CPU time vs. predictability.



- Develop a robust approach for estimating the parameters of physics-based lithium-ion battery model.

The p2D Model For Lithium-ion Batteries



The p2D Li-ion Battery Model

- Coupled Conservation Laws :**

Electronic Charge :

$$\sigma_{\text{eff,p}} \frac{\partial^2 \Phi_1}{\partial x^2} = a_p F j_p; \Phi_1 \text{ is the solid-phase potential}$$

Electrolyte Mass:

$$\varepsilon_p \frac{\partial c}{\partial t} = D_{\text{eff,p}} \frac{\partial^2 c}{\partial x^2} + a_p (1 - t_+) j_p;$$

c is the electrolyte concentration

Solid Phase Mass:

$$\frac{\partial c_p^s}{\partial t} = D_{s,p} \left(\frac{\partial^2 c_p^s}{\partial r^2} + \frac{2}{r} \frac{\partial c_p^s}{\partial r} \right); c_p^s \text{ is the solid phase concentration}$$

Overall Charge :

$$-\sigma_{\text{eff,p}} \frac{\partial \Phi_1}{\partial x} - \kappa_{\text{eff,p}} \frac{\partial \Phi_2}{\partial x} + \frac{2\kappa_{\text{eff,p}} RT}{F} (1 - t_+) \frac{\partial \ln c}{\partial x} = I;$$

Φ_2 is the liquid phase potential

Parameters and Constitutive Equations

$$\kappa_{\text{eff,p}} = \varepsilon_p^{\text{brugg,p}} \kappa = 0.01775 \times \begin{pmatrix} 4.1253 \times 10^{-2} + 5.007 \times 10^{-1} c - 4.7212 \times 10^{-1} c^2 \\ + 1.5094 \times 10^{-1} c^3 - 1.6018 \times 10^{-2} c^4 \end{pmatrix}$$

$$j_p = 2k_p \left(c_{s,\text{max,p}} - c_p^s \right)^{0.5} c_p^s 0.5 c^{0.5} \sinh \left[\frac{0.5F}{RT} (\Phi_1 - \Phi_2 - U_p) \right]$$

$$U_p = f(SOC)$$

Computational Complexity has spurred active investigation of different simplification and reformulation techniques

The Single Particle Model

Neglect Electrolyte Effects:

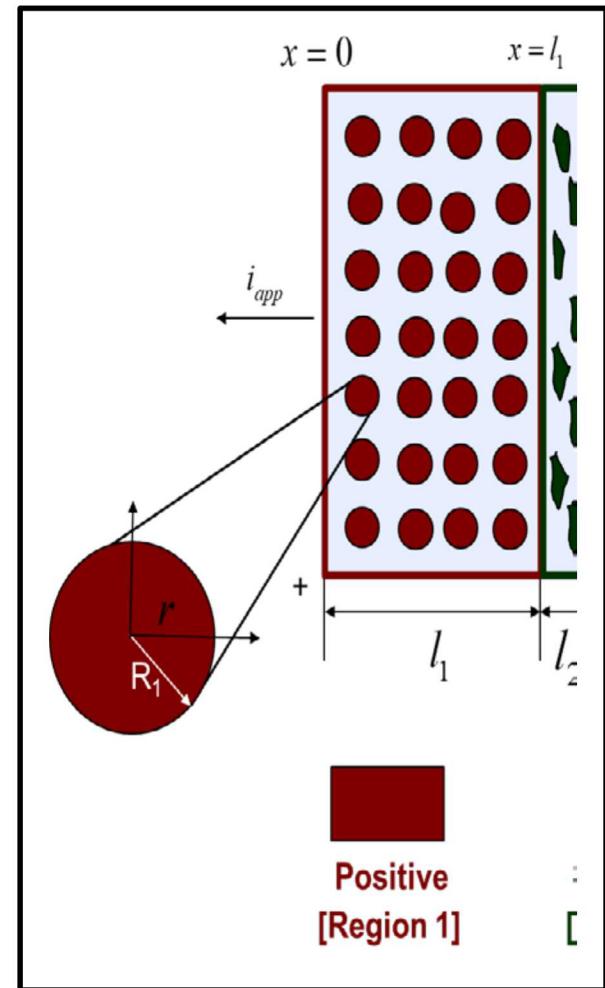
$$\frac{\partial c_1^s}{\partial t} = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 D_1^s \frac{\partial c_1^s}{\partial r} \right)$$



$$\frac{\partial \int_{x=0}^{x=l_1} c_1^s dx}{\partial t} = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 D_1^s \frac{\partial \int_{x=0}^{x=l_1} c_1^s dx}{\partial r} \right)$$

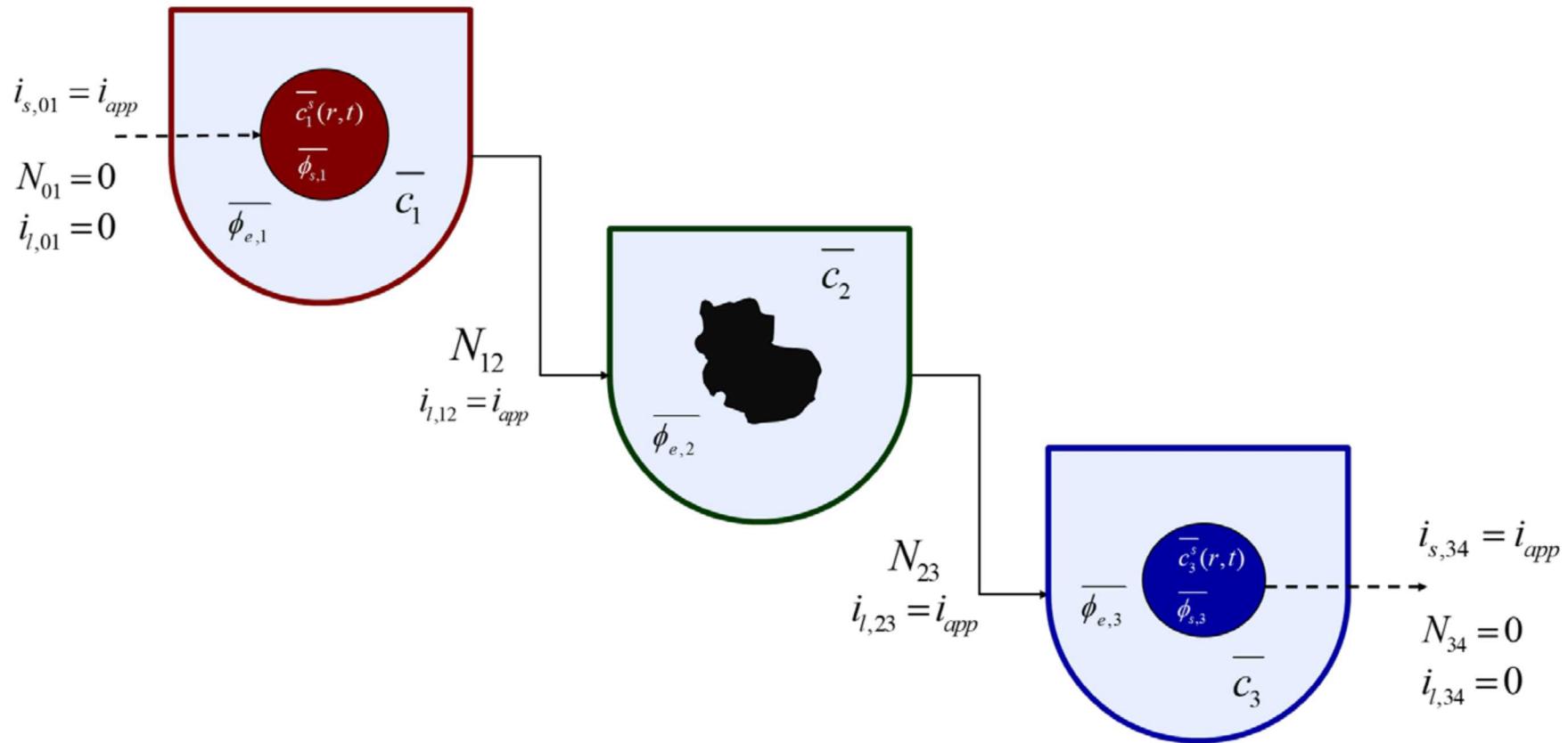


$$\frac{\partial \bar{c}_1^s}{\partial t} = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 D_1^s \frac{\partial \bar{c}_1^s}{\partial r} \right)$$



Restricted to moderate current scenarios, where liquid phase polarizations aren't significant

Lithium-ion Battery as Tanks in Series

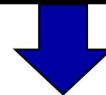


Can we incorporate average electrolyte dynamics by a similar volume-averaging approach?

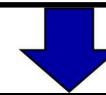
Generating the Tank Model

Example Conservation Equation:

$$\varepsilon_1 \frac{\partial c_1}{\partial t} = -\frac{\partial N_1}{\partial x} + a_1 (1 - t_+^0) j_1$$

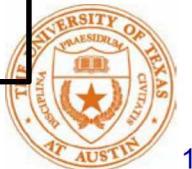


$$\frac{d \int_{V_1} \varepsilon_1 c_1 dV}{dt} = - \int_{V_1} \frac{\partial N_1}{\partial x} dV + \int_{V_1} a_1 (1 - t_+^0) j_1 dV$$

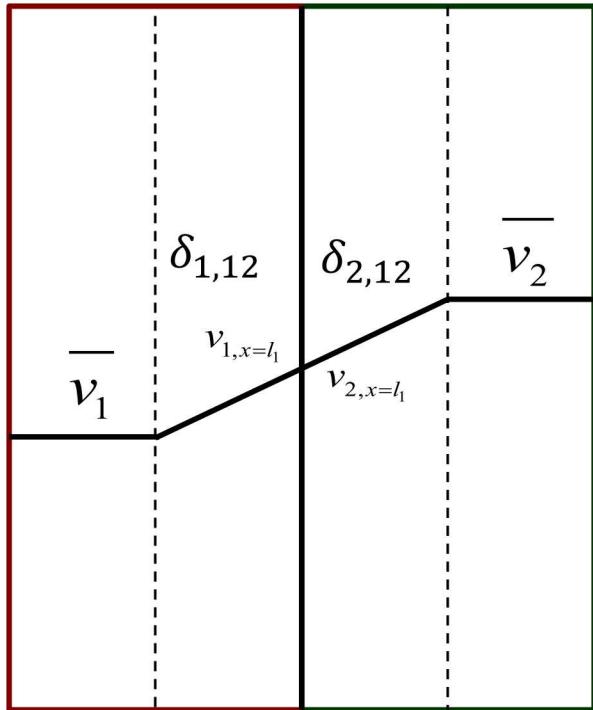


$$\varepsilon_1 \frac{d \bar{c}_1}{dt} = - \frac{\int_{x=0}^{x=l_1} \frac{\partial N_1}{\partial x} dx}{l_1} + a_1 (1 - t_+^0) \bar{j}_1$$

$\frac{N_{1,x=0} - N_{1,x=l_1}}{l_1} + a_1 (1 - t_+^0) \bar{j}_1$



Approximating Interfacial Fluxes



$$N_{1,x=l_1} = -D(c_{1,x=l_1})\varepsilon_1^{b_1} \frac{\partial c_1}{\partial x} \Big|_{x=l_1} \quad \square \quad D(c_{1,x=l_1})\varepsilon_1^{b_1} \left(\frac{\Delta c_1}{\delta_{1,12}} \right) = D(c_{1,x=l_1})\varepsilon_1^{b_1} \left(\frac{\bar{c}_1 - c_{1,x=l_1}}{\frac{l_1}{2}} \right)$$

$$N_{2,x=l_1} = -D(c_{2,x=l_1})\varepsilon_2^{b_2} \frac{\partial c_2}{\partial x} \Big|_{x=l_2} \quad \square \quad D(c_{2,x=l_1})\varepsilon_2^{b_2} \left(\frac{\Delta c_2}{\delta_{2,12}} \right) = D(c_{2,x=l_1})\varepsilon_2^{b_2} \left(\frac{-\bar{c}_2 + c_{2,x=l_1}}{\frac{l_2}{2}} \right)$$

$$D(c_{1,x=l_1})\varepsilon_1^{b_1} \left(\frac{\bar{c}_1 - c_{1,x=l_1}}{\frac{l_1}{2}} \right) = D(c_{2,x=l_1})\varepsilon_2^{b_2} \left(\frac{-\bar{c}_2 + c_{2,x=l_1}}{\frac{l_2}{2}} \right)$$

Naive assumptions of 'film thickness', can be regarded an adjustable parameter



Tank Model DAE System

Electronic Charge :

$$\bar{j}_1 = \frac{i_{app}}{a_1 Fl_1} \quad \bar{j}_3 = -\frac{i_{app}}{a_3 Fl_3}$$

Electrolyte Mass:

$$\begin{aligned} \varepsilon_1 \frac{d\bar{c}_1}{dt} &= \frac{\frac{2D(c_{12})(\bar{c}_2 - \bar{c}_1)}{\frac{l_1}{\varepsilon_1^b} + \frac{l_2}{\varepsilon_2^b}} + a_1(1-t_+) \bar{j}_1}{l_1} \\ &\quad + \frac{\frac{-2D(c_{12})(\bar{c}_2 - \bar{c}_1)}{\frac{l_1}{\varepsilon_1^b} + \frac{l_2}{\varepsilon_2^b}} + \frac{2D(c_{23})(\bar{c}_3 - \bar{c}_2)}{\frac{l_2}{\varepsilon_2^b} + \frac{l_3}{\varepsilon_3^b}}}{l_2} \\ \varepsilon_2 \frac{d\bar{c}_2}{dt} &= \frac{\frac{-2D(c_{23})(\bar{c}_3 - \bar{c}_2)}{\frac{l_2}{\varepsilon_2^b} + \frac{l_3}{\varepsilon_3^b}} + a_3(1-t_+) \bar{j}_3}{l_3} \end{aligned}$$

Liquid Charge (plus reference) :

$$i_{l,1,x=l_1} = -2\kappa(c_{12}) \left(\frac{\bar{\phi}_{l,2} - \bar{\phi}_{l,1}}{\frac{l_1}{\varepsilon_1^b} + \frac{l_2}{\varepsilon_2^b}} \right) + \frac{4RT(1-t_+^0)}{F} \kappa(c_{12}) \frac{1}{c_{12}} \left(\frac{\bar{c}_2 - \bar{c}_1}{\frac{l_1}{\varepsilon_1^b} + \frac{l_2}{\varepsilon_2^b}} \right) = i_{app}$$

$$i_{l,3,x=l_1+l_2} = -2\kappa(c_{23}) \left(\frac{\bar{\phi}_{l,3} - \bar{\phi}_{l,2}}{\frac{l_3}{\varepsilon_3^b} + \frac{l_2}{\varepsilon_2^b}} \right) + \frac{4RT(1-t_+^0)}{F} \kappa(c_{23}) \frac{1}{c_{23}} \left(\frac{\bar{c}_3 - \bar{c}_2}{\frac{l_2}{\varepsilon_2^b} + \frac{l_3}{\varepsilon_3^b}} \right) = i_{app}$$

$$\bar{\phi}_{l,12} = \left(\frac{\frac{\varepsilon_1^b}{l_1} \bar{\phi}_{l,1} + \frac{\varepsilon_2^b}{l_2} \bar{\phi}_{l,2}}{\frac{\varepsilon_1^b}{l_1} + \frac{\varepsilon_2^b}{l_2}} \right) = 0$$

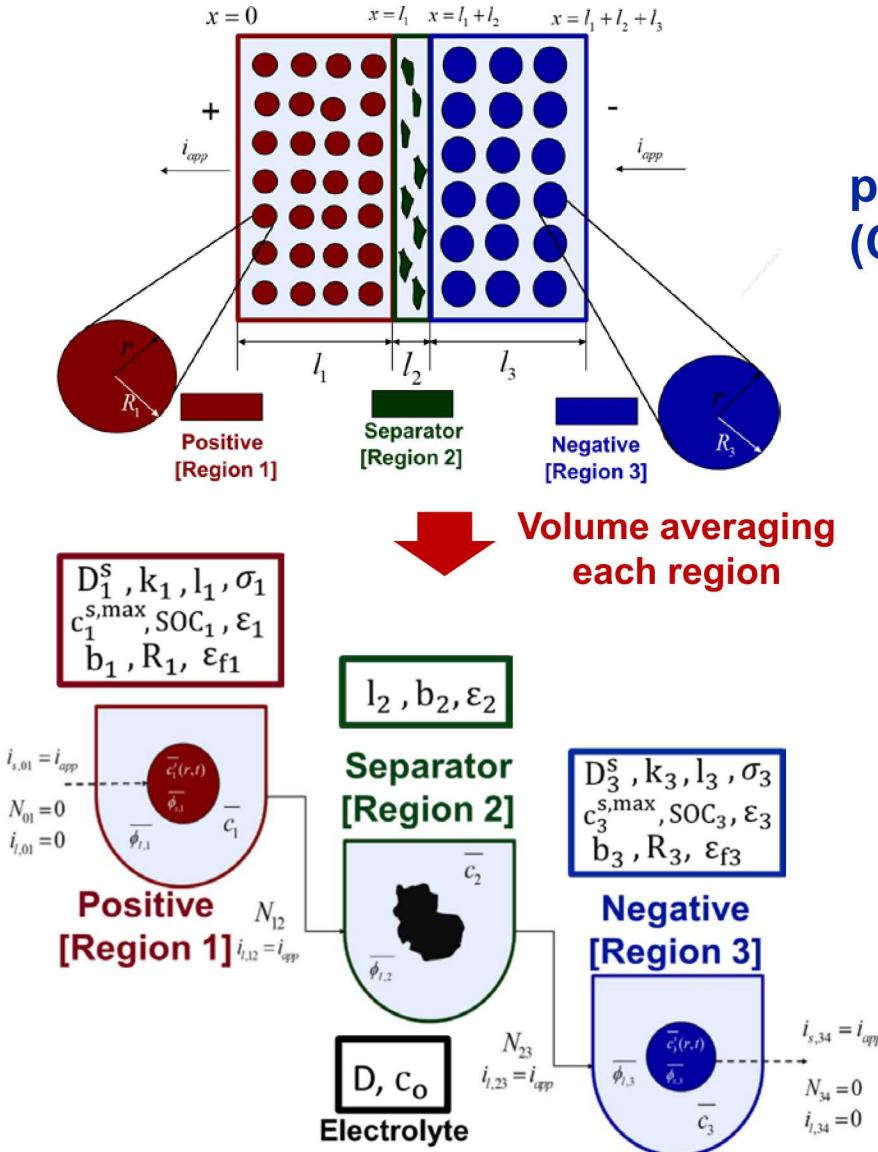
Solid Phase Mass (SPM):

$$\frac{\partial \bar{c}_1^s}{\partial t} = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 D_1^s \frac{\partial \bar{c}_1^s}{\partial r} \right)$$

PDE Model is thus reduced to simplified DAE system for liquid phase.



Novel Tanks-in-Series Model



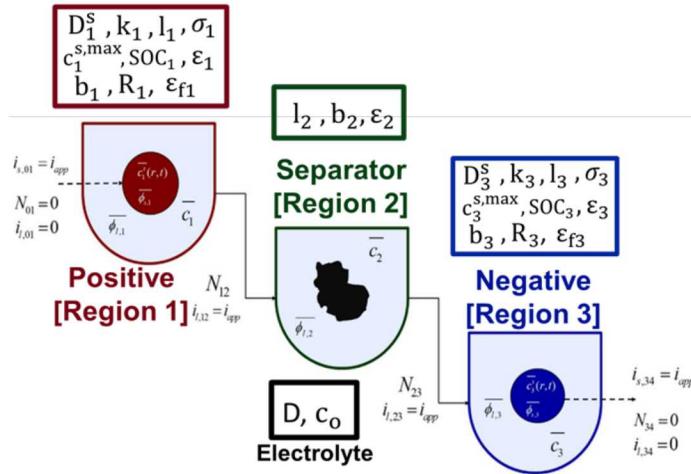
p2D model > 1000 DAEs (complex)
(Computation time, C++: 1493ms)

Tanks-in-Series model ~10 DAEs
(Computation time, C++: 2.1ms)

<1% error observed as compared
to existing physics-based battery
models (SPM, P2D, etc.)



Novel Tanks-in-Series Model

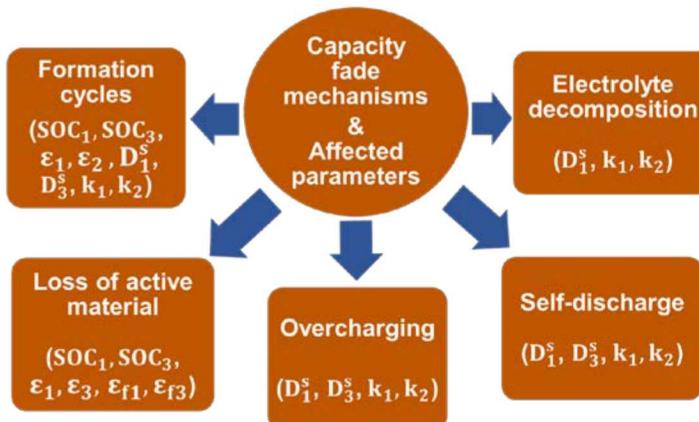


List of Tanks-in-Series model parameters

Parameter	Symbol
Electrolyte Diffusivity	D
Solid Phase Diffusivity	D_1^s, D_3^s
Reaction Rate Constant	k_1, k_3
Solid Phase Conductivity	σ_1, σ_3
Electrolyte Conductivity	κ
Electrolyte Concentration	c_0
Maximum Solid Phase Concentration	$c_1^{s,\max}, c_3^{s,\max}$
Initial SOC	SOC_1, SOC_3
Transference Number	t_+^0
Porosity	ϵ_1, ϵ_2
Filler Fraction	$\epsilon_{f1}, \epsilon_{f3}$
Bruggeman Constant	b_1, b_2, b_3
Thickness	l_1, l_2, l_3
Particle Size	R_1, R_3

index: 1 (+ve electrode), 2 (separator), 3 (-ve electrode)

Parameters to be estimated to model Battery aging



Ref: Subramaniam, A. et. al. (2020). JES, 167(1), 013534.



Optimization Formulation

Objective:
$$\min \sqrt{\frac{\left[\sum_{t_0=0}^{t_n=t_f} (V_{\text{exp}}(t_i) - V_{\text{model}}(t_i))^2 \right]}{N_{\text{exp}}}}$$

where, V_{exp} : experimental voltage profile

V_{model} : Tanks – in – Series model voltage profile

Subject to:

$$\frac{\partial}{\partial x} g_j(\bar{x}) = 0, \quad j = 1, \dots, J \quad \text{Ordinary differential equations (ODEs)}$$

$$h_k(\bar{x}) = 0, \quad k = 1, \dots, K \quad \text{Algebraic equations (AEs)}$$

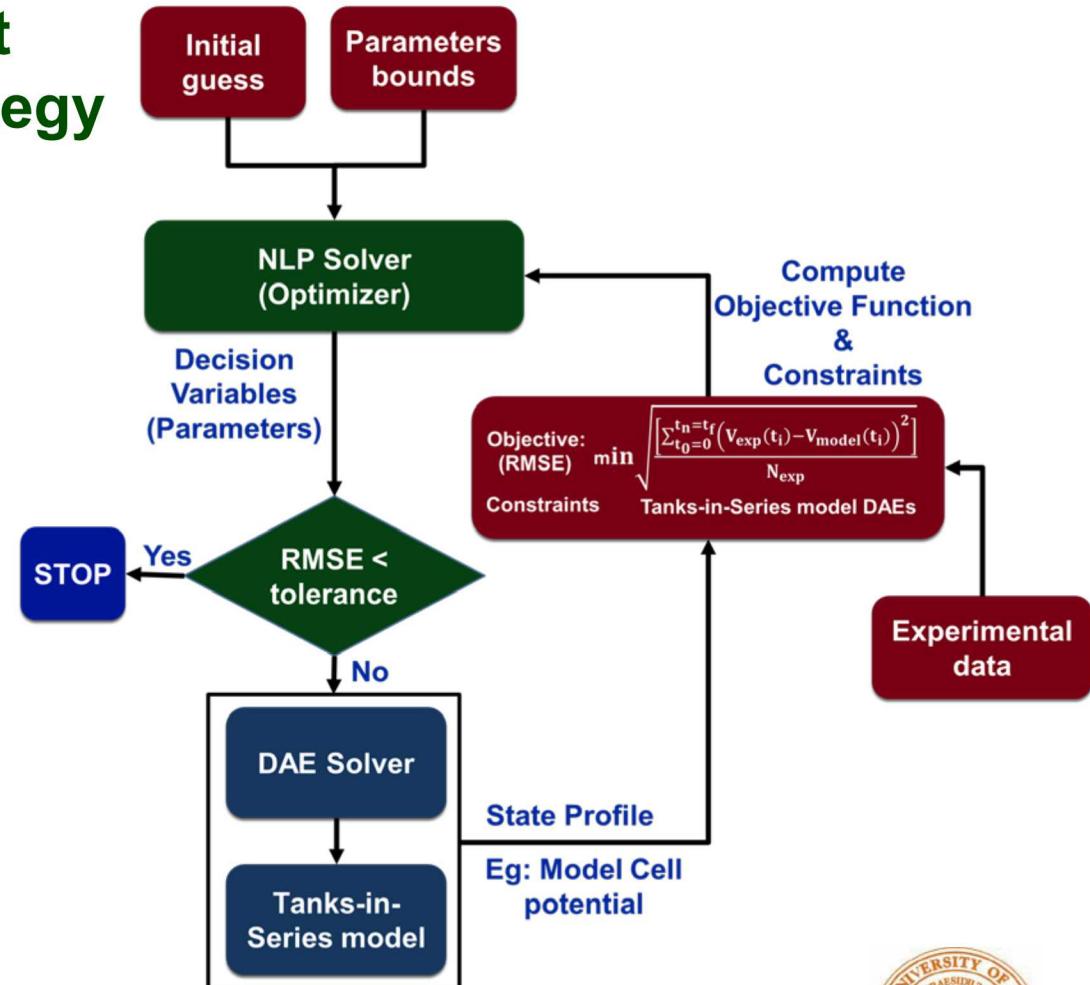
Differential algebraic equations (DAEs)

Variables: $\bar{x} \in \Omega$ $lb \leq \bar{x} \leq ub$

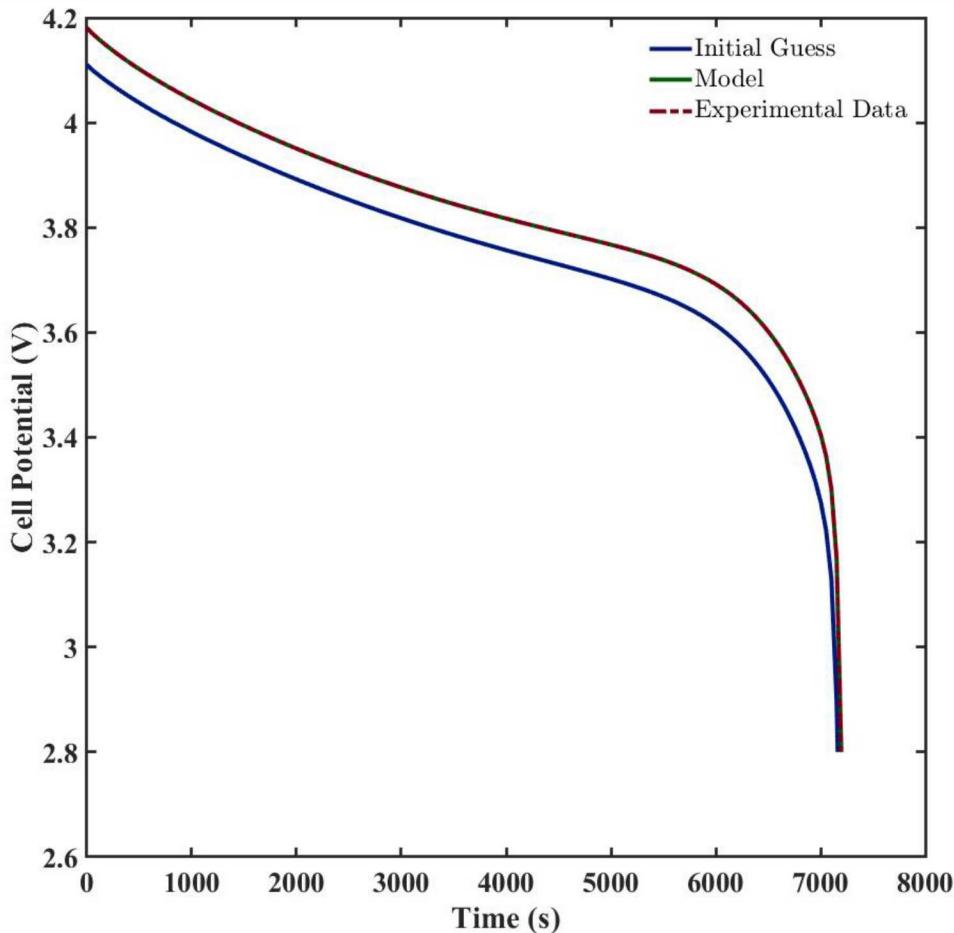


Proposed Methodology

Sequential parameter estimation using robust dynamic optimization strategy



Fast Computation



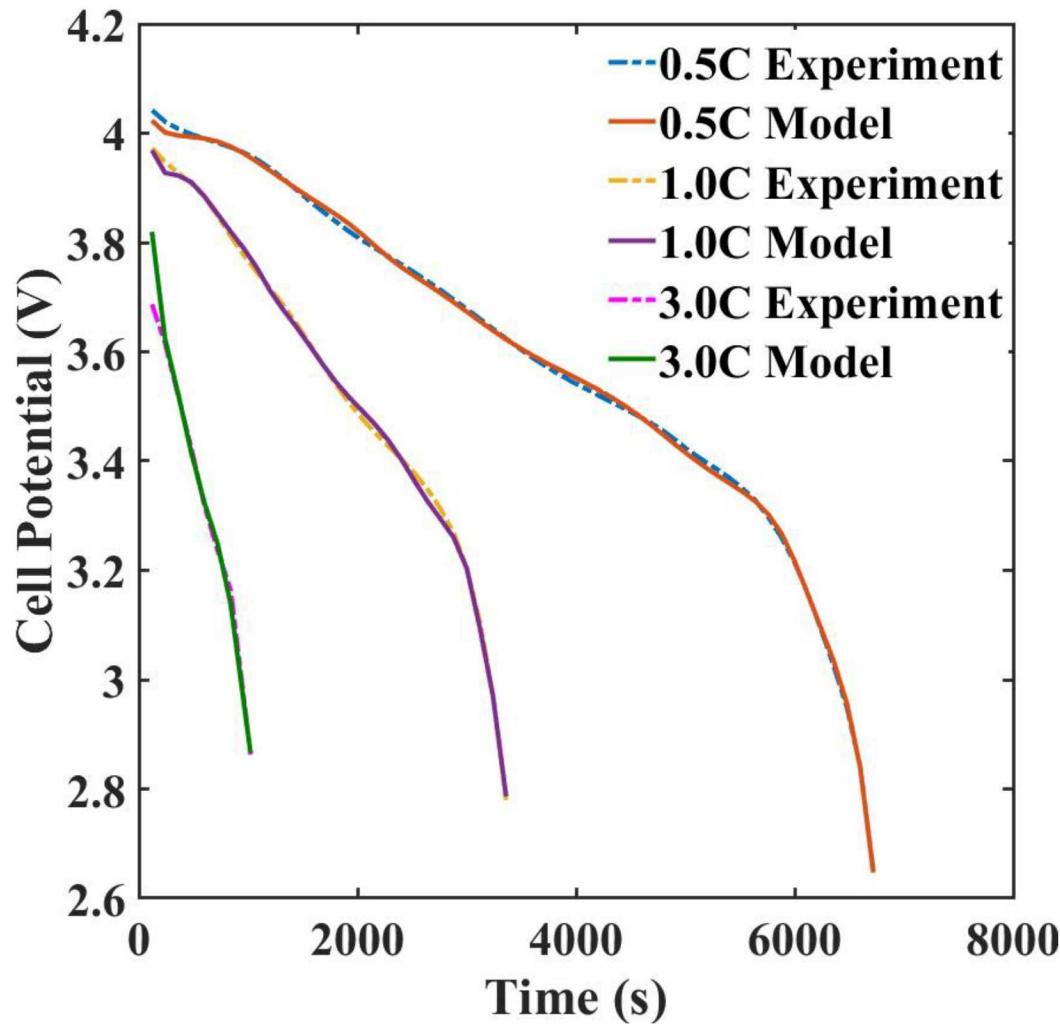
► **Tanks-in-Series model enables
Fast Computation in MATLAB**

Battery Model	Total Estimation Time (s)
SPM	~40
Tanks-in-Series	~45
p2D	~900

Comparison plot between Initial Guess and Optimal parameters of Tanks-in-Series model for 0.5C synthetic discharge data.



Results and Discussions



RMSE error values (in mV)

Crate	RMSE (mV)
0.5	7.22
1.0	10.13
3.0	11.64

Note: Experiments are performed on 3Ah 18650 cylindrical cell (Cathode: NMC811 and Anode: Si-C) at different discharge rates (C/2, 1C and 3C) at Sandia National Laboratories.

Conclusions

- ▶ Novel Tanks-in-Series approach is adopted to model the cell behavior at different discharge rates (0.5C, 1C, and 3C).
- ▶ The adopted model competes with SPM (in terms of simulation time) and complies with the synthetic experimental data within 15 mV error.
- ▶ A robust sequential optimization approach is proposed to estimate the parameters of the battery model with <12 mV error.



Acknowledgements

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THANK YOU!

Any Questions?

