BNL-222260-2021-TECH
Booster Technical Note No. 137 (Revised)

Damping the transverse resistive wall instability in the AGS Booster

E. Raka, E. Raka

August 1989

Collider Accelerator Department
Brookhaven National Laboratory

U.S. Department of Energy
USDOE Office of Science (SC), Nuclear Physics (NP) (SC-26)

DISCLAIMER

DAMPING THE TRANSVERSE RESISTIVE WALL INSTABILITY IN THE AGS BOOSTER

AD
BOOSTER TECHNICAL NOTE
NO. 137 (Revised)

E. RAKA

AUGUST 25, 1989

ACCELERATOR DEVELOPMENT DEPARTMENT
BROOKHAVEN NATIONAL LABORATORY
UPTON, NEW YORK 11973

DAMPING THE TRANSVERSE RESISTIVE WALL INSTABILITY IN THE AGS BOOSTER

E. RAKA

I. Introduction

When accelerating protons it is expected that due to the large peak circulating currents (> 3 amp), the resistive wall impedance of the vacuum chamber will cause the beam to become unstable against coherent coupled bunch oscillations in both the horizontal and vertical planes. Theoretical calculations[1] estimate the growth rate of the lowest order coupled bunch mode to be $\approx 300\ \text{sec}^{-1}$ assuming that the beam is well above the stability threshold. However, if we scale from observations of this instability in the AGS, whose vacuum chamber is made of the same type of stainless steel and has essentially the same cross-section, then the growth rate at 1.5 GeV kinetic energy and 1.5×10^{13} protons would be $1500\ \text{sec}^{-1}$ for the vertical plane.

In order to control this instability transverse feedback damping systems for both planes will be required. A conceptual design of such a system employing digital signal processing and bunch to bunch correction signals will be presented. In addition, the scaling of growth rates between the AGS and its Booster will be discussed. Finally, formulas for calculating the effective damping rate of a digital feedback system will be derived. These are applied to the cases of zero chromaticity ($\chi = 0$) for $m = 0$ and non-zero chromaticity ($\chi = \pi$) for $m = 0, 1$, and for the two unstable coupled bunch modes $(-5+Q)$ and $(-6+Q)$.

II. Growth Rate Scaling (AGS, Booster)

We use the following expression due to Sacherer[2] for the growth rate due to the resistive wall impedance.

$$\Delta\omega_m = \frac{j}{(1+m)} \frac{e\beta I}{2Q\omega_o \gamma m_o 2\pi R} \left[\sqrt{\frac{\pi}{MB}}\, Z_\perp(\omega_o)\, F_m(\chi) + Z_\perp(\omega_p)\, F_m'(\chi - \omega_p \tau_\ell) \right] \quad (1)$$

Here I = total current $= Nef_o$; ω_o the rotation (angular) frequency; N the number of protons; M the number of bunches; $B = lM/2\pi R$ with l the bunch length and R the machine radius; $\chi = \dfrac{\xi Q \omega_o \tau_\ell}{\eta}$ with τ_ℓ the bunch length and $\xi = \Delta Q/Q/\Delta p/p$ the chromaticity. $Z_\perp \propto 1/\sqrt{\omega}$, is the resistive wall impedance in ohm/meter and $F_m'(x)$, $F_m(x)$ are form factors with

$$F_m' = \frac{1}{B} \frac{h_m(\omega_\xi)}{\sum_p h_m(\omega)} \quad (2)$$

where $\omega_\xi = \chi/\tau_\ell$ and

$$F_m(\chi) = \frac{1}{B} \frac{\int_{-\infty}^{\infty} Z_\perp(\omega) h_m(\omega - \omega_\xi)\, d\omega}{\int_{-\infty}^{\infty} h_m(\omega)\, d\omega} \quad (3)$$

Now $h_m(\omega) = |\tilde{p}_m(\omega)|^2$ where $\tilde{p}_m(\omega)$ is the Fourier transform of $p_m(t)$ the oscillating part of the charge distribution. It is what one would observe by viewing the output of a position sensitive detector (from which any orbit offset has been removed) for a given bunch on an oscilloscope. The signal is of the form

$$\Delta y \propto p_m(t) e^{j(\omega_\xi t + 2\pi k Q)} \tag{4}$$

for the kth revolution. Sacherer assumes that the $p_m(t)$ are approximately sines or cosines where the $(m+1)$ refers to the number of half wavelength along the bunch or m refers to the number of nodes along the bunch. Then one can write

$$p_m = \begin{cases} \cos (m+1)\, \pi\, t/\tau_\ell & m = 0, 2, 4 \ldots \\ \sin (m+1)\, \pi t/\tau_\ell & m = 1, 3, 5 \ldots \end{cases} \tag{5}$$

and

$$h_m = (m+1)^2 \; \frac{\tau_\ell^2}{\pi^2} \; \frac{2\,[1 \pm \cos \pi y]}{[y^2 - (m+1)^2]^2} \tag{6}$$

with the plus sign for m even and the minus for m odd and $y = \omega \tau_\ell / \pi$.

We assume that the resistive wall impedance is the sole source of any instability and rewrite equation (1) for the Booster as

$$\Delta\omega_m = \frac{j e^2 \beta N_B f_o}{(m+1) 4\pi Q_B f_o 2\pi R_B \gamma m_o} [\qquad]_B$$

Now we also assume that $N_B/R_B = N_{AGS}/R_{AGS}$ and that $\chi = 0$ so that $F_m(0) = 0$. Then the $m = 0$ mode has by far the largest growth rate and we can write

$$\Delta\omega_o(\mathrm{AGS}) \approx \frac{[Z_\perp(\omega_p) F'(\omega_p \tau_\ell)]_{AGS}}{Q_{AGS}}$$

$$\Delta\omega_o(\mathrm{Booster}) \approx \frac{[Z_\perp(\omega_p) F'(\omega_p \tau_\ell)]_B}{Q_B}$$

For the resistive wall impedance the coupled bunch mode giving rise to the lowest value of ω_p will have the largest growth rate. Sacherer defines ω_p as $\omega_p = (p+Q)\,\omega_o$, $-\infty < p < \infty$ for a single bunch or independent bunch motion. For M bunches there are M coupled bunch modes so that only every M^{th} line occurs with $p = n + kM$, $-\infty < k < \infty$. If $p < -Q$ then ω_p is negative and $Z_\perp$ is also negative, i.e., $Z_\perp \approx RK\sqrt{|\omega_p|/\omega_p}$ where R is the machine radius and K depends upon the vacuum chamber composition and dimensions. It is the negative frequencies in the coupled bunch spectrum that produce negative contributions to $\Delta\omega_m$ and hence growth while the positive frequencies contribute damping. In general, there are M/2 unstable coupled bunch modes so that the lowest frequency line is for the $n = 1$ mode i.e., $p = -5$ in the Booster and the $n = 3$ mode or $P = -9$ for the AGS. Here $2\pi n/M$ is the phase shift between bunches of the coupled bunch motion.

Thus, for $Q_B = 4.8$ and $Q_{AGS} = 8.8$ we have $\omega_p = .2\omega_o$ for both machines and we can write

$$\frac{\Delta\omega_o(\text{Booster})}{\Delta\omega_o(\text{AGS})} \alpha \frac{F_B' R_B}{Q_B\sqrt{.2f_B}} \frac{Q_{AGS}\sqrt{.2f_{AGS}}}{F_{AGS}' R_{AGS}} = \frac{8.8}{9.6} \frac{F_B'}{F_{AGS}'} \frac{1}{4}$$

where we have used $R_B = R_{AGS}/4$ and $f_B = 4f_{AGS}$. Now assuming the same τ_ℓ in both machines we note that $\omega_p \tau_\ell$ in F will be four times larger for the Booster than the AGS so that $F_B' < F_{AGS}' \cong 0.8$ for the lowest frequency line $.2\omega_o$. Also the first pair of lines of the spectrum, at $f_{rf} \pm (9 - Q) f_o$ in the AGS and at $f_{rf} \pm (5 - Q) f_o$ in the Booster, produce a greater net reduction in the growth rate for the Booster than for the AGS. This is because the lower sideband corresponds to a positive frequency while the upper line adds to the growth rate. We note here that for the $n = 2$ mode the first negative frequency line in the Booster spectrum would be $(-7 + 4.8) f_o = -2.2 f_o$ while the first positive frequency would be $0.8 f_o$ so that this mode is the only stable mode of the three ($n = 0, 1, 2$).

Returning to the $n = 1$ mode we conclude that for the same resistive wall impedance per unit length the maximum growth rate in the Booster would be $< 1/4$ that of the AGS for the same line change density N/R. Or for the same number of particles in both machines $\Delta\omega_o(\text{Booster}) < \Delta\omega_o(\text{AGS})$ at the same energy. Now in the AGS the measured growth rate of the $n = 3$ mode on a 1.5 GeV kinetic energy flat top is 900 sec^{-1} at 9×10^{12} protons in the vertical plane where $\xi \cong 0$ so that $\chi \approx 0$ also. We remark that the growth rate also scales as $(\beta/\gamma) \times 1/\sqrt{\beta}$ for fixed Q so that if the beam were unstable at 200 MeV the growth rate would be 1.68 times greater. This, of course, assumes the same F_0' and zero χ hence, the Booster growth rate at 1.5 GeV and 1.5×10^{13} proton should be $< 1500\ \text{sec}^{-1}$ at zero χ and the same τ_ℓ. It will be shown that the type of kicker proposed for the feedback system produces a $(\Delta p/p)_\perp$ that is proportional to $(1 + \beta)/\beta^2\gamma$ while the overall damping rate is $\sim \beta\ (\Delta p/p)_\perp$. This results in an increase of about 2.5 at 200 MeV over the 1.5 GeV damping rate for the same position error. Thus, the growth rate at 1.5 GeV should be used to determine the required damping rate.

III. Description of the Damping system

The position error of each bunch will be processed in such a manner that the corresponding correction signal will be applied to the same bunch. It is not feasible to employ narrow band analogue feedback as presently used in the AGS[3] to the Booster (this will be discussed in the appendix). We assume that the pickup electrode signals from the vertical pair at QD-8 and a pair halfway between QE-2 and QE-3 and from a horizontal pair at the same position and the pair at QE-5 will be available for separate processing. Since they are 90^o apart at the nominal tune of 4.8 one can obtain any phase of the bunch oscillation by a linear combination of the measured displacements. For tunes between 4.5625 and 4.95 the phase difference from $\pi/2$ is $< \pm 4^o$.

The combined correction signal from each bunch will be delayed 4 revolution periods (T_o) before being applied to a 50Ω travelling wave deflector (50 Ω strip line kicker) located at the upstream end of SS E-3. In order to obtain damping the phase of

the correction must be in quadruture with the phase of the bunch oscillation as it passes the kicker. For a fixed tune the phase of the correction signal also remains a constant. However, due to the long delay between measurement and kick (4 turns!) small changes in tune call for large changes in the correction signal phase. Hence, the need for being able to generate a wide range of phase variation for the feedback signal.

Now the total loop delay should be a multiple of the rotation period, if the pickup and kicker are at the same location, in order that the signal derived from a given bunch is applied to the same bunch. When this criterion is satisfied then, assuming the quadruture phase relation is also satisfied, the system can damp in principal all the unstable modes of a multi-bunch ring without exciting the stable modes. Since the rotation period changes with energy it is necessary to vary the time delay between pickup and kicker. In a pure analogue feedback system this is done by switching cable lengths in the signal loop. In a digital system the digitized correction signal is stored in a memory whose clock is related to the rotation frequency. The latter type of system, i.e., digital processing of the pickup (or error) signal, digital delay, and D/A conversion prior to the 50Ω wideband power amplifiers that drive the deflectors is being developed for the AGS. Here only a one turn delay is needed since the rotation period is always $\geq 2.7\mu$sec which is sufficient to perform the digital processing and transport the pickup signals to a remote location and return it to the kicker on the ring (F-20). In order to minimize the amount of development time needed to produce a suitable damping system for the Booster it has been suggested to consider using a modification of the AGS design. This is why a 3 or 4 turn delay is required for the feedback signal, i.e., in order to do the digital processing. Since there is space enough in the E-3 straight section for a one meter long kicker the design used in the AGS system can be copied. Both horizontal and vertical units will be contained in the same chamber as shown in Figure 1. They will be driven by four 100W power amplifiers that must have a linear phase response (i.e. constant time delay) between 100 KC and 16 MHz.

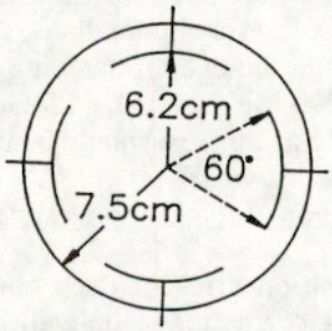

Figure 1

The individual bunch difference signals will be integrated on a turn by turn basis, digitized, normalized, combined and stored in a serial memory. Thus, any within the bunch amplitude variation due to non-zero χ or higher order modes m ≥ 1 will not be detected. Only net dipole motion of the entire bunch will be sensed. How this affects the damping rate for $\chi \neq 0$ and for the m = 0, 1 modes will be described below. We note here that the phase of the correction signal could be determined by sensing the quadrupole currents of the machine and computing the required combination of the two position signals on a real time basis.

Damping Rate Calculations

For ideal damping one has $a = a_0 e^{-\epsilon f_0 t/2}$ where

$$\epsilon = \sqrt{\frac{\beta_k}{\beta_p}} \quad \frac{(\Delta p/p)_\perp}{\Delta y/\beta_p} \tag{7}$$

is a measure of the open loop (linear) gain of the feedback system. Here $f_0 = \beta f_\infty$ is the rotation frequency and β_k, β_p are the beta functions at the kicker and pickup. In the vertical plane $\beta_k \cong 11$m and we take $\beta_p = 13.5$m the value at the D-8 pickup so that $\epsilon = 12.2\,(\Delta p/p)_\perp / \Delta y(\text{m})$. At 1.5 GeV $f_0 = 1.367$MHz hence we should have

$$\epsilon > \frac{1.5\text{x}10^3\text{x}2}{1.367\text{x}10^6} = 2.2\text{x}10^{-3}$$

in order to obtain a damping rate $\epsilon f_0/2$, greater than the maximum expected growth rate. For the $\Delta p_\perp$ produced by a pair of kicker plates we can write

$$\Delta p_\perp(\omega) = \frac{(1+\beta)}{\beta} \; e\sqrt{Z_o \hat{P}} \; \frac{\ell k}{c} \; \frac{\sin\theta}{\theta} \tag{8}$$

where $Z_o = 377\Omega$, $\hat{P}$ is the peak power at a frequency ω delivered to the 50 impedance of the plate(s), ℓ their length, $\theta = \omega\ell/c$ and k is a geometrical factor that includes the effect of image currents in the vacuum chamber.[4].

$$k = \frac{1}{\pi b}\sqrt{\frac{Z_o}{Z_c}} \; \frac{(1-b^2)}{a^2} \; \frac{\sin\phi}{\phi} \tag{9}$$

Here $Z_c = 50\Omega$, a is the outer radius of the kicker chamber b the radius of the deflection plates and ϕ their azimathal extent. We shall assume that $k = 4\text{m}^{-1}$ the design goal of the AGS deflector. Then for $\omega = 0.25\omega_0$, $(\sin\theta/\theta) \approx 1$ and we obtain at 1.5 GeV with $\ell = 1$m.

$$\left(\frac{\Delta p}{p}\right)_\perp = \frac{1.923e\sqrt{377\text{x}200}}{.923\text{x}2.4m_o c} \quad \frac{1\text{x}4}{c} = 1.02\text{x}10^{-6}$$

so that a

$$\Delta y = \frac{1.02\text{x}10^{-6} \text{ x } 12.2}{2.2 \text{ x } 10^{-3}} = 5.65 \text{ mm}$$

should produce full power out of the amplifier ($\hat{P} = 2$ P the average power).

Now we can write $\Delta p_\perp$ in the following form

$$\Delta p_\perp = \frac{e}{\beta c} \int_0^{2\pi R} (E + v \text{ x } B)_\perp ds \tag{10}$$

where E and B are the deflection fields of the kicker. From Sacherer we have the definition

$$Z_\perp = \frac{j \int_o^{2\pi R} [E + v \times B]_\perp ds}{\beta I \Delta y} \qquad (11)$$

where E and B are the fields due to the wall currents induced by the displacement of the current I by an amount Δy. Hence, for the deflector we can write

$$Z_\perp = \frac{\beta c}{e} \quad \frac{\Delta p_\perp(\omega)}{\beta I \Delta y(\omega)} \qquad (12)$$

where the j drops out since we assume a 90^o phase shift between the displacement Δy and the kick $\Delta p_\perp$. Now one can also express $\Delta\omega_m$ as

$$\Delta\omega_m = \frac{j}{(m+1)} \quad \frac{e\ I\beta}{\gamma m_o 2Q\omega_o 2\pi R} \quad \frac{\sum_p Z_\perp\ (\omega_p)\ h_m\ (\omega_p - \omega_\xi)}{B \sum_p h_m\ (\omega_p - \omega_\xi)} \qquad (13)$$

assuming $\chi = 0$ so that $\omega_\xi = 0$ and inserting our expression for $Z_\perp$ we obtain

$$\Delta\omega_m = \frac{j}{(m+1)} \quad \frac{e\beta^2 cI \quad \beta c}{\beta\gamma m_o c 2Q\omega_o 2\pi R} \quad \sum_p \frac{\Delta p_\perp(\omega_p) h_m(\omega_p)}{e\beta I \Delta y(\omega_p) B \sum_p h_m(\omega_p)}$$

or

$$\Delta\omega_m = \frac{j}{(m+1)} \quad \frac{\beta c}{2\pi R} \quad \frac{\sum_p \Delta p_\perp h_m}{\frac{2p\Delta y}{\beta}} \quad \frac{1}{B \sum_p h_m}$$

Finally then we can write

$$\Delta\omega_m = \frac{j}{(1+m)} \quad \frac{f_o}{2} \quad \sum_p \epsilon(\omega_p)\ F_m'\ (\omega_p) \qquad (14)$$

Hence, if we know the transfer function between $\Delta p_\perp(\omega)$ and $\Delta y(\omega)$ we can calculate the net damping rate by summing the terms $\epsilon(\omega_p)\ F_m(\omega_p)$. In the case of pure analog feedback and for $m = 0$ one can generally have $\epsilon(\omega)$ a constant for all ω_p up to where $F_o(\omega_p) \to 0$. Then we obtain $\Delta\omega_o = jf_o\epsilon/2B$ for the damping rate.

Recalling equations 4, 5 and putting $m = 0$ we can write

$$\Delta y_o \sim \mathrm{Re}\ [e^{j\omega^+ t + j\phi} + e^{-j\omega^- t + j\phi}] = 1/2\ [\cos\ (\omega^+ t + \phi) + \cos\ (\omega^- t - \phi)] \qquad (15)$$

where $\omega^+ = \omega + \omega_\xi$; $\omega^- = \omega - \omega_\xi$; $\omega = \pi/\tau_\ell$; $\phi = 2\pi kQ$. This becomes then

$$\Delta y_o \sim 1/2\ [(\cos\omega^+ t + \cos\omega^- t)\cos\phi - (\sin\omega^+ t - \sin\omega^- t)\sin\phi] \qquad (15a)$$

which for $\chi = 0$ gives

$$\Delta y_o \sim \cos\ \frac{\pi t}{\tau_\ell}\quad \cos\ \phi$$

For $m = 1$ we obtain

$$\Delta y_1 \sim 1/2\,[(\sin\omega^+ t + \sin\omega^- t)\cos\phi + (\cos\omega^+ t - \cos\omega^- t)\sin\phi] \qquad (16)$$

and for $\chi = 0$, with $\omega = 2\pi/\tau_\ell$, $\Delta y_1 \sim (\sin 2\pi t/\tau_\ell)\cos\phi$.

Now we have assumed that the difference signal $\Delta y(t)$ is integrated on a bunch by bunch turn by turn basis. Hence, for $m = 0$, $\chi = 0$ we obtain

$$\delta y_o \sim \int_{\frac{-\tau_\ell}{2}}^{\frac{\tau_\ell}{2}} \cos\frac{\pi t}{\tau_\ell}\, dt\, \cos\phi$$

Thus, we have $\delta y_o \sim (2\tau_\ell/\pi\tau_{rf})\cos\phi$. From now on we shall assume $\tau_\ell = 1/2\, f_{rf}$ or $\phi_\ell = \pi$ which is approximately true in the Booster at 1.5 GeV so that $\delta y_o \sim \cos\phi/\pi$. Next we assume that the voltage that is applied to the deflectors is a series of pulses of duration τ_{rf} whose amplitude is $\sim \delta y_o$ as shown in Figure 2.

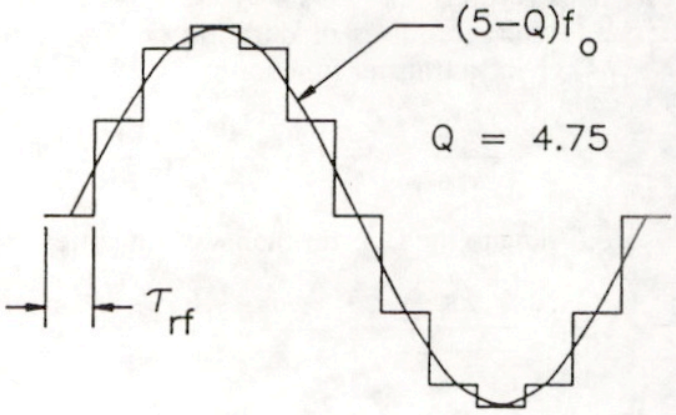

Figure 2

We must now find the transfer function for this process.

If we assume that the coherent mode $(-5+Q)$ is present in a <u>continuous</u> beam then one would see a signal at $(5\text{-}Q)f_o$ when measuring $\Delta y\,(t)$ at a position sensitive pickup. If we were to sample that signal at f_{rf} and locked to the bunch center in phase then one would obtain a similar $\delta y_o(t)$.

Hence one can consider that the bunches constitute a sampling of the coherent signal $(5\text{-}Q)f_o$ and that the sampling function is $p_o(t)$. In the feedback loop δy_o is digitized and stored in memory for $3T_o$ or $4T_o$ before being retrieved (see later about a correction to this) and converted into a voltage pulse of duration τ_{rf}. For the case $m = 0$, $\chi = 0$ the δy_o signal is equivalent to sampling the signal with a δ function since the integral is always proportional to the peak amplitude of $\Delta y(t)$. Thus, the output pulse can be thought of as the "impulse" response of a "sampled data system", containing a zero order data hold, that is used to reconstruct the signal $(5\text{-}Q)f_o t$. It can be written as

$$\frac{1-e^{-s\tau_{rf}}}{s} = \frac{\tau_{rf}}{(\omega t_{rf}/2)} \sin\left(\frac{\omega t_{rf}}{2}\right) / \frac{(-\omega t_{rf}/2)}{} \qquad (17)$$

for $s=j\omega$. By definition this is the transfer function from $\Delta y(\omega)$ to $\Delta p(\omega)$. Actually, the full transfer function would be

$$\frac{\cos\phi}{\pi} \quad \frac{(1-e)^{-s\tau_{rf}}}{s} \quad e^{-s\ell T_o} \qquad (17a)$$

where ℓT_o is the overall loop delay and it is assumed that the output voltage level is changed at the center of the bunch. If now we reduce the delay by $\tau_{rf}/2$ (as shown in Figure 2) then one should multiply (17a) by $e^{s\tau_{rf}/2}$.

Finally, then, for $s = j\omega$ we obtain

$$\frac{\cos\phi}{\pi} \quad \frac{\tau_{rf}}{} \quad \frac{\sin(\omega\tau_{rf}/2)}{\omega\tau_{rf}/2} \quad e^{-j\omega\ell T_o} \qquad (18)$$

i.e., a sin x/x response ($x=\omega\tau_\ell=\omega\tau_{rf}/2$). We note here that in the AGS damping system we assume voltage pulses of duration ($\tau_{rf}/2$) so that the actual time delay should be ($T_o-\tau_{rf}/4$) giving a transfer function

$$\frac{\cos\phi}{\pi} \quad \frac{\tau_{rf}}{2} \quad \frac{\sin(x/2)}{(x/2)} \quad e^{-j\omega T_o} \qquad (18a)$$

Returning to the Booster then, we can write for $\epsilon(\omega)$

$$\epsilon(\omega) \sim \frac{A\beta_{max}}{p} \quad \frac{\cos\phi}{\pi} \quad \frac{\tau_{rf}}{} \quad \frac{\sin x}{x} \quad \frac{\tilde{e}^{j\omega\ell T_o}}{\Delta\tilde{y}_o(\omega)\cos\phi}$$

with $\Delta\tilde{y}_o(\omega)$ being the Fourier (Laplace for $s=j\omega$) transform of $\Delta y(t)$ and A a gain factor. Since $h_o(\omega) = |p_o(\omega)|^2$ we can write the summation in equation 14 as

$$\sum_p \frac{\sin(\omega_p\tau_{rf}/2)}{(\omega_p\tau_{rf}/2)} \tilde{p}_o(\omega_p)e^{-j\omega\ell T_o}$$
$$\overline{B\sum_p h_o(\omega_p)}$$

where

$$\tilde{\Delta y}_o(\omega) = \tilde{p}_o(\omega) = \frac{2\tau\ell}{\pi} \left| \frac{\cos(\omega\tau_\ell/2)}{1-\left(\frac{\omega\tau_\ell}{\pi}\right)^2} \right| \qquad (19)$$

and it can be shown that[5]

$$\sum_p h_o(\omega_p) = \frac{2\tau_\ell^2}{\pi^2} \quad \frac{\pi^2}{4B}$$

so that $\epsilon_{eff} = \sum_p \epsilon(\omega_p)\, F_m'(\omega_p)$ can be written as

$$\frac{8A}{\tau_{rf}^2}\;\frac{\tau_{rf}^2}{\pi^2}\;\frac{\beta_{max}}{p}\;\sum_p\;\frac{\sin x}{x}\;\frac{\cos(x/2)\; e^{-j\omega_p \ell T_o}}{\left[1-\left(\frac{x}{\pi}\right)^2\right]}$$

where $x = (\omega_p \tau_{rf}/2)$ and we have assumed $\tau_\ell = \tau_{rf}/2$. We show in Figure 3 a plot of $\tilde{p}_o(x)$ and of $(\sin x/x)$ and $\sin(x/2)/(x/2)$. The summation out to 3 f_{rf} gives ≈ 1 so that we obtain

$$\epsilon_{eff} = \frac{8}{\pi^2}\;\frac{\beta_{max}}{p}\;A\,(\Delta p_\perp/\Delta\hat{y}) \qquad (20)$$

Relative to an ideal analogue system giving the same damping rate the gain A would have to be 2/.834 = 2.4 times greater. Hence, the gain should be such that a displacement of .834x5.65 = 4.7mm peak will produce full output power since our initial calculation of the damping rate and hence ϵ did not include the 1/B factor. We note here that in the summation for the AGS (the sin x/2/x/2 transfer function) we obtain a factor of 2 but since there is factor of 1/2 between equations 18 and 18a the ϵ_{eff} remains essentially the same. Now the $e^{-j\omega_p \ell T_o}$ phase factor should really be written as $\exp j(\psi - \omega_p T_o \ell)$ where ψ is the phase of the correction signal. This can be written as

$$(\psi - \ell\omega_p T_o) = (\psi - 2\pi\delta Q f_o T_o \ell) = \psi - 2\pi\,\delta Q \ell = (2n+1)\pi/2$$

where δQ = (5-Q) or (6-Q), since ψ will track any changes in tune i.e., δQ.

Next we consider the other potentially unstable coupled bunch mode (-6+Q) and evaluate the summation for m=0, $\chi = 0$. It turns out to be 0.975 for Q=4.75 which is the value used above (rather than 4.8). Hence, the damping rate would be the same but the growth rate for this mode would be only 0.12 times that of the (-5+Q) mode. Thus, the loop gain is determined by the mode (n=1).

Finally, let us consider what happens for $\chi \neq 0$ both for the m = 0 and the m = 1 modes since the latter is unstable in this case also (n=1 still). We can show that for m=0 and $\chi = \pi$, $\delta y_o \sim (\cos\phi/4)$ rather than $(\cos\phi/\pi)$ the $\chi = 0$ value, due to the integration. Also, we can show using equation 16 that $\delta y_1 \sim -2\sin\phi/3\pi$ for $\chi = \pi$, $\tau_\ell = \tau_{rf}/2$. In Figure 4 we show plots of $p_o(x)$ and $p_1(x)$ for $\chi = +\pi$ as well as $\pm \sin x/x$ from which we can readily obtain approximate values for the summation needed to find ϵ_{eff} for these two cases. We obtain 0.71 for the $\chi = \pi$, m = 0 case rather than ≈ 1 as in the $\chi = 0$ case. The m=1, $\chi = \pi$ sum is 0.67 relative to the $\chi = 0$, m=0 value of 1. Thus, if A remains the same the damping rate for the m=0, $\chi = \pi$ case becomes $(\pi/4)$ x 0.71 = 0.56 of the $\chi = 0$ rate. However, the growth rate for this mode decreases by a factor of $\approx$ 10. This result can be obtained by using either equation 1 or 13. Hence, a finite amount of negative chromaticity is desirable to control the growth rate of this mode.

Now for the m=1 mode with $\chi = \pi$ the growth rate would be $\approx 0.5/(1+1)$ or one quarter of the m=0, $\chi = 0$ value if it is unstable. On the other hand, the damping rate would be $(2\pi/3\pi)$x.67÷(1+1)=.222 of the $\chi = 0$ m=0 value if the loop parameters were

unchanged. For $\chi = \pi/2$ we find also that the loop gain is still less than the growth rate for the m = 1 case. Here the m = 0 growth rate is still 58% of the $\chi = 0$ value. We conclude that operating at small values of negative chromaticity would be helpful if the m = 1 mode is near the intensity threshold for instability. This is because the growth rate would be less than the values calculated by equation 1 which is only valid well above the intensity threshold.

Acknowledgement: The possibility of using a digital signal processing system similar to that being designed for the AGS plus a four turn delay for the Booster damper was suggested by Y. Y. Lee and W. Weng.

References

1. AGS Booster Design Manual (October 1988) pg. 2-37
2. F. Sacherer, "Transverse Bunched Beam Instabilities Theory", Proc. IX Intl. Conf. on High Energy Accelerator, Stanford 1974 pg. 374-351
3. E. Raka, "Damping Coherent Oscillations in the AGS", IEEE Trans. on Nu Sci. Vol. NS-14 #3 June 1967, pg. 1091-1095
4. J. L. Pellegrin, "Design of an Electrode System for Beam Transverse Excitation", SPEAR-19, PEP-120 August 1975
5. B. Zotter, "The Effective Coupling Impedance for Bunched Beam Instabilities", CERN/ISR - TH/78-16.

Appendix

In the AGS narrow band analogue feedback system[3] a vertical difference signal is obtained at one point on the ring and this signal after some filtering is applied to a pair of deflecting coils located downstream at an azimathal angle θ. It can be shown that for a given value of p (ignoring the filter delay) the damping rate is proportional to

$$\text{Sin}\,[|p|\theta - (|p|-Q)\omega_0\tau]$$

where τ is the time delay of the cables and electronics (assumed wideband). For a vertical pickup at D-8 and a pair of deflection coils centered between QE-2 and QE-3 we have $\theta = 18.75^0$ while the phase advance between the pickup and kicker is 90^0 at Q = 4.8. We estimate about 50 nsec cable delay and 35 nsec for the electronics. Then for |p| = 5 and Q = 4.6 we obtain Sin $(93.75 - 14)^0 = .984$ at 1.5 GeV where $f_0 = 1.336$ MHz. for |p| = 6 the result is Sin $62.2^0 = .88$ and |p| = 4 gives Sin $93.8^0 = .997$. Now the loop filter would have to transmit the lowest frequencies present in the bunch spectrum for each of these modes, i.e., (|p| - Q) f_0 or .4 f_0, .6 f_0 1.4 f_0, and reject all the other sidebands and 3 f_0, 6 f_0 etc. In order to do this a tuneable filter that had a very sharp cutoff at $f_{rf}/2$ would be required. This would entail a multistage design that over the passband would introduce large additional contributions to the time delay τ. Hence, such a system is not feasible in the Booster.

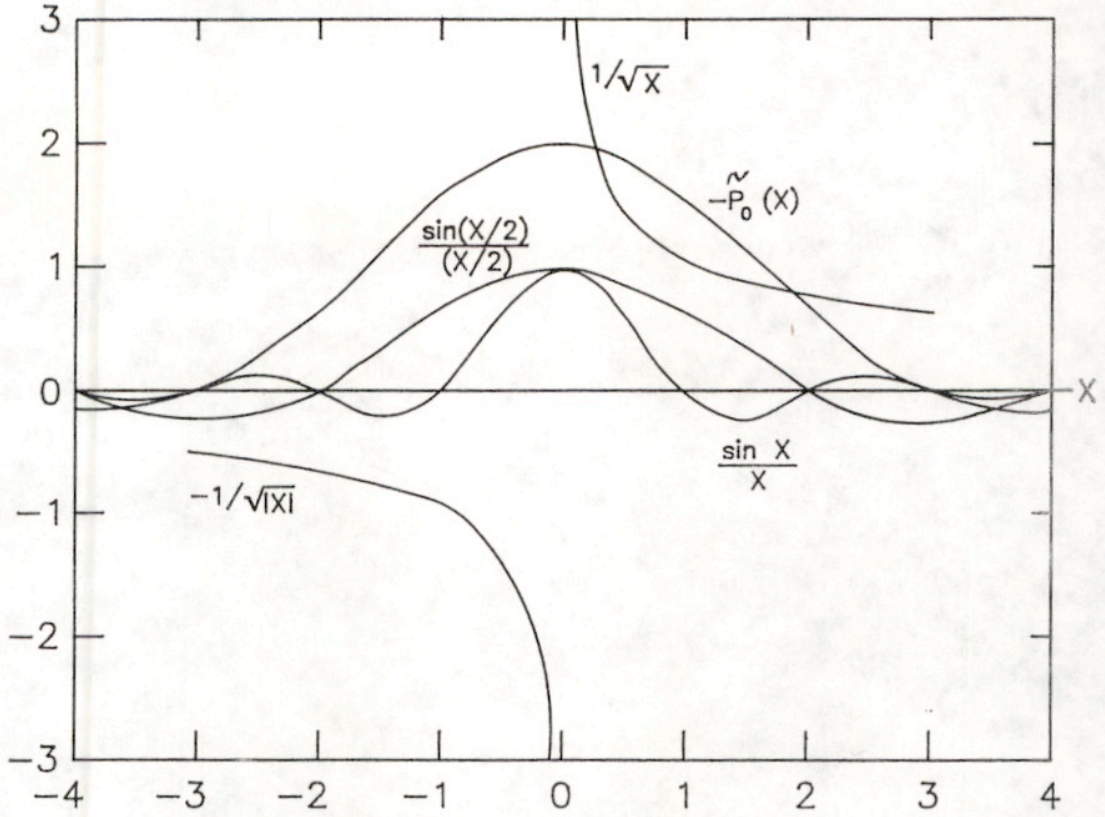

Figure 3. $\chi = 0$
$(X = \omega\tau_{rf}/2)$

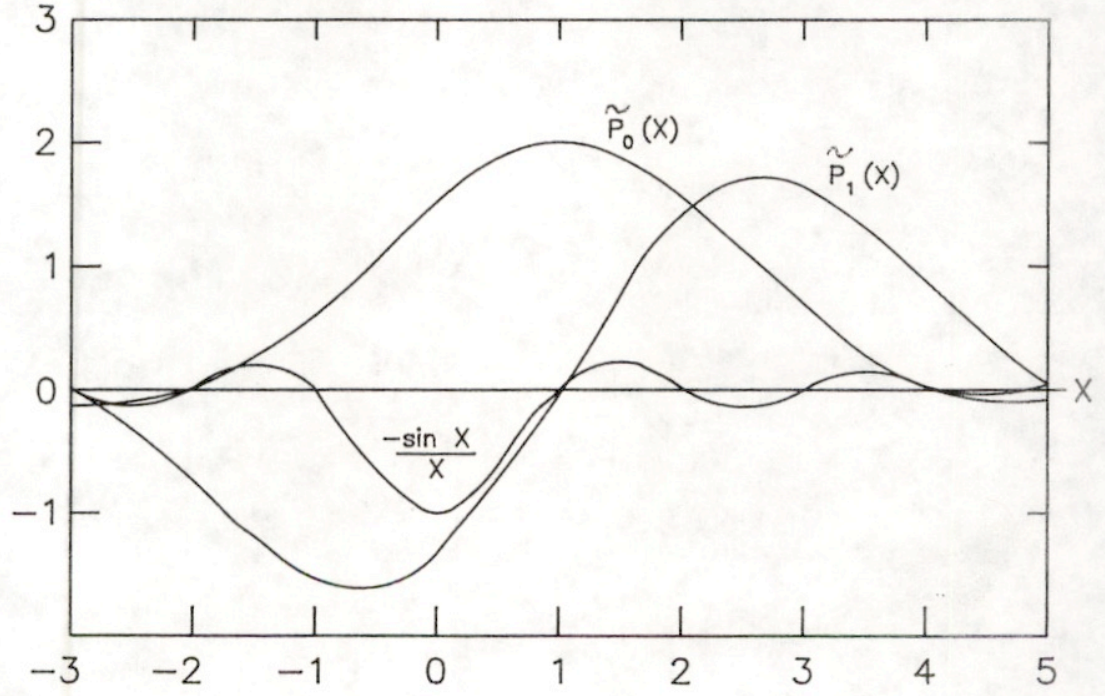

Figure 4. $\chi = \pi$
$(X = \omega\tau_{rf}/2)$

P. Hughes 911B