



Distributed Graph Coloring on Multiple GPUs

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Main contributions and results

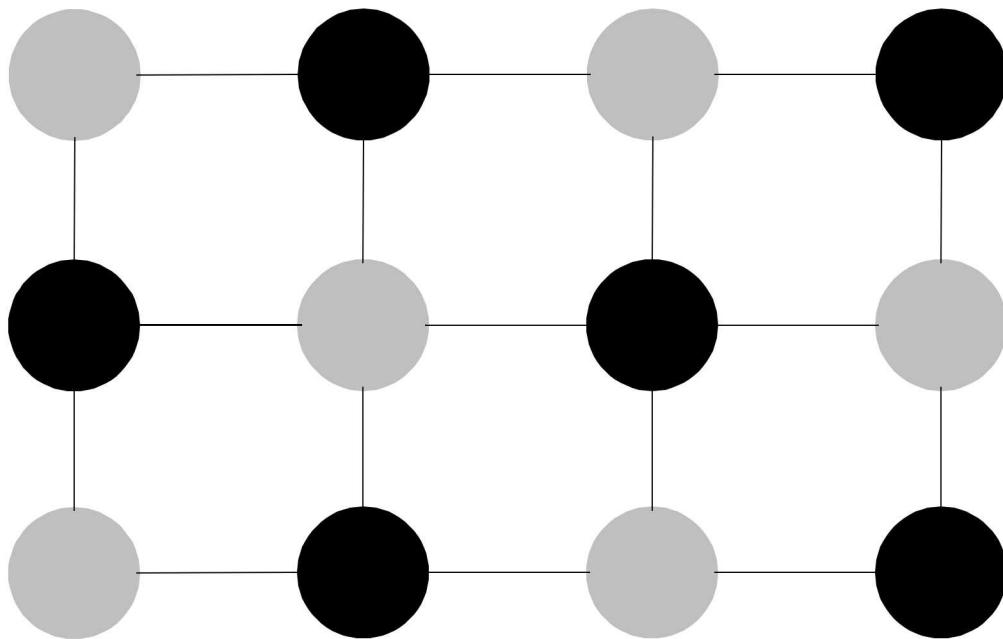
- We present the first distributed memory multi-GPU graph coloring implementation, to our knowledge
- Our distributed distance-1 coloring implementation sees up to a **28x speedup** on 128 GPUs over a single GPU, and **only a 7.5% increase** in colors on average
- Our distributed distance-2 coloring implementation also sees up to a **28x speedup** on 128 GPUs over a single GPU, and a **4.9% increase** in colors in the **worst** case
- Our approach is able to color a 12.8 Billion vertex mesh with **76.7 Billion edges** in **under half a second**
- We also present an approach that reduces the number of collective communications by increasing the cost of each communication.

Graph Coloring is Useful for Finding Concurrency

- Computations with specific data access patterns can be parallelized with colorings
- Compiler parallelization, register allocation
- Preprocessing Jacobian and Hessian matrix computations
- Also useful for finding short-circuits in circuit designs

Distance-1 Graph Coloring

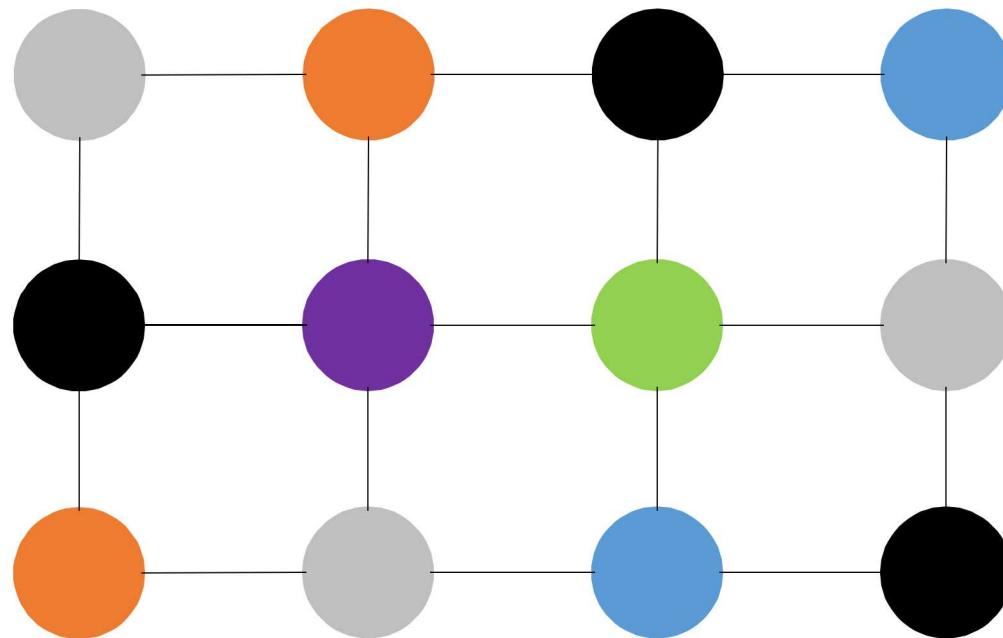
Each vertex ends up with a different color from its neighbors.



Minimizing the number of colors used in the coloring is known to be NP-Hard.

Distance-2 Graph Coloring

A valid distance-2 coloring assigns each vertex a color not used by any vertex at most two edges away



Minimizing the number of colors used in the coloring is known to be NP-Hard.

Related Work

- Coloring heuristics use few enough colors to be useful to applications
- Two main parallelization approaches
 - Jones & Plassmann (1993) uses independent sets to color vertices in parallel
 - “Speculate & Iterate” approaches use heuristics in parallel and fix conflicts
 - Gebremedhin and Manne (2000)
 - More popular approach recently
- Shared Memory & GPU Implementations
 - Catalyurek et al. (2012) and Rokos et al. (2015) present shared-memory implementations
 - Devenci et al. (2016) and Grosset et al. (2011) present coloring implementations on the GPU
- Distributed approaches
 - Bozdag et al. (2008) adapt the “Speculate & Iterate” approach in distributed memory, subsequent works (2010) include a distance-2 implementation.
 - Sariyüce et al. (2012) presents a hybrid MPI+OpenMP implementation.

We Leverage KokkosKernels to Run on GPUs

- KokkosKernels is a package that implements various linear algebra and graph operations, using the Kokkos shared-memory parallel programming model
 - <https://github.com/kokkos/kokkos-kernels>
- The distance-1 graph coloring algorithms we use are described by Deveci et al. (2016)
- The distance-2 graph coloring algorithm in KokkosKernels uses the same Net-Based approach described by Taç et al. (2017)
- We vary the distance-1 algorithms based on how skewed the inputs are.

Distributed “Speculate and Iterate” Coloring

- Based on Gebremedhin and Manne (2000)

procedure PARALLEL-COLOR(Graph $G = (V, E)$)

 Color all local vertices

 Communicate colors of boundary vertices

do

 Detect conflicts

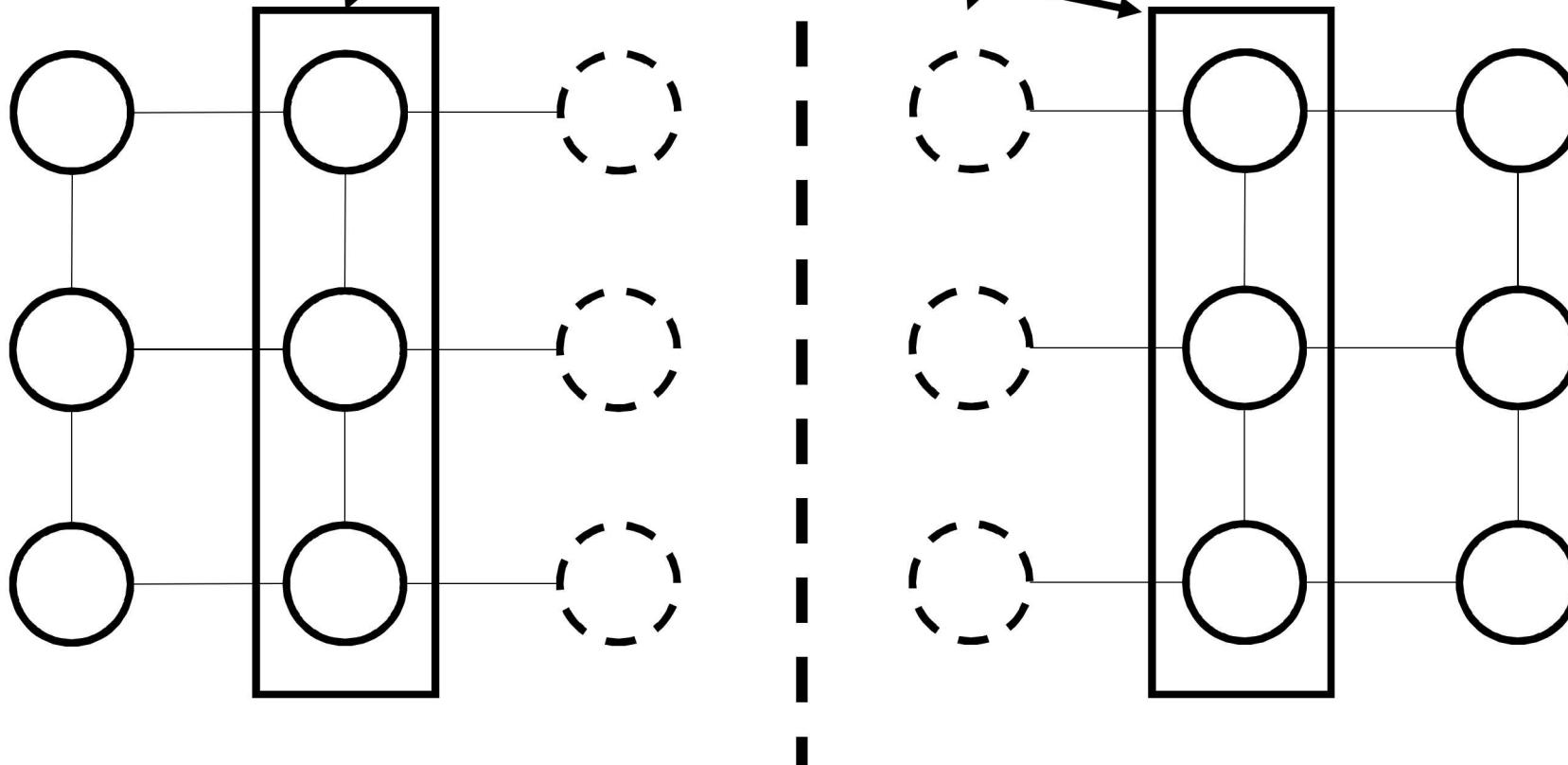
 Recolor conflicting vertices

 Communicate updated boundary colors

while Conflicts exist

Distributed Distance-1 Coloring

The vertices in the local boundary form vertices, boundary vertices to remote ghosts

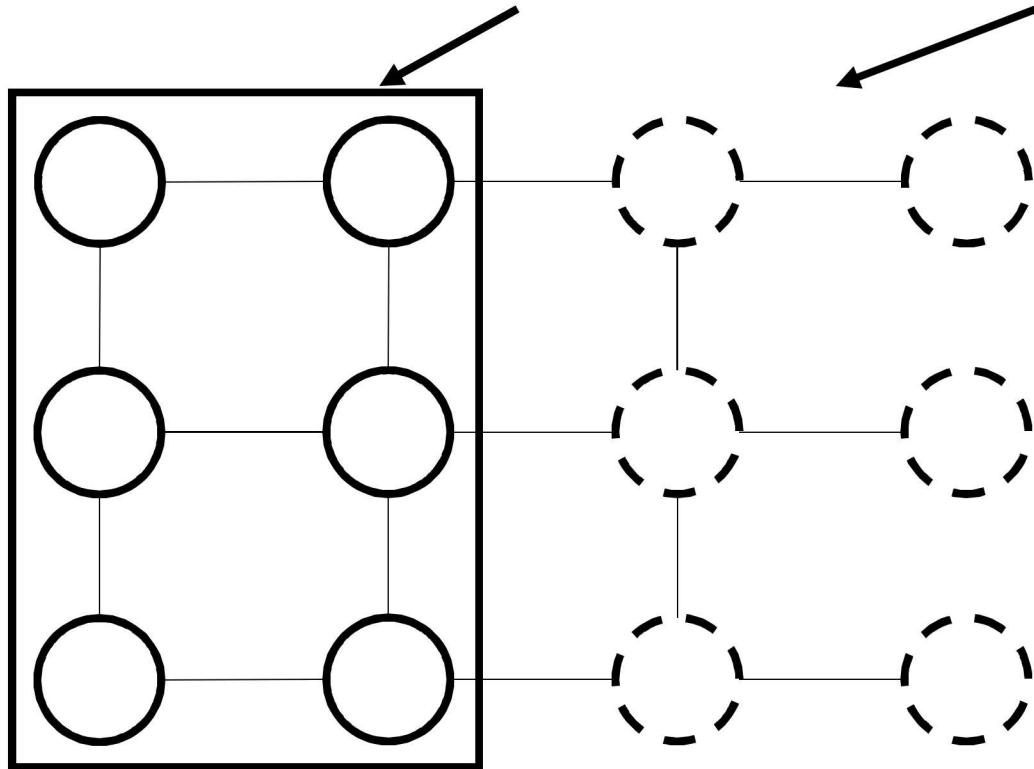


These ghost vertices are copies of the boundary vertices on other processes. We detect conflicts consistently across processes. And repeat this process until no conflicts exist.

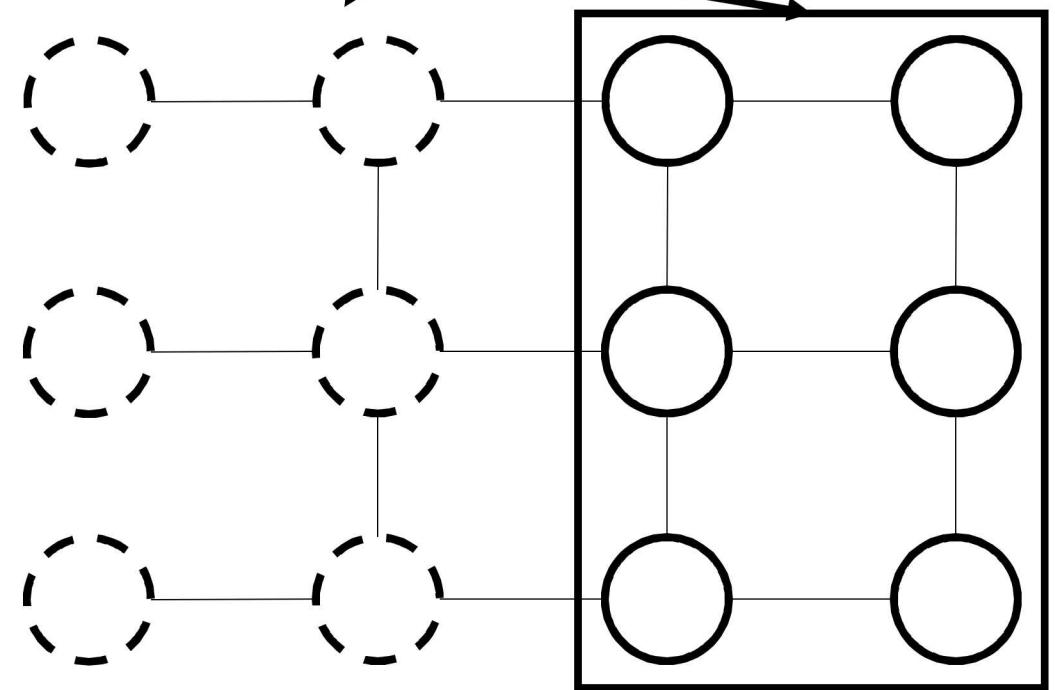
They only have edges to local boundary vertices

Distributed Distance-2 Coloring

The ghost vertices, which are the vertices with degree 2, are the vertices they are adjacent to at most edges away from a ghost vertex

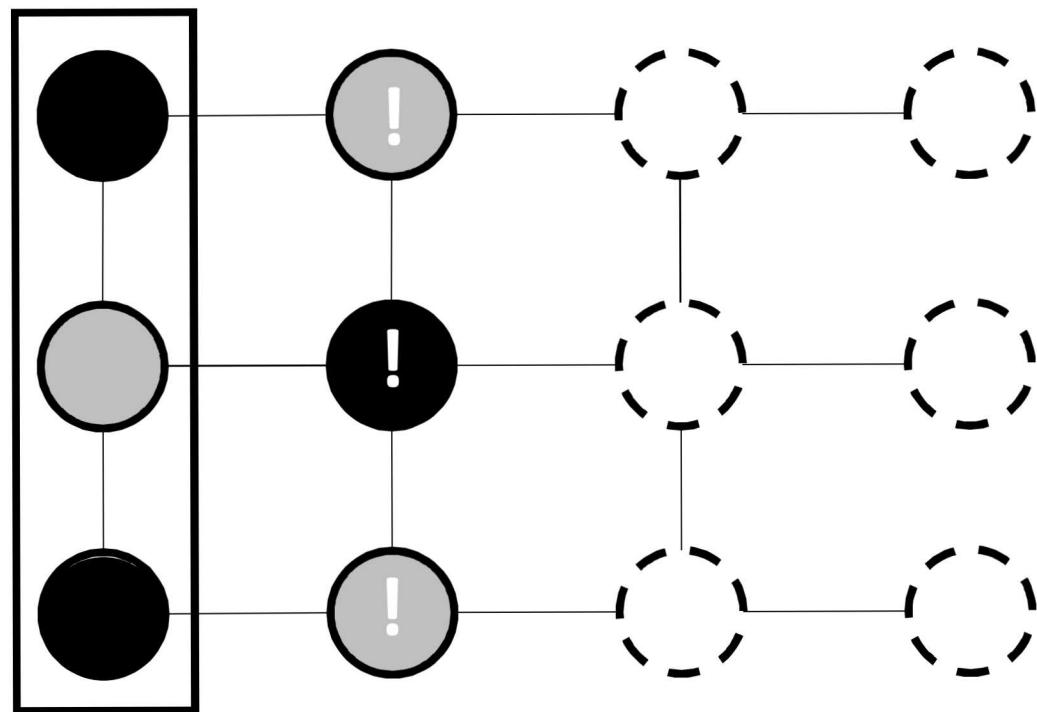


We add another layer of **ghost vertices** by copying the first ghost layer's neighbors. Second layer ghosts only have edges to the first ghost layer

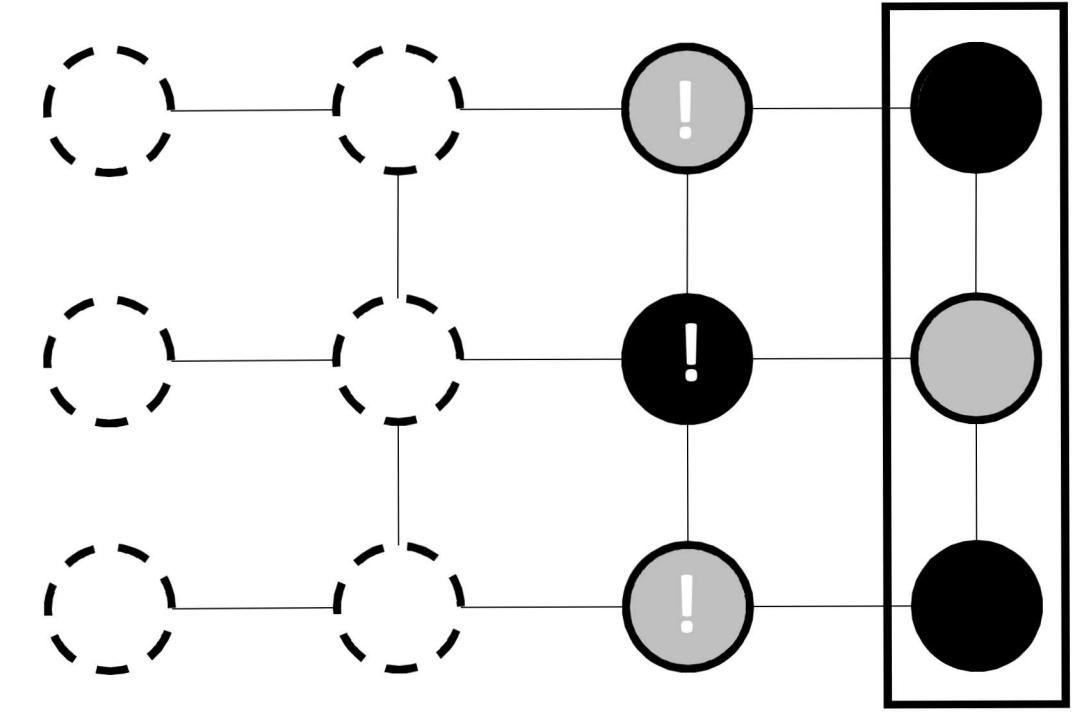


Distance-1 with Two Ghost Layers (D1-2GL)

These ghost sites will not change after the initial ghost communication



Each process can solve its conflicts independently without communicating (in this example)

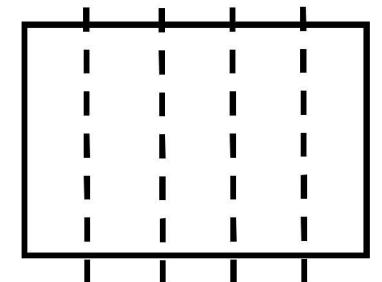


D1-2GL Aims To Reduce the Total Number of Collective Communications

- Second ghost layer vertices can get recolored, in general
- Local colorings need to make the same choices independently
- Each communication costs more
 - Ghosts can be on more than one process
 - Each ghost copy also copies its neighbors
 - Very expensive for dense inputs

Our Results Include Real and Synthetic Inputs

- Our real inputs come from a variety of domains
 - Including: PDEs, social networks, road networks
 - Range in size from 0.9 M vtx, 21 M edges to 30 M vtx, 3.3 B edges
 - Max degrees vary from 13 to 2.9 M
 - Use XtraPuLP graph partitioner developed by Slota et al. (2017)
- Our synthetic inputs are uniform hexahedral meshes
 - Vary only one dimension to keep communication costs constant
 - Partitioned in slabs
 - Range in size from 12.5 M vtx, 75 M edges to 12.8 B vtx, 76.7 B edges

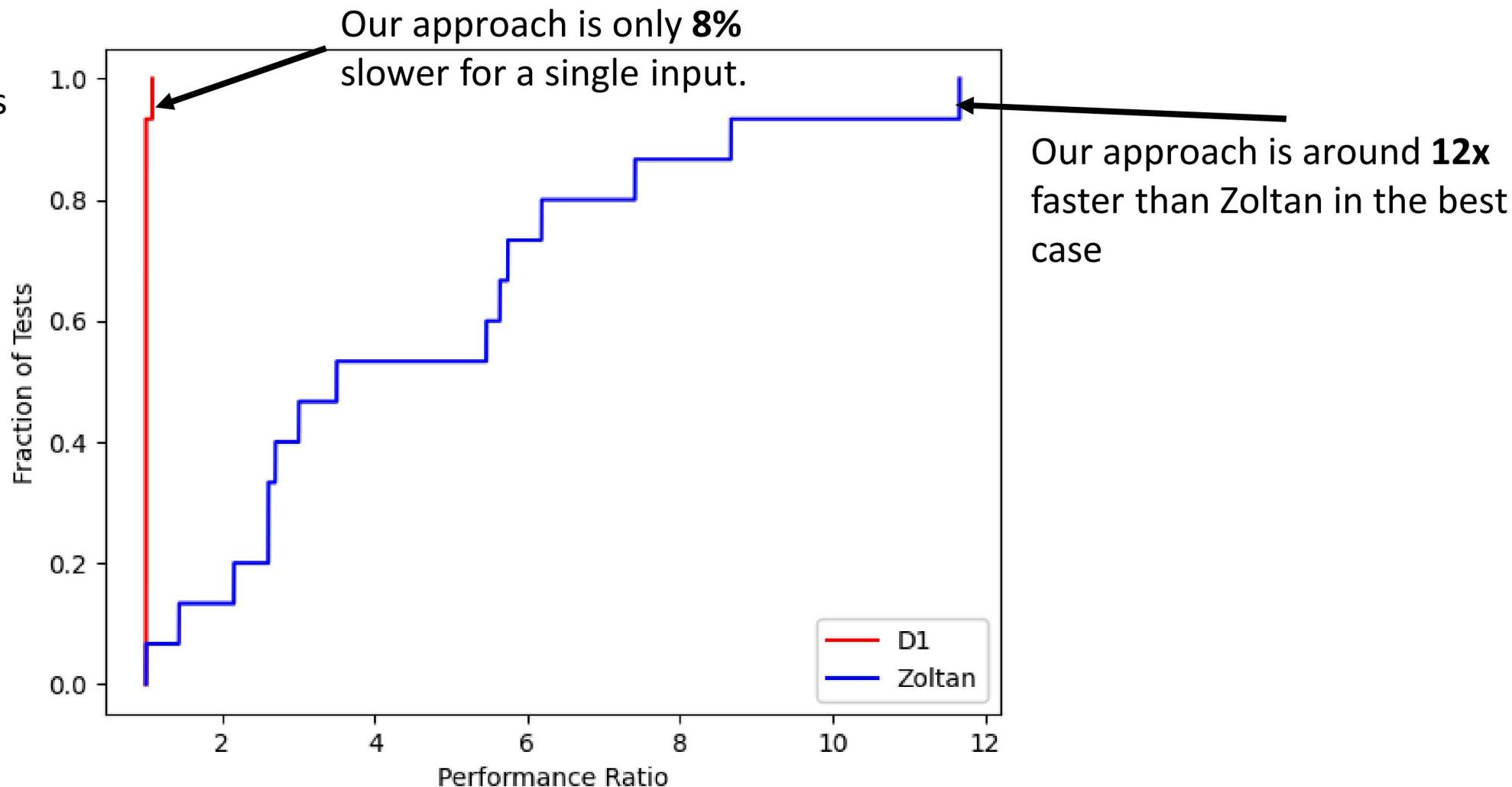


Experimental Setup

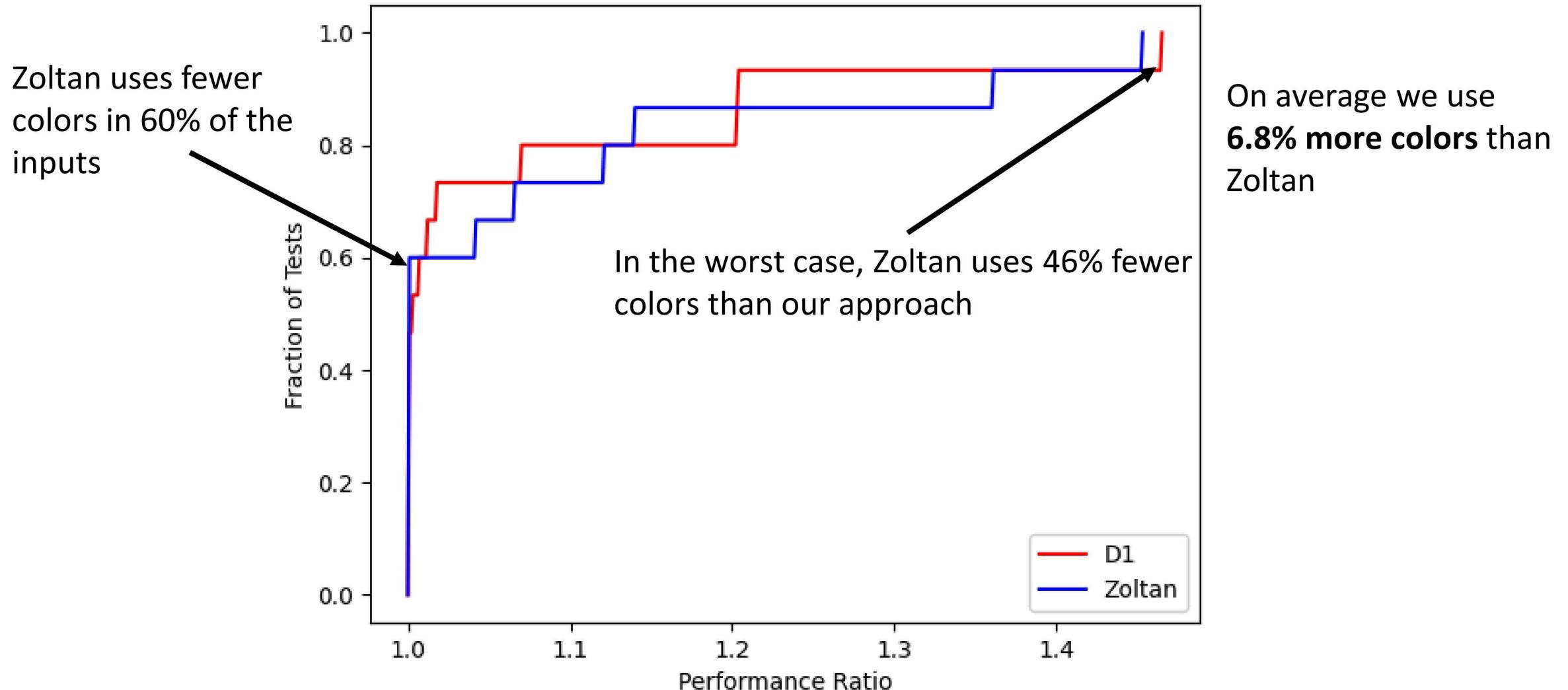
- We performed tests on AiMOS, housed at RPI.
 - 268 nodes with 2 IBM Power9 3.15GHz processors
 - 4 NVIDIA Tesla V100 GPUs with 16GB of memory connected with NVLink
 - Infiniband interconnect
 - Compiled with xIC 16.1.1, and Spectrum MPI with GPU-Direct disabled
- We run all experiments with 4 ranks per node
 - On GPU runs each rank gets an exclusive GPU
 - For performance profiles we use 128 ranks, or the largest run for which all approaches completed a run
- Zoltan is a package of the Trilinos Library containing distributed combinatorial algorithms, has MPI-only distance-1 and distance-2 implementations
 - <http://cs.sandia.gov/zoltan>
 - Run with 4 ranks per node, no shared-memory parallelism

Distance-1 Runtime Performance Profile

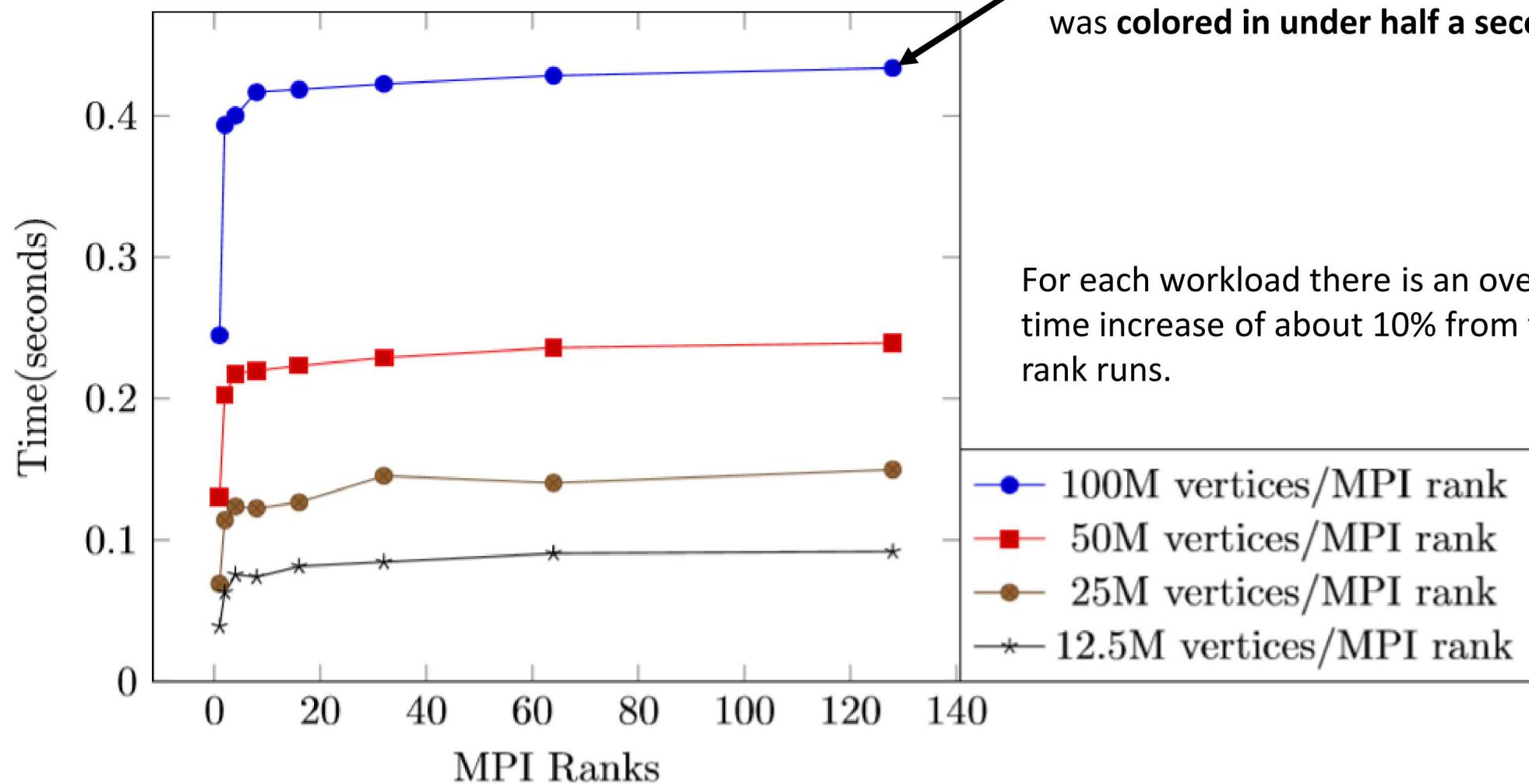
Performance ratio
can be thought of
as how many times
slower one
approach is than
the other



Distance-1 Color Usage Performance Profile

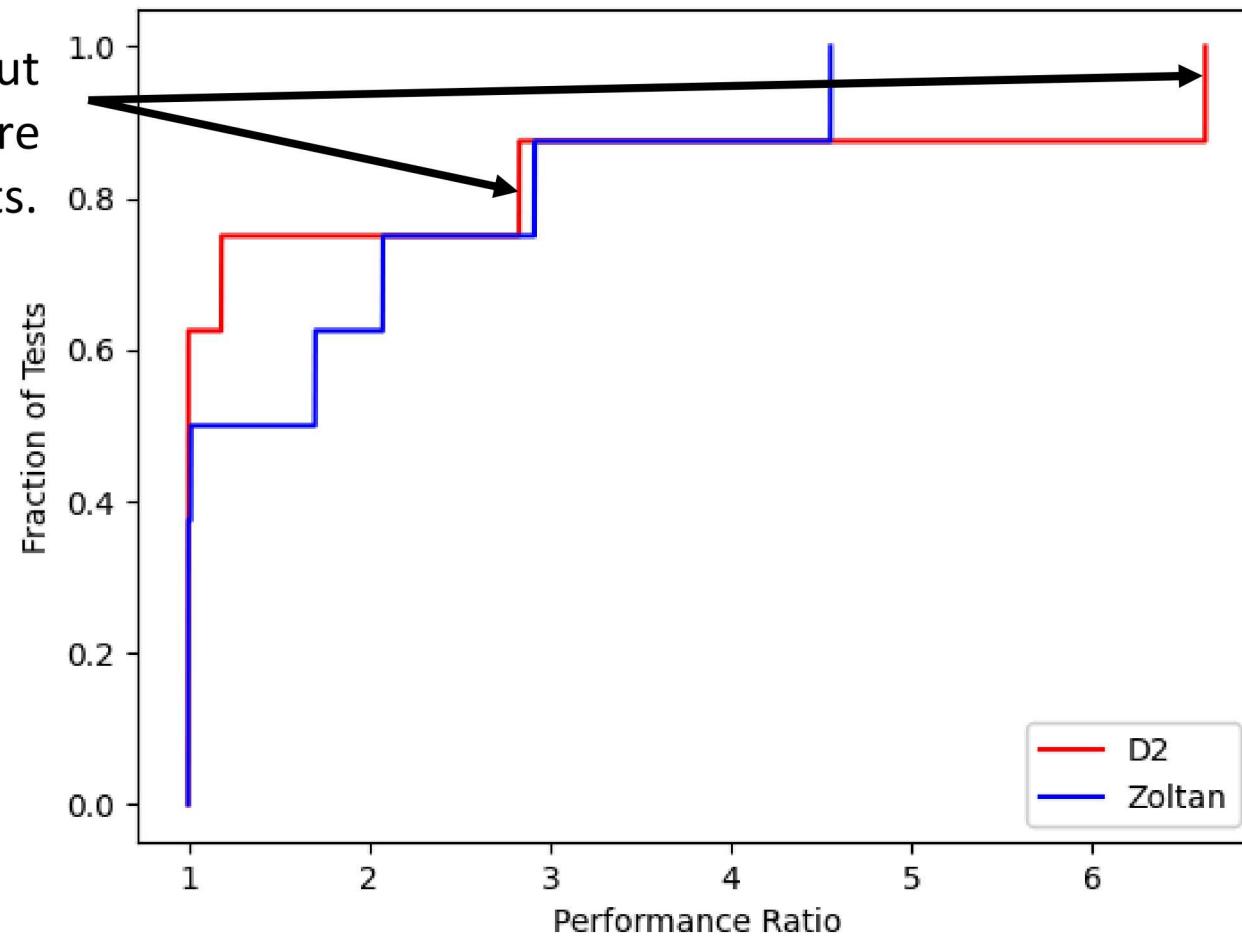


Distance-1 Weak Scaling



Distance-2 Runtime Performance Profile

We are competitive with Zoltan for all but two inputs, which are highly skewed inputs.

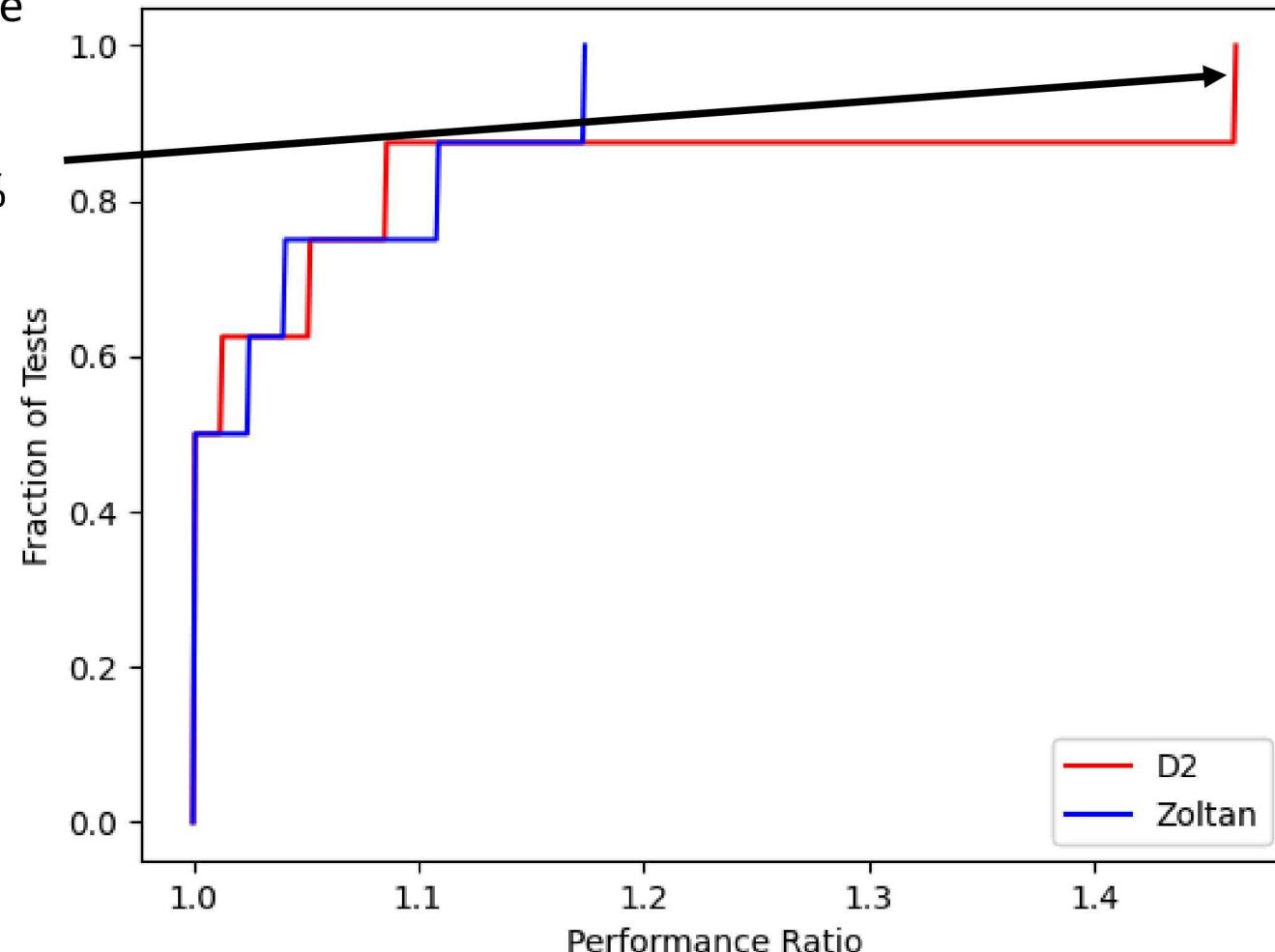


We use a smaller set of inputs for these profiles

Our approach sees at most a **4.5x speedup** relative to Zoltan

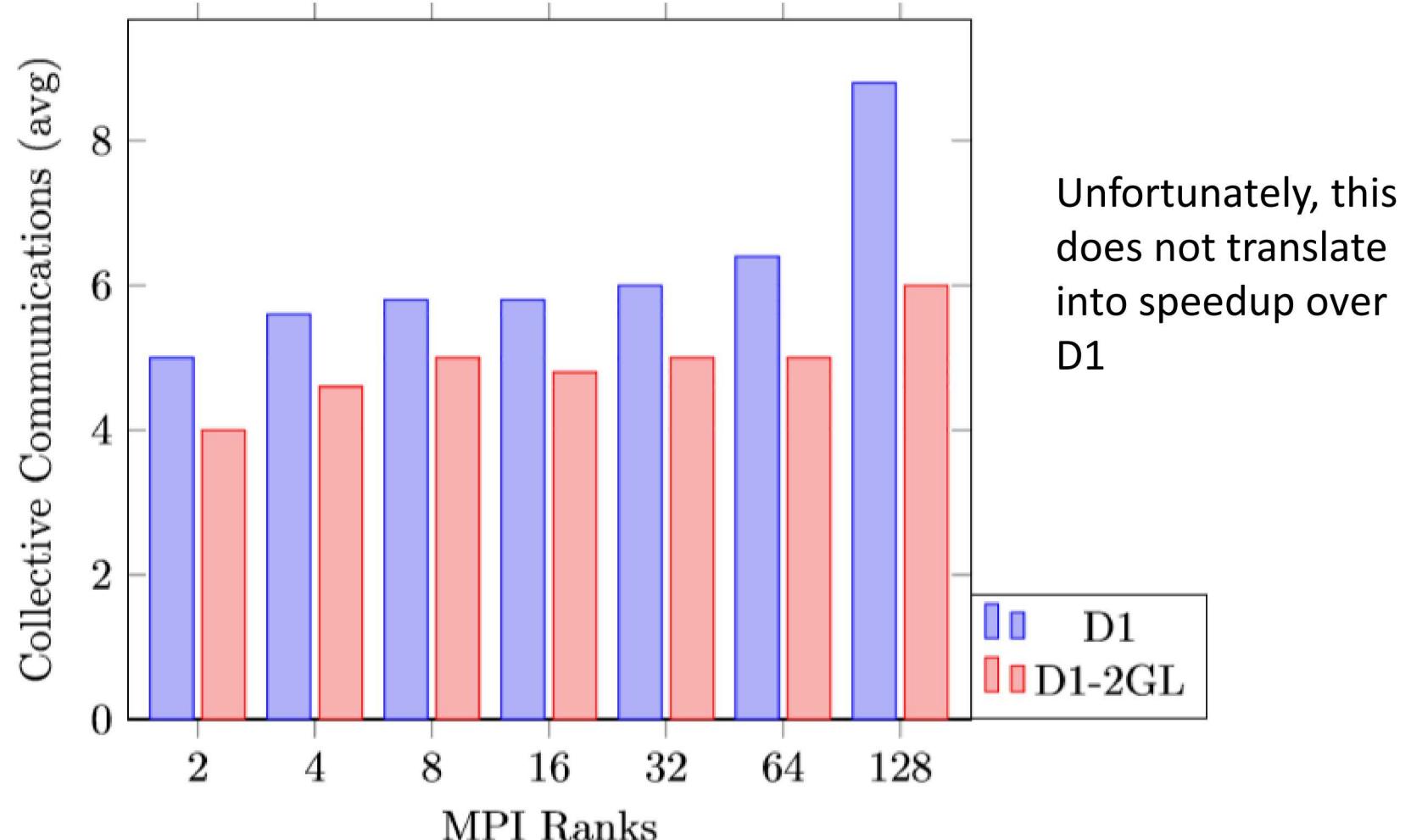
Distance-2 Color Usage Performance Profile

We are competitive with Zoltan on all but one input, where we use 46% more colors than Zoltan.



Our approach uses at most 10% more colors than Zoltan in all but one case

Two Ghost Layers Reduce the Number of Collective Communications



Future Work

- Optimizing distance-2 communication
- Extending our approach to solve different coloring problems
- Exploring optimizations to D1-2GL communication

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