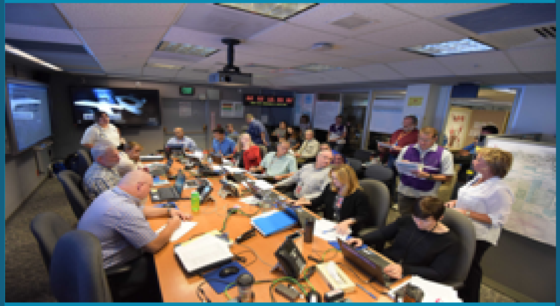
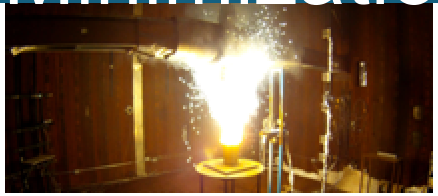
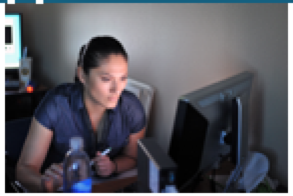
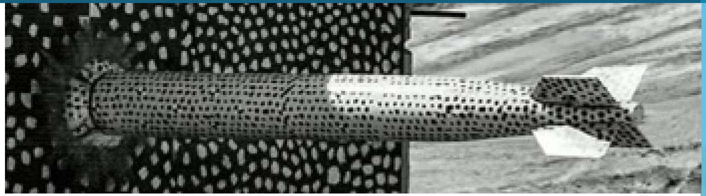
SAND2020-12031PE

Gradient-based Optimization for Structural Isolation and Acoustic Scattering Minimization









Presented by

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Outline

Metamaterials and gradient-based methods

Sierra/SD structural dynamics code

Theoretical framework for optimization

Example problems

- Split-ring resonator for vibration isolation
- Two-phase design to minimize scattered field
- Optimizable channels to minimize scattered field

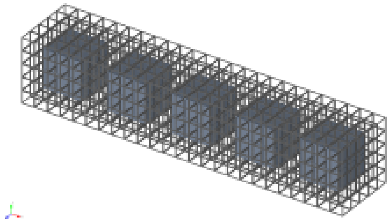
Conclusions and future work

Metamaterials and Gradient-based Methods

Metamaterials and Gradient-Based Methods

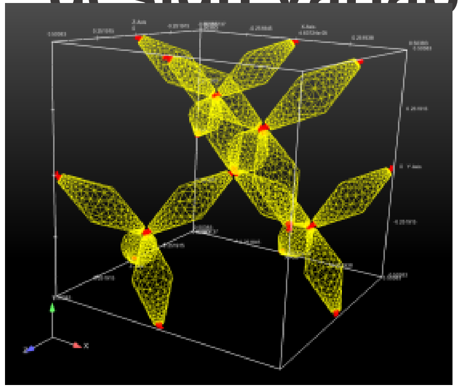
Acoustic metamaterials are useful in a wide variety of applications

- Can de difficult to manufacture (e.g., pentamode)
- May have a large number of parameters (potentially thousands to millions), presenting challenges for global search-based optimization

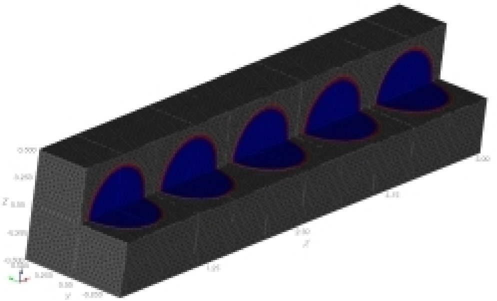


Lattice with embedded masses

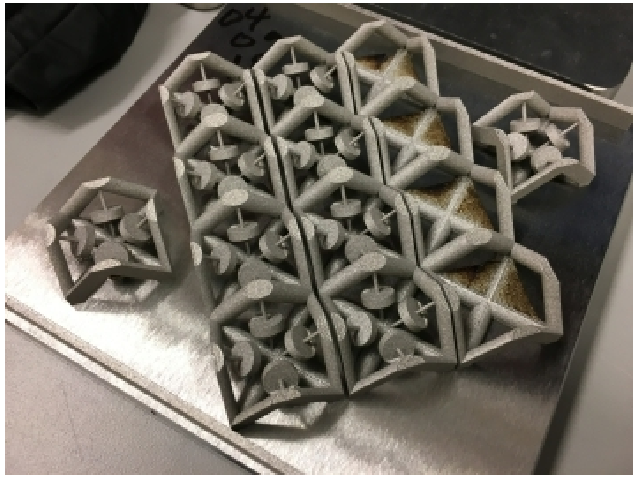
Gradient (adjoint) based optimization allows for sensitivity computations independent of number of design variables



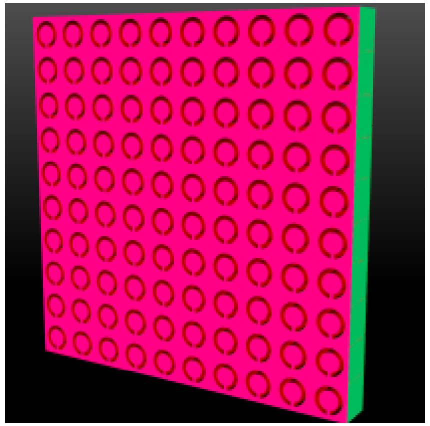
Pentamode lattice



Multiphase composite



Stainless steel print with embedded resonators



Split-ring resonator

Sierra/SD Structural Dynamics Code

Overview of Sierra Mechanics

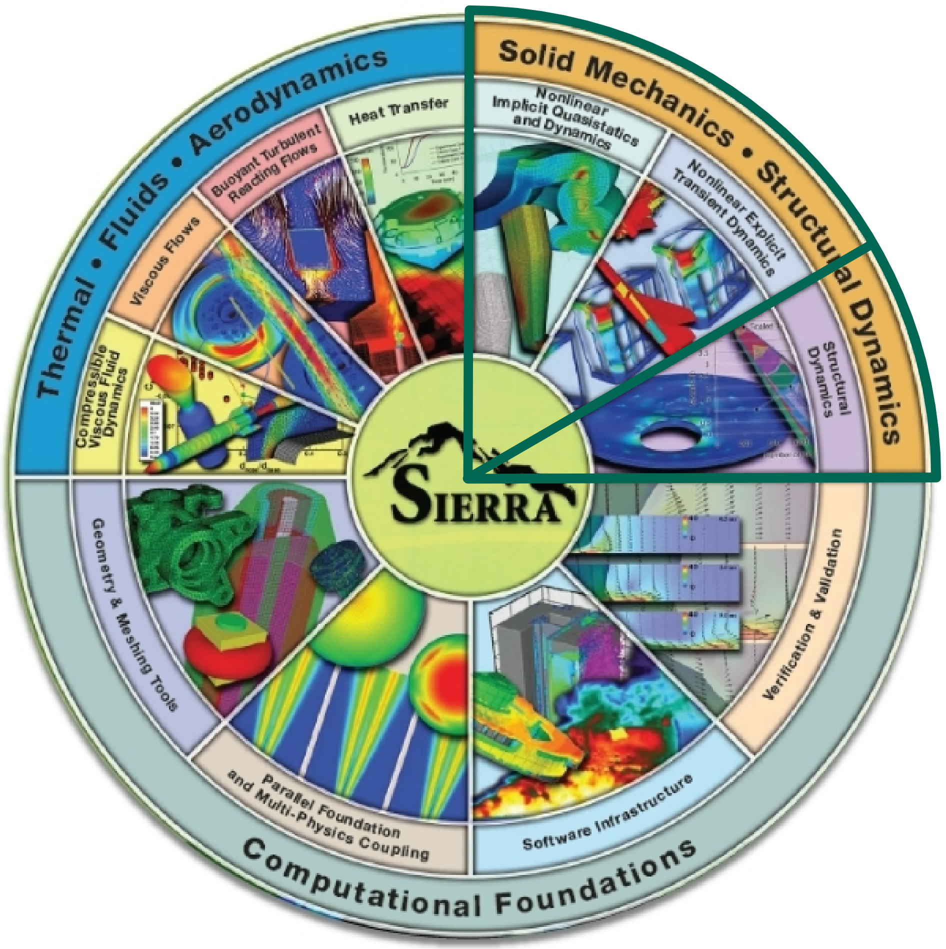
Massively parallel, coupled multiphysics simulations

Physics modules
- Structural dynamics
- Solid mechanics
- Fluids
- Thermal

Geometry and meshing tools

Funded by Department of Energy
- Advanced Simulation and Computing Program

Available for federal government use



Sierra/SD Structural Acoustics Capabilities

Massively parallel

Various acoustic elements (up to order $p = 6$)

- Hexahedral
- Wedge
- Tetrahedral

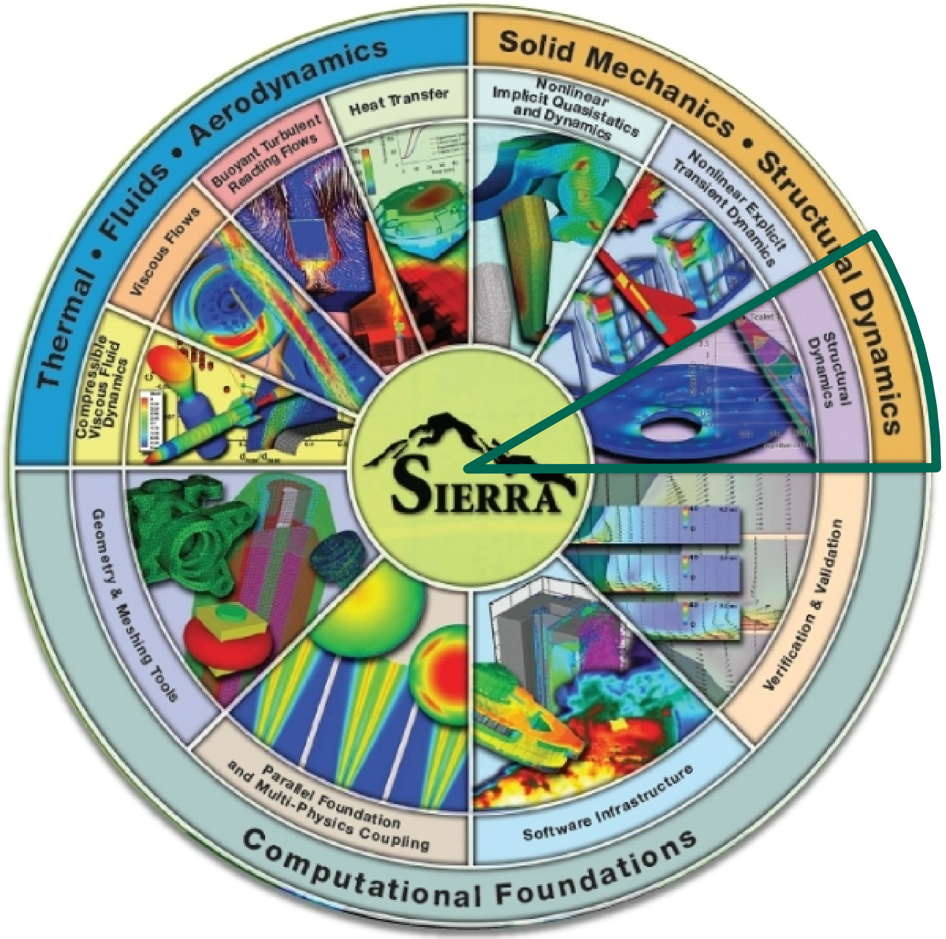
Coupling between acoustic elements and 2D/3D structural elements

Mismatched acoustic/solid meshes

Support for absorbing boundary conditions

- Infinite elements
- Perfectly matched layers (PMLs)

Frequency domain, time domain, and modal analysis



Example Sierra/SD Simulation: Orion Capsule

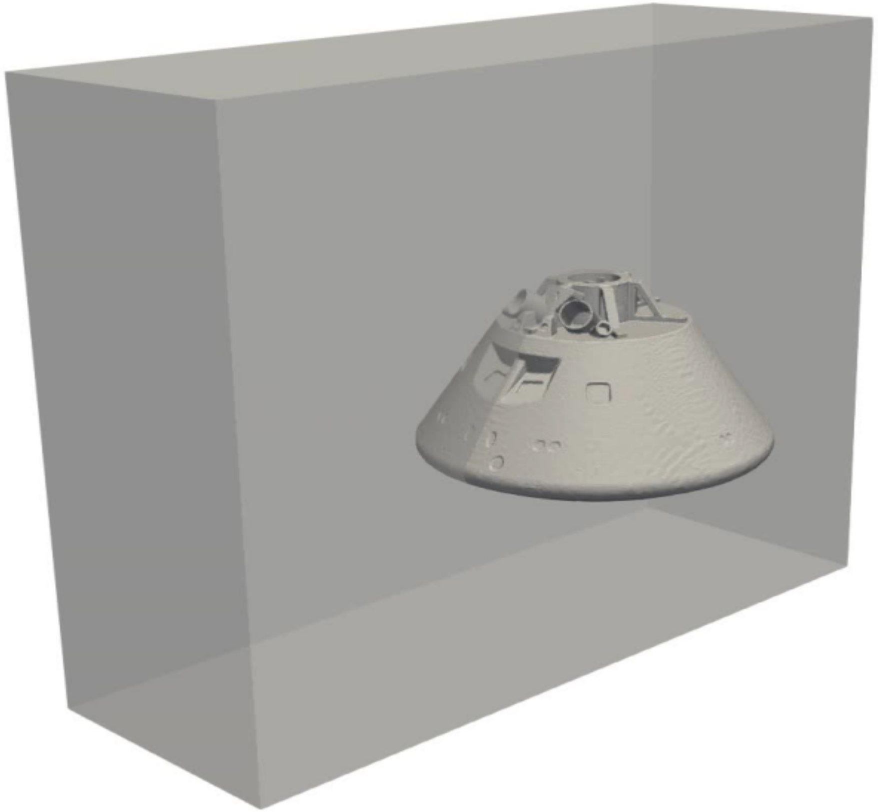
Publicly available model from NASA

Vibroacoustic Test Facility (VATF)

- Rectangular box: 6.58m by 7.50m by 9.17m

Simulation

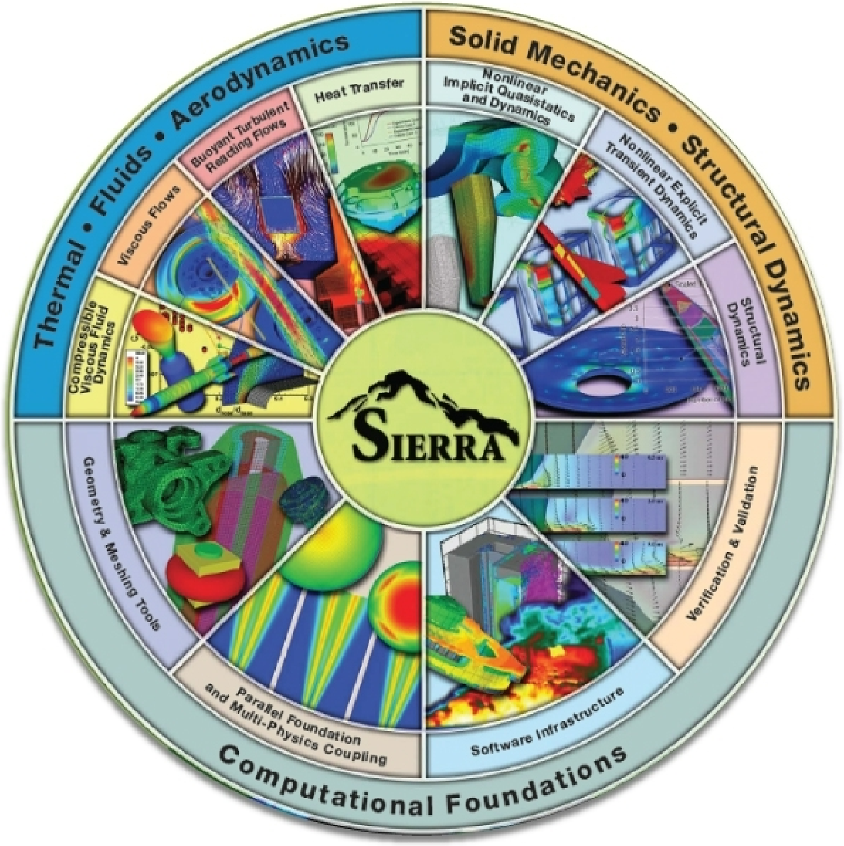
- Acoustic excitation from loudspeaker in bottom corner room (140 dB re 20 μPa)
- 2.5 million Hex20 elements, 10 millions degrees of freedom
- 1,000 time steps
- 2 hrs simulation time on 256 processors



Theoretical Framework for Optimization

Design Optimization in Sierra/SD

Inverse solution types via Sierra/SD linked to Rapid Optimization Library (ROL)



Objective function,
Derivative operators

→

←

Next iteration
of design variables

RAPID OPTIMIZATION LIBRARY

PDE-Constrained Optimization

Abstract optimizatio n formulation

$$\underset{\boldsymbol{u},\boldsymbol{p}}{\text{minimize}} \quad J(\boldsymbol{u},\boldsymbol{p})$$

Objective function

$$\text{subject to} \quad \boldsymbol{g}(\boldsymbol{u},\boldsymbol{p}) = 0$$

PDE constraint

$$\mathcal{L}(\boldsymbol{u},\boldsymbol{p},\boldsymbol{w}) := J + \boldsymbol{w}^T \boldsymbol{g}$$

Lagrangian

$$\begin{Bmatrix} \mathcal{L}_u \\ \mathcal{L}_p \\ \mathcal{L}_w \end{Bmatrix} = \begin{Bmatrix} J_u + \boldsymbol{g}_u^T \boldsymbol{w} \\ J_p + \boldsymbol{g}_p^T \boldsymbol{w} \\ \boldsymbol{g} \end{Bmatrix} = \{0\}$$

First order optimality conditions

$$\begin{bmatrix} \mathcal{L}_{uu} & \mathcal{L}_{up} & \boldsymbol{g}_u^T \\ \mathcal{L}_{pu} & \mathcal{L}_{pp} & \boldsymbol{g}_p^T \\ \boldsymbol{g}_u & \boldsymbol{g}_p & 0 \end{bmatrix} \begin{Bmatrix} \delta \boldsymbol{u} \\ \delta \boldsymbol{p} \\ \boldsymbol{w}^\star \end{Bmatrix} = - \begin{Bmatrix} J_u \\ J_p \\ \boldsymbol{g} \end{Bmatrix}$$

Newton iteration

$$\boldsymbol{W} \Delta \boldsymbol{p} = -\hat{J}',$$

$$\boldsymbol{W} = \boldsymbol{g}_p^T \boldsymbol{g}_u^{-T} (\mathcal{L}_{uu} \boldsymbol{g}_u^{-1} \boldsymbol{g}_p - \mathcal{L}_{up}) - \mathcal{L}_{pu} \boldsymbol{g}_u^{-1} \boldsymbol{g}_p + \mathcal{L}_{pp}$$

Hessian calculation

$\boldsymbol{u}$ ~ response
$\boldsymbol{p}$ ~ design parameters
$\boldsymbol{w}$ ~ Lagrange multipliers

Discrete Equations for Inverse Problem

	Objective Functions	Governing Equations
Time	$J(\{\boldsymbol{u}\},\{\boldsymbol{p}\}) = \frac{\kappa}{2}\left(\{\boldsymbol{u}\} - \{\boldsymbol{u}_m\}\right)^{\mathrm{T}}[Q]\left(\{\boldsymbol{u}\} - \{\boldsymbol{u}_m\}\right) + \mathscr{R}(\{\boldsymbol{p}\})$	$\boldsymbol{g}(\boldsymbol{u},\boldsymbol{p}) = \boldsymbol{M}(\boldsymbol{p})\ddot{\boldsymbol{u}} + \boldsymbol{C}(\boldsymbol{p})\dot{\boldsymbol{u}} + \boldsymbol{K}(\boldsymbol{p})\boldsymbol{u} - \boldsymbol{f}(\boldsymbol{p})$
Frequency	$J(\{\tilde{\boldsymbol{u}}\},\{\boldsymbol{p}\}) = \frac{\kappa}{2}\sum_{k=1}^{N}\left(\tilde{\boldsymbol{u}}_k - \tilde{\boldsymbol{u}}_{mk}\right)^{\mathrm{H}}[Q]\left(\tilde{\boldsymbol{u}}_k - \tilde{\boldsymbol{u}}_{mk}\right) + \mathscr{R}(\{\boldsymbol{p}\})$	$\boldsymbol{g}(\boldsymbol{u},\boldsymbol{p}) = \left[\boldsymbol{K}(\boldsymbol{p}) + i\omega\boldsymbol{C}(\boldsymbol{p}) - \omega^2\boldsymbol{M}(\boldsymbol{p})\right]\boldsymbol{u} - \boldsymbol{f}(\boldsymbol{p})$
Eigen	$J(\{\lambda_i\},\{\boldsymbol{u}_i\},\{\boldsymbol{p}\}) = \frac{\beta_i}{2}\left\|\{r_i\}\right\|^2 + \frac{\kappa_i}{2}\mathscr{G}\left(\{\boldsymbol{u}_i\},\{\boldsymbol{u}_{mi}\}\right) + \mathscr{R}(\{\boldsymbol{p}\})$ $r_i = \frac{\lambda_i - \lambda_{mi}}{\lambda_{mi}}$	$\boldsymbol{g}(\lambda_i,\boldsymbol{u}_i,\boldsymbol{p}) = \boldsymbol{K}(\boldsymbol{p})\boldsymbol{u}_i - \lambda_i\boldsymbol{M}(\boldsymbol{p})\boldsymbol{u}_i$

Frequency-Domain Material or Force Optimization

Equations of motion

$$g(u, p) = \left[K(p) + i\omega C(p) - \omega^2 M(p)\right] u - f(p)$$
$$\sigma(x, \omega) \cdot n = t(x, \omega) \quad x \in \Gamma_N$$
$$u(x, \omega) = u_D \,, \quad x \in \Gamma_D$$

$$\hat{u}(x, \omega) = \sum_{i=1}^{N} u_i(\omega) \Phi_i(x)$$

Mass and stiffness matrices

$$K = \int_\Omega B^T D B \, d\Omega = \int_\Omega B^T \left(K D_K + \mu D_\mu\right) B \, d\Omega$$
$$M = \int_\Omega \rho \Phi \Phi^T \, d\Omega$$

$$\sigma = D\varepsilon = \left(K D_K + \mu D_\mu\right) \varepsilon$$

Force vector

$$f(p) = -\int_{\partial\Omega} v \cdot n \Phi^T \, dS$$

Gradients with respect to unknown material properties or unknown forces can be determined analytically

Optimization Process

PDE and Objective

$$\boldsymbol{g}(\boldsymbol{u},\boldsymbol{p}) = \left[\boldsymbol{K}(\boldsymbol{p}) + i\omega \boldsymbol{C}(\boldsymbol{p}) - \omega^2 \boldsymbol{M}(\boldsymbol{p})\right] \boldsymbol{u} - \boldsymbol{f}(\boldsymbol{p})$$

$$J(\hat{\boldsymbol{u}}) = \frac{1}{2}\left|\hat{\boldsymbol{u}} - \hat{\boldsymbol{u}}_m\right|^2$$

1. Solve PDE for $\hat{\boldsymbol{u}}$ (forward solution)
2. Use forward solution with J and $J_{\hat{\boldsymbol{u}}}$ (known) to find $\boldsymbol{w}$ (adjoint solution)
3. Use adjoint solution along with $J_{\boldsymbol{p}}$ and $\boldsymbol{g}_{\boldsymbol{p}}$ (both known) to find current gradient
4. Pass current objective, gradient to ROL

Optimality conditions

$$\mathcal{L}_{\hat{\boldsymbol{u}}} = J_{\hat{\boldsymbol{u}}} + \boldsymbol{g}_{\hat{\boldsymbol{u}}}^T \boldsymbol{w} = 0$$

$$\mathcal{L}_{\boldsymbol{p}} = J_{\boldsymbol{p}} + \boldsymbol{g}_{\boldsymbol{p}}^T \boldsymbol{w} = 0$$

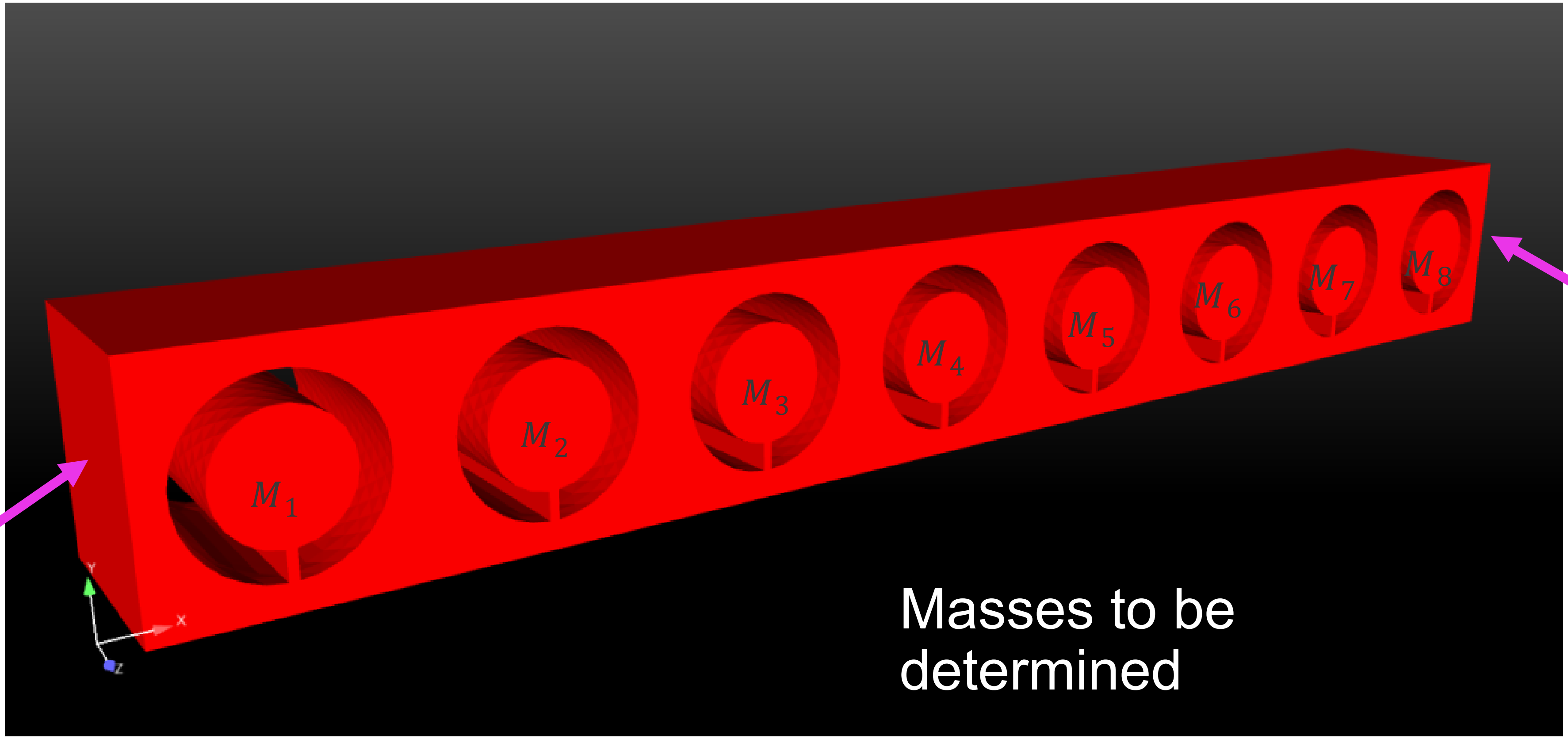
$$\mathcal{L}_{\boldsymbol{w}} = \boldsymbol{g} = 0$$

RAPID OPTIMIZATION LIBRARY

Example Problems

Example 1: Split-Ring Resonator for Vibration Isolation

Single material sample with embedded split-ring resonators



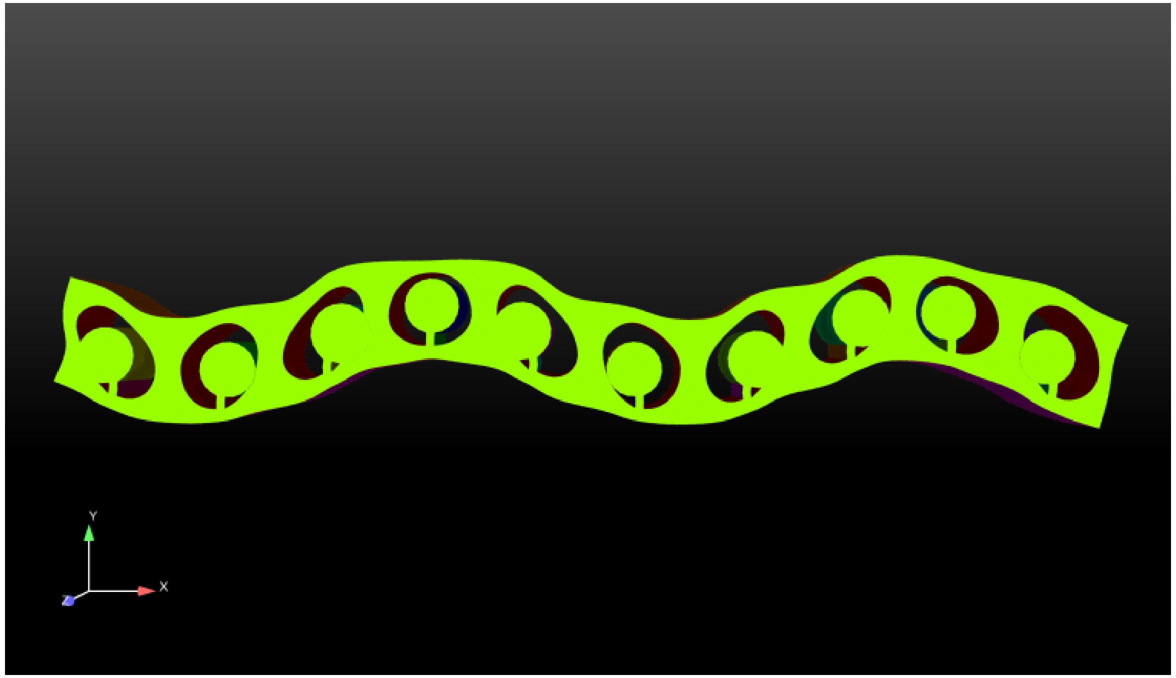
Steady-state input

$a_0 \sin \omega t$

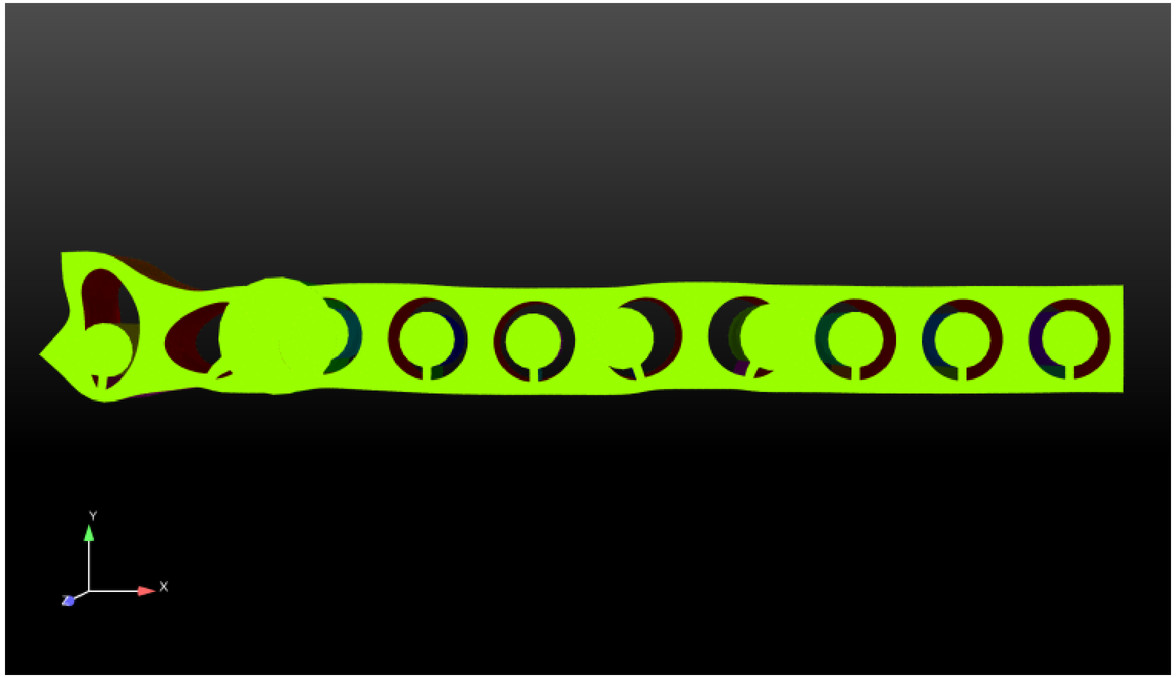
Measured output

Example 1: Single-Frequency Results (2.1 kHz)

Substantial reduction in displacement at right end when masses are optimized

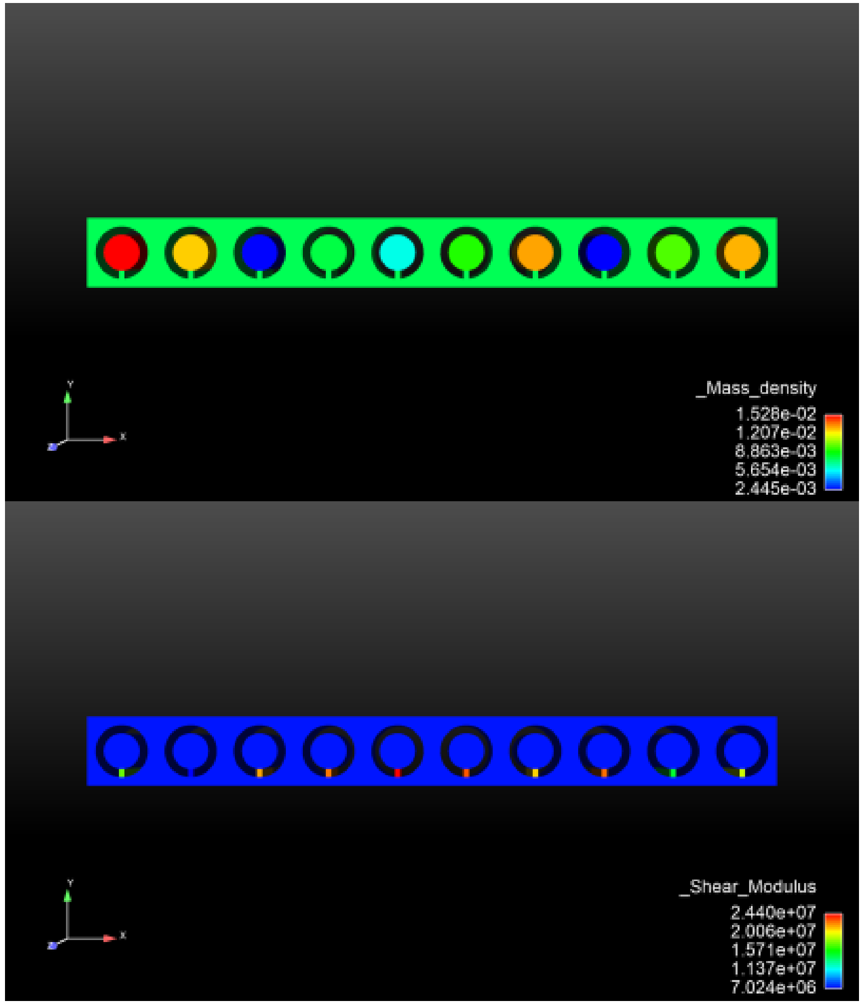


Initial guess: uniform material

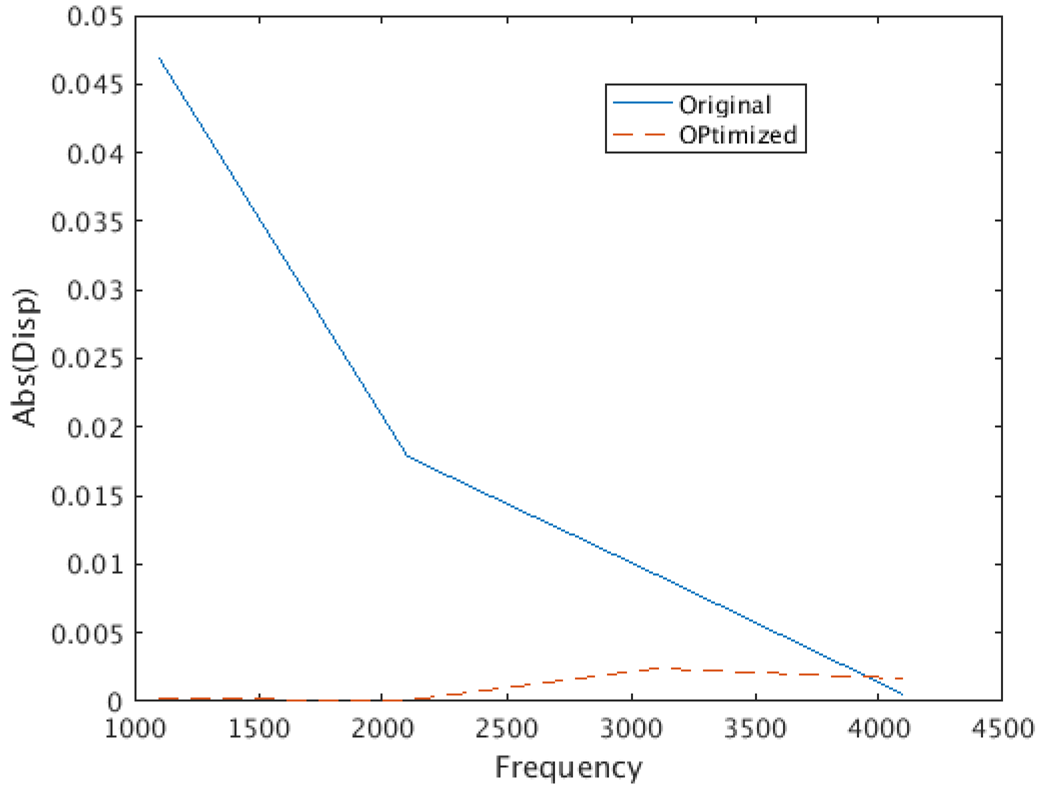


Optimized structure

Example 1: Multi-Frequency Results (1.1-4.1 kHz)



Optimized mass and stiffness distribution



Minimized displacement at right end

Example 1: 3D Printing Optimized Mass Resonators

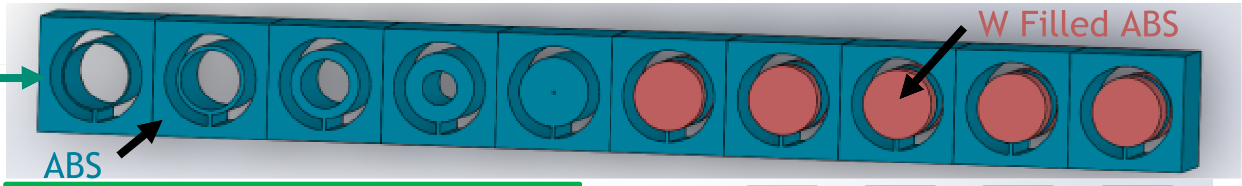
Multi-material additive manufacturing enables optimized mass distributions

1. Input optimized resonator mass values

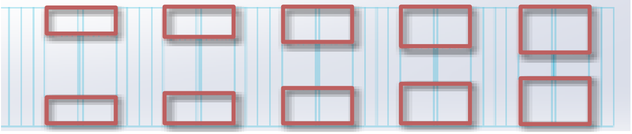
GlobalVariables.txt - Notepad

File Edit Format View Help

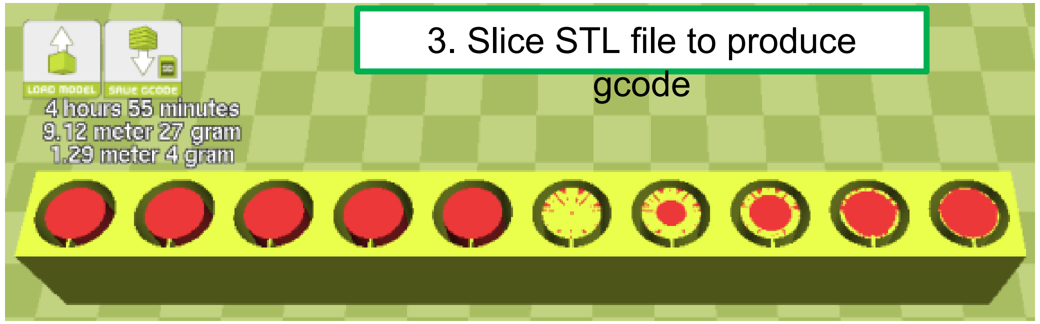
```
"m1"=0.1
"m2"=0.25
"m3"=0.5
"m4"=0.75
"m5"=1
"m6"=1.25
"m7"=2
"m8"=3
"m9"=4
"m10"=5
"rho_abs"=1.05
"rho_Wf"=8
"t"=1.5
"D1"=0.9
```



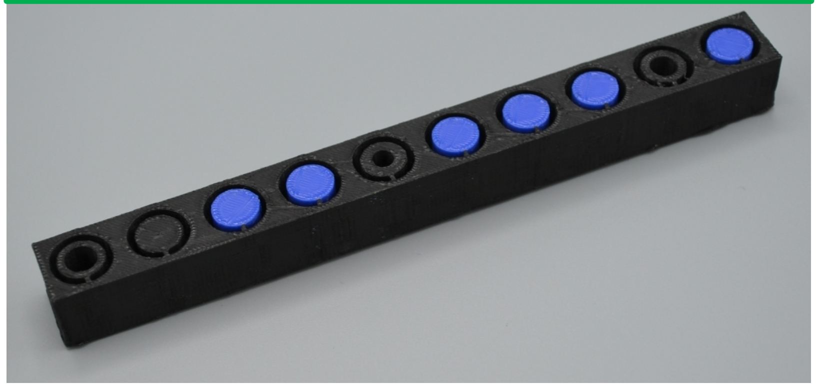
2. Global variables modify resonator geometry in SolidWorks



3. Slice STL file to produce gcode



4. 3D print optimized resonator masses



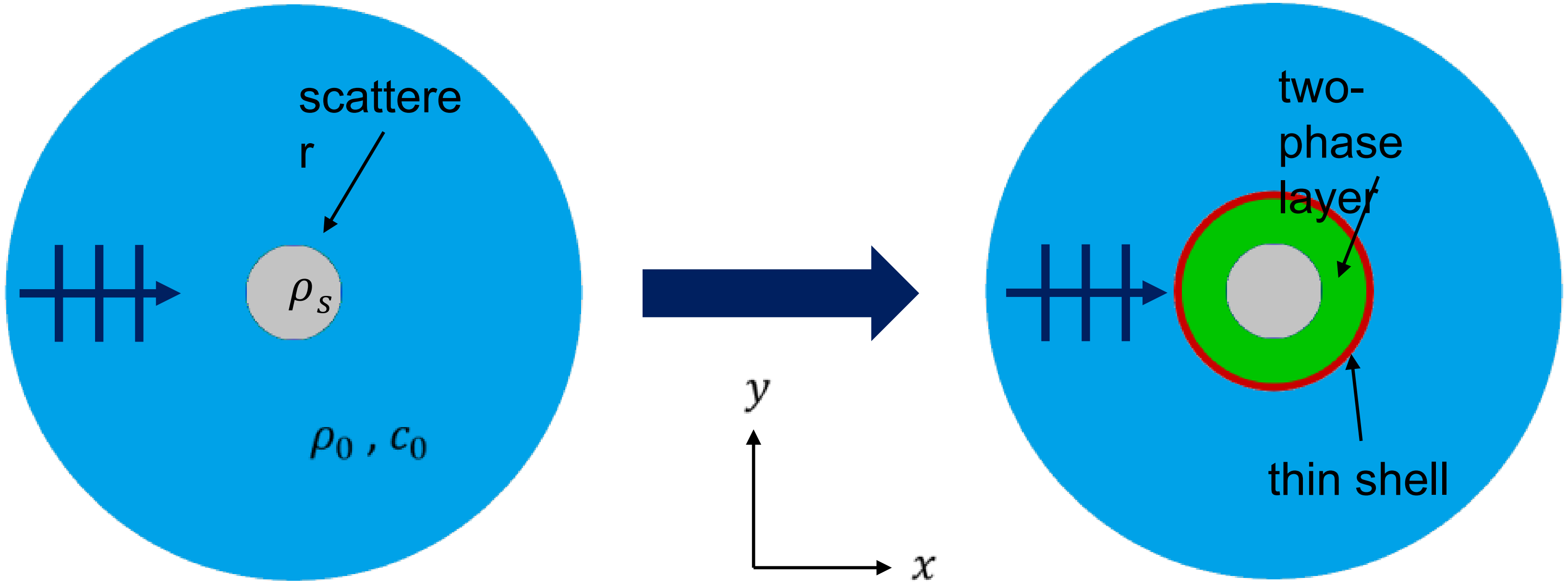
Example 2: Two-phase Design to Minimize Scattering

Time-harmonic plane wave (kHz range) incident on cylindrical scatterer

Scattered pressure formulation

$$\nabla^2\psi_{\mathrm{sc}} - \frac{1}{c_0^2}\frac{\partial^2\psi_{\mathrm{sc}}}{\partial t^2} = 0$$ in flui $$\rho_s\frac{\partial^2\mathbf{u}}{\partial t^2} - \boldsymbol{\nabla}\cdot\boldsymbol{\sigma} = \mathbf{F}$$ in scatterer

Two-phase surrounding layer: printable material either present or absent

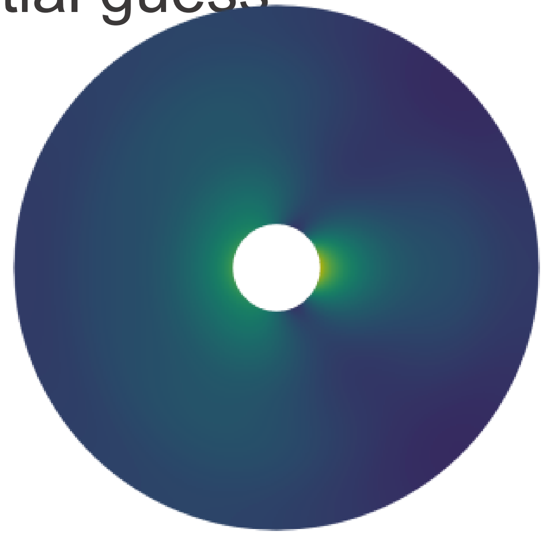


Example 2: Single-Frequency Optimization

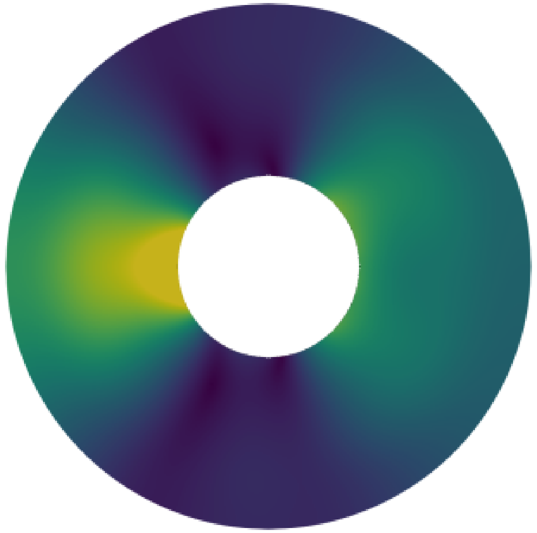
No annulus: scattering from cylinder alone

Initial guess: uniform material properties in surrounding layer

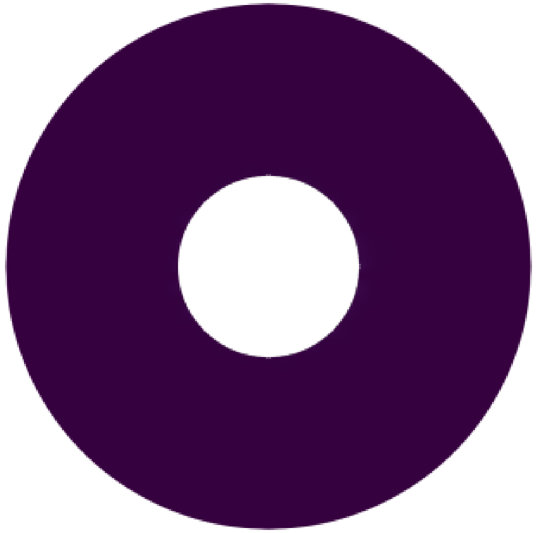
Optimized solution: scattered pressure objective function **reduced by ~4 orders of magnitude** versus initial guess



No annulus



Initial guess



Optimized solution

Example 2: Multi-Frequency Optimization

Operators for multi-frequency optimization problems straightforward to modify

$$\begin{aligned} \underset{\boldsymbol{u},\boldsymbol{p}}{\text{minimize}} \quad & J(\boldsymbol{u},\boldsymbol{p}) \\ \text{subject to} \quad & \boldsymbol{g}_i(\boldsymbol{u}_i,\boldsymbol{p}) = 0 \end{aligned}$$

Lagrangian modified accordingly, summing from $i = 1$ to N_{freqs}

$$\mathcal{L}(\boldsymbol{u},\boldsymbol{p},\boldsymbol{w}) = J + \sum_{i=1}^{N_{\text{freqs}}} \boldsymbol{w}_i^{\mathrm{T}} \boldsymbol{g}_i(\boldsymbol{u}_i,\boldsymbol{p})$$

Multiple directions can also be investigated with another sum from $j = 1$ to N_{dirs}

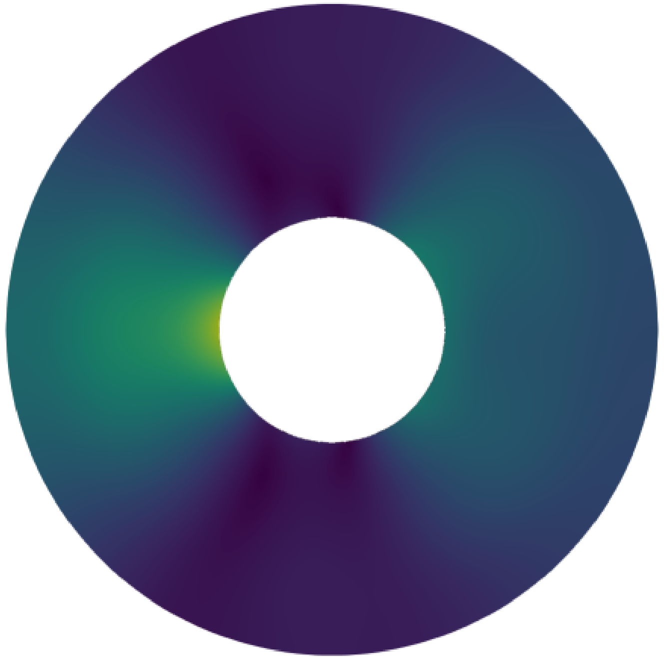
No further adjustments required in ROL!

Example 2: Multi-Frequency Optimization Results

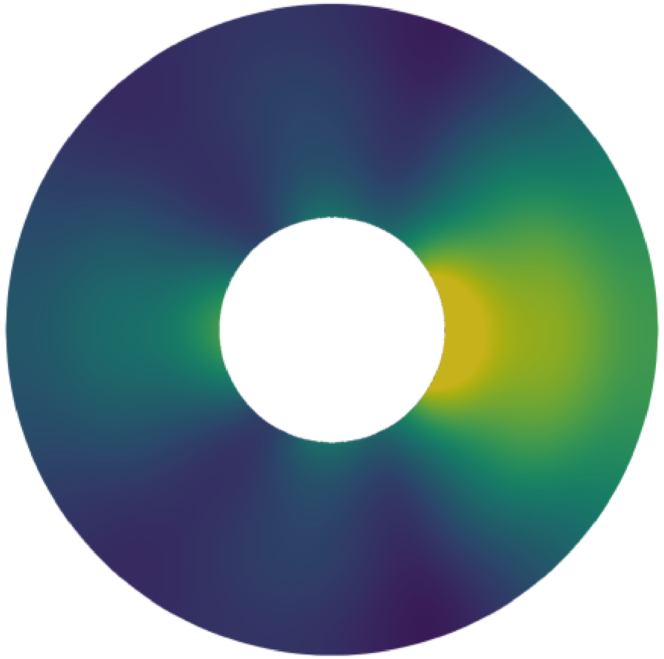
Two frequencies (kHz range) differing by ~10%

Same annulus design for both frequencies (simultaneous optimization)

Scattered pressure for **initial guess** (uniform properties in layer):



First frequency



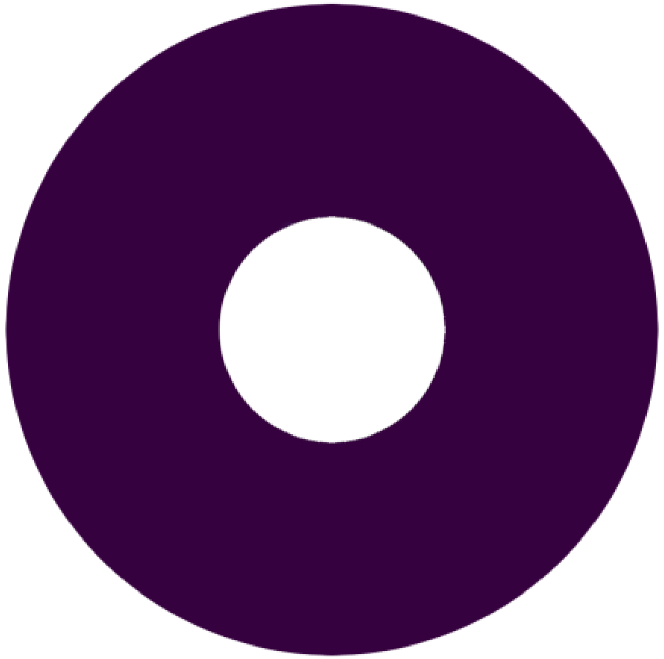
Second

Example 2: Multi-Frequency Optimization Results

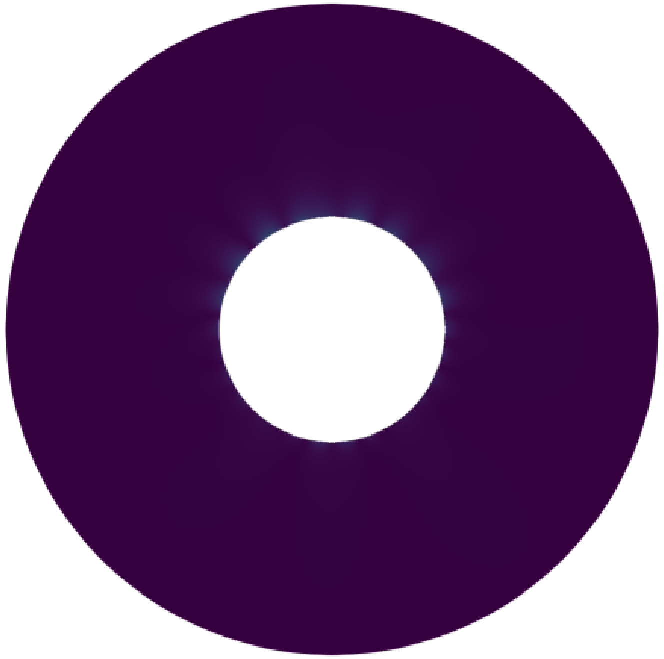
Two frequencies (kHz range) differing by ~10%

Same annulus design for both frequencies (simultaneous optimization)

Scattered pressure objective **reduced by ~3 orders of magnitude** for **optimized solution** (non-uniform properties in layer):



First frequency



Second

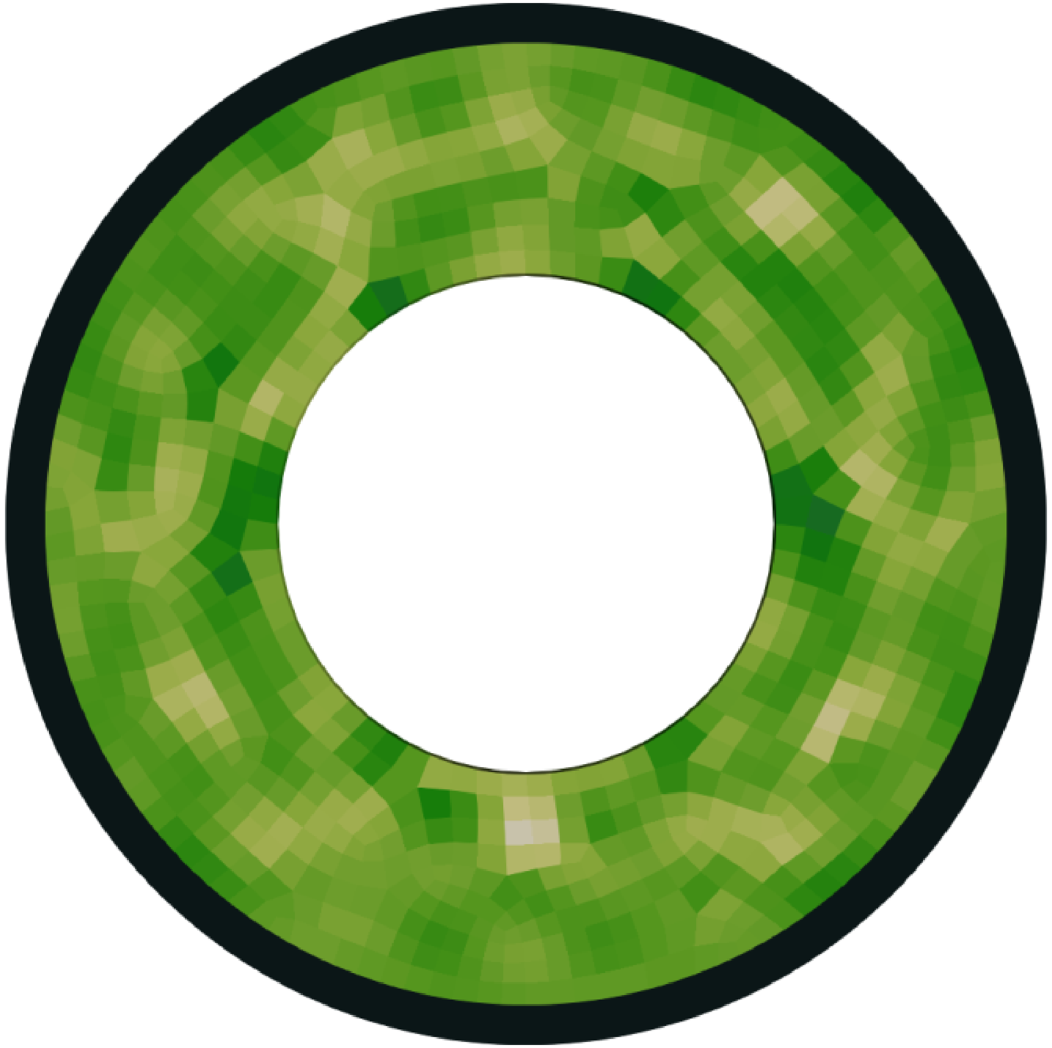
Example 2: Design for Two Frequencies

Material distribution in annulus for two-frequency optimization

Dark green = printable material

Light green = no material

Much of the optimized design contains material “in-between”, presenting manufacturing difficulties



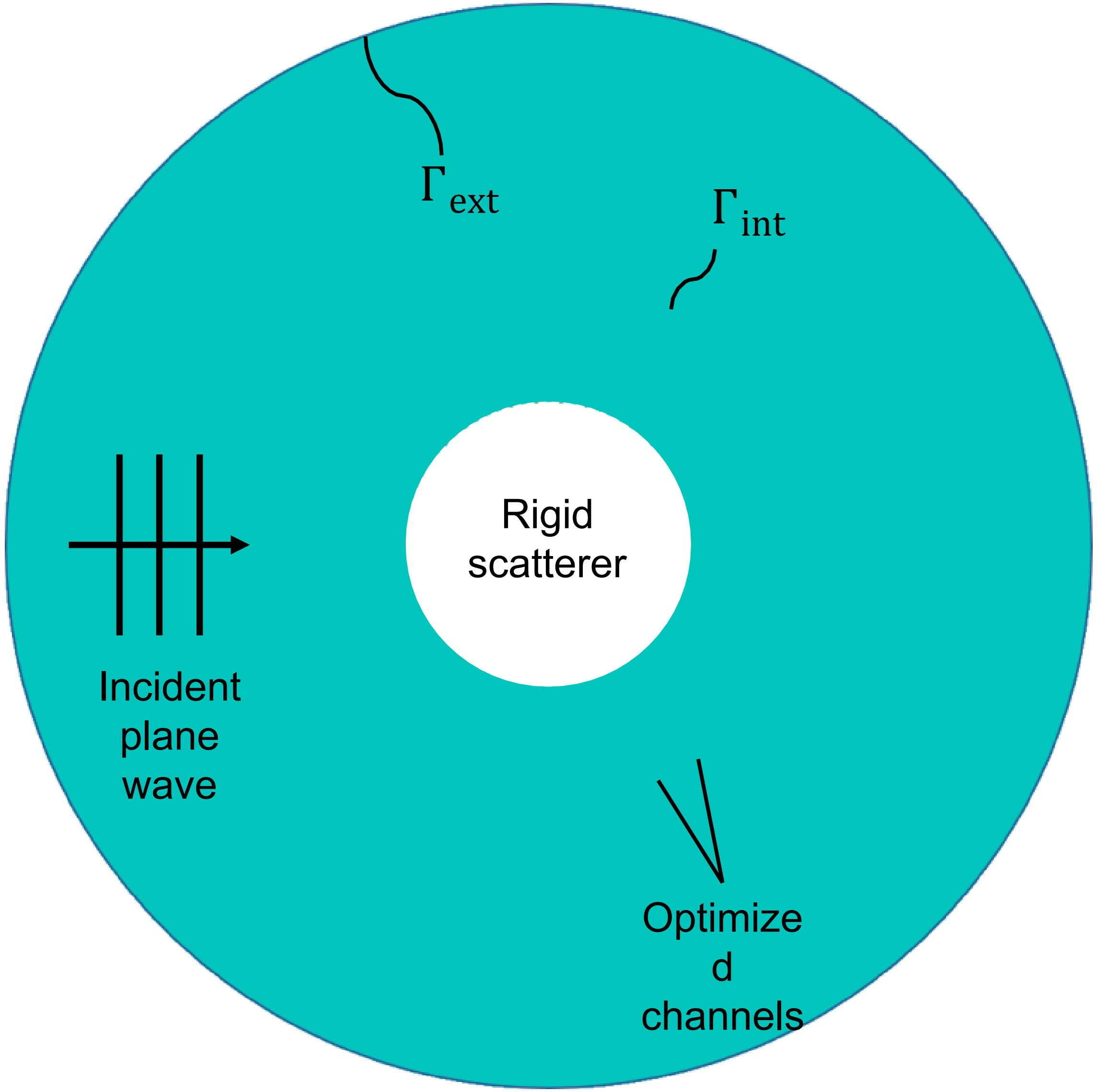
Example 3: Optimized Channels to Minimize Scattering

Time-harmonic plane wave incident on rigid cylindrical scatterer

Surrounding annulus is also rigid, but has channels carved out

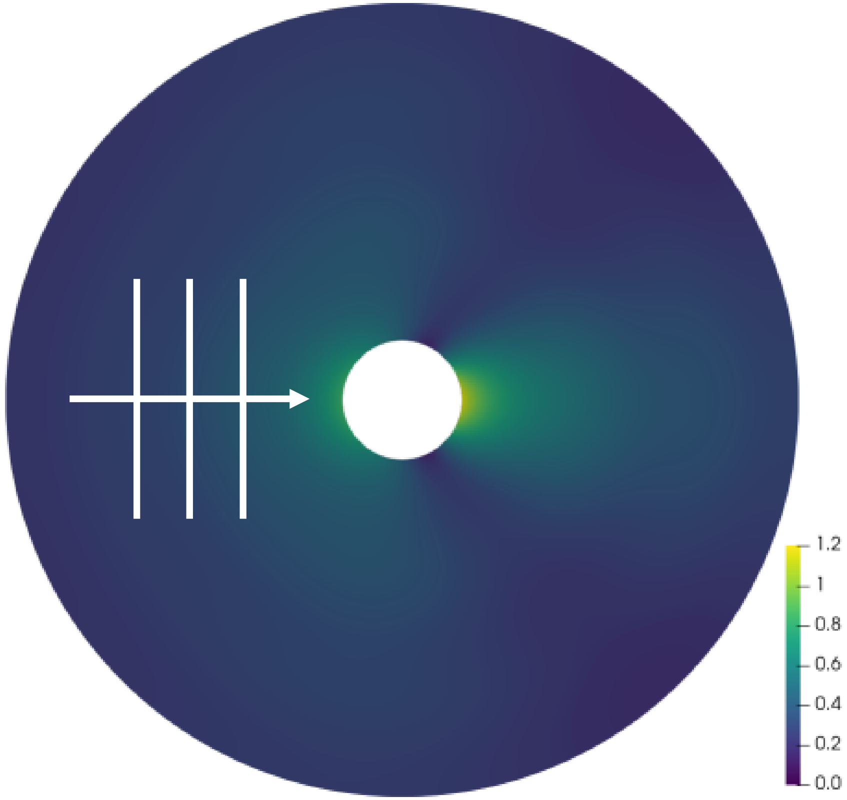
Material in each channel is unknown, but impedance matches that in the surrounding medium

- Facilitates transmission into each channel
- Allows for optimization of sound speed OR density, with the other being derived (fewer unknowns)
- Optimized sound speed serves as a proxy for optimized path length (e.g., coiled space)

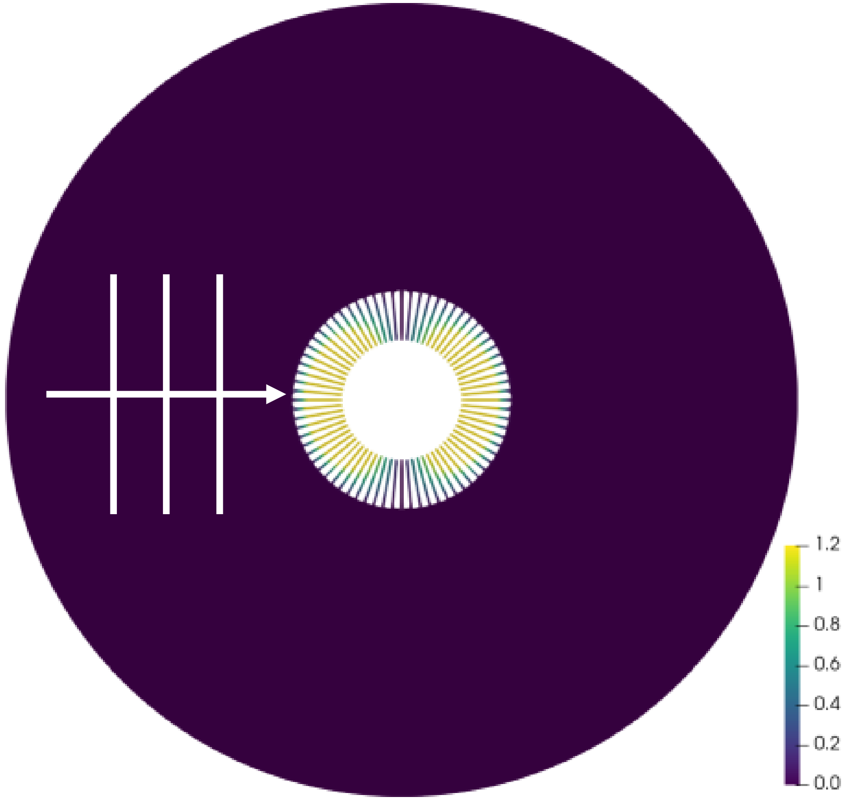


Example 3: One Frequency, One Direction

Scattered pressure objective function **reduced by ~4 orders of magnitude**



Cylinder alone

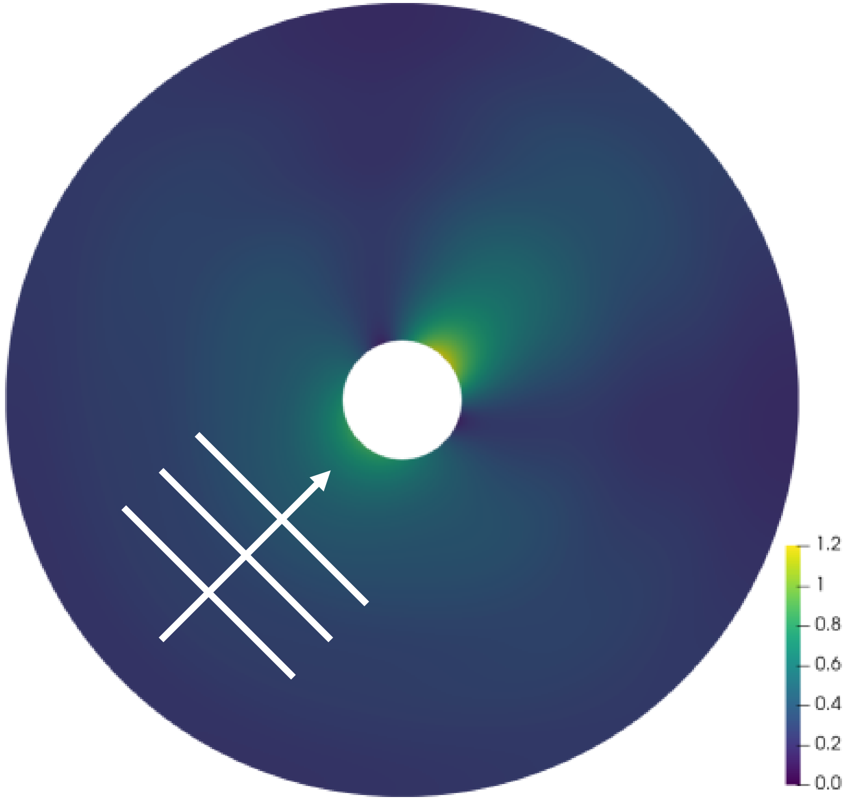


Optimized channels

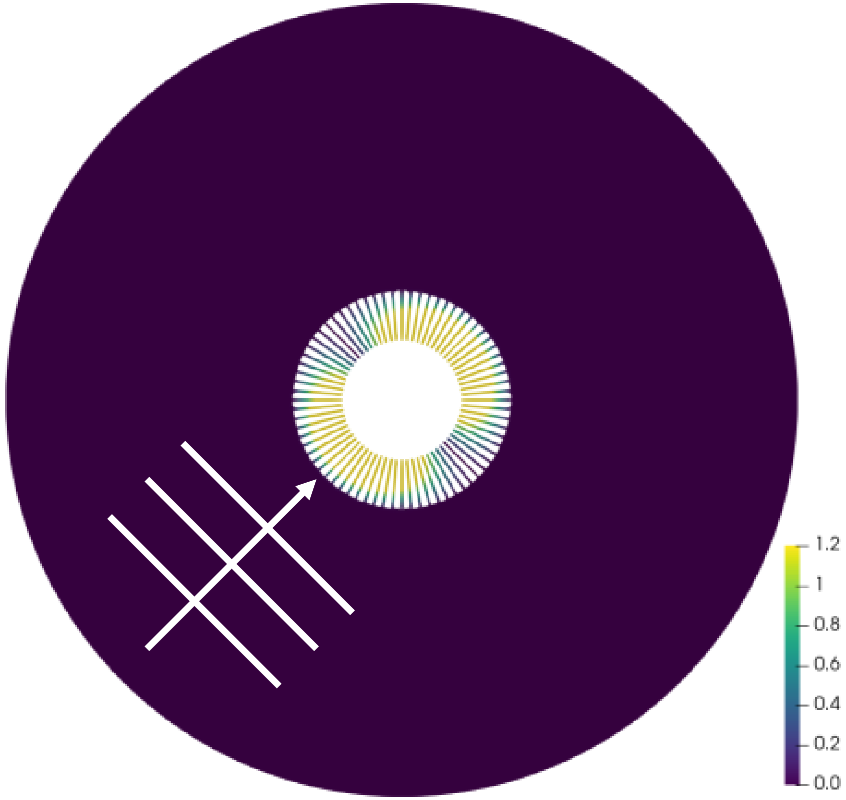
Example 3: One Frequency, Three Directions

Incident plane waves at 0, 45, 90 degrees

~4 orders of magnitude reduction



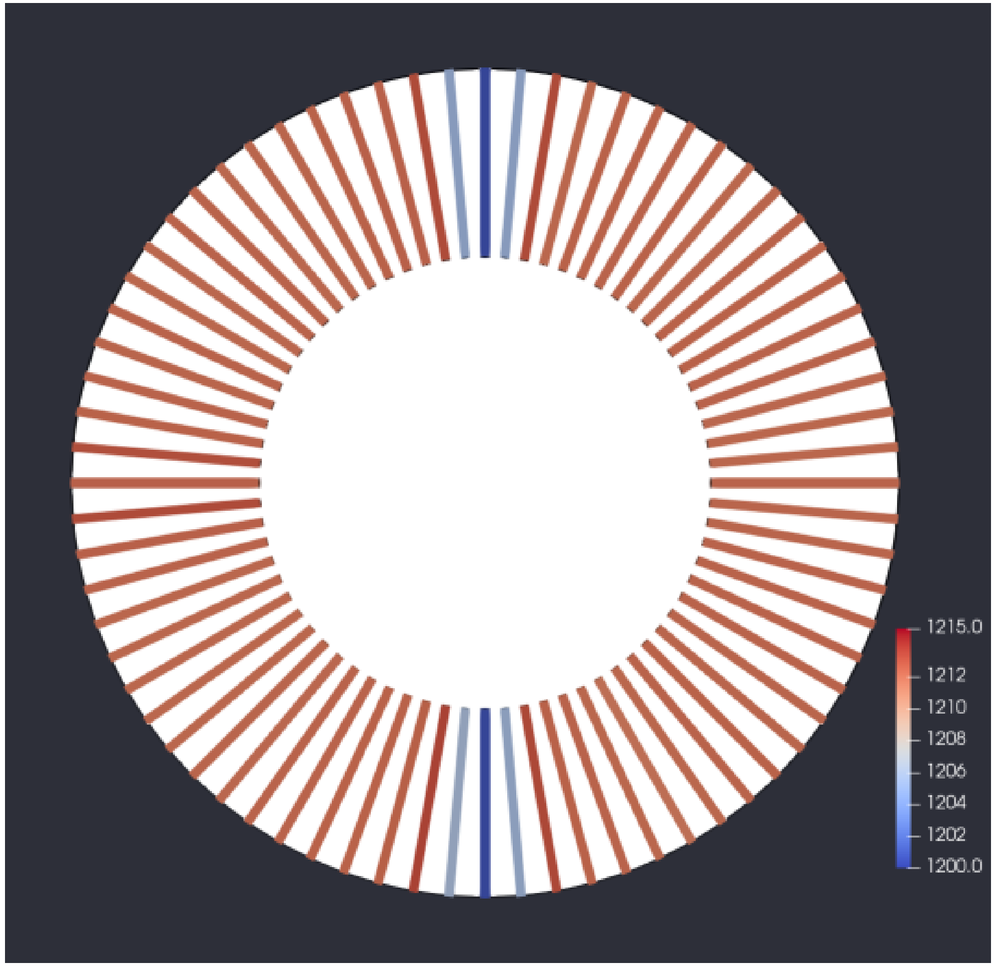
Cylinder alone



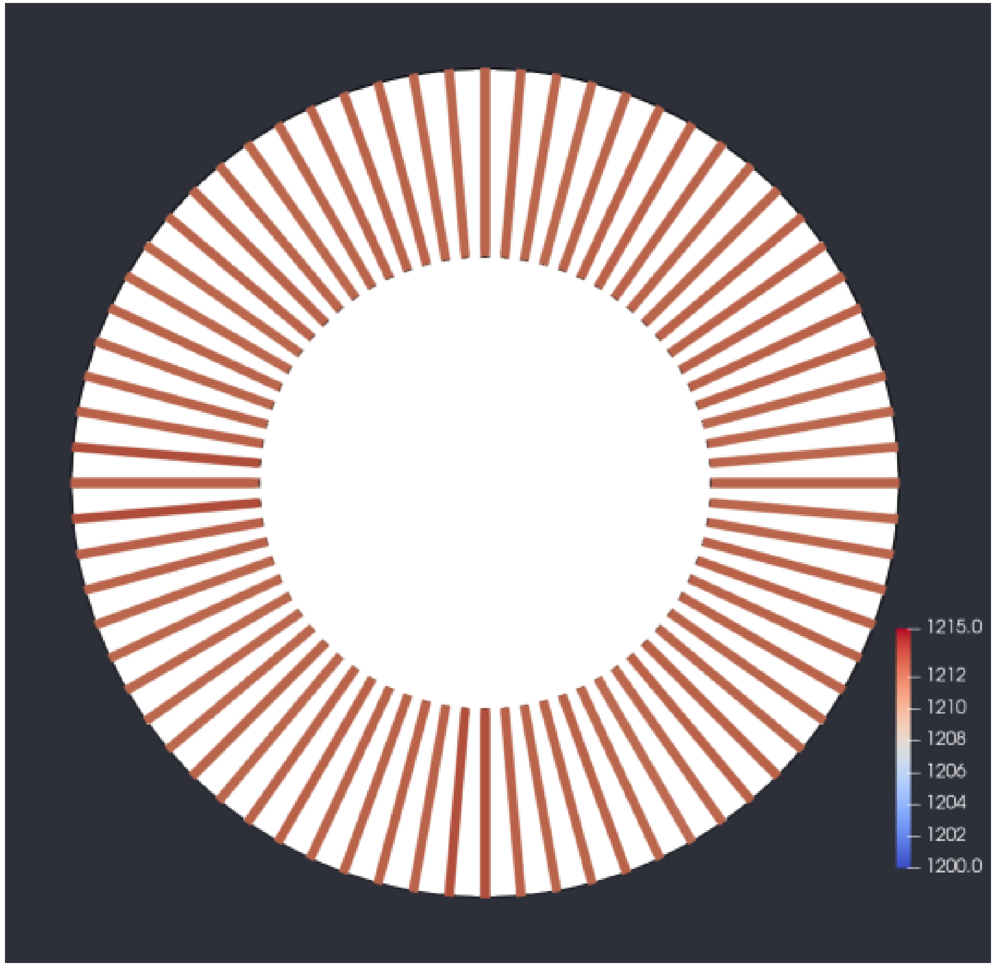
Optimized channels

Example 3: One Frequency, Various Directions

Optimized sound speeds more uniform for three directions than for one di



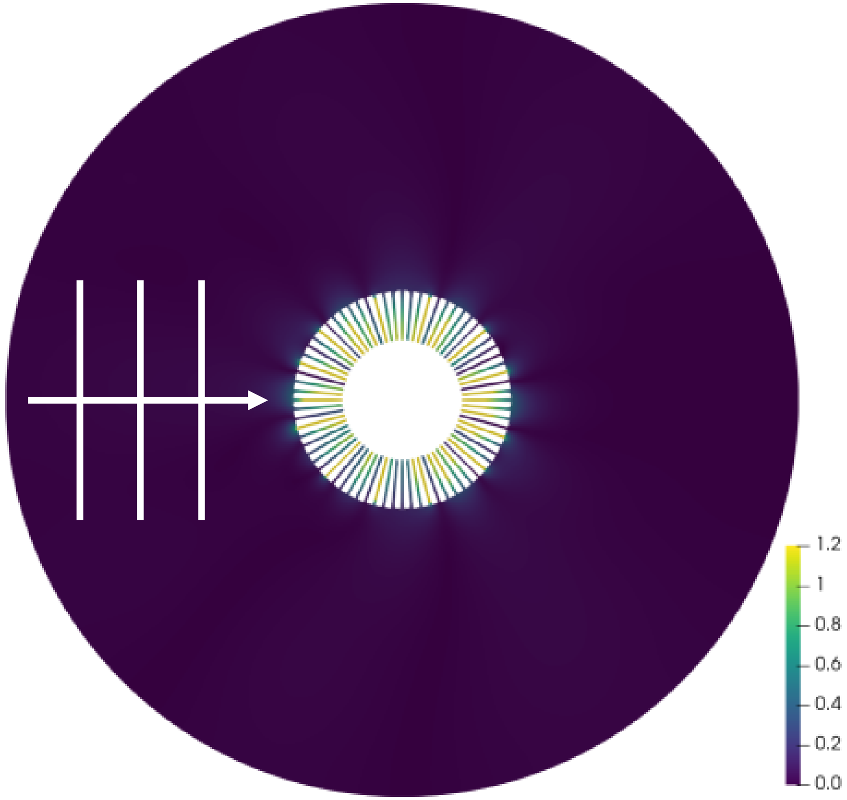
One direction



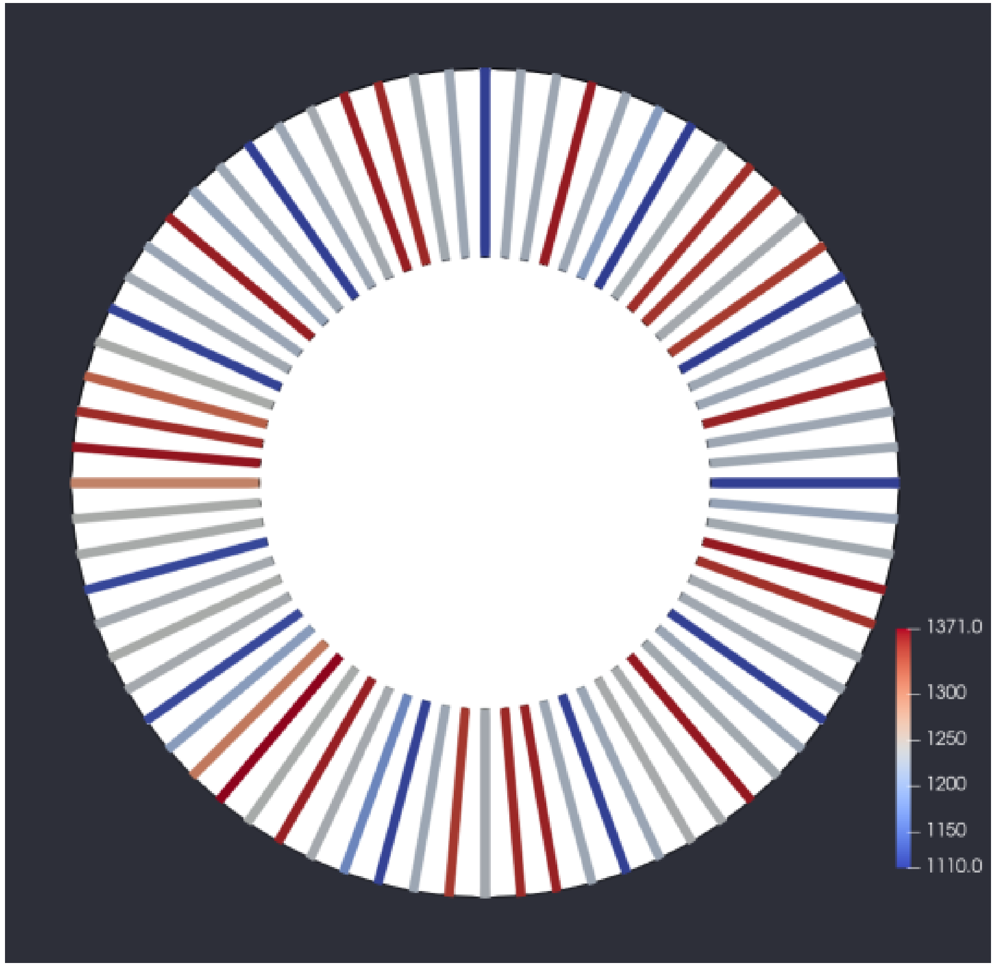
Three directions

Example 3: Three Frequencies, Three Directions

0, 45, 90 degrees; $f_c \pm 10\%$; ~2 orders of magnitude reduction



Scattered pressure



Channel sound speeds

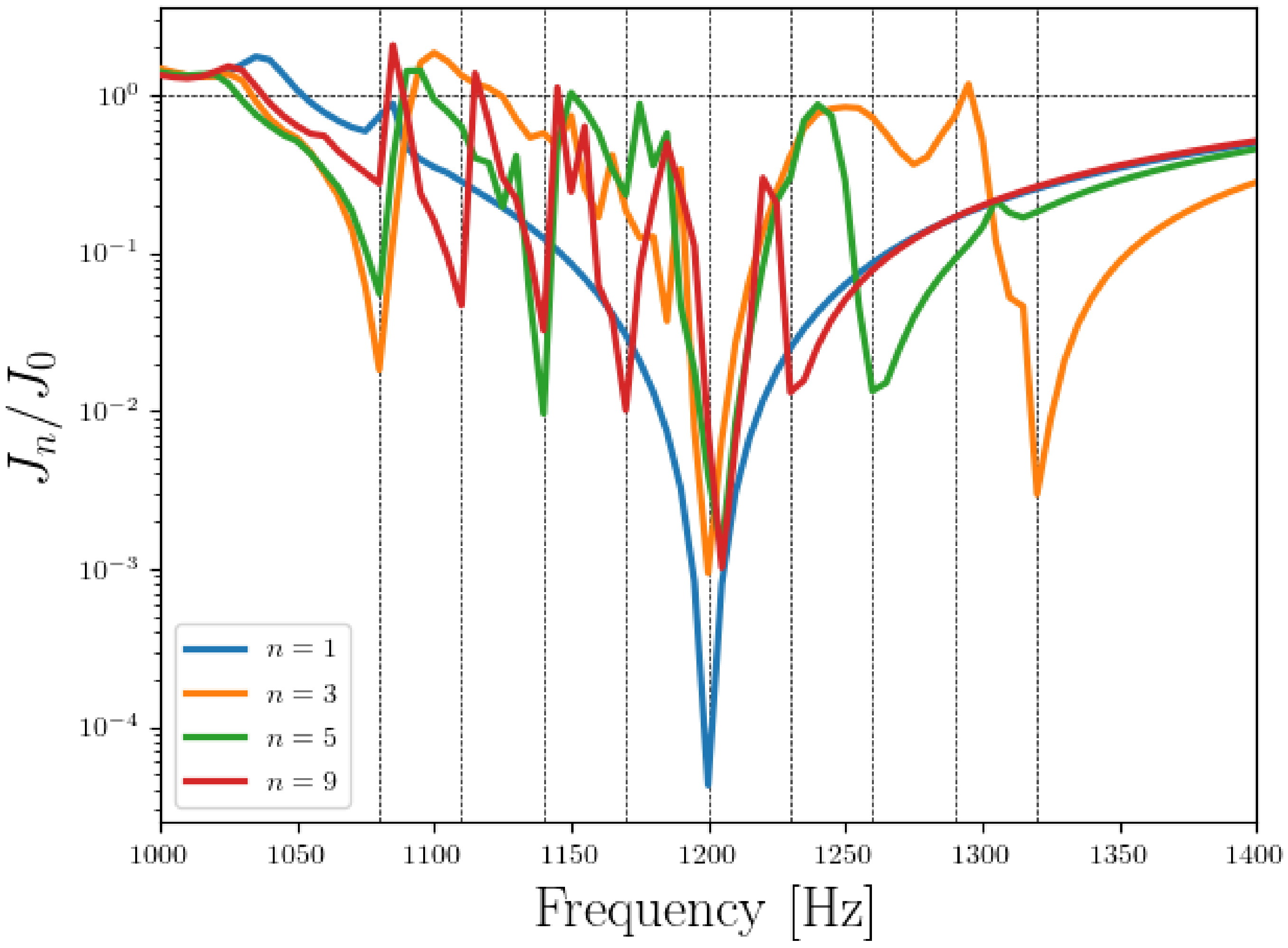
Example 3: N Frequencies, One Direction

All designs obtained with the same initial guess

Noticeable reduction of scattered pressure at frequencies prescribed in optimization

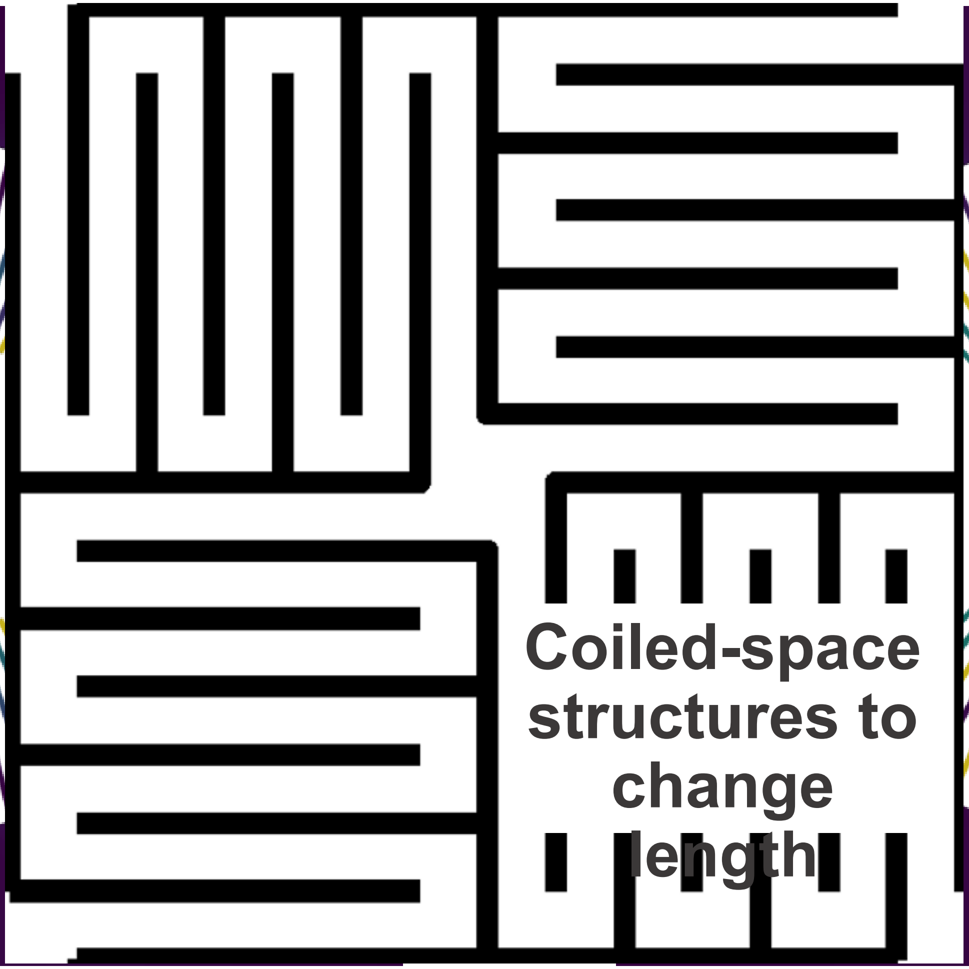
Boost in scattered pressure field at other frequencies within the band

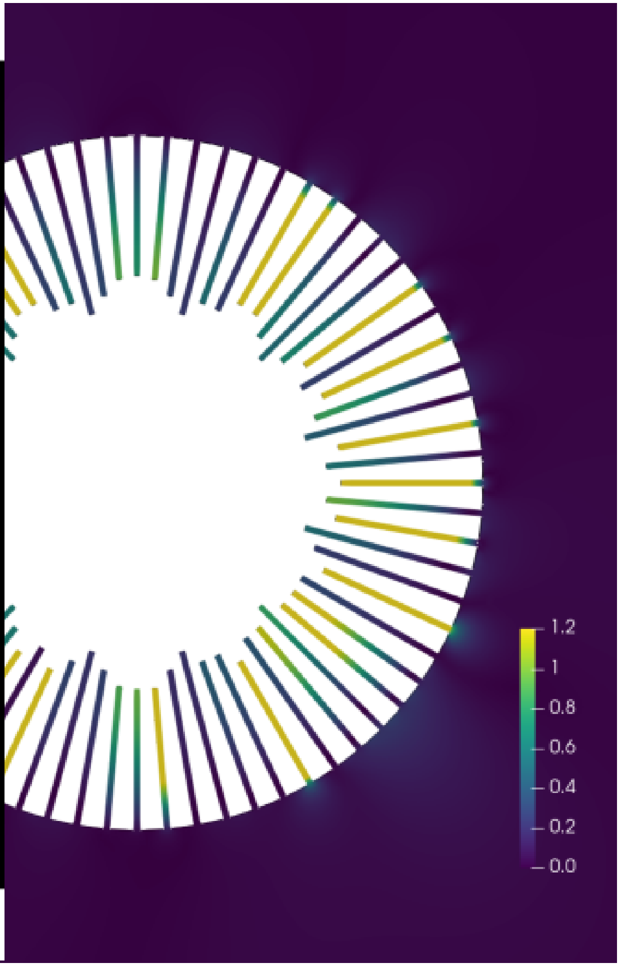
More frequencies makes optimization problem harder to solve due to non-convexity



Example 3: Sound Speed vs Channel Length

Scattered pressure identical between sound speed and channel length op





Sound speed optimized **Channel length optimized**

Conclusions and Future Work

Conclusions and Future Work

Conclusions

- Gradient-based optimization can help determine ideal material properties
  - Large number of tunable parameters does not substantially increase difficulty
  - Not as necessary to rely on physical intuition for a good design
- Printable designs can be achieved with additive manufacturing techniques

Future Work

- Shape optimization (parametrized or arbitrary) for scattering reduction
- Designs with less acoustic contrast between annulus and surrounding medium
- Techniques to improve optimization convexity
  - Force optimization with constraints across frequencies (i.e., different frequencies not independent)
  - Modified error in constitutive equations (MECE)

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Questions?