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Convex Hull Pricing for the AC OPF Problem

Non-Convex Electricity Markets

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M. Garcia, H. Nagarajan, and R. Baldick. "Generalized Convex Hull Pricing for the AC Optimal Power Flow Problem". In: *IEEE Transactions on Control of Network Systems* 7.3 (2020), pp. 1500–1510

Optimal Power Flow Problem

The *Optimal Power Flow (OPF) Problem* is the

myopic social welfare max. problem (no horizon considered)

Significant Research Efforts to Solve Non-Convex OPF Problems

- Fully Detailed AC OPF Problem (with reactive power/voltage magnitudes)
  - NP-hard in general[1]
  - Iterative methods converge to local minima
  - SDP relaxation is exact under certain criteria[2]
  - Other relaxations proposed to approximate solution[3]

[1] Daniel Bienstock and Abhinav Verma. "Strong NP-hardness of AC power flows feasibility". In: *arXiv preprint arXiv:1512.07315* (2015).

[2] Javad Lavaei and Steven H Low. "Zero duality gap in optimal power flow problem". In: *IEEE Trans. on Power Systems* 27 (2012), pp. 92–107.

[3] Carleton Coffrin, Hassan L Hijazi, and Pascal Van Hentenryck. "The QC relaxation: A theoretical and computational study on optimal power flow". In: *IEEE Trans. on Power Systems* 31.4 (2016), pp. 3008–3018.

Optimal Power Flow Problem

Optimal Power Flow (OPF) Problem: myopic S.W. Max. Problem

Little Research Addresses the
Economic Problems Associated with Non-Convexity

- Identifying Revenue Inadequacy caused by Non-Convexity
  - Standard Locational Marginal Prices
    - Each bus has different price
  - Congestion Revenue Shortfall in presence of FTRs[4and5]
    - Congestion revenue shortfall is typically caused by line outages
- Pricing Structures Addressing this Problem
  - Discriminatory pricing structure suggested[6]
    - Each generator sees different price
  - Convex Hull Pricing (Topic of this work)

[4] Andy Philpott and Geoffrey Pritchard. "Financial transmission rights in convex pool markets". In: *Operations Research Letters* 32.2 (2004), pp. 109–113.

[5] Bernard C Lesieutre and Ian A Hiskens. "Convexity of the set of feasible injections and revenue adequacy in FTR markets". In: *IEEE Trans. on Power Systems* 20.4 (2005), pp. 1790–1798.

[6] Javad Lavaei and Somayeh Sojoudi. "Competitive equilibria in electricity markets with nonlinearities". In: *American Control Conference (ACC), 2012*. IEEE. 2012, pp. 3081–3088.

Review of Convex Hull Pricing

Convex Hull Prices (CHPs) minimize worst case shortfall of ISO.
The various sources of shortfall are referred to as *uplift*.

Minimize Various Uplift Quantities[7 and 8]

- Generator Uplift
- Financial Transmission Right (FTR) Uplift
- Reserve Related Uplift (Future work)

Typical Setting

- UC Problem with linear transmission constraints[7 and 9]
  - Observe that CHPs decrease side-payments as compared to LMPs
- Generalization to AC OPF problem does not exist

[7] Dane A Schiro et al. "Convex hull pricing in electricity markets: Formulation, analysis, and implementation challenges". In: *IEEE Trans. on Power Systems* 31.5 (2016), pp. 4068–4075.

[8] Paul R Gribik, William W Hogan, and Susan L Pope. "Market-clearing electricity prices and energy uplift". In: *Cambridge, MA* (2007).

[9] Bowen Hua and Ross Baldick. "A Convex Primal Formulation for Convex Hull Pricing". In: *IEEE Trans. on Power Systems* (2016).

Outline

1. Background and Introduction
2. CHP Problem Formulation (Multi-Objective Minimum Uplift)
3. Computing Approximate CHPs
4. Examples
5. Conclusion

Electricity Market Setting

Underlying Graph

$\mathcal{G} = (\mathcal{N}, \mathcal{E})$: Directed graph

$\mathcal{N}$: Set of n buses

$\mathcal{E}$: Set of m trans. lines

Uniform Nodal Prices

$\pi \in \mathbb{R}^n$: nodal price for real power

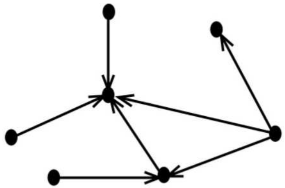

Figure: Arbitrary Directed Graph.
(Arrows represent edges and dots represent nodes)

Market Participants
(More Details Later)

Demand:

- One for each node $i \in \mathcal{V}$
- D_i: fixed p.u. demand
- Charged amount $\pi_i D_i$

Generators:

- One for each node $i \in \mathcal{V}$
- G_i: variable p.u. generation
- paid amount $\pi_i G_i$

FTR holders:

- Awarded FTRs through auction
- Paid/charged based on nodal price difference $\pi_i - \pi_j$

Feasible Set of Net Power Injections

General AC Transmission Model

$\mathcal{P} \subset \mathbb{R}^n$: Feasible net power injections

$G - D \in \mathcal{P}$: physical network constraint

Possible Forms of $\mathcal{P}$

(Arbitrary paramters M_k and b_k)

- DC Approximation (Convex)

$$\mathcal{P} = \{P \in \mathbb{R}^n : \; b_1 \leq M_2 P \leq b_2\}$$

- General Quadratic (Potentially non-convex)

$$\mathcal{P} = \{P \in \mathbb{R}^n : \exists x \in \mathbb{R}^\xi \text{ where } \begin{bmatrix} P^T & x^T \end{bmatrix} M_k \begin{bmatrix} P \\ x \end{bmatrix} \leq b_k \; \forall k\}$$

Generator Uplift

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<u>Generators are Profit Maximizers</u>

Similar to other models in literature[10 and 11]

- Paid amount $\pi_i G_i$ accrue costs of $C_i(G_i)$, where $C_i(\,\cdot\,)$ is convex
- *Private constraints* represent generation limits

$$G_i \in \mathcal{X}^i := \{G_i : G_i^{\text{min}} \leq G_i \leq G_i^{\text{max}}\}$$

- Max profit as a function of nodal price

$$\Pi_i(\pi_i) := \max_{G_i \in \mathcal{X}^i} \left(\pi_i G_i - C_i(G_i)\right). \quad (1)$$

[10] Paul R Gribik, William W Hogan, and Susan L Pope. "Market-clearing electricity prices and energy uplift". In: *Cambridge, MA* (2007).

[11] Bowen Hua and Ross Baldick. "A Convex Primal Formulation for Convex Hull Pricing". In: *IEEE Trans. on Power Systems* (2016).

Generator Uplift

Dispatched Generation

The ISO dispatches generators at amount $G_i^d \in \mathcal{X}^i$.
The dispatch may not maximize the generator's profits.

Lost Opportunity Cost/Side Payments

- Realized profit is $\pi_i G_i^d - C_i(G_i^d)$
- *Lost Opportunity Cost* is

$$C_i^o(\pi_i, G_i^d) := \Pi_i(\pi_i) - \left(\pi_i G_i^d - C_i(G_i^d)\right). \tag{2}$$

- *Side Payments* in the amount of $C_i^o(\pi_i, G_i^d)$
  - Conditioned on generator following dispatch
  - Neutralize incentive to deviate from dispatch
  - Not covered by another revenue stream
- Side-payments are non-negative, e.g. $C_i^o(\pi_i, G_i^d) \geq 0$

FTR Uplift

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Maximum FTR Payoff

(Market must be revenue adequate for any FTR allocation)

$$\Psi(\pi) := \max_{f \in \mathcal{P}} \quad -\pi^T f. \tag{3}$$

Modeling the Aggregate Effect of FTRs[12]

- $-\pi^T f$: *total FTR payoff* (FTR Obligation)
- $f \in \mathbb{R}^n$: Aggregate FTR allocation vector
  - Element f_k represents a megawatt value of injection into node k

Simultaneous Feasibility Conditions (SFCs)

- FTR auction occurs far in advance of market clearing
- FTR auction ensures that SFCs are satisfied
  - The aggregate FTR allocation vector represents a feasible vector of net power injections, eg. $f \in \mathcal{P}$.

[12] M. Garcia, H. Nagarajan, and R. Baldick. "Generalized Convex Hull Pricing for the AC Optimal Power Flow Problem". In: *IEEE Transactions on Control of Network Systems* 7.3 (2020), pp. 1500–1510.

FTR Uplift

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Potential Congestion Revenue Shortfall (PCRS)

(Worst case shortfall of congestion revenue)

- FTRs are funded using congestion revenue
- Congestion revenue: $\pi^T\left(D - G^d\right)$
- PCRS (also known as FTR uplift):

$$C^s(\pi, G^d) := \Psi(\pi) - \pi^T\left(D - G^d\right). \tag{4}$$

- PCRS is non-negative, eg. $C^s(\pi, G^d) \geq 0$ (Assuming that $G^d - D \in \mathcal{P}$)
- If $C^s(\pi, G^d) = 0$ then congestion revenue adequacy is guaranteed

Multi-Objective Minimum Uplift Problem

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Convex Hull Pricing (CHP) Problem

The *Convex Hull Prices* (CHPs) minimize the weighted sum of PCRS and total side-payments and are denoted $\pi^\star$.

$$\pi^\star \in \underset{\pi \in \mathbb{R}^n}{\text{argmin}} \left(\alpha C^s(\pi, G^d) + \sum_{i \in \mathcal{V}} C_i^o(\pi_i, G_i^d) \right) \tag{5}$$

- PCRS represents only a potential shortfall
  - Weight $\alpha > 0$ is likely less than 1
- Generalization of typical CHP definition
  - Typical definition sets weight parameter to $\alpha = 1$
- Difficult to solve
  - Bi-level optimization problem
  - Inner optimization problem is non-convex

Primal CHP Problem

Convex Primal Counterpart (Primal CHP Problem)

Equivalent to the AC OPF problem with $\mathcal{P}$ replaced by its convex hull conv($\mathcal{P}$).
(Note: $P^d := G^d - D$)

$$\min_{G \in \mathcal{X}, P \in \text{conv}(\mathcal{P})} \sum_{i \in \mathcal{V}} C_i(G_i) \tag{6}$$

$$st : D_i - G_i + \alpha P_i + (1-\alpha) P_i^d = 0 \quad \forall i \in \mathcal{V} \tag{6a}$$

Theorem

Optimal Lagrange multipliers of constraints (6a) minimize the CHP problem (5) and thus represent CHPs.

Proof: Contained in reference.[13] □

[13] M. Garcia, H. Nagarajan, and R. Baldick. "Generalized Convex Hull Pricing for the AC Optimal Power Flow Problem". In: *IEEE Transactions on Control of Network Systems* 7.3 (2020), pp. 1500–1510.

Approximating CHPs

Approximating CHPs

Conv($\mathcal{P}$) may be intractable to evaluate!

Approximate using convex relaxation relax($\mathcal{P}$)$\supseteq$conv($\mathcal{P}$).

Relaxation produces *approximate CHPs* $\bar{\pi}$

Relaxed Primal CHP Problem

Replace $\mathcal{P}$ with a convex relaxation relax($\mathcal{P}$)

$$\min_{G \in \mathcal{X},\ P \in \text{relax}(\mathcal{P})} \quad \sum_{i \in \mathcal{V}} C_i(G_i) \tag{7}$$

$$st : D_i - G_i + \alpha P_i + (1-\alpha) P_i^d = 0 \quad \forall i \in \mathcal{V} \tag{7a}$$

Approximate CHPs

Approximate CHPs $\bar{\pi}$ are Lagrange multipliers for constraint (7a)

Outline

Examples

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Overview of All Test Cases

- Full AC transmission model[14]
  - Accounts for reactive power and voltage magnitudes
  - Used to define $\mathcal{P}$
- This presentation will only analyze the SDP relaxation[15].
-
  - Other options: QC[16] and SOCP[17] relaxations
- Part I: Weight parameter is set to $\alpha = 1$
- Part II: Analyze impact of varying weight parameter α

[14] Daniel K Molzahn and Ian A Hiskens. "Convex relaxations of optimal power flow problems: An illustrative example". In: *IEEE Transactions on Circuits and Systems I: Regular Papers* 63.5 (2016), pp. 650–660.

[15] Daniel K Molzahn et al. "Implementation of a large-scale optimal power flow solver based on semidefinite programming". In: *IEEE Trans. on Power Systems* 28.4 (2013), pp. 3987–3998.

[16] Carleton Coffrin, Hassan L Hijazi, and Pascal Van Hentenryck. "The QC relaxation: A theoretical and computational study on optimal power flow". In: *IEEE Trans. on Power Systems* 31.4 (2016), pp. 3008–3018.

[17] Rabih A Jabr. "Radial distribution load flow using conic programming". In: *IEEE Trans. on power systems* 21.3 (2006), pp. 1458–1459.

LMP Analysis

Overview

- Three test cases available by NESTA[18]
  - Strong duality does not hold so Total Uplift is non-zero.
- Must solve the max FTR payoff problem (3) to compute FTR Uplift.
  - Local minimum found with interior point solver (Conservative).

LMPs and Shortfall

FTR Uplift is significant for some test cases

$> 30\%$ of cost for 162 bus case

Table: Results with LMPs (USD)

Test Case	LMPs (for AC OPF) Generator Uplift	FTR Uplift	Total Operating Cost
162_ieee_dtc	~ 0	1,352.92	4,230.23
189_edin	~ 0	1.22	849.29
300_ieee	~ 0	36.87	16,891.27

[18] Carleton Coffrin, Dan Gordon, and Paul Scott. "NESTA, the NICTA energy system test case archive". In: *arXiv preprint arXiv:1411.0359* (2014).

Approximate CHPs with SDP

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Approximate CHPs and Shortfall

- Relaxed primal CHP problem formulated using SDP relaxation.
  - Implimented using MATPOWER toolbox in MATLAB[19]
- Significantly lower FTR Uplift, slightly larger Generator Uplift.
  - Particularly effective for 162 bus test case

Table: Results with SDP Relaxation (USD)

Test Case	LMPs (for AC OPF)		Approximate CHPs with SDP Relaxation		Total Operating Cost
	Generator Uplift	FTR Uplift	Generator Uplift	FTR Uplift	
162_ieee_dtc	~ 0	1,352.92	0.11	42.55	4,230.23
189_edin	~ 0	1.22	0.05	0.74	849.29
300_ieee	~ 0	36.87	0.03	14.77	16,891.27

[19] Daniel K Molzahn et al. "Implementation of a large-scale optimal power flow solver based on semidefinite programming". In: *IEEE Trans. on Power Systems* 28.4 (2013), pp. 3987–3998.

Choice of Weight Parameter α (Case 162_ieee_dtc)

Varying Weight α

- Decreasing α from 1
  - Decreases side-payments
  - Increases PCRS
- When $\alpha \approx .7$
  - Generator Uplift is zero
  - FTR Uplift is approx. $\$42$ down from approx $\$1352$

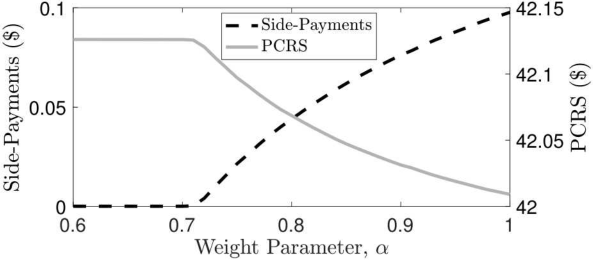

Figure: Varying the weight Parameter α

Conclusions and Future Work

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Conclusions

- The SDP relaxation can be used to effectively approximate CHPs.
  - Computational restrictions limit us to small test cases.
- The trade-off between generator uplift and FTR uplift can be adjusted by the weight parameter α

Future Work

- Extend work to UC problem with simple quadratic losses.
- Analyze weight constant α using empirical results
- Include reserve uplift in the formulation