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Gimballed Tracking Mount Pointing Angle Qualification

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ABSTRACT

Tonopah Test Range (TTR), in support of its testing mission and modernization effort acquired a fleet of new gimballed tracking mounts (GTMs) manufactured by BAE Systems. The new GTMs can be operated remotely during flight tests and provide near real-time target tracking data. Furthermore, test vehicle Time-Space-Position-Information (TSPI) is evaluated using post-test synchronized imagery and pointing angle measurements acquired from each tracking mount. To comply with the Nuclear Enterprise Assurance Program (NEAP), all measurements devices must be certified. In keeping with the NEAP program, qualification of the new GTMs have been assessed to confirm that their pointing angle measurements produce acceptable TSPI results. This study only evaluated the four GTMs as a stand-alone solution and found that the GTMs meet their performance requirement of 0.006 degrees RMS error (or less) for post-processed pointing angles and produced TSPI solution with error volumes on the order of one meter or less. The new GTMs will be utilized in combination with existing optical tracking mounts, which will only improve the accuracy of the resulting TSPI data product. Details regarding the approach, analysis, summary results, and conclusions are presented.

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EXECUTIVE SUMMARY

As part of the Tonopah Test Range (TTR) modernization effort, a fleet of new gimballed tracking mounts (GTMs) manufactured by BAE Systems were acquired in support of its testing mission. The new GTMs can be operated remotely during flight tests and provide near real-time target tracking data. Synchronized imagery and pointing angle measurements are acquired from each tracking mount for post-test data processing to evaluate flight test vehicle Time-Space-Position-Information (TSPI).

To comply with the Nuclear Enterprise Assurance Program, all measurements devices must be certified. In 2015, TTR and the Measurement Science and Engineering Department (01535) initiated a project to quantify the uncertainty in TTR's TSPI data products, which lead to the certification of the updated Contraves Cinetheodolite Model F and TTR's TSPI data reduction suite. This work is documented in SAND report, SAND2018-6952R, "Tonopah Test Range Optical Tracking TSPI Uncertainty Quantification Analysis" [1]. In keeping with the Nuclear Enterprise Assurance Program (NEAP), the pointing angle accuracy of the new GTMs has been assessed based on a combination of star-based and ground-based targets. The GTM qualification criteria is based on their collective ability to acquire pointing angle measurements such that the post-processed TSPI solution data product is consistent with an error volume of one meter cubed or less.

Using TTR's TSPI data processing software, a generalized pointing angle requirement for a post-processed TSPI solution having a volume error of one meter cubed or less was evaluated to determine the maximum allowable Root-Mean-Squared (RMS) error. This pointing angle requirement states:

> The absolute value of the individual residual errors, from the post-processed solution, and disregarding atmospheric effects, shall be no larger than 0.006 degrees RMS, at any gimbal angle, and when tracking a high SNR point target.

A GTM is considered qualified when the post-processed solution evaluated by TTR's TSPI data processing software meets the requirement stated above.

The GTM qualification process was accomplished by tracking two target types (star-based and ground-based) and evaluating the resulting pointing angles and TSPI solutions. The star-based targets provided a large sample size of high Signal-to-Noise-Ratio (SNR) targets over a wide set of gimbal angles; however, the pointing angles generated were acquired from each mount individually, consisting of corrected azimuth and elevation results and cannot be used to evaluate a TSPI solution. Whereas the corrected pointing angles acquired from the ground-based targets, were obtained simultaneously by all four GTMs and enabled the evaluation of post-processed pointing angles from each GTM based on the resulting TSPI solution for each ground-based target.

A TSPI solution is evaluated using corrected pointing angles where the angles are corrected to account for each mount's calibration parameters and the pixel location of the target in the GTM's image. Once a TSPI solution is evaluated, the post-processed pointing angles can be calculated based on the mount's location and the results of the TSPI solution. This is important because (a) the "Davis Solution" [2] used by the TSPI code slightly adjusts the pointing angles from each mount to minimize the volume error in the TSPI solution and (b) the pointing angle requirement used to qualify the GTMs is based on the 'processed' pointing angle resulting from the TSPI solution.

The corrected pointing angle error values indicate the amount of random errors and unaccounted for bias errors present in both the star-based and ground-based measurement that cannot be addressed by mount calibration. However, the unaccounted-for pointing angle errors of the corrected angles are significantly reduced as a result of the "Davis Solution" encoded in the TSPI software. Although there are much fewer ground targets and their corrected pointing angles errors are similar (or in some instances greater) in magnitude to the corrected star-based pointing angle errors, it reasonable to expect that the "Davis Solution" will also minimize the pointing angle errors for flight test targets as well.

To show the relative error magnitude of both target types, the errors reported here were evaluated on the corrected pointing angles, not the post-processed pointing angles resulting from a TSPI solution. The star-based target assessment showed that the GTMs produced corrected pointing angle errors of 0.0374 ± 0.0309 (1σ) degrees in azimuth and 0.0221 ± 0.0141 (1σ) degrees in elevation. The azimuth and elevation RMS errors were 0.0443 and 0.0255 degrees, respectively. The ground-based target assessment showed that the GTMs produced corrected pointing angle errors of 0.0789 ± 0.0647 (1σ) degrees in azimuth, and 0.1313 ±0.0821 (1σ) degrees in elevation. The azimuth and elevation RMS errors were 0.0968 and 0.1493 degrees, respectively.

The post-processed ground-based pointing angle errors resulting from the ensemble of TSPI solutions were evaluated and compared to the true pointing angles based on surveyed ground targets. The aggregated post-processed pointing angle errors of all four GTMs on all ground-based targets was 0.0016 ± 0.0007 (1σ) degrees in azimuth, and 0.0054 ±0.0027 (1σ) degrees in elevation. The azimuth and elevation RMS errors were 0.0017 and 0.0059 degrees, respectively. These results meet the GTM performance requirement of 0.006 degrees RMS error (or less). In addition, the TSPI solution generated from the GTMs resulted in error volumes of one meter cubed or less and meet TTR's data quality requirements.

For this study, only the four new GTMs were evaluated together as a stand-alone solution and it was determined that the GTMs meet the performance requirement of 0.006 degrees RMS error (or less) for post-processed pointing angles, while producing acceptable TSPI solutions for each ground-based target. It is important to note that the new GTMs will be used in conjunction with several of TTR's existing optical tracking mounts, which will only improve the accuracy of the TSPI data product. Therefore, the four new GTMs are considered qualified to acquire pointing angle measurements suitable for a TSPI data product consistent with an error volume of one meter cubed or less.

ACRONYMS AND DEFINITIONS

Abbreviation	Definition
2-D	Two-dimensional
3-D	Three-dimensional
DVR	Digital Video Recorder
ERFA	Essential Routines for Fundamental Astronomy
FOR	Field-of-Regard
FOV	Field-of-View
GTM	Gimballed Tracking Mount
ICRS	International Celestial Reference System
IERS	International Earth Rotation Reference Services
m	meters
MM	Mount Model
NEAP	Nuclear Enterprise Assurance Program
RFQ	Request for Quote
RMS	Root Mean Squared
SF	Starfind.py
SNR	Signal to Noise Ratio
SOFA	Standards of Fundamental Astronomy
STA	Station
STD	Standard Deviation
TE	TrackEye
TSPI	Time-Space-Position-Information
TTR	Tonopah Test Range
TTR-TSPI-UQ	Tonopah Test Range Optical Tracking TSPI Uncertainty Quantification Analysis
UQ	Uncertainty Quantification
WT	Water Tower

1. INTRODUCTION

The Tonopah Test Range (TTR) has acquired and upgraded range instrumentation equipment as part of their range modernization effort. TTR utilizes optical tracking mounts to evaluate the performance of aerial test vehicles. These optical tracking mounts use synchronized images and associated angular measurements to record the path and altitude of the test vehicle. The tracking data is post processed and combined to produce a solution utilizing data reduction software. The final data product is commonly referred to as Time-Space-Position-Information (TSPI).

To comply with the Nuclear Enterprise Assurance Program, all measurements devices must be certified. Since 2010, TTR has been modernizing its measurement instrumentation and associated post processing analysis tools. In 2015, TTR and the Measurement Science and Engineering Department (01535) initiated a project to quantify the uncertainty in TTR's TSPI data products, which lead to the certification of the digitally upgraded Contraves Cinetheodolite Model F and TrackEye TSPI data reduction suite.

In 2016, TTR began the acquisition process for a new fleet of gimballed tracking mounts (GTMs) manufactured by BAE Systems, shown in Figure 1-1. The new GTMs can be operated remotely during flight tests. Imagery and pointing angle measurements consisting of azimuth and elevation encoder readings are acquired from each tracking mount for post-test data processing to evaluate test-unit TSPI. The GTMs and the associated software additions to the TrackEye data reduction suite recently came online and require certification before the TSPI results can be considered qualified. TTR reached out to department 1535 to perform an independent evaluation of the performance of the newly acquired GTMs. The Measurement Science and Engineering Department evaluated the performance of the new GTMs to determine the accuracy of the pointing angles via measured star targets and ground-based targets, and to assess suitability for TTR's TSPI data product. Data collection and reduction activities were aided by TTR staff.

Figure 1-1. Gimballed Tracking Mount (GTM)

2. BACKGROUND

An in-depth analysis of TTR's optical tracking TSPI uncertainty was completed and documented in a June 2018 report titled "Tonopah Test Range Optical Tracking TSPI Uncertainty Quantification Analysis" [1] (aka, TTR-TSPI-UQ report). For this analysis, a mathematical model of TTR's Contraves Cinetheodolites optical tracking mounts was developed based on real measurements acquired from (a) Stars, (b) fixed ground-based calibration targets, and (c) an optical target board. The mount-model (MM) along with core functions from TTR's Legacy TSPI data reduction software, including program OPTXYZ, were implemented in MATLAB®. The new simulation capability enables uncertainty estimates to be evaluated for various optical tracking mount configurations along TTR's flight path. The MM simulation capability and resulting TSPI uncertainty quantification results are documented in the TTR-TSPI-UQ report [1].

When TTR began the acquisition process for a new fleet of GTMs, the new MM simulation capability and the transcribed OPTXYZ Pascal code[1] were used to generate pointing angle requirements for the GTMs. The OPTXYZ code calculates the X, Y, Z locations relative to TTR's right-handed Cartesian coordinate system based on each mount's location and their measured and corrected pointing angles. The pointing angle correction process is performed on each mount's measured angles to remove any bias and apply scaling errors[2] before the OPTXYZ program is used to evaluate an optimized location of the test object. The OPTXYZ program assumes corrected pointing angles, regardless of the angle measurement device. Therefore, a set of hypothetical target locations above the test range were selected along with a set of nominal mount locations. Next, random zero-mean noise values were added to each mount's theoretically true pointing angles to create a set of simulated pointing angles, where the pointing angle errors were known and quantified. The simulated mount locations and randomized pointing angles were processed by the OPTXYZ MATLAB® function and the output results consisting of three-dimensional (3-D) point locations and error-volumes were evaluated. This process was repeated until the optimal pointing angle requirement corresponding to error-volumes of one meter cubed or less was determined. The resulting pointing angle performance requirement is stated below:

> The absolute value of the individual residual errors, from the post-processed solution, and disregarding atmospheric effects, shall be no larger than 0.006 degrees RMS, at any gimbal angle, and when tracking a high SNR point target.

Consequently, qualification of the GTM's pointing angle performance will be based on data acquired from semi-static and static targets (i.e., Stars and ground targets) where the pointing angles of each target can be evaluated independently. Furthermore, a GTM will be considered *qualified* for post-processed TSPI results if the absolute value of the individual residual errors, disregarding atmospheric effects, are equal to or less than 0.006 degrees RMS, when tracking a high SNR target, at any gimbal angle, based on a post-processed pointing angle solution evaluated by TrackEye.

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[1] The TTR Legacy program, OPTXYZ transcribed into MALAB® evaluates X-Y-Z position coordinates based on mount locations and corrected azimuth and elevation pointing angles using triangulation followed by a geometric weighting algorithm commonly call the 'Davis' solution developed by R.C. Davis [3]. The 'Davis' solution calculates an 'error-volume' for the final X-Y-Z position and indicating the overall quality of the solution. The Legacy TTR software is comprised of several command line programs that are issued in a pipeline fashion to generate a TSPI data product. The MATLAB implementation only consists of the transcribed codes that were needed for the Cinetheodolite based TSPI uncertainty analysis [1].

[2] An optical mount 'calibration' (i.e., scaling) process is conducted before and/or after each test mission to evaluate the mount's internal and external calibration parameters. The resulting parameters are used to remove offset biases and to scale the optical tracking measurements during the angle measurement correction process.

2.1. Systematic and Random Errors

When qualifying or calibrating an instrument the, systematic errors, those originating from the measurement system (e.g., biases, telescope droop, mislevel, sensor plane skew, encoder offsets & scale, image scaling, etc.), should be understood and accounted for before processing data. Thus, leaving the random errors, those originating from environmental and measurement system changes during the measurement (e.g., atmospheric refraction, scintillation, thermal, humidity, turbulence, timing, measurement noise, SNR, etc.), as the remaining error sources. Systematic errors are typically accounted for through a calibration process, while the random errors remain unknown and unpredictable.

For the GTMs, the quality of the calibration parameters for each mount depend on many factors, such as: (a) the GTM's "Star Calibration" procedure implemented by the internal GTM hardware, (b) the accuracy of the surveyed target boards located around the GTM that are used by TrackEye's calibration procedure, (c) the true time signal provided to the mount during the internal "Star Calibration", etc. These systematic errors affect the corrected azimuth and elevation values.

The evaluated RMS residual error values include the mount's systematic errors, unaccounted for by the mount's calibration, in addition to the random errors originating from environment and the measurement system during data collection. The random errors (e.g., scintillation, thermal, humidity, atmospheric turbulence, measurement noise, SNR, etc.) are typically unbiased and are assessed by taking the standard deviation (STD) error between the true and corrected pointing angles. Whereas the unaccounted-for systematic errors (e.g., atmospheric refraction, timing, mislevel, alignment, etc.) are biased and are evaluated by taking the mean of the corrected pointing angle errors. The RMS error is an unsigned measure of both the mean and STD errors.

The STD residual error values do not include bias errors and provide a measure of the random errors in the mount. Random errors are caused by noise sources in electronics, vibrations, wind, image processing tracking errors, and other unknown random error sources. The amount of random errors in a measurement are an indication of the *precision* of the instrument. The mean and RMS residual errors do include bias and are an induction of how *true* the instrument is. The *accuracy* of the instrument is based on how *true* and *precise* it is [3].

3. TECHNICAL APPROACH

A multi-step process was developed to qualify the GTMs to the requirements outlined in section 2. This multi-step process is depicted in Figure 3-1 and a more in-depth procedure is document in Appendix A.1. First, data was collected using the GTMs on two different target types, stars and ground-based targets.

Stars provided a large number of high SNR targets and the ability to evaluate the residual errors of the GTM's pointing angles over a wide set of gimbal angles. The expected pointing angles for the various star targets were determined based on the time of tracking and the GTM's location. The star data was generated by locating the star of interest in the center of the FOV and then steering the mount around the star so that the image of the star appears throughout the FOV of the image. Acquiring the Star data in this manner ensures pointing angle corrections based on image location of the star relative to the pointing angle of the mount are evaluated.

Ground-based targets were located around the test range and allowed for the generation of a TSPI solution that could be compared to measured surveyed data. The measured surveyed data was used as the true or reference position location. This also provided a way to evaluate and qualify the GTM generated data to the TSPI solution process.

While the star data provided a large sample size of targets, the pointing angles were acquired from each mount individually and consisted of, corrected azimuth and elevation measurements. Although these angles were corrected[3] and compared with their expected values, they were not suitable for evaluating TSPI solutions. Whereas the pointing angles generated from the ground-based targets were acquired simultaneously by all four GTMs allowing a TSPI solution to be created and the post-processed pointing angles from each GTM to be obtained. The post-processed point angle data was then used to evaluate the GTMs against their performance requirements.

The data collected from the GTMs was processed using a combination of Python scripts and TrackEye to generate the pointing angles and the TSPI solutions. The processed data was then analyzed and compared to the appropriate references. The data processing procedure is outlined below in Section 3.1.

[3] Based on mount calibration parameters and location of the target in the image.

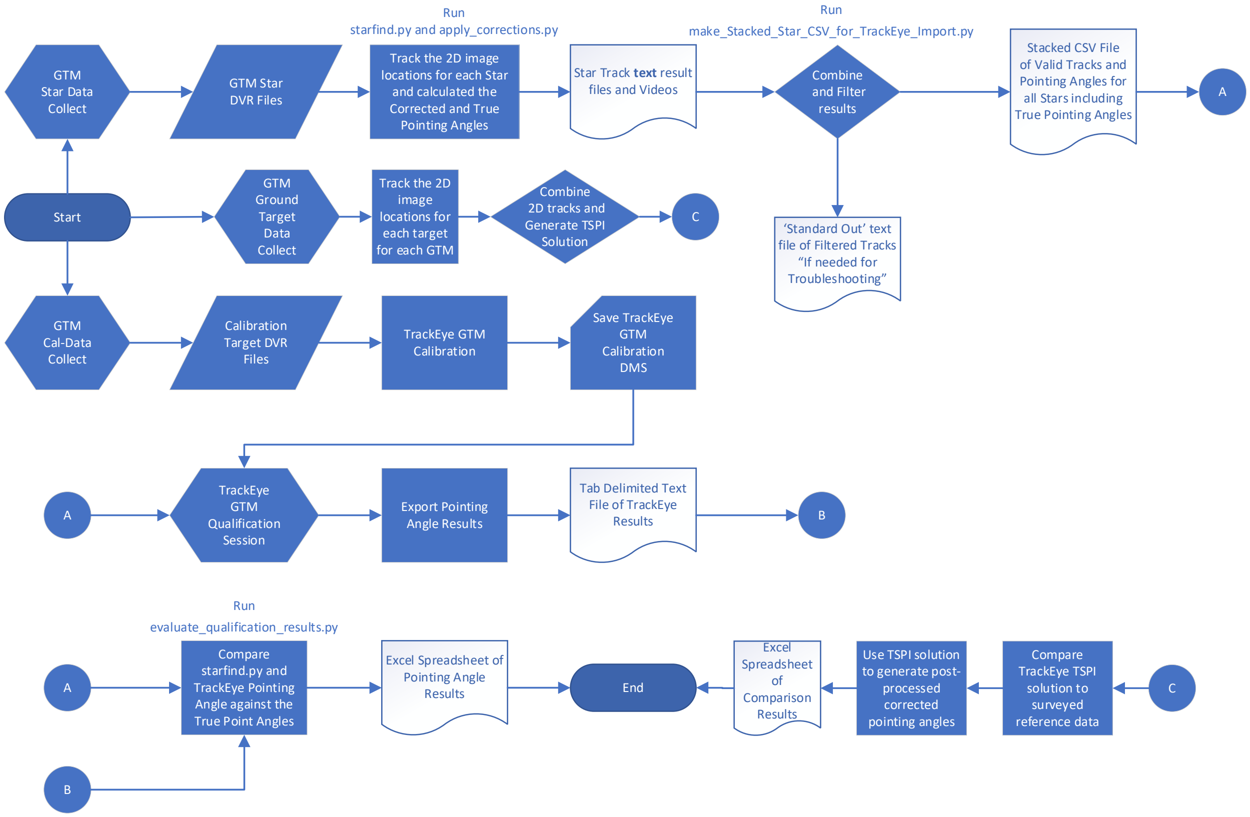

Figure 3-1. GTM Qualification Data Flow Diagram

In addition, a mount-model was created based off the model described in TTR-TSPI-UQ report [1] to help understand the amount of error that may remain in the corrected pointing-angles and their effects on the evaluated position and error-volume. The mount-model was not critical to the qualification of the GTMs, but provided useful information about the magnitude of errors in the GTM and TSPI solution process and how the corrected pointing angles for the star-based and ground-based targets are consistent with observed results. A description of the model and the results are presented in Appendix C.

3.1. Data Processing

Data processing of the GTM generated data utilized a combination of TrackEye and Python scripts. TrackEye is a motion analysis software used at TTR to perform 2-D tracking of targets and to generate the 3D TSPI solution. Python scripts were developed for this qualification study to aid in processing the large amounts of data generated and to provide the reference pointing angles for the star targets. Reference data for the ground targets were generated from survey data of the ground targets.

The basic data processing steps are as follows:

1.) Take the collected target datasets and track the image location of the target,

2.) Evaluate the corrected pointing angles for all tracked locations of each target based on the mount's calibration and the tracked image location of the target,

3.) Evaluate the true pointing angles of the target based on the calibrated position of the mount and associated mount parameters,

4.) Evaluate the pointing angle residual errors,

5.) Evaluate the residual error RMS, mean, STD, maximum and minimum values,

6.) Summarize results, and

7.) Evaluate results against the requirements.

The multi-step process relies on a mixture of Python scripts and TrackEye for data processing. Four Python scripts were developed for the data processing. The Python scripts were called: DVRReader.py, Starfind.py, apply_corrections.py, make_Stacked_Star_CSV_for_TrackEye_Import.py (Herein: TrackEye_Import.py), and evaluate_qualification_results.py. Python script, DVRReader.py, was developed to read the raw DVR files created by the GTMs and was able to extract the mount's calibration parameters and the individual image frames with the corresponding raw pointing angles for the entire track.

Starfind.py and apply_corrections.py were developed to (a) automatically find and track the image location of each star, (b) apply azimuth and elevation corrections based on calibration parameters, (c) use the image acquisition time and pointing angles to determine which celestial body is being pointed at and calculate the true position in space relative to the mount, and (d) evaluate the reference or true azimuth and elevation pointing angles of the target star based on the mount's location. The calculation of the reference or true pointing angles for the target star were performed using the AstroPy Python library. AstroPy performs the calculation by pulling information about the target star, and the earth's rotation and nutation from reference databases, and using the time when the tracking was performed to calculate the position of the star in the sky. AstroPy then uses the mount location to determine the pointing angle from the mount location to the target star in the sky. The reference databases are the International Earth Rotation Reference Services (IERS), the International Celestial Reference System (ICRS), and SIMBAD Astronomical Database. The Starfind.py script then exports (a) image frame number, (b) frame time, (c) the image location of the star, (d) raw, corrected, and true, pointing angles, (e) the difference between the true and corrected azimuth and elevation angles, (f) horizontal and vertical pixels/degree scale factor, and (g) azimuth and elevation offset correction values for each tracked frame.

TrackEye_Import.py was developed to filter the output files from Starfind.py and combine all the outputs into a single file, or stacked star file, for import into the TrackEye software. The filtering applied was to remove two-dimensional (2-D) image tracking failures caused when the image of the star is very poor and cannot easily be found, but instead tracks an image artifact. Thus, resulting in a reported image position that is very different than the previous tracked location of the star. Consequently, erroneous frames were discarded from the analysis if the location of the star deviated by more than 100 pixels in the x-axis or y-axis image coordinates from the previous frame. Additional noise filtering was applied to remove noise spikes.

TrackEye has similar capabilities to the Python codes described above but requires human interaction to run the 2-D image tracking processes such as those needed to track the locations of the celestial body. To better automate the process, the Python codes were used to track the image locations for each star and provide the raw pointing angle data to be processed by TrackEye. The TrackEye code (i.e. a TrackEye Session) was then setup up to read the image locations and pointing angles of each star from the input file and couple it with a mount calibration performed separately in TrackEye for the corresponding mount. The TrackEye calibration procedure updates (or fine tunes) the calibration parameters provided in the DVR file by tracking a set of four target boards erected around the mount. Next, The TrackEye Session evaluates the corrected pointing angles and the results are exported.

The evaluate_qualification_results.py Python script reads the stacked star file and exported TrackEye results file and merges both datasets based on the image frame times. The results allow the TrackEye corrected pointing angels and Python (aka, Starfind.py) corrected pointing angles to be matched with the true pointing angles on a frame by frame basis. The script then evaluates the RMS, mean, STD, maximum, and minimum residual error values for each star and exports the results as a Microsoft® Excel spreadsheet file.

The exported spreadsheet files for each GTM contain three sheets where the summary results consisting of combined RMS, Mean, Standard Deviation, Maximum, and Minimum values of the RMS, Mean, Standard Deviation, Maximum, and Minimum error values for each star, are stored in the "Summary Results" sheet for the TrackEye and Starfind data reduction methods. The star data used for the analysis is saved to the "Data" sheet, and the "Star Results" tab contains the individual statistics for each star along with four plots that contain the following information: (a)True and Corrected pointing angles; (b)True, Corrected, and Encoder pointing angles; (c) True, Corrected, and Encoders Azimuth angles with corresponding X-pixel image locations of the star; and (d) True, Corrected, and Encoders Elevation angles with corresponding Y-pixel image locations of the star. Pointing angle error statics (RMS, mean, STD, minimum, maximum) were calculated by subtracting the corrected pointing angles of each star from the true pointing angles.

For the ground target portions, data processing was performed using strictly TrackEye to mimic how a TSPI solution is generated at TTR. The 2-D image tracking capability within TrackEye was used to generate corrected pointing angles. TrackEye is then used to calculate a 3-D TSPI solution of the target locations on the range. Using the 3-D solution, a post-processed corrected pointing angle is generated. The 3-D solution of the targets were compared to previously obtained surveyed data of the ground target locations. The reference or true pointing angles from the mount to the targets were calculated from the surveyed data of the ground targets.

4. RESULTS AND ASSESMENTS

For the star dataset, a total of about 29 stars were tracked. A list of stars tracked can be found in Appendix A.2. For the ground targets, four different targets were tracked on 3 March 2021. The four ground targets were Pedro Beacon, Station 30, Station 40, and the New Water Tower.

4.1. Star Target Validation Results

The star validation provides information on the unprocessed pointing angle errors the GTMs produced in a statistically significant manner. While the point angles are unprocessed, they do have corrections applied from the calibration process. Table 4-1 shows the resulting pointing angle error from the star validation data and Table 4-2 provides the star validation pointing angle residual error statistics.

The mean errors and STD indicate the amount of random errors and unaccounted for bias errors present in the measurement that cannot be addressed by calibration. The low values in the "Mean of STD of Error among All Stars" as well as the "STD of STD Errors among All Stars" sub-tables in Table 4-2 indicates that the GTMs have a high precision.

The star validation data does show some variation between the TrackEye processed pointing angles and the Starfind.py processed pointing angles. This is due to the slight differences in the calibration process used by each. TrackEye uses the tracked image locations of four target boards placed around the GTM in combination with the GTM's internal calibration parameters, to evaluate corrections for raw pointing angles. Starfind.py only uses the GTM's internal calibration parameters to apply corrections to the pointing angles.

Table 4-1. Summary of Star Validation Errors

Star Validation Errors [Degrees]				
GTM	TrackEye		Starfind.py	
	Azimuth	*Elevation*	*Azimuth*	*Elevation*
1	0.0099	0.0127	0.0201	0.0543
2	0.0677	0.0126	0.0856	0.0339
3	0.0177	0.0324	0.0023	0.0184
4	0.0544	0.0307	0.0665	0.0390
Mean	**0.0374**	**0.0221**	**0.0436**	**0.0364**
STD	**0.0309**	**0.0141**	**0.0086**	**0.0055**
RMS	**0.0443**	**0.0255**	**0.0448**	**0.0364**

Table 4-2. Star Validation Data Pointing Angle Residual Error Statistics

Overall Combined Mean Values

Mean RMS of Errors among All Stars				
	TrackEye		Starfind	
GTM	Azimuth	Elevation	Azimuth	Elevation
1	0.0227	0.0246	0.0201	0.0543
2	0.0737	0.0143	0.0856	0.0340
3	0.0228	0.0324	0.0068	0.0184
4	0.0578	0.0307	0.0666	0.0390
Mean of Mean Errors among All Stars				
	TrackEye		Starfind	
GTM	Azimuth	Elevation	Azimuth	Elevation
1	-0.0099	-0.0127	0.0201	-0.0543
2	0.0677	0.0126	0.0856	-0.0339
3	-0.0177	0.0324	-0.0023	-0.0184
4	0.0544	0.0307	0.0665	0.0390
Mean of STD of Errors among All Stars				
	TrackEye		Starfind	
GTM	Azimuth	Elevation	Azimuth	Elevation
1	0.0008	0.0006	0.0007	0.0004
2	0.0012	0.0009	0.0012	0.0008
3	0.0010	0.0006	0.0006	0.0003
4	0.0010	0.0007	0.0010	0.0006

Overall Combined Standard Deviation Values

STD RMS of Errors among All Stars				
	TrackEye		Starfind	
GTM	Azimuth	Elevation	Azimuth	Elevation
1	0.0233	0.0093	0.0032	0.0049
2	0.0117	0.0139	0.0041	0.0050
3	0.0360	0.0105	0.0117	0.0048
4	0.0081	0.0070	0.0136	0.0074
STD of Mean of Errors among All Stars				
	TrackEye		Starfind	
GTM	Azimuth	Elevation	Azimuth	Elevation
1	0.0312	0.0234	0.0033	0.0049
2	0.0321	0.0155	0.0041	0.0050
3	0.0389	0.0105	0.0133	0.0048
4	0.0216	0.0070	0.0139	0.0074
STD of STD Errors among All Stars				
	TrackEye		Starfind	
GTM	Azimuth	Elevation	Azimuth	Elevation
1	0.0004	0.0001	0.0004	0.0001
2	0.0010	0.0001	0.0010	0.0001
3	0.0007	0.0001	0.0006	0.0002
4	0.0006	0.0001	0.0010	0.0001

It should be noted that some of the error sources could be from the calculation of the true or reference pointing angles. The reference pointing angles are calculated values based on the reported tracking times and GPS reported mount locations from the GTMs. Errors in these reported values can affect the calculated reference pointing angles.

Atmospheric refraction was accounted for when calculating the reference pointing angles. The calculations apply a refraction model based on that implemented in the Essential Routines for Fundamental Astronomy (ERFA) established by the Standards of Fundamental Astronomy (SOFA) [4]. However, for the calculations performed for the star validation, only a standard temperature and pressure was used when accounting for atmospheric refraction, since weather data during the star tracking was not collected.

4.2. Ground Target Validation Results

The star validation provided data showing the unprocessed pointing angle errors that the GTMs produced. However, the data did not provide insight into how the data generated by the GTMs would translate into the TSPI solution. The ground-based targets, with known locations, were then tracked to evaluate the GTMs in producing a TSPI solution. The ground-based targets' locations were surveyed to determine their 3-D location within TTR coordinate system. Layout of the GTMs and target location are shown in Figure 4-1.

The TSPI solutions generated from the GTM data were compared to the surveyed locations of the ground targets. Table 4-3 shows the mean error, STD, and RMS from the comparison between the TSPI solution and the surveyed data. The Z-axis has the highest mean error and is primarily driven by atmospheric refraction, but remains less than one-meter of error. The STD shows the variation in the errors and represents the error volume in the TSPI solution [1]. The TSPI solution generated from the GTMs shows the errors and error volumes are well within TTR's stated accuracy of one-meter cubed. The TSPI solution and surveyed data for each ground target can be found in Appendix B.2.

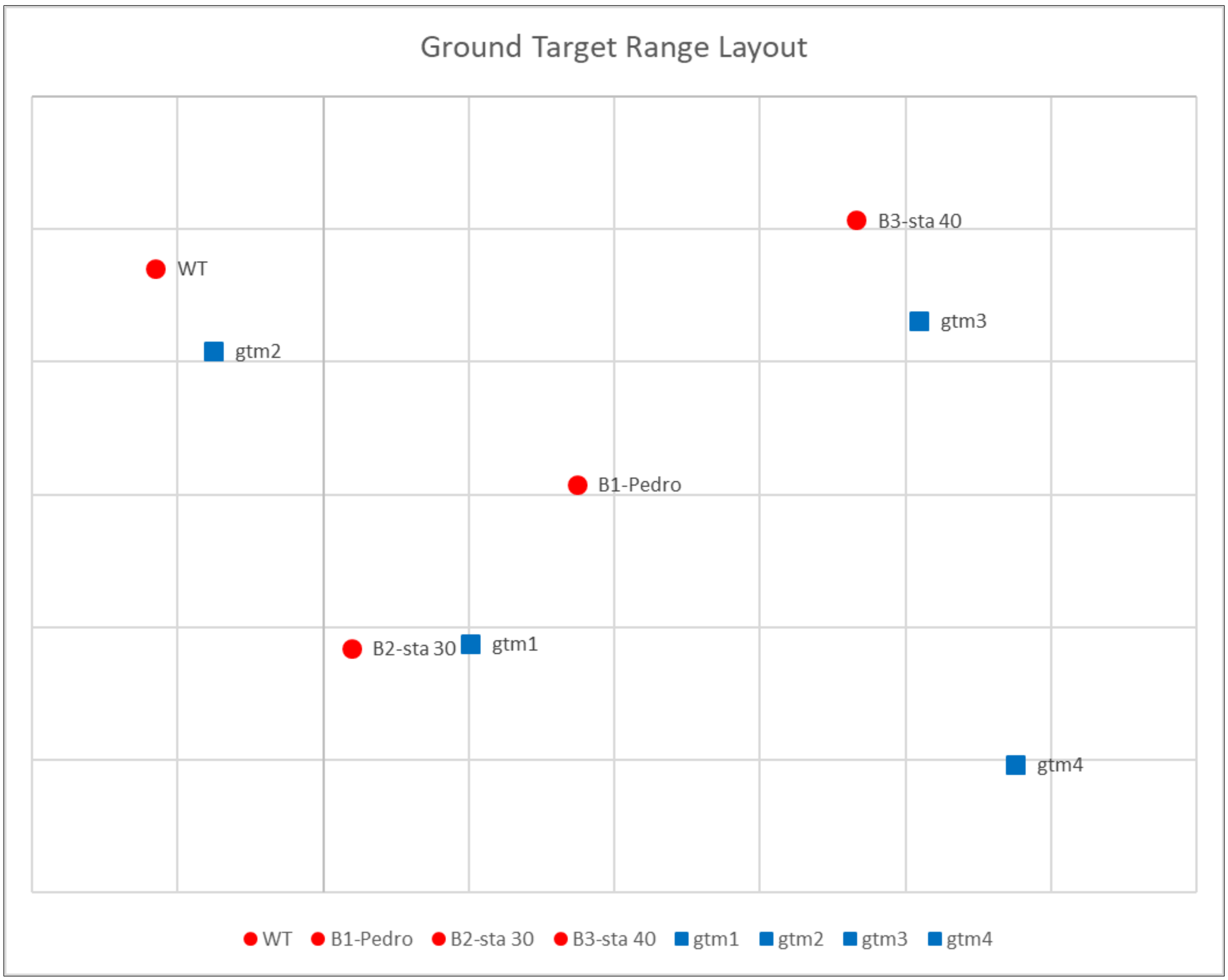

Figure 4-1. Range Layout for Ground Target Validation

Table 4-3. TSPI Solution Errors

TrackEye 3-D Solution			
	Average X (m)	**Average Y (m)**	**Average Z (m)**
Mean Error	0.3210	0.1896	0.6085
STD	0.1709	0.1409	0.4017
RMS	0.3534	0.2255	0.7010

Using the TSPI generated solution, post-processed pointing angle errors of the GTMs can be evaluated. Table 4-4 shows the unprocessed pointing angle errors and Table 4-5 shows the post-processed pointing angle errors. The unprocessed pointing angle errors of the ground targets show similar magnitude to the unprocessed pointing angle errors from the star validation. Although there are much fewer ground targets, the similar magnitudes of the errors indicate that the errors present in the star validation data are similar to that in the ground targets data. The Elevation error is greater than the star validation, but can be attributed to a greater effect of atmospheric refraction at lower altitudes [1]. The post-processed pointing angle errors are significantly reduced and show a mean error and RMS of 0.0016 degrees and 0.0017 degrees respectively in azimuth; and 0.0054 degrees and 0.0059 degrees respectively in Elevation. This indicates that post-processing of the pointing angle data reduces much of the errors present in the pointing angles. This result shows that the GTMs meet the performance requirement stated in section 2 of 0.006 degrees RMS for post-processed pointing angles. The pointing angles for each target and GTM is shown in Appendix B.2.

Table 4-4. Corrected Pointing Angles Errors

Average Pointing Angles		
Mount	**Azimuth [degrees]**	**Elevation [degrees]**
GTM1	0.0295	0.1584
GTM2	0.0510	0.0670
GTM3	0.1739	0.0642
GTM4	0.0614	0.2355
Mean Error	**0.0789**	**0.1313**
STD	**0.0647**	**0.0821**
RMS	**0.0968**	**0.1493**

Table 4-5. Post-Processed Pointing Angle Errors

Average Pointing Angles Post-Processing		
Mount	**Azimuth [degrees]**	**Elevation [degrees]**
GTM1	0.0019	0.0086
GTM2	0.0017	0.0064
GTM3	0.0023	0.0045
GTM4	0.0006	0.0022
Mean Error	**0.0016**	**0.0054**
STD	**0.0007**	**0.0027**
RMS	**0.0017**	**0.0059**

5. SUMMARY AND CONCLUSIONS

The primary objective of the qualification activities was to evaluate the performance of TTR's new GTMs and determine if they meet the performance qualifications required for TTR's TSPI data product using TrackEye post processing software. This was accomplished by tracking two different target types, stars and ground targets, and evaluating the generated pointing angles and TSPI solution. Stars provided a large sample size of high SNR targets over a wide set of gimbal angles however, the pointing angles generated were unprocessed, or pre-processed, pointing angles. The ground targets allowed for the generation of a TSPI solution and provided the post-processed pointing angle data that was used to compare to the GTM performance requirements, using the TSPI solution. Furthermore, the generation of a TSPI solution provided a way to evaluate and qualify the GTM generated data to the TSPI solution process.

The star validation showed the GTMs produced corrected pointing angle errors of 0.0374 ± 0.0309 (1σ) degrees in azimuth and 0.0221 ± 0.0141 (1σ) degrees in elevation. The azimuth and elevation RMS errors are 0.0443 and 0.0255 degrees, respectively. And, the ground-based target assessment showed that the GTMs produced corrected pointing angle errors of 0.0789 ± 0.0647 (1σ) degrees in azimuth, and 0.1313 ± 0.0821 (1σ) degrees in elevation. The azimuth and elevation RMS errors are 0.0968 and 0.1493 degrees, respectively. The corrected pointing angles error values indicate the amount of random errors and unaccounted for bias errors present in the measurement that cannot be addressed by calibration. The data also indicated that the GTMs have high precision based on the low mean of the STD of errors of 0.0010 in azimuth and 0.0007 in elevation.

The ground target validation showed that the post-processed pointing angle errors are significantly reduced. The mean error and RMS for the post-processed pointing angles were 0.0016 degrees and 0.0017 degrees respectively in azimuth, and 0.0054 degrees and 0.0059 degrees respectively in Elevation. The corrected pointing angles showed errors similar in magnitude to the corrected pointing angle errors from the star validation. Although there are much fewer ground targets, the similar magnitudes of the errors indicate that the errors present in the star validation data are similar to those in the ground targets data. The TSPI solution generated from these pointing angles yielded X, Y, Z-axes errors of 0.3210 meters (m), 0.1896 m, and 0.6085 m with an STD of 0.1709 m, 0.1409 m, 0.4017 m. The TSPI solution generated from the GTMs shows the errors and error volumes are well within TTR's stated accuracy of one-meter cubed.

This study only evaluated the four GTMs as a stand-alone solution and found that the GTMs meet their performance requirement of 0.006 degrees RMS for post-processed pointing angles and produced acceptable results for the resulting TSPI solution. TTR will be utilizing the four GTMs in conjunction with several existing optical tracking mounts, which will only improve the accuracy of the resulting TSPI data. The increased number in tracking cameras generates a more accurate TSPI solution with increased numbers of optical tracking mounts utilized [5]. This potentially yields an error volume that is less than stand-alone four GTM setup presented in this study.

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APPENDIX A. DATA PROCESING

A.1. GTM UQ Data Processing Procedure

Conducting a GTM qualification requires the following:

1.) GTM qualification dataset consisting of:
   a. A set of DVR files from several stars were the GTM data of each star was collected such that the GTM was moved around the star so that the image of star traversed as much of the field-of-view (FOV) as possible. In addition, the first few seconds and last few seconds of the star data should be acquired with the star positioned near the center of the image.
   b. Calibration DVR files of the target boards located around the GTM suitable for a standard GTM TrackEye calibration dataset.
2.) Python environment for running the following python codes:
   a. Starfind.py
   b. apply_corrections.py
   c. make_Stacked_Star_CSV_for_TrackEye_Import.py
   d. evaluate_qualification_results.py
3.) TrackEye environment running TrackEye_5.8-003K.

The GTM Qualification data flow diagram is shown in Figure A-5-1 and the GTM UQ Data Processing Procedure steps are outlined below:

1.) Use the Starfind.py python script to track the two-dimensional (2D) image locations of each star. The Starfind.py script exports a video file that can used to observe the operation of the star tracker as well as text file ('.out' extension) of the tracking results, including the true pointing angles of the star.

2.) Use the apply_corrections.py script to update the results of the *.out text files exported by Starfind.py.

3.) Run the make_Stacked_Star_CSV_for_TrackEye_Import.py script on the directory containing all of the corrected Starfind *.out text files to create a comma-separated-value (CSV) file containing the tracked star results from all of the *.out files format for import into TrackEye. The make_Stacked_Star_CSV_for_TrackEye_Import.py script reads each *.out file and filters out bad track results and stacks the results from each tracked frame from all of the Starfind.py correct .out files. Bad tracks are those where the tracked pixel location of the star changes by more than 100 pixels in the X or Y directions.

4.) Process the GTM calibration DVR files in TrackEye and save the results to new TrackEye 'sensor' that will contain the mounts calibration parameters.

5.) Setup a TrackEye session that reads in the stacked CSV file containing the tracked star data like the one shown in Figure 1 below. Notes captured while setting the TrackEye session were captured using Microsoft's Office OneNote program which have been exported to the TrackEye GTM Qualification Setup webpage. The initial TrackEye session used to document the steps assumed that the Tracking Mount Correction icon should come after the Offset Angles icon. However, when the final exported results were evaluated, it was determined that the Total Angles icon should follow Offset Angles instead.

6.) To save time creating sessions for each GTM a template of the session was saved to the TTR-Tools icon folder in TrackEye. In order to not have to reset the Text Diagram parameters, generic names for the mount and its sensor were used. This time saving changes has the adverse property in that it is imperative that the correct mount calibration used by the Ascii Import icon.

   At some point the notes may be redone to exclude the diversion caused by the Tracking Mount Correction vs. Total Angles error and will include the generic mount name used by the GTM Qualification Session Template located in the TTR-Tools icon folder.

   Once the TrackEye session has been setup and correctly associated the GTM calibration, the Text Diagram icon can be double clicked and its results exported as a tab delimited text file.

7.) Run the evaluate_qualification_results.py python program and follow the prompts to select the following files:
   a. The Stacked Star CSV file made by the make_Stacked_Star_CSV_for_TrackEye_Import.py script
   b. The tab delimited results file exported by GTM Qualification Session Text Diagram icon
   c. A filename for the GTM Qualification results Microsoft Excel file.

The resulting Excel file created by step 6 above contains the tabs noted below:
1) **Summary Results** shows the GTM's qualification results for all of the stars evaluated.
2) **Data** contains all of the process data used to evaluate the results and header legend.
3) **Star Results** provides statistics for each individual star along with plots of the results.

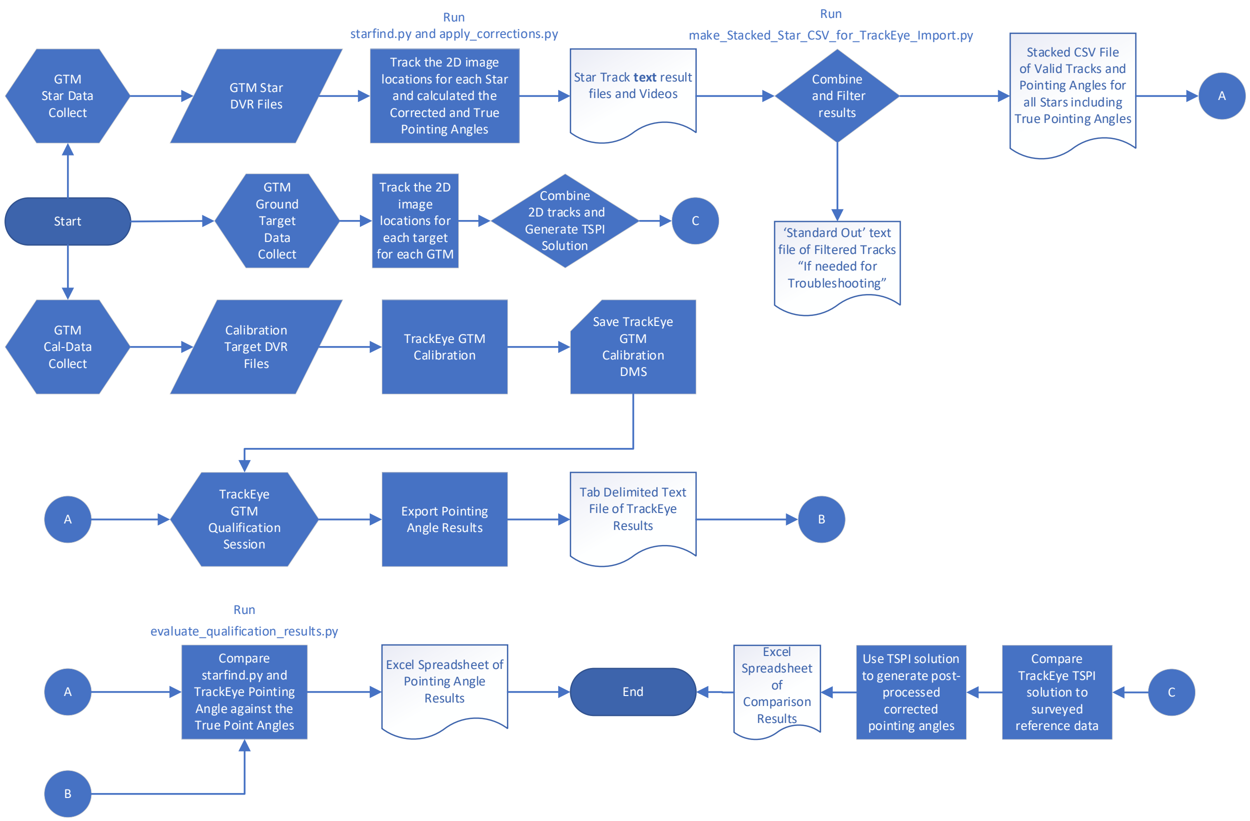

Figure A-5-1. GTM Qualification Data Flow Diagram

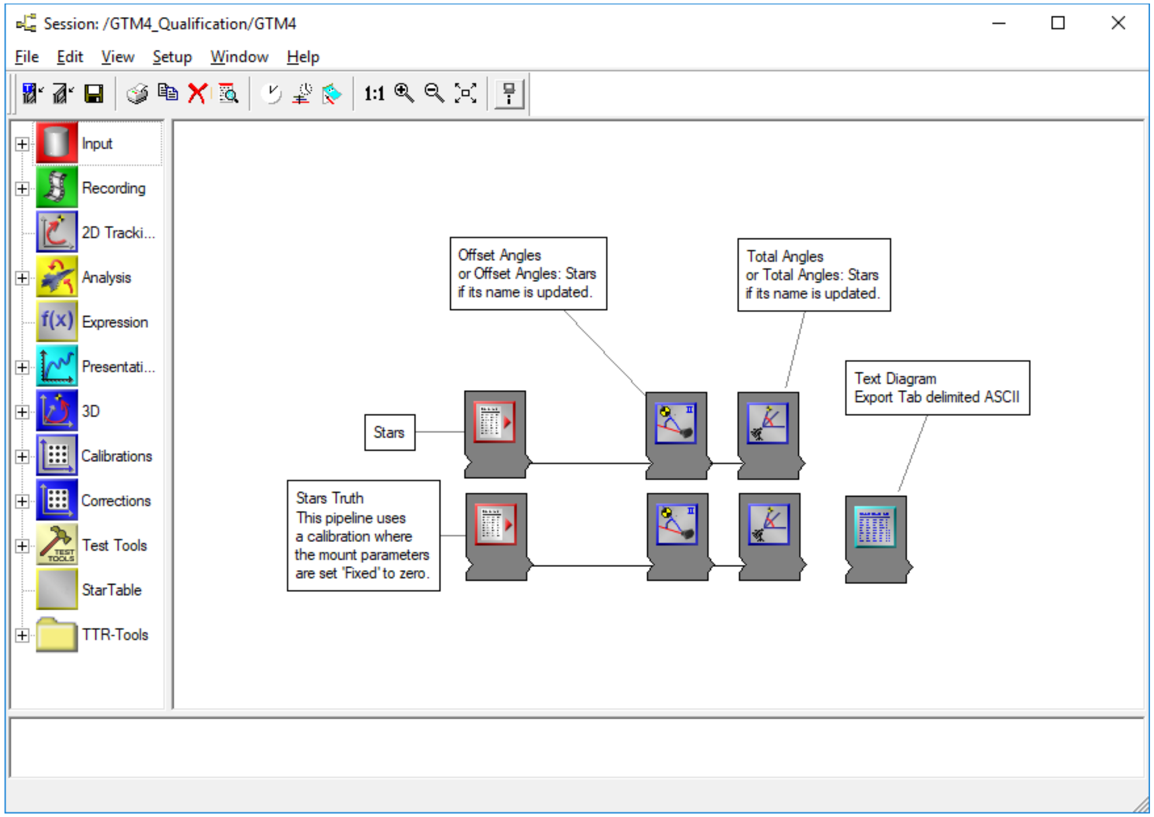

Figure A-5-2. TrackEye session used to process GTM Qualification results

This sheet contains the overall Mean, Standard Deviation, Max, and Min values for each of the the combined statistics listed in the tables below.

Mean Values

method	err_type	az_cor_err_val	el_cor_err_val	az_enc_err_val	el_enc_err_val
StarFind	Max	0.0223	-0.053003846	0.207815385	0.095184615
	Mean	0.020097878	-0.054325242	-0.021558673	-0.004746928
	Min	0.017376923	-0.055811538	-0.242219231	-0.121384615
	RMS	0.020120516	0.054326795	0.085419674	0.048553881
	STD	0.000686332	0.000405095	0.077508426	0.042334443
TrackEye	Max	-0.007219231	-0.010761538	0.207819231	0.095219231
	Mean	-0.009947701	-0.012660037	-0.021557231	-0.00473146
	Min	-0.012880769	-0.014938462	-0.242203846	-0.121473077
	RMS	0.022688397	0.024601882	0.085421336	0.048545277
	STD	0.000820852	0.000629115	0.077517212	0.042336714

Standard Deviation Values

method	err_type	az_cor_err_val	el_cor_err_val	az_enc_err_val	el_enc_err_val
StarFind	Max	0.002346231	0.004788693	0.101143046	0.031874626
	Mean	0.003272374	0.004850675	0.032277936	0.024735466
	Min	0.005039628	0.004806856	0.137273027	0.038756818
	RMS	0.003225334	0.004850604	0.040140724	0.011832523
	STD	0.000389763	6.5801E-05	0.037857846	0.009712749
TrackEye	Max	0.030346704	0.023364059	0.101149944	0.031863434
	Mean	0.031230002	0.023417011	0.032283845	0.024738261
	Min	0.032063649	0.023611609	0.137223944	0.038653916
	RMS	0.023317358	0.009273509	0.040139309	0.011847544
	STD	0.000353517	7.3627E-05	0.03783735	0.009676668

Maximum Values

method	err_type	az_cor_err_val	el_cor_err_val	az_enc_err_val	el_enc_err_val
StarFind	Max	0.0257	-0.0417	0.4895	0.1698
	Mean	0.023610493	-0.042968803	0.022994386	0.050759009
	Min	0.0223	-0.0445	-0.065	0.0052
	RMS	0.023614958	0.060208964	0.208555845	0.068556879
	STD	0.002005725	0.00049608	0.188931188	0.053197818
TrackEye	Max	0.038	0.0388	0.4896	0.1699
	Mean	0.03510614	0.036882206	0.022923684	0.050879325
	Min	0.0324	0.0352	-0.0654	0.0047
	RMS	0.099666259	0.036887634	0.208571057	0.068668951
	STD	0.001962103	0.000705311	0.188934559	0.05316824

Minimum Values

method	err_type	az_cor_err_val	el_cor_err_val	az_enc_err_val	el_enc_err_val
StarFind	Max	0.0128	-0.0584	-0.0645	0.0056
	Mean	0.006981234	-0.060207083	-0.103513539	-0.034448171
	Min	-0.0011	-0.0615	-0.7308	-0.1679
	RMS	0.007231437	0.042971109	0.040833684	0.005356068
	STD	0.000324319	0.000216944	8.94742E-05	7.8212E-05
TrackEye	Max	-0.0933	-0.0298	-0.0642	0.0059
	Mean	-0.09964901	-0.031776955	-0.10355004	-0.034533003
	Min	-0.1068	-0.0346	-0.7305	-0.1677
	RMS	0.001146928	0.007663455	0.040872441	0.005343565
	STD	0.000454629	0.000368401	0.000314439	0.000297567

Summary Results | Data | Star Results

Figure A-5-3. GTM Qualification results spreadsheet example of the Summary Results tab

	id	Datetime	star_file	image_id	pxl_x	pxl_y	az_enc	el_enc	az_cor	el_cor	az_cor_te	el_cor_te	az_cor_te_fp	el_cor_te_fp	az_star	el_star	az_err	el_err	az_enc_err	el_enc_err	az_err_te	el_err_te	az_err_te_fp	el_err_te_fp
0	0	2020-10-01 15:47	data/GTM	0	571.14	518.3	335.3934	15.9451	335.3562	16	335.371	15.947	335.393	15.945	335.3798	15.9567	0.0236	-0.0433	-0.0136	0.0116	0.0088	0.0097	-0.0132	0.0117
1	1	2020-10-01 15:47	data/GTM	1	571.35	518.34	335.3934	15.9451	335.3562	16	335.371	15.947	335.393	15.945	335.3799	15.9566	0.0237	-0.0434	-0.0135	0.0115	0.0089	0.0096	-0.0131	0.0116
2	2	2020-10-01 15:47	data/GTM	2	571.13	518.35	335.3934	15.9449	335.3562	15.9998	335.371	15.947	335.393	15.945	335.3799	15.9566	0.0237	-0.0432	-0.0135	0.0117	0.0089	0.0096	-0.0131	0.0116
3	3	2020-10-01 15:47	data/GTM	3	571.77	518.27	335.3934	15.9447	335.3564	15.9996	335.371	15.947	335.393	15.945	335.38	15.9565	0.0236	-0.0431	-0.0134	0.0118	0.009	0.0095	-0.013	0.0115
4	4	2020-10-01 15:47	data/GTM	4	571.79	517.95	335.3935	15.9446	335.3565	15.9994	335.371	15.946	335.393	15.945	335.38	15.9565	0.0235	-0.0429	-0.0135	0.0119	0.009	0.0105	-0.013	0.0115
5	5	2020-10-01 15:47	data/GTM	7	571.79	517.35	335.3935	15.9444	335.3565	15.999	335.371	15.946	335.393	15.944	335.3802	15.9564	0.0237	-0.0426	-0.0133	0.012	0.0092	0.0104	-0.0128	0.0124
6	6	2020-10-01 15:47	data/GTM	8	572.12	516.87	335.3937	15.9446	335.3568	15.999	335.372	15.946	335.394	15.945	335.3803	15.9563	0.0235	-0.0427	-0.0134	0.0117	0.0083	0.0103	-0.0137	0.0113
7	7	2020-10-01 15:47	data/GTM	9	572.18	516.16	335.3937	15.9446	335.3568	15.9988	335.372	15.946	335.394	15.945	335.3803	15.9563	0.0235	-0.0425	-0.0134	0.0117	0.0083	0.0103	-0.0137	0.0113
8	8	2020-10-01 15:47	data/GTM	10	572.27	515.58	335.3939	15.9447	335.357	15.9987	335.372	15.946	335.394	15.945	335.3804	15.9562	0.0234	-0.0425	-0.0135	0.0115	0.0084	0.0102	-0.0136	0.0112
9	9	2020-10-01 15:47	data/GTM	11	572.51	515.28	335.3939	15.9447	335.3571	15.9986	335.372	15.946	335.394	15.945	335.3804	15.9562	0.0233	-0.0424	-0.0135	0.0115	0.0084	0.0102	-0.0136	0.0112
10	10	2020-10-01 15:47	data/GTM	12	572.65	515.01	335.3939	15.9446	335.3572	15.9984	335.372	15.946	335.394	15.945	335.3805	15.9561	0.0233	-0.0423	-0.0134	0.0115	0.0085	0.0101	-0.0135	0.0111
11	11	2020-10-01 15:47	data/GTM	13	572.36	514.93	335.394	15.9444	335.3572	15.9982	335.372	15.945	335.394	15.944	335.3805	15.9561	0.0233	-0.0421	-0.0135	0.0117	0.0085	0.0111	-0.0135	0.0121
12	12	2020-10-01 15:47	data/GTM	14	571.87	515.4	335.394	15.9442	335.357	15.9982	335.372	15.945	335.394	15.944	335.3806	15.956	0.0236	-0.0422	-0.0134	0.0118	0.0086	0.011	-0.0134	0.012
13	13	2020-10-01 15:47	data/GTM	15	571.68	515.72	335.394	15.9441	335.357	15.9982	335.372	15.945	335.394	15.944	335.3807	15.956	0.0237	-0.0422	-0.0133	0.0119	0.0087	0.011	-0.0133	0.012
14	14	2020-10-01 15:47	data/GTM	16	571.23	515.41	335.394	15.9439	335.3568	15.9979	335.372	15.945	335.394	15.944	335.3807	15.9559	0.0239	-0.042	-0.0133	0.012	0.0087	0.0109	-0.0133	0.0119
15	15	2020-10-01 15:47	data/GTM	17	572.2	515.76	335.3942	15.9441	335.3573	15.9982	335.372	15.945	335.394	15.944	335.3808	15.9559	0.0235	-0.0423	-0.0134	0.0118	0.0088	0.0109	-0.0132	0.0119
16	16	2020-10-01 15:47	data/GTM	18	571.13	515.42	335.3942	15.9441	335.357	15.9981	335.372	15.945	335.394	15.944	335.3808	15.9559	0.0238	-0.0422	-0.0134	0.0118	0.0088	0.0109	-0.0132	0.0119
17	17	2020-10-01 15:47	data/GTM	19	571.25	514.75	335.3944	15.9442	335.3572	15.998	335.372	15.945	335.394	15.944	335.3809	15.9558	0.0237	-0.0422	-0.0135	0.0116	0.0089	0.0108	-0.0131	0.0118
18	18	2020-10-01 15:47	data/GTM	20	569.98	514.28	335.3946	15.9442	335.357	15.9978	335.372	15.945	335.395	15.944	335.3809	15.9558	0.0239	-0.042	-0.0137	0.0116	0.0089	0.0108	-0.0141	0.0118
19	19	2020-10-01 15:47	data/GTM	21	570.7	514.49	335.3946	15.9442	335.3572	15.9979	335.372	15.945	335.395	15.944	335.381	15.9557	0.0238	-0.0422	-0.0136	0.0115	0.009	0.0107	-0.014	0.0117
20	20	2020-10-01 15:47	data/GTM	22	571.4	515.27	335.3946	15.9441	335.3575	15.998	335.372	15.945	335.395	15.944	335.3811	15.9557	0.0236	-0.0423	-0.0135	0.0116	0.0091	0.0107	-0.0139	0.0117
21	21	2020-10-01 15:47	data/GTM	23	571.67	515.23	335.3946	15.9439	335.3576	15.9978	335.372	15.945	335.395	15.944	335.3811	15.9556	0.0235	-0.0422	-0.0135	0.0117	0.0091	0.0106	-0.0139	0.0116
22	22	2020-10-01 15:47	data/GTM	24	572.15	516.18	335.3946	15.9437	335.3577	15.9979	335.373	15.945	335.395	15.944	335.3812	15.9556	0.0235	-0.0423	-0.0134	0.0119	0.0082	0.0106	-0.0138	0.0116
23	23	2020-10-01 15:47	data/GTM	25	571.46	516.33	335.3947	15.9435	335.3576	15.9978	335.372	15.945	335.395	15.944	335.3812	15.9555	0.0236	-0.0423	-0.0135	0.012	0.0092	0.0105	-0.0138	0.0115
24	24	2020-10-01 15:47	data/GTM	26	572.17	517.08	335.3947	15.9435	335.3578	15.998	335.373	15.945	335.395	15.944	335.3813	15.9555	0.0235	-0.0425	-0.0134	0.012	0.0083	0.0105	-0.0137	0.0115
25	25	2020-10-01 15:47	data/GTM	27	572.48	516.67	335.3947	15.9435	335.3579	15.9979	335.373	15.945	335.395	15.944	335.3813	15.9554	0.0234	-0.0425	-0.0134	0.0119	0.0083	0.0104	-0.0137	0.0114
26	26	2020-10-01 15:47	data/GTM	28	572.24	516.21	335.3949	15.9437	335.358	15.9979	335.373	15.945	335.395	15.944	335.3814	15.9554	0.0234	-0.0425	-0.0135	0.0117	0.0084	0.0104	-0.0136	0.0114
27	27	2020-10-01 15:47	data/GTM	29	571.9	515.77	335.3949	15.9437	335.3579	15.9978	335.373	15.945	335.395	15.944	335.3815	15.9554	0.0236	-0.0424	-0.0134	0.0117	0.0085	0.0104	-0.0135	0.0114
28	28	2020-10-01 15:47	data/GTM	30	571.31	515.84	335.3949	15.9439	335.3577	15.998	335.373	15.945	335.395	15.944	335.3815	15.9553	0.0238	-0.0427	-0.0134	0.0114	0.0085	0.0103	-0.0135	0.0113
29	29	2020-10-01 15:47	data/GTM	31	571.96	516.36	335.3951	15.9437	335.3581	15.998	335.373	15.945	335.395	15.944	335.3816	15.9553	0.0235	-0.0427	-0.0135	0.0116	0.0086	0.0103	-0.0134	0.0113
30	30	2020-10-01 15:47	data/GTM	32	571.73	516.15	335.3951	15.9437	335.3581	15.9979	335.373	15.945	335.395	15.944	335.3816	15.9552	0.0235	-0.0427	-0.0135	0.0115	0.0086	0.0102	-0.0134	0.0112
31	31	2020-10-01 15:47	data/GTM	33	572.31	516.39	335.3951	15.9435	335.3583	15.9978	335.373	15.945	335.395	15.944	335.3817	15.9552	0.0234	-0.0426	-0.0134	0.0117	0.0087	0.0102	-0.0133	0.0112
32	32	2020-10-01 15:47	data/GTM	34	572.04	517.5	335.3952	15.9432	335.3583	15.9978	335.373	15.945	335.395	15.943	335.3817	15.9551	0.0234	-0.0427	-0.0135	0.0119	0.0087	0.0101	-0.0133	0.0121
33	33	2020-10-01 15:47	data/GTM	35	572.62	517.65	335.3952	15.9432	335.3585	15.9979	335.373	15.945	335.395	15.943	335.3818	15.9551	0.0233	-0.0428	-0.0134	0.0119	0.0088	0.0101	-0.0132	0.0121
34	34	2020-10-01 15:47	data/GTM	36	571.62	517.5	335.3954	15.9432	335.3583	15.9978	335.373	15.945	335.395	15.943	335.3818	15.955	0.0235	-0.0428	-0.0136	0.0118	0.0088	0.01	-0.0132	0.012
35	35	2020-10-01 15:47	data/GTM	37	571.03	517.54	335.3954	15.9432	335.3581	15.9978	335.373	15.945	335.395	15.943	335.3819	15.955	0.0238	-0.0428	-0.0135	0.0118	0.0089	0.01	-0.0131	0.012
36	36	2020-10-01 15:47	data/GTM	38	570.87	516.83	335.3956	15.9434	335.3583	15.9978	335.373	15.945	335.396	15.943	335.382	15.9549	0.0237	-0.0429	-0.0136	0.0115	0.009	0.0099	-0.014	0.0119
37	37	2020-10-01 15:47	data/GTM	39	570.64	516.41	335.3956	15.9434	335.3582	15.9977	335.373	15.945	335.396	15.943	335.382	15.9549	0.0238	-0.0428	-0.0136	0.0115	0.009	0.0099	-0.014	0.0119
38	38	2020-10-01 15:47	data/GTM	40	571.67	515.47	335.3958	15.9435	335.3588	15.9975	335.374	15.945	335.396	15.944	335.3821	15.9549	0.0233	-0.0426	-0.0137	0.0114	0.0081	0.0099	-0.0139	0.0109
39	39	2020-10-01 15:47	data/GTM	41	571.58	515.57	335.3958	15.9434	335.3587	15.9974	335.374	15.945	335.396	15.943	335.3821	15.9548	0.0234	-0.0426	-0.0137	0.0114	0.0081	0.0098	-0.0139	0.0118
40	40	2020-10-01 15:47	data/GTM	42	571.2	515.86	335.3958	15.9434	335.3586	15.9975	335.373	15.945	335.396	15.943	335.3822	15.9548	0.0236	-0.0427	-0.0136	0.0114	0.0092	0.0098	-0.0138	0.0118
41	41	2020-10-01 15:47	data/GTM	43	571.36	516.61	335.3959	15.943	335.3588	15.9973	335.374	15.944	335.396	15.943	335.3822	15.9547	0.0234	-0.0426	-0.0137	0.0117	0.0082	0.0107	-0.0138	0.0117
42	42	2020-10-01 15:47	data/GTM	44	572.12	516.32	335.3959	15.9428	335.359	15.9971	335.374	15.944	335.396	15.943	335.3823	15.9547	0.0233	-0.0424	-0.0136	0.0119	0.0083	0.0107	-0.0137	0.0117

Legend	
id	Index
Datetime	Date and Time
star_file	StarFind output filename
image_id	Image Index
pxl_x	X-pixel Location of Star in Image
pxl_y	Y-pixel Location of Star in Image
az_enc	Azimuth Encoder
el_enc	Elevation Encoder
az_cor	StarFind Azimuth Correction
el_cor	StarFind Elevation Correction
az_cor_te	TrackEye Azimuth Correction
el_cor_te	TrackEye Elevation Correction
az_cor_te_fp	TrackEye Azimuth Correction FP
el_cor_te_fp	TrackEye Elevation Correction FP
az_star	True Azimuth of Star
el_star	True Elevation of Star
az_err	StarFind Azimuth Error
el_err	StarFind Elevation Error
az_enc_err	Azimuth Encoder Error
el_enc_err	Elevation Encoder Error
az_err_te	TrackEye Azimuth Error
el_err_te	TrackEye Elevation Error
az_err_te_fp	TrackEye Azimuth Error FP
el_err_te_fp	TrackEye Elevation Error FP

Summary Results | Data | Star Results

Figure A-5-4. GTM Qualification results spreadsheet example of the Data tab

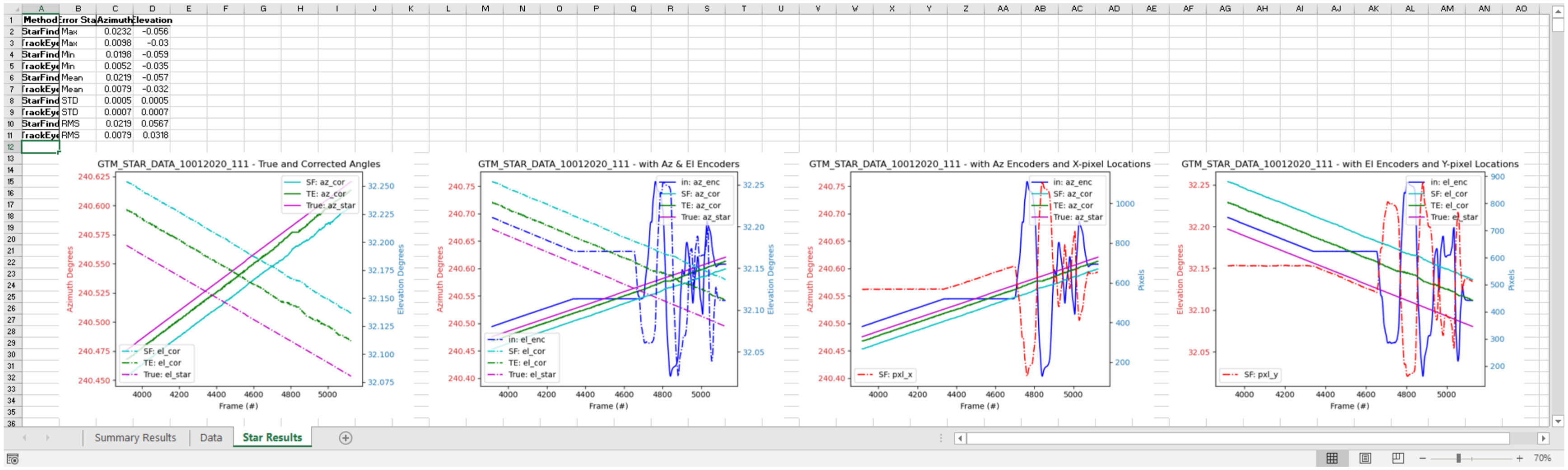

Figure A-5-5. GTM Qualification results spreadsheet example of the Star Results tab

A.2. Star Tacking Information

The star calibration routine on the GTM user interface was used to point to currently visible stars at the time data was collected. The GTM star database just provides a number for the various stars, but doesn't provide the star name or any other identifying information of the star currently being pointed at. However, the Starfind.py python script associates the star's name with the numeric code used by the GTM before tracking the star's image location. The star number and corresponding star name are shown in Table A 5-1. A list of the stars tracked used for the validation study by GTM star code are shown in Table A 5-2.

Table A 5-1. GTM Star Code List

Star Name	GTM Star Code	Star Name	GTM Star Code
Caph	2	Betelgeuse	224
Algenib	7	Menkalinan	227
Schedar	21	Mirzam	243
Mirach	42	Alhena	251
Almach	73	Sirius	257
Hamal	74	Castor	287
Menkar	107	Procyon	291
Algol	111	Pollux	295
Aldebaran	168	Alphard	354
Capella	193	Regulus	380
Rigel	194	Merak	416
Alnilam	201	Phecda	447
Bellatrix	201	Alioth	483
Elnath	202	Mizar	497
Mintaka	206	Kochab	550
Arneb	207	Polaris	907
Saiph	220		

Table A 5-2. List of Stars Tracked

GTM Star Code of Tracked Star		
2	202	291
21	206	295
42	207	354
73	220	380
74	224	416
107	227	447
111	243	497
168	251	550
193	257	907
194	287	

APPENDIX B. RESULTS

B.1. Summary Results for Star Validation

Summary results consisting of the combined mean and STD values based on the individual RMS, mean, STD residual error values for each star using the TrackEye and Starfind.py data reduction methods are provided in Table A 5-4. These summary statistics were calculated to create cohesive Mean and STD metrics for each GTM using the Star Validation results for each. If the RMS, mean and STD error statistics were evaluated across all stars at one time, the larger azimuth and elevation changes between each star would skew the statistic. Therefore, the mean, STD, and RMS residual errors evaluated for each star are amalgamated to determine the mean and STDs for all of the stars. For example, the mean of the *Mean Values* for all stars is the mean of the individual mean error values for each star, the mean of the *Standard Deviation* for all stars is the mean of the individual STD values for each star, the STD of the *Mean Values* for all stars is the STD of the individual mean error values for each star, etc. Table A 5-3 shows the functional expression and description for each of the summary statistics.

Table A 5-3. Summary of Combined Statistic Functions

Statistic	Function Form	Description
Mean Values for all Stars Combined		
RMS	mean(individual star RMSs values)	Mean of RMSs
Mean	mean(individual star mean values)	Mean of Means
Standard Deviation	mean(individual star STDs values)	Mean of Standard Deviations
Standard Deviation Values for all Stars Combined		
RMS	std(individual star RMSs values)	STD of RMSs
Mean	std(individual star Means values)	STD of Means
Standard Deviation	std(individual star STDs values)	STD of Standard Deviations

Table A 5-4. Star Validation Data Pointing Angle Residual Error Statistics All Units in Degrees

Overall Combined Mean Values

Mean RMS of Errors among All Stars				
	TrackEye		Starfind	
GTM	Azimuth	*Elevation*	*Azimuth*	*Elevation*
1	0.0227	0.0246	0.0201	0.0543
2	0.0737	0.0143	0.0856	0.0340
3	0.0228	0.0324	0.0068	0.0184
4	0.0578	0.0307	0.0666	0.0390

Mean of Mean Errors among All Stars				
	TrackEye		Starfind	
GTM	Azimuth	*Elevation*	*Azimuth*	*Elevation*
1	-0.0099	-0.0127	0.0201	-0.0543
2	0.0677	0.0126	0.0856	-0.0339
3	-0.0177	0.0324	-0.0023	-0.0184
4	0.0544	0.0307	0.0665	0.0390

Mean of STD of Errors among All Stars				
	TrackEye		Starfind	
GTM	Azimuth	*Elevation*	*Azimuth*	*Elevation*
1	0.0008	0.0006	0.0007	0.0004
2	0.0012	0.0009	0.0012	0.0008
3	0.0010	0.0006	0.0006	0.0003
4	0.0010	0.0007	0.0010	0.0006

Overall Combined Standard Deviation Values

STD RMS of Errors among All Stars				
	TrackEye		Starfind	
GTM	Azimuth	*Elevation*	*Azimuth*	*Elevation*
1	0.0233	0.0093	0.0032	0.0049
2	0.0117	0.0139	0.0041	0.0050
3	0.0360	0.0105	0.0117	0.0048
4	0.0081	0.0070	0.0136	0.0074

STD of Mean of Errors among All Stars				
	TrackEye		Starfind	
GTM	Azimuth	*Elevation*	*Azimuth*	*Elevation*
1	0.0312	0.0234	0.0033	0.0049
2	0.0321	0.0155	0.0041	0.0050
3	0.0389	0.0105	0.0133	0.0048
4	0.0216	0.0070	0.0139	0.0074

STD of STD Errors among All Stars				
	TrackEye		Starfind	
GTM	Azimuth	*Elevation*	*Azimuth*	*Elevation*
1	0.0004	0.0001	0.0004	0.0001
2	0.0010	0.0001	0.0010	0.0001
3	0.0007	0.0001	0.0006	0.0002
4	0.0006	0.0001	0.0010	0.0001

Summary results consisting of combined Mean, STD, Maximum, and Minimum values of the individual RMS, Mean, STD, Maximum, and Minimum error values for each star, for the TrackEye and Starfind data reduction methods are provided below. These summary results were evaluated based on the combined Mean, STD, Maximum, Minimum, and RMS error values for each star. This was done to create cohesive set of metrics for each value. If the error statistic were evaluated across all stars at one time, the larger errors for some of the stars would dominate the statistic. Therefore, the Mean, STD, and RMS residual errors for each star are evaluated and amalgamated into a combined statistic. For example, the Mean of the *Mean Values* for all stars is the mean of the individual mean values for each star, and the Mean of the *Standard Deviation* for all stars is the mean of the individual STD values for each star. Table A 5-5 shows the functional expression and description for each of the summary statistics.

Table A 5-5. Summary of Statistics Functions

Statistic	Function Form	Description
Mean Values for all Stars Combined		
Max	mean(individual star Max values)	Mean of Maximums
Mean	mean(individual star Mean values)	Mean of Means
Min	mean(individual star Min values)	Mean of Minimums
Standard Deviation	mean(individual star STDs values)	Mean of Standard Deviations
RMS	mean(individual star RMSs values)	Mean of RMSs
Standard Deviation Values for all Stars Combined		
Max	std(individual star Max values)	STD of Maximums
Mean	std(individual star Means values)	STD of Means
Min	std(individual star Min values)	STD of Minimums
Standard Deviation	std(individual star STDs values)	STD of Standard Deviations
RMS	std(individual star RMSs values)	STD of RMSs
Maximum Values for all Stars Combined		
Max	max(individual star Max values)	Max of Maximums
Mean	max(individual star Means values)	Max of Means
Min	max(individual star Min values)	Max of Minimums
Standard Deviation	max(individual star STDs values)	Max of Standard Deviations
RMS	max(individual star RMSs values)	Max of RMSs
Minimum for all Stars Combined		
Min	min(individual star Max values)	Min of Maximums
Mean	min(individual star Means values)	Min of Means
Min	min(individual star Min values)	Min of Minimums
Standard Deviation	min(individual star STDs values)	Min of Standard Deviations
RMS	min(individual star RMSs values)	Min of RMSs

Table A 5-6. Corrected Pointing Angle Residual Error Statistics for GTM-1. All units in degrees.

Mean Values

Method	Statistic	Azimuth	Elevation
Starfind	Max	0.0223	-0.0530
	Mean	0.0201	-0.0543
	Min	0.0174	-0.0558
	RMS	0.0201	0.0543
	STD	0.0007	0.0004
TrackEye	Max	-0.0072	-0.0108
	Mean	-0.0099	-0.0127
	Min	-0.0129	-0.0149
	RMS	0.0227	0.0246
	STD	0.0008	0.0006

Standard Deviation Values

Method	Statistic	Azimuth	Elevation
Starfind	Max	0.0023	0.0048
	Mean	0.0033	0.0049
	Min	0.0050	0.0048
	RMS	0.0032	0.0049
	STD	0.0004	0.0001
TrackEye	Max	0.0303	0.0234
	Mean	0.0312	0.0234
	Min	0.0321	0.0236
	RMS	0.0233	0.0093
	STD	0.0004	0.0001

Maximum Values

Method	Statistic	Azimuth	Elevation
Starfind	Max	0.0257	-0.0417
	Mean	0.0236	-0.0430
	Min	0.0223	-0.0445
	RMS	0.0236	0.0602
	STD	0.0020	0.0005
TrackEye	Max	0.0380	0.0388
	Mean	0.0351	0.0369
	Min	0.0324	0.0352
	RMS	0.0997	0.0369
	STD	0.0020	0.0007

Minimum Values

Method	Statistic	Azimuth	Elevation
Starfind	Max	0.0128	-0.0584
	Mean	0.0070	-0.0602
	Min	-0.0011	-0.0615
	RMS	0.0072	0.0430
	STD	0.0003	0.0002
TrackEye	Max	-0.0933	-0.0298
	Mean	-0.0996	-0.0318
	Min	-0.1068	-0.0346
	RMS	0.0011	0.0077
	STD	0.0005	0.0004

Table A 5-7. Corrected Pointing Angle Residual Error Statistics for GTM-2. All units in degrees.

Mean Values

Method	Statistic	Azimuth	Elevation
Starfind	**Max**	0.0887	-0.0312
	Mean	0.0856	-0.0339
	Min	0.0816	-0.0362
	RMS	0.0856	0.0340
	STD	0.0012	0.0008
TrackEye	**Max**	0.0711	0.0157
	Mean	0.0677	0.0126
	Min	0.0634	0.0101
	RMS	0.0737	0.0143
	STD	0.0012	0.0009

Standard Deviation Values

Method	Statistic	Azimuth	Elevation
Starfind	**Max**	0.0058	0.0050
	Mean	0.0041	0.0050
	Min	0.0045	0.0051
	RMS	0.0041	0.0050
	STD	0.0010	0.0001
TrackEye	**Max**	0.0296	0.0159
	Mean	0.0321	0.0155
	Min	0.0338	0.0155
	RMS	0.0117	0.0139
	STD	0.0010	0.0001

Maximum Values

Method	Statistic	Azimuth	Elevation
Starfind	**Max**	0.1076	-0.0213
	Mean	0.0953	-0.0234
	Min	0.0881	-0.0252
	RMS	0.0955	0.0425
	STD	0.0050	0.0011
TrackEye	**Max**	0.0946	0.0406
	Mean	0.0910	0.0377
	Min	0.0876	0.0355
	RMS	0.0910	0.0378
	STD	0.0051	0.0010

Minimum Values

Method	Statistic	Azimuth	Elevation
Starfind	**Max**	0.0784	-0.0401
	Mean	0.0767	-0.0425
	Min	0.0731	-0.0447
	RMS	0.0767	0.0234
	STD	0.0006	0.0004
TrackEye	**Max**	-0.0399	-0.0016
	Mean	-0.0538	-0.0028
	Min	-0.0647	-0.0059
	RMS	0.0531	0.0010
	STD	0.0007	0.0005

Table A 5-8. Corrected Pointing Angle Residual Error Statistics for GTM-3. All units in degrees.

Mean Values

Method	Statistic	Azimuth	Elevation
Starfind	Max	0.0023	-0.0172
	Mean	-0.0023	-0.0184
	Min	-0.0053	-0.0213
	RMS	0.0068	0.0184
	STD	0.0006	0.0003
TrackEye	Max	-0.0122	0.0342
	Mean	-0.0177	0.0324
	Min	-0.0215	0.0290
	RMS	0.0228	0.0324
	STD	0.0010	0.0006

Standard Deviation Values

Method	Statistic	Azimuth	Elevation
Starfind	Max	0.0133	0.0047
	Mean	0.0133	0.0048
	Min	0.0162	0.0085
	RMS	0.0117	0.0048
	STD	0.0006	0.0002
TrackEye	Max	0.0370	0.0105
	Mean	0.0389	0.0105
	Min	0.0414	0.0116
	RMS	0.0360	0.0105
	STD	0.0007	0.0001

Maximum Values

Method	Statistic	Azimuth	Elevation
Starfind	Max	0.0304	-0.0048
	Mean	0.0072	-0.0062
	Min	0.0063	-0.0077
	RMS	0.0604	0.0249
	STD	0.0030	0.0010
TrackEye	Max	0.0150	0.0494
	Mean	0.0122	0.0477
	Min	0.0097	0.0456
	RMS	0.1800	0.0477
	STD	0.0038	0.0011

Minimum Values

Method	Statistic	Azimuth	Elevation
Starfind	Max	-0.0516	-0.0239
	Mean	-0.0603	-0.0249
	Min	-0.0714	-0.0483
	RMS	0.0007	0.0062
	STD	0.0003	0.0002
TrackEye	Max	-0.1722	0.0088
	Mean	-0.1799	0.0072
	Min	-0.1912	0.0053
	RMS	0.0006	0.0072
	STD	0.0003	0.0003

Table A 5-9. Corrected Pointing Angle Residual Error Statistics for GTM-4. All units in degrees.

Mean Values

Method	Statistic	Azimuth	Elevation
Starfind	Max	0.0697	0.0408
	Mean	0.0665	0.0390
	Min	0.0637	0.0374
	RMS	0.0666	0.0390
	STD	0.0010	0.0006
TrackEye	Max	0.0577	0.0326
	Mean	0.0544	0.0307
	Min	0.0517	0.0288
	RMS	0.0578	0.0307
	STD	0.0010	0.0007

Standard Deviation Values

Method	Statistic	Azimuth	Elevation
Starfind	Max	0.0120	0.0076
	Mean	0.0139	0.0074
	Min	0.0162	0.0073
	RMS	0.0136	0.0074
	STD	0.0010	0.0001
TrackEye	Max	0.0201	0.0071
	Mean	0.0216	0.0070
	Min	0.0228	0.0069
	RMS	0.0081	0.0070
	STD	0.0006	0.0001

Maximum Values

Method	Statistic	Azimuth	Elevation
Starfind	Max	0.0821	0.0583
	Mean	0.0801	0.0560
	Min	0.0781	0.0537
	RMS	0.0801	0.0560
	STD	0.0053	0.0008
TrackEye	Max	0.0722	0.0461
	Mean	0.0685	0.0439
	Min	0.0668	0.0422
	RMS	0.0685	0.0439
	STD	0.0033	0.0008

Minimum Values

Method	Statistic	Azimuth	Elevation
Starfind	Max	0.0197	0.0248
	Mean	0.0072	0.0235
	Min	-0.0057	0.0221
	RMS	0.0089	0.0235
	STD	0.0005	0.0004
TrackEye	Max	-0.0273	0.0122
	Mean	-0.0375	0.0106
	Min	-0.0450	0.0091
	RMS	0.0376	0.0106
	STD	0.0006	0.0006

The table on the next page contains a summary of the overall RMS, Mean and Standard Deviation results from each GTM.

Maximum RMS of Errors among All Stars				
	TrackEye		Starfind	
GTM	Azimuth	*Elevation*	*Azimuth*	*Elevation*
1	0.0997	0.0369	0.0236	0.0602
2	0.0910	0.0378	0.0955	0.0425
3	0.1800	0.0477	0.0604	0.0249
4	0.0685	0.0439	0.0801	0.0560

Mean RMS of Errors among All Stars				
	TrackEye		Starfind	
GTM	Azimuth	*Elevation*	*Azimuth*	*Elevation*
1	0.0227	0.0246	0.0201	0.0543
2	0.0737	0.0143	0.0856	0.0340
3	0.0228	0.0324	0.0068	0.0184
4	0.0578	0.0307	0.0666	0.0390

STD RMS of Errors among All Stars				
	TrackEye		Starfind	
GTM	Azimuth	*Elevation*	*Azimuth*	*Elevation*
1	0.0233	0.0093	0.0032	0.0049
2	0.0117	0.0139	0.0041	0.0050
3	0.0360	0.0105	0.0117	0.0048
4	0.0081	0.0070	0.0136	0.0074

Minimum RMS of Errors among All Stars				
	TrackEye		Starfind	
GTM	Azimuth	*Elevation*	*Azimuth*	*Elevation*
1	0.0011	0.0077	0.0072	0.0430
2	0.0531	0.0010	0.0767	0.0234
3	0.0006	0.0072	0.0007	0.0062
4	0.0376	0.0106	0.0089	0.0235

Maximum Mean of Errors among All Stars				
	TrackEye		Starfind	
GTM	Azimuth	*Elevation*	*Azimuth*	*Elevation*
1	0.0351	0.0369	0.0236	-0.0430
2	0.0910	0.0377	0.0953	-0.0234
3	0.0122	0.0477	0.0072	-0.0062
4	0.0685	0.0439	0.0801	0.0560

Mean of Mean of Errors among All Stars				
	TrackEye		Starfind	
GTM	Azimuth	*Elevation*	*Azimuth*	*Elevation*
1	-0.0099	-0.0127	0.0201	-0.0543
2	0.0677	0.0126	0.0856	-0.0339
3	-0.0177	0.0324	-0.0023	-0.0184
4	0.0544	0.0307	0.0665	0.0390

STD of Mean of Errors among All Stars				
	TrackEye		Starfind	
GTM	Azimuth	*Elevation*	*Azimuth*	*Elevation*
1	0.0312	0.0234	0.0033	0.0049
2	0.0321	0.0155	0.0041	0.0050
3	0.0389	0.0105	0.0133	0.0048
4	0.0216	0.0070	0.0139	0.0074

Minimum Mean of Errors among All Stars				
	TrackEye		Starfind	
GTM	Azimuth	*Elevation*	*Azimuth*	*Elevation*
1	-0.0996	-0.0318	0.0070	-0.0602
2	-0.0538	-0.0028	0.0767	-0.0425
3	-0.1799	0.0072	-0.0603	-0.0249
4	-0.0375	0.0106	0.0072	0.0235

Maximum STD of Errors among All Stars				
	TrackEye		Starfind	
GTM	Azimuth	*Elevation*	*Azimuth*	*Elevation*
1	0.0020	0.0007	0.0020	0.0005
2	0.0051	0.0010	0.0050	0.0011
3	0.0038	0.0011	0.0030	0.0010
4	0.0033	0.0008	0.0053	0.0008

Mean of STD of Errors among All Stars				
	TrackEye		Starfind	
GTM	Azimuth	*Elevation*	*Azimuth*	*Elevation*
1	0.0008	0.0006	0.0007	0.0004
2	0.0012	0.0009	0.0012	0.0008
3	0.0010	0.0006	0.0006	0.0003
4	0.0010	0.0007	0.0010	0.0006

STD of STD of Errors among All Stars				
	TrackEye		Starfind	
GTM	Azimuth	*Elevation*	*Azimuth*	*Elevation*
1	0.0004	0.0001	0.0004	0.0001
2	0.0010	0.0001	0.0010	0.0001
3	0.0007	0.0001	0.0006	0.0002
4	0.0006	0.0001	0.0010	0.0001

Minimum STD of Errors among All Stars				
	TrackEye		Starfind	
GTM	Azimuth	*Elevation*	*Azimuth*	*Elevation*
1	0.0005	0.0004	0.0003	0.0002
2	0.0007	0.0005	0.0006	0.0004
3	0.0003	0.0003	0.0003	0.0002
4	0.0006	0.0006	0.0005	0.0004

B.2. Summary Results for Ground Targets

Table B 5-10. 3-D TSPI Comparison

3-D TSPI Results Comparison				
Target		X (m)	Y (m)	Z (m)
B1 (Pedro)	TrackEye	3484.47	-14606.19	1634.13
	Surveyed	3484.14	-14605.82	1634.08
	Error	0.33	0.37	0.05
B2 (STA 30)	TrackEye	383.91	-20789.42	1834.46
	Surveyed	383.36	-20789.39	1833.70
	Error	0.56	0.03	0.76
B3 (STA 40)	TrackEye	7315.24	-4641.40	1691.16
	Surveyed	7315.08	-4641.59	1690.52
	Error	0.16	0.19	0.64
WT (New WT)	TrackEye	-2310.92	-6470.79	1740.85
	Surveyed	-2311.16	-6470.62	1739.86
	Error	0.23	0.17	0.99
Mean Error		**0.32**	**0.19**	**0.61**
STD		**0.17**	**0.14**	**0.40**

Table B 5-11. Corrected Pointing Angles

Pre-Processed Pointing Angles									
Mount		WT		B3 (Sta 40)		B2 (sta 30)		B1 (Pedro)	
		Azimuth [degrees]	Elevation [degrees]	Azimuth [degrees]	Elevation [degrees]	Azimuth [degrees]	Elevation [degrees]	Azimuth [degrees]	Elevation [degrees]
GTM1	Measured	343.0210	-0.0710	18.3890	-0.2530	263.8372	4.4387	13.7976	-0.8860
	Expected	343.0011	0.1248	18.3690	-0.0583	263.7849	4.4000	13.7718	-0.6815
	Error	0.0199	0.1958	0.0200	0.1947	0.0523	0.0387	0.0258	0.2045
GTM2	Measured	345.5597	0.7893	60.7010	0.0092	170.3469	0.7991	135.0964	-0.3650
	Expected	345.6021	0.8834	60.7569	0.0013	170.3998	0.7229	135.1492	-0.4547
	Error	0.0424	0.0941	0.0559	0.0080	0.0530	0.0762	0.0528	0.0897
GTM3	Measured	280.7834	0.2966	347.3789	0.3190	212.4179	0.6890	217.4267	-0.2040
	Expected	280.6105	0.4229	347.1998	0.4333	212.2463	0.6770	217.2548	-0.1997
	Error	0.1729	0.1263	0.1790	0.1143	0.1715	0.0120	0.1720	0.0043
GTM4	Measured	327.7580	0.1056	353.9656	-0.0052	295.6770	1.1160	330.3595	-0.1120
	Expected	327.6987	0.3539	353.9063	0.2423	295.6111	1.3049	330.2985	0.1453
	Error	0.0593	0.2483	0.0593	0.2474	0.0659	0.1889	0.0610	0.2573
Mean Error		**0.0736**	**0.1661**	**0.0786**	**0.1411**	**0.0857**	**0.0790**	**0.0779**	**0.1389**
STD		**0.0681**	**0.0693**	**0.0693**	**0.1043**	**0.0576**	**0.0779**	**0.0645**	**0.1138**
RMS		**0.0943**	**0.1766**	**0.0989**	**0.1675**	**0.0991**	**0.1038**	**0.0958**	**0.1703**

Table B 5-12. Post-Processed Pointing Angles

Post-Processed Pointing Angles									
Mount		WT		B3 (Sta 40)		B2 (sta 30)		B1 (Pedro)	
		Azimuth [degrees]	Elevation [degrees]	Azimuth [degrees]	Elevation [degrees]	Azimuth [degrees]	Elevation [degrees]	Azimuth [degrees]	Elevation [degrees]
GTM1	Measured	343.0018	0.1287	18.3693	-0.0561	263.7819	4.4278	13.7756	-0.6811
	Expected	343.0011	0.1248	18.3690	-0.0583	263.7849	4.4000	13.7718	-0.6815
	Error	0.0007	0.0038	0.0003	0.0022	0.0030	0.0278	0.0038	0.0004
GTM2	Measured	345.6053	0.9011	60.7564	0.0049	170.3971	0.7267	135.1495	-0.4543
	Expected	345.6021	0.8834	60.7569	0.0013	170.3998	0.7229	135.1492	-0.4547
	Error	0.0033	0.0177	0.0005	0.0036	0.0027	0.0038	0.0002	0.0004
GTM3	Measured	280.6098	0.4282	347.2028	0.4426	212.2444	0.6800	217.2512	-0.1994
	Expected	280.6105	0.4229	347.1998	0.4333	212.2463	0.6770	217.2548	-0.1997
	Error	0.0007	0.0053	0.0030	0.0094	0.0019	0.0030	0.0036	0.0004
GTM4	Measured	327.6990	0.3565	353.9068	0.2440	295.6123	1.3092	330.2989	0.1455
	Expected	327.6987	0.3539	353.9063	0.2423	295.6111	1.3049	330.2985	0.1453
	Error	0.0003	0.0026	0.0005	0.0018	0.0012	0.0044	0.0005	0.0002
Mean Error		**0.0012**	**0.0074**	**0.0011**	**0.0042**	**0.0022**	**0.0097**	**0.0020**	**0.0004**
STD		**0.0014**	**0.0070**	**0.0013**	**0.0035**	**0.0008**	**0.0121**	**0.0019**	**0.0001**
RMS		**0.0017**	**0.0095**	**0.0015**	**0.0052**	**0.0023**	**0.0143**	**0.0026**	**0.0004**

APPENDIX C. MOUNT-MODEL SIMULATION

The calibration process is responsible for removing systematic errors; however, the unknowable random errors remain. It is important to quantify the magnitude of the uncertainty expected due to random errors in the assessment of any accuracy criteria. For example, errors due to parallactic refraction are caused by atmospheric conditions between the observer and the target and are path dependent, so the error must be estimated based on the environmental conditions during the experiment. Even though the estimated refraction errors are corrected for, some unknown refraction uncertainty will remain. Atmospheric scintillation is another random error affecting image quality and tracking SNR. Another random error affecting pointing angle accuracy originates from the overall quality of the system's calibration such that not all bias offsets and scaling factors are accurate or accounted for, especially due to environmental changes between the time of calibration and test time.

To get a sense of the magnitude of these 'unaccounted-for' errors and their effects on the evaluated position of the object being tracked, and how effectively the TSPI solution accounts for pointing angle errors, a computer simulation was conducted using the Mount-Model (MM) simulation capability developed for the TTR-TSPI-UQ [1] report based on the Contraves Cinetheodolite Model F. The simulation was configured using four Cinetheodolite[4] Mount-Models located at the same positions occupied by the GTMs during the acquisition of the Star Validation datasets that were used for the qualification process. In so doing, the relative pointing angle errors based on four Cinetheodolites could be assessed. Although Cinetheodolites were simulated and the GTMs were not, the relative pointing angle error magnitude for each may still be compared because the TSPI code has been developed to account for pointing angle errors, regardless of the angle measurement device.

The MM simulation results are provided below in Section C.1 and include simulated input and output results of the OPTXYZ function for each mount location.

The main points demonstrated by MM simulation study are:

1. The simulated pointing angles generated by MM Simulation reveal RMS, Mean and STD errors that are on par with those found for the GTMs based on the star validation data results.
2. Out of 1,000 simulated TSPI solutions, 485 meet the one meter cubed or less error volume expectation for TTR TSPI results. For all 1,000 simulations, the mean error volume evaluated to 3.7 meters with a standard deviation of 7.8 meters. The mean error volume drops to 1.0 meters with a standard deviation of 0.9 meters for 750 the simulated solutions having error volumes of 3.5 meters or less.
3. The OPTXYZ function contains the Davis Solution [2] and evaluates a TSPI solution based on mount locations and the corresponding pointing angles for each mount, regardless of the angle measuring instrument. In other words, the OPTXYZ function is agnostic to where its input angles come from – they can be corrected angels from a Cinetheodolite, a radar, or a GTM.
4. If the simulated pointing angles error statistics based on a Cinetheodolite MM are on par with the real error statistics found for the GTMs based on the star-based angle assessment, then it may be reasonable to assumed that GTMs will produce TSPI results consistent with error volumes of one meter cubed or less.

[4] A mount-model based on the GTMs does not exist.

C.1. Mount -Model Simulation

Mount-Model (MM) simulation input and output results to the OPTXYZ function, based on the GTM locations used for the Star Validation data collection, are provided below. The top plot in Figure C 5-6 shows the relative position of each GTM location (i.e., station) relative to the TTR origin (i.e., (0,0,1648.04)) and the collection of simulated triangulate positions. The bottom plot in Figure C 5-6 has been scaled (i.e., zoomed in) to show the individual target realization for each of 1,000 simulations. The mean triangulated results from all simulations is provided on each plot.

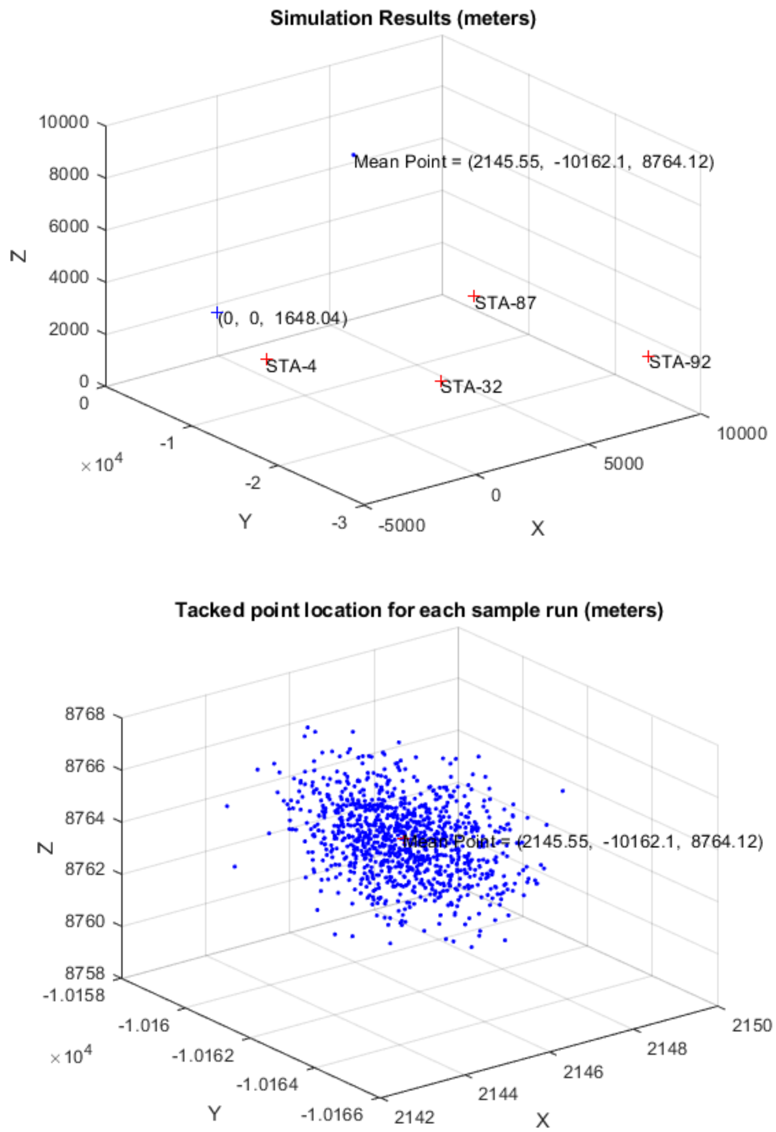

Figure C 5-6. Mount-Model Simulation Results.
Top: GTM stations with the ensemble of triangulated target points including the mean value.
Bottom: Zoomed version of top plot, showing each of the 1,000 simulated TSPI solution results.

The azimuth and elevation input RMS, mean, and STD error statistics to the OPTXYZ triangulation function from the MM simulation are provided in Table C 5-13 and Table C 5-14, respectively. The resulting triangulated OPTXYZ output statistics are provided in Table C 5-15.

Table C 5-13. Mount-Model OPTXYZ() Azimuth Input Error Statistics

Station / MM	Azimuth RMS Error (degrees)	Azimuth Mean Error (degrees)	Azimuth STD Error (degrees)
STA-32 / MM-1	0.0077	0.0003	0.0077
STA-4 / MM-2	0.0225	0.0007	0.0225
STA-87 / MM-3	0.0138	0.0002	0.0138
STA-92 / MM-4	0.0045	-0.0009	0.0044

Table C 5-14. Mount-Model OPTXYZ() Elevation Input Error Statistics

Station / MM	Elevation RMS Error (degrees)	Elevation Mean Error (degrees)	Elevation STD Error (degrees)
STA-32 / MM-1	0.0080	-0.0057	0.0056
STA-4 / MM-2	0.0115	-0.0024	0.0113
STA-87 / MM-3	0.0073	-0.0034	0.0065
STA-92 / MM-4	0.0099	-0.0091	0.0040

Table C 5-15. Mount-Model OPTXYZ() Output Statistics

Axis	True Location (m)	Mean (m)	STD (m)	1-Sigma (m)	2-Sigma (m)
X	2145.5	2145.6	1.0	2144.6 to 2146.6	2141.6 to 2147.6
Y	-10162.0	-10162.1	1.0	-10163.1 to -10161.1	-10164.1 to -10160.1
Z	8762.4	8762.1	1.1	8761.0 to 8763.2	8759.9 to 8764.3
Altitude	8770.08	8766.7	1.1	8765.6 to 8767.8	8764.5 to 8768.9
Error-Volume	N/A	3.7	7.8	N/A	N/A

Note that the Error-Volume Mean and STD in Table C 5-15 are 3.7 and 7.8 meters respectively, showing an Error-Volume greater than the one-meter cubed or less expectation for TTR TSPI results. It was determined that the error volumes for 250 of the 1,000 simulated results were greater than 3.50 meters when sorted. When these 250 simulated results were discarded, the mean error volume for the remaining 750 results evaluated to 1.0 meters with a standard deviation of 0.9 meters. The mean error volume for the 250 discarded values evaluated to 11.8 meters with a standard deviation of 12.5 meters. Plots of the Error Volume vs. Simulation Run Number are shown below in Figure C 5-7, where the error volumes for the bottom two plots have been sorted in ascending order.

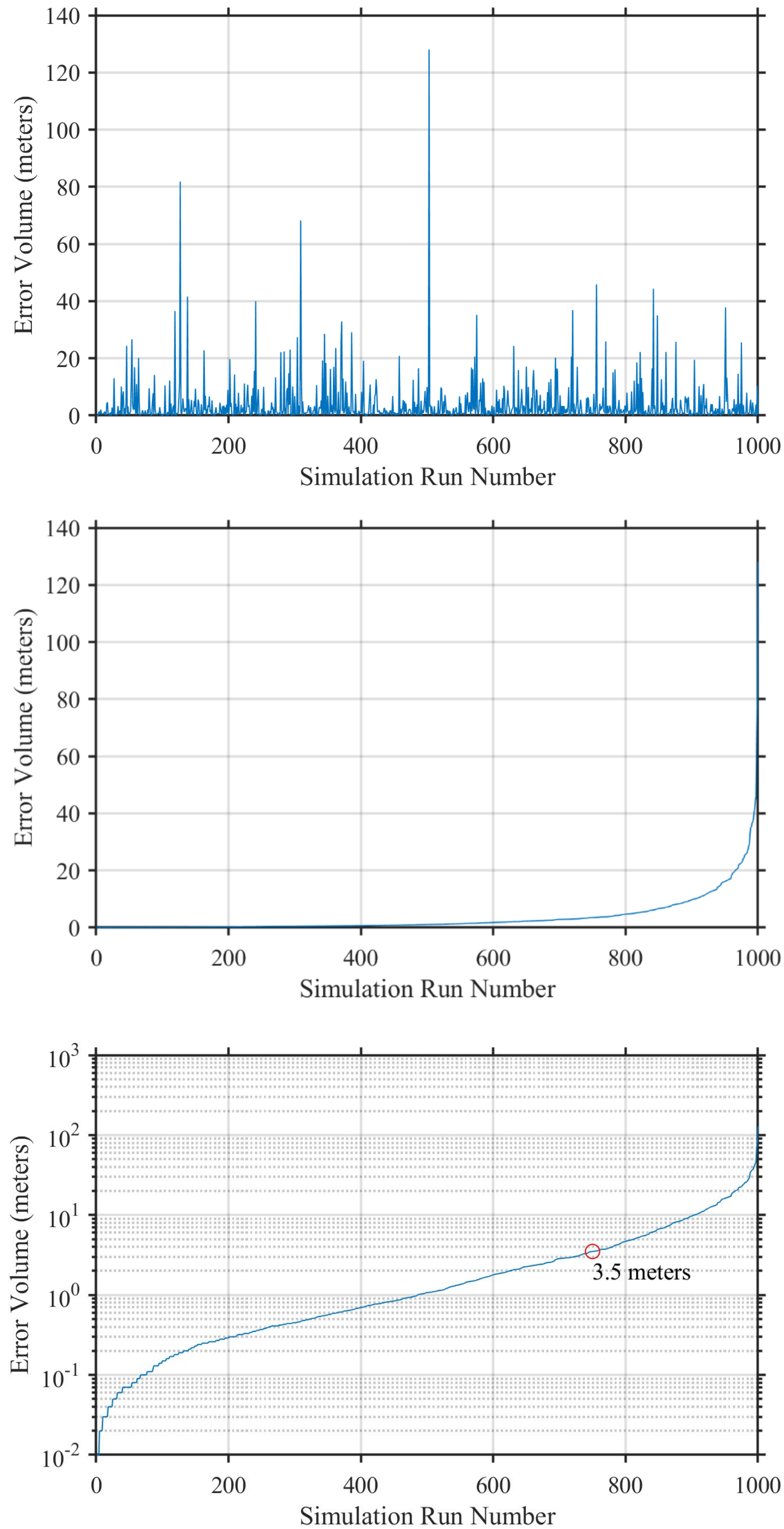

Figure C 5-7. Error Volume vs Simulation Run Number plots.
Note: Error volumes for Linear (middle) and Semi-Log (bottom) are sorted in ascending order.

The azimuth and elevation RMS errors for all 1000 simulation runs, the 750 runs having runs with volume errors less than 3.5 meters, and the 250 runs with volume errors greater than 3.5 meters are reported in tables Table C 5-16 and Table C 5-17, respectively.

Table C 5-16. Mount-Model OPTXYZ() Azimuth Input RMS Error Statistics

Station / MM	Azimuth RMS Error (degrees) All 1000 Runs	Azimuth RMS Error (degrees) 750 Runs Error-Volumes less than 3.5m	Azimuth RMS Error (degrees) 250 Runs Error-Volumes greater than 3.5m
STA-32 / MM-1	0.0077	0.0075	0.0085
STA-4 / MM-2	0.0225	0.0185	0.0312
STA-87 / MM-3	0.0138	0.0127	0.0164
STA-92 / MM-4	0.0045	0.0044	0.0049
Error-Volume Mean (meters)	3.7	1.0	11.8
Error-Volume STD (meters)	7.8	0.9	12.5

Table C 5-17. Mount-Model OPTXYZ() Elevation Input RMS Error Statistics

Station / MM	Elevation RMS Error (degrees) All 1000 Runs	Elevation RMS Error (degrees) 750 Runs Error-Volumes less than 3.5m	Elevation RMS Error (degrees) 250 Runs Error-Volumes greater than 3.5m
STA-32 / MM-1	0.0080	0.0077	0.0089
STA-4 / MM-2	0.0115	0.0108	0.0134
STA-87 / MM-3	0.0073	0.0071	0.0079
STA-92 / MM-4	0.0099	0.0100	0.0098
Error-Volume Mean (meters)	3.7	1.0	11.8
Error-Volume STD (meters)	7.8	0.9	12.5

Note that the azimuth and elevation RMS errors for the all 1,000 simulated runs and the retained 750 runs are relatively close, while the RMS errors for the 250 discarded runs are greater.

The azimuth and elevation input RMS, mean, and STD error statistics to the OPTXYZ triangulation function from the MM simulation for the 750 simulation results with error volumes less than 3.5 meters are provided in Table C 5-18 and Table C 5-19, respectively. The corresponding triangulated OPTXYZ output statistics are provided in Table C 5-20.

Table C 5-18. Mount-Model OPTXYZ() Azimuth Input Error Statistics for 750 results with Error Volumes less than 3.5 meters

Station / MM	Azimuth RMS Error (degrees)	Azimuth Mean Error (degrees)	Azimuth STD Error (degrees)
STA-32 / MM-1	0.0075	0.0001	0.0075
STA-4 / MM-2	0.0185	0.0006	0.0185
STA-87 / MM-3	0.0127	0.0000	0.0128
STA-92 / MM-4	0.0044	-0.0007	0.0043

Table C 5-19. Mount-Model OPTXYZ() Elevation Input Error Statistics for 750 results with Error Volumes less than 3.5 meters

Station / MM	Elevation RMS Error (degrees)	Elevation Mean Error (degrees)	Elevation STD Error (degrees)
STA-32 / MM-1	0.0077	-0.0055	0.0053
STA-4 / MM-2	0.0108	-0.0022	0.0105
STA-87 / MM-3	0.0071	-0.0034	0.0063
STA-92 / MM-4	0.0100	-0.0091	0.0040

Table C 5-20. Mount-Model OPTXYZ() Output Statistics for 750 results with Error Volumes less than 3.5 meters

Axis	True Location (m)	Mean (m)	STD (m)	1-Sigma (m)	2-Sigma (m)
X	2145.5	2145.6	1.0	2144.6 to 2146.6	2141.6 to 2147.6
Y	-10162.0	-10162.1	1.0	-10163.1 to -10161.1	-10164.1 to -10160.1
Z	8762.4	8762.1	1.0	8761.1 to 8763.1	8759.1 to 8764.1
Altitude	8770.08	8766.7	1.0	8765.7 to 8767.1	8764.7 to 8768.7
Error-Volume	N/A	1.0	0.9	N/A	N/A

The azimuth and elevation input RMS, mean, and STD error statistics to the OPTXYZ triangulation function from the MM simulation for the 250 discarded simulation results are provided in **Error! Not a valid bookmark self-reference.** and Table C 5-22, respectively. The corresponding triangulated OPTXYZ output statistics are provided in Table C 5-23.

Table C 5-21. Mount-Model OPTXYZ() Azimuth Input Error Statistics for 250 results with Error Volumes greater than 3.5 meters

Station / MM	Azimuth RMS Error (degrees)	Azimuth Mean Error (degrees)	Azimuth STD Error (degrees)
STA-32 / MM-1	0.0085	0.0009	0.0085
STA-4 / MM-2	0.0312	0.0009	0.0317
STA-87 / MM-3	0.0164	0.0008	0.0165
STA-92 / MM-4	0.0049	-0.0015	0.0047

Table C 5-22. Mount-Model OPTXYZ() Elevation Input Error Statistics for 250 results with Error Volumes greater than 3.5 meters

Station / MM	Elevation RMS Error (degrees)	Elevation Mean Error (degrees)	Elevation STD Error (degrees)
STA-32 / MM-1	0.0089	-0.0062	0.0064
STA-4 / MM-2	0.0134	-0.0029	0.0131
STA-87 / MM-3	0.0079	-0.0036	0.0070
STA-92 / MM-4	0.0098	-0.0090	0.0039

Table C 5-23. Mount-Model OPTXYZ() Output Statistics for 250 results with Error Volumes greater than 3.5 meters

Axis	True Location (m)	Mean (m)	STD (m)	1-Sigma (m)	2-Sigma (m)
X	2145.5	2145.5	1.1	2144.4 to 2146.6	2143.3 to 2147.7
Y	-10162.0	-10162.1	1.2	-10163.3 to -10160.9	-10164.5 to -10159.7
Z	8762.4	8764.2	1.2	8763.0 to 8765.2	8761.8 to 8766.4
Altitude	8770.08	8766.8	1.2	8765.6 to 8768.0	8764.4 to 8769.2
Error-Volume	N/A	11.8	12.5	N/A	N/A

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