



# Estimating electron density, electron temperature, and signal-to-noise ratio from laser-collision induced fluorescence data by treating the measurement as a stochastic process

Brian Z. Bentz<sup>1</sup>, Zachary K. White<sup>2</sup>, Ryan P. Gott<sup>2</sup>, Kunning G. Xu<sup>2</sup>, and Edward V. Barnat<sup>3</sup>

<sup>1</sup>Applied Science and Technology Maturation, Sandia National Laboratories  
<sup>2</sup>Department of Mechanical and Aerospace Engineering, University of Alabama in Huntsville  
<sup>3</sup>Applied Optical and Plasma Science, Sandia National Laboratories

## I. Abstract

Diagnostic measurements are inherently stochastic. Using Gaussian statistics, we demonstrate estimation of electron density ( $n_e$ ), temperature ( $T_e$ ), and the signal-to-noise ratio (SNR) in 2-D in a cathodic arc (30 mTorr He) and the positive column of a DC discharge (500 mTorr He) using Laser-collision induced fluorescence (LCIF) data.

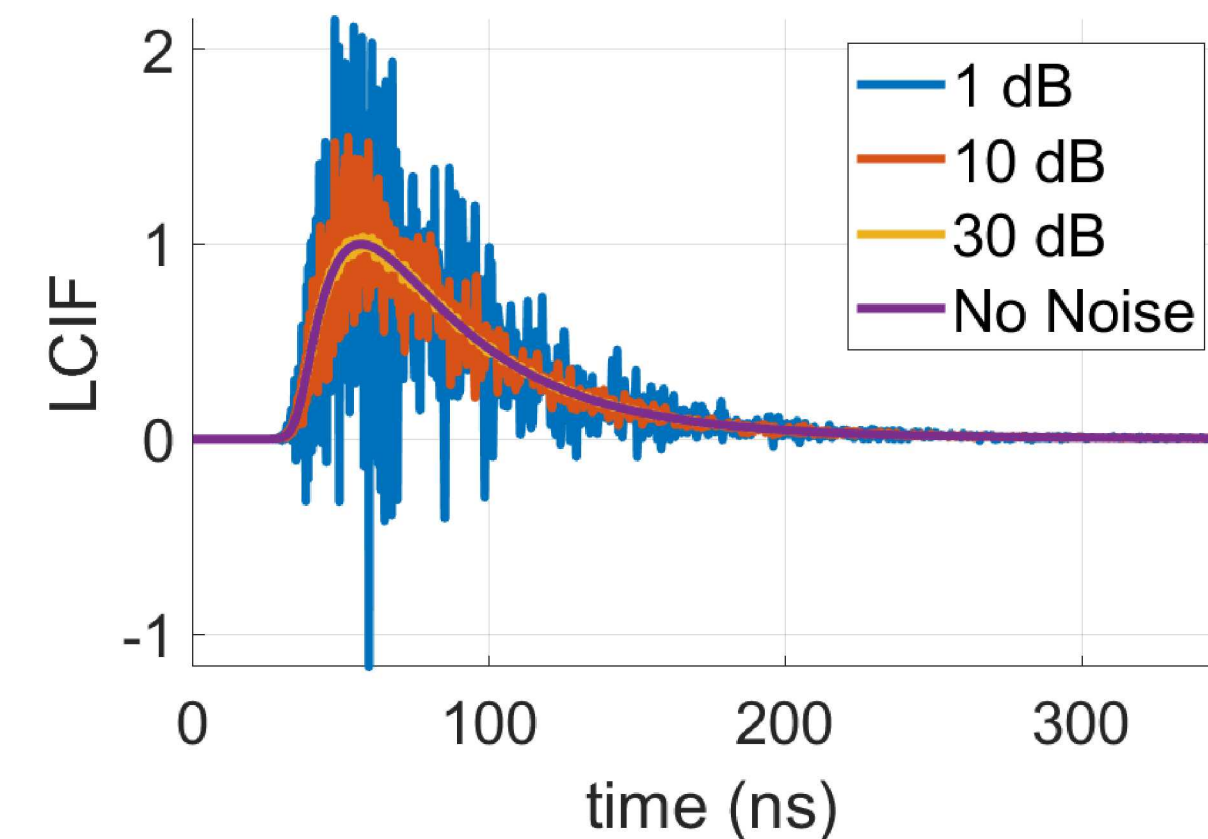
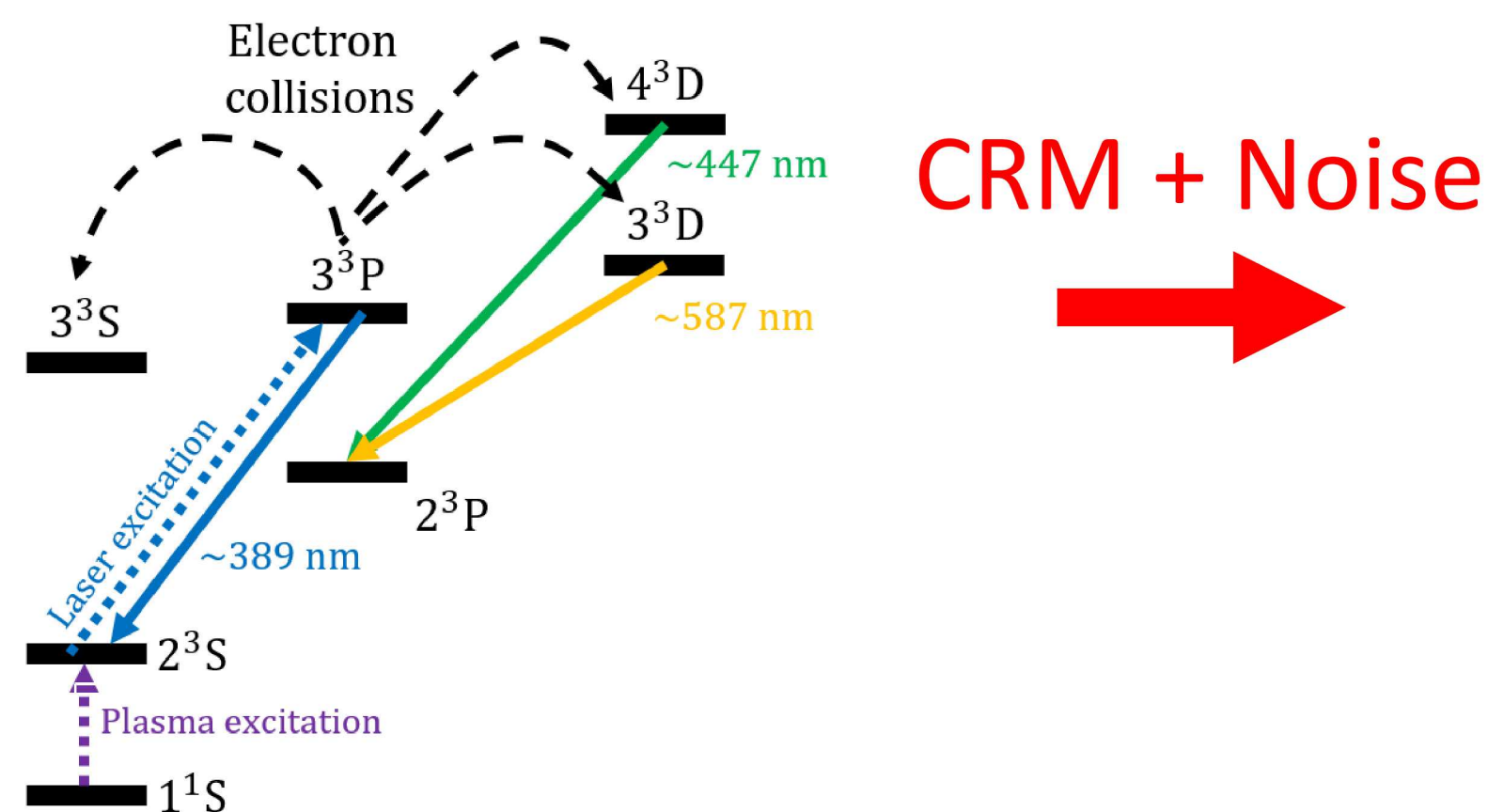
## II. LCIF Diagnostic Measurement Model

- He atomic system described by collisional radiative model (CRM),  $f(n_e, T_e)$  [1]
- Shot noise for measured photodetector current,  $y$ , is described by Gaussian statistics [2]

$$p(y) = \frac{1}{[(2\pi)^P |Y|]^{1/2}} \exp \left[ -\frac{1}{2} \|y - wf(n_e, T_e)\|_{Y^{-1}}^2 \right]$$

- Noisy measurements can be simulated from covariance,  $Y_{ii} = \sigma^2$ , and SNR

$$y = y + \sigma N(0,1) \quad \text{SNR} = \frac{[f(n_e, T_e)]^2}{\sigma^2}$$



## III. Parameter Estimation and Simulated Detection Limits

- Model enables estimation of  $x = [n_e, T_e]$  [3]

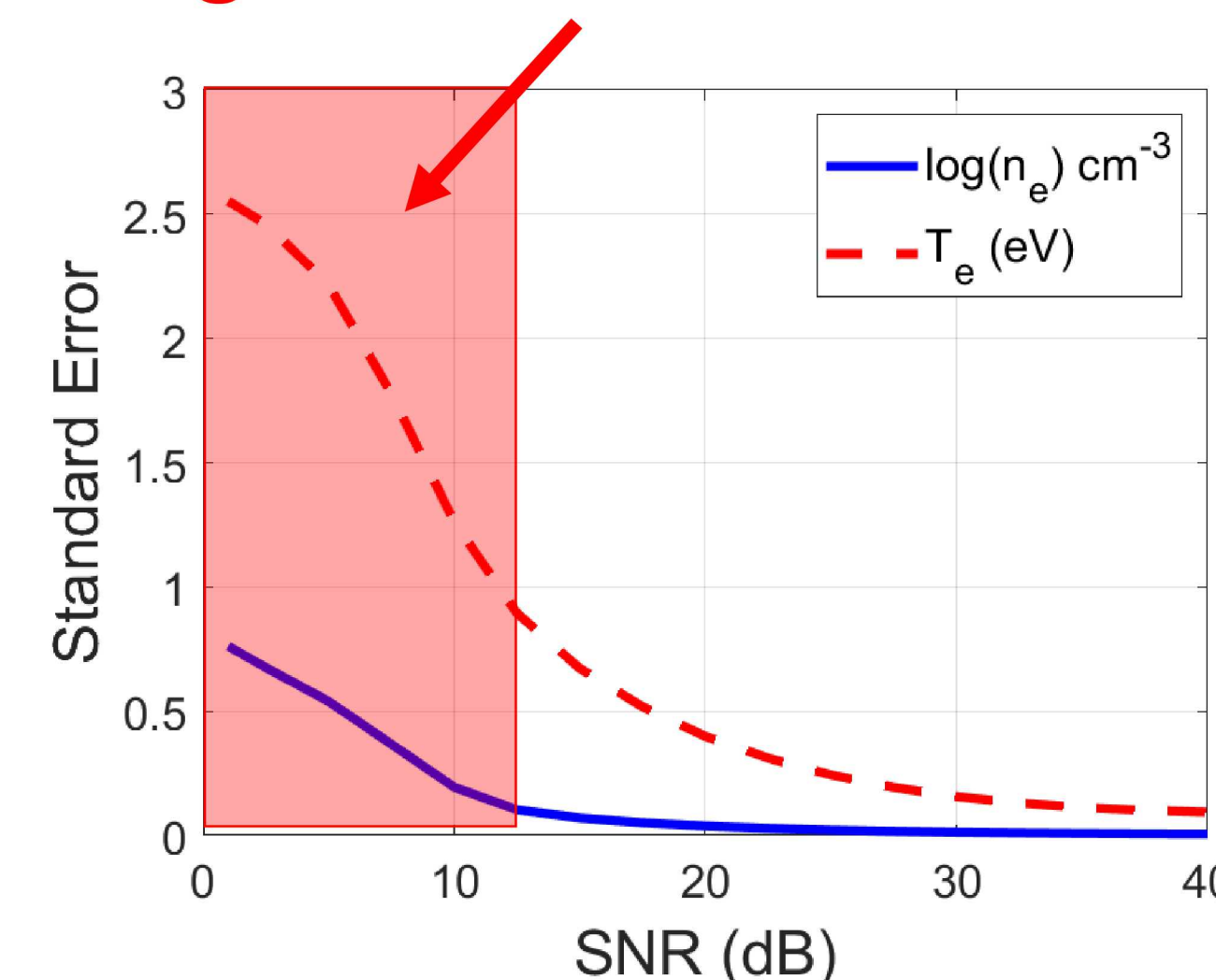
$$c(x) = \|y - \tilde{w}f(x)\|_{Y^{-1}}^2 \quad \tilde{w} = \frac{f^T(x)Y^{-1}y}{f^T(x)Y^{-1}f(x)}$$

$$\hat{x} = \arg \min_x c(x) \quad \hat{w} = \tilde{w}(\hat{x})$$

- SNR determined by estimating  $\sigma^2$

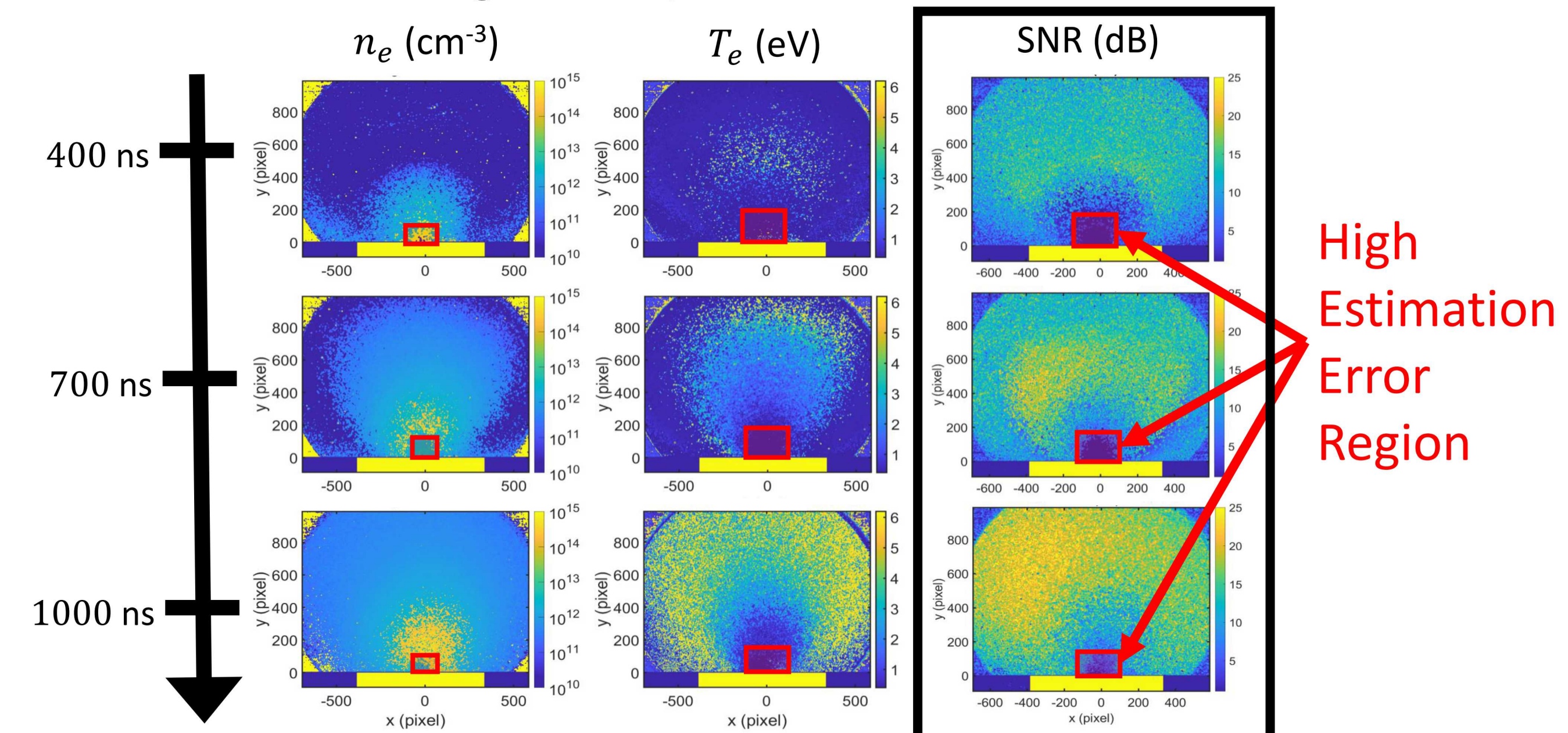
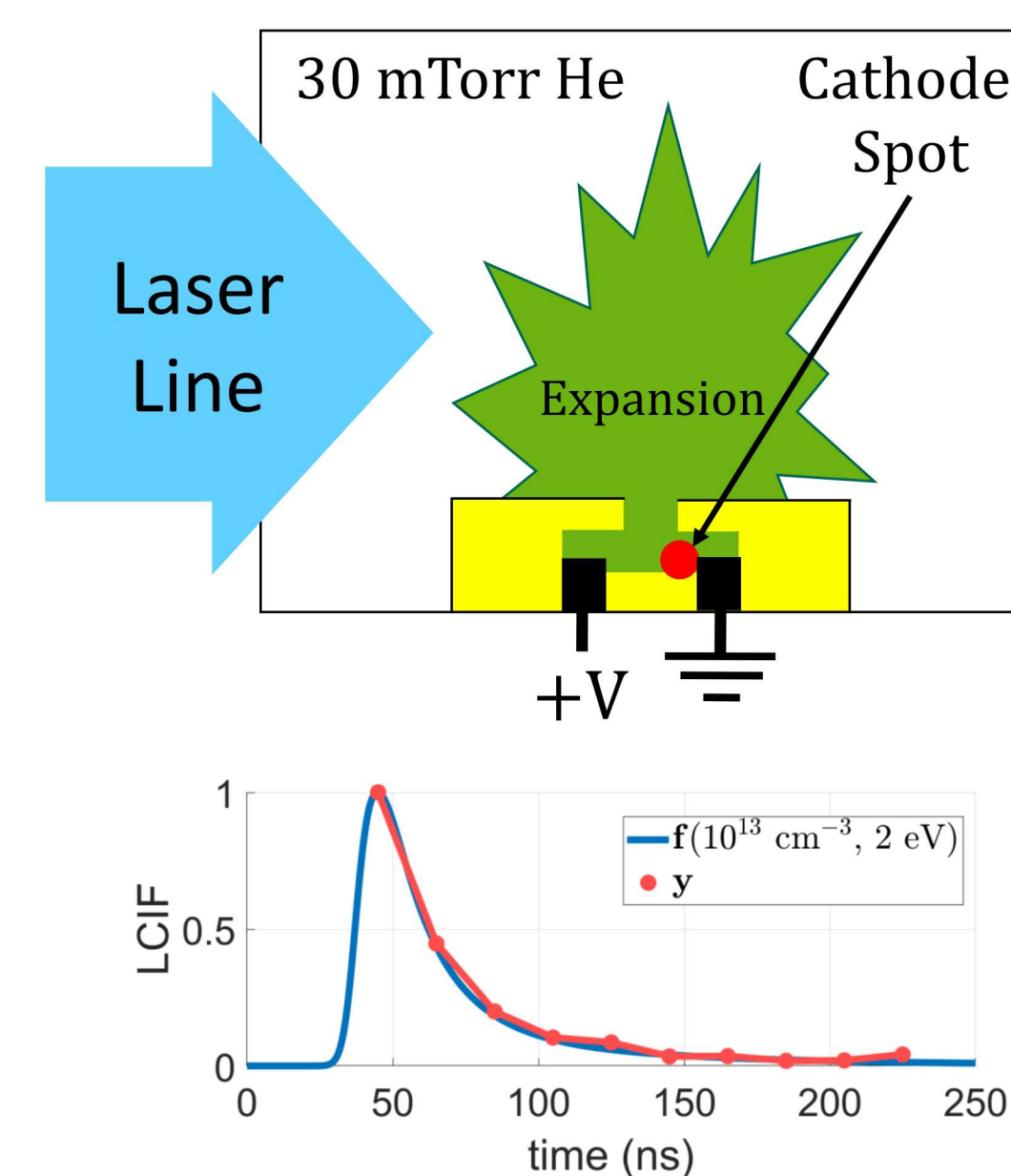
$$\text{SNR} = \frac{f(x)}{\left[ \frac{1}{N} \|y - \hat{w}f(\hat{x})\|_{Y^{-1}}^2 \right]}$$

## High Estimation Error



## IV. Results

- Localized estimation of  $n_e$  and  $T_e$  in a cathodic arc expanding into 30 mTorr of He
- Fast electrons at expansion front and cooler high density core observed



- Estimation of  $n_e$  and  $T_e$  in the positive column of a DC discharge (500 mTorr He)
- Ionization wave propagation and radial expansion observed

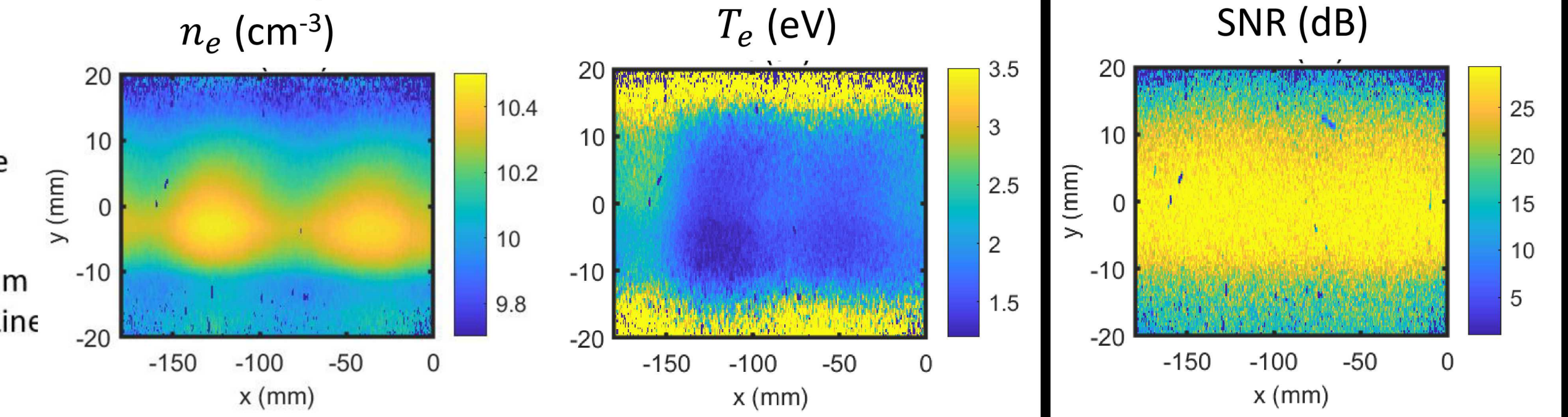
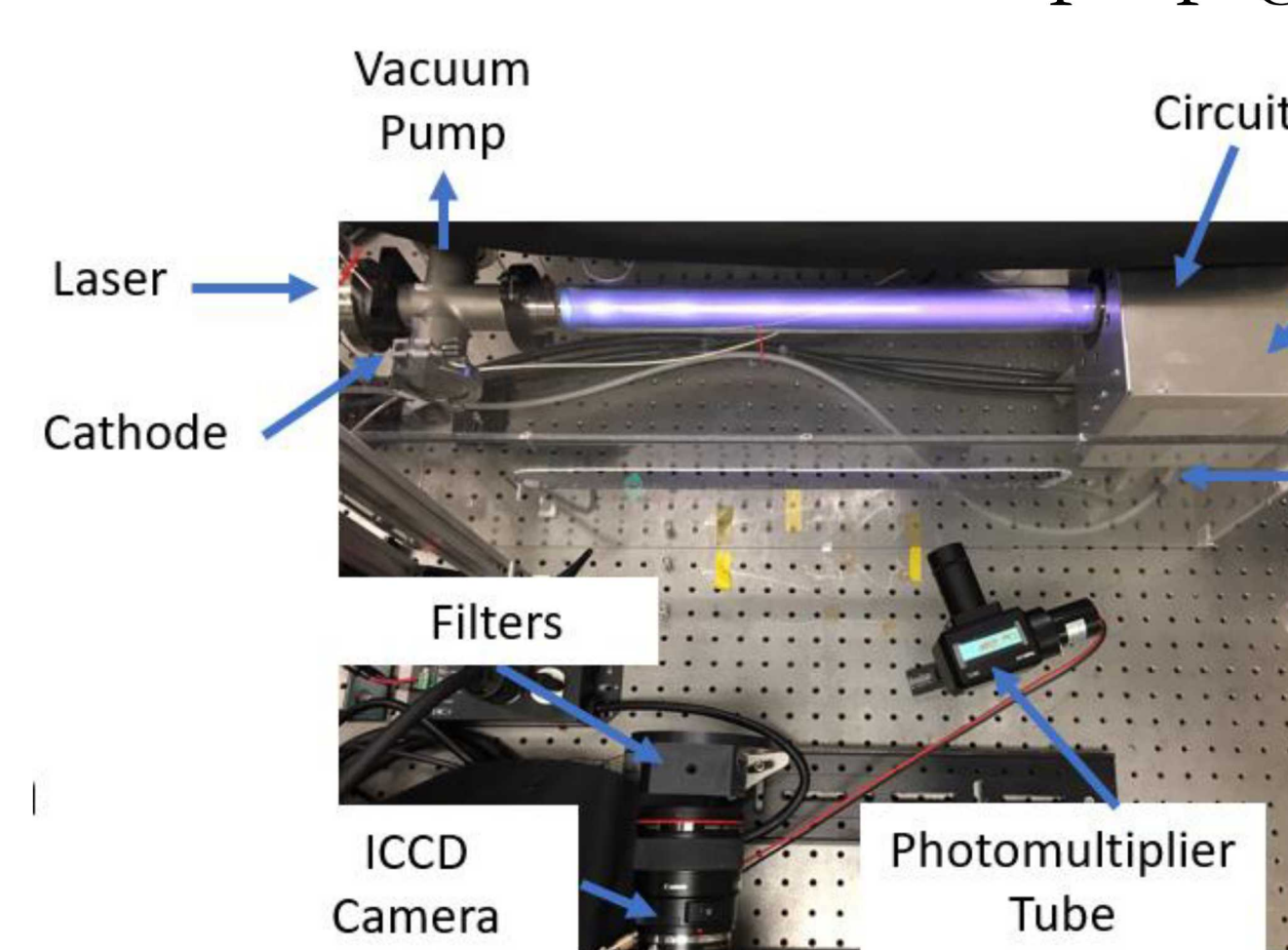
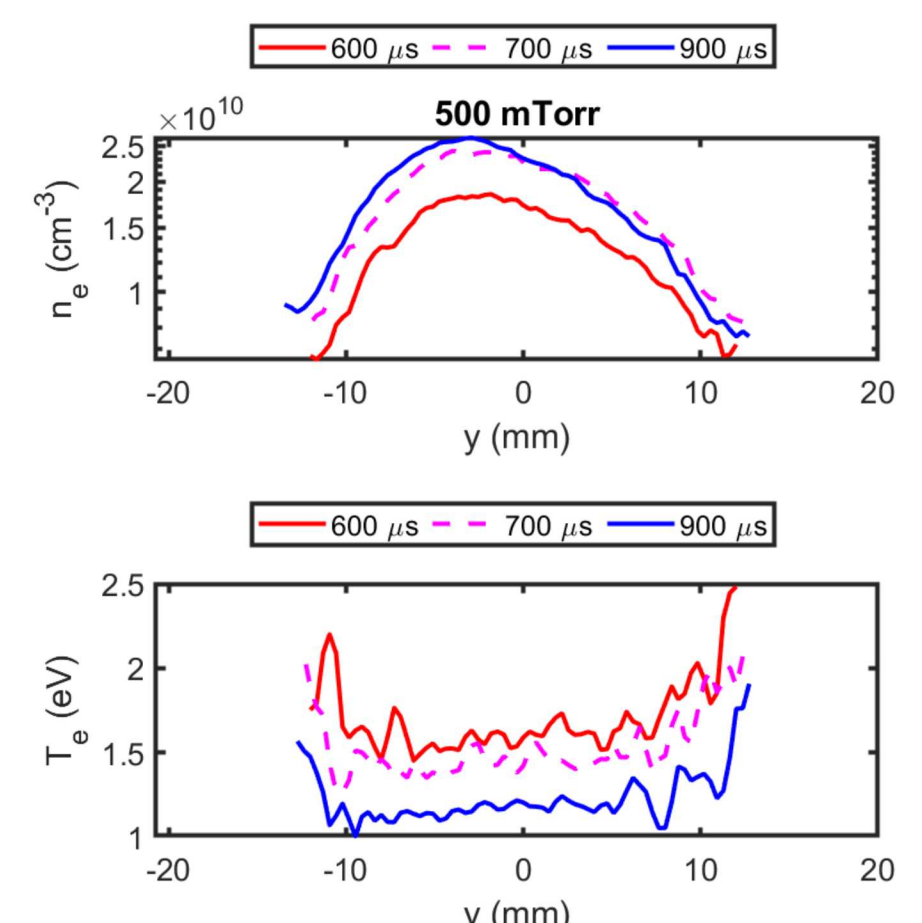


Table 1. Measured Wave Velocities

Pressure (Torr)	Group Velocity (m/s)	Phase Velocity (m/s)
1	277	-175
.75	514	-125
.5	1200	-125



## V. Conclusions

Maximum likelihood estimation of  $n_e$ ,  $T_e$ , and SNR in 2-D is possible using a CRM and Gaussian statistics. The SNR provides a useful metric for establishing regions of diagnostic validity.

## References

- [1] E. V. Barnat and K. Frederickson, *Plasma Sources Sci. Technol.* 19(5), 055015 (2010)
- [2] J.-C. Ye, K. J. Webb, C. A. Bouman, and R. P. Millane, *J. Opt. Soc. Am. A* 16(10), 2400 (1999)
- [3] B. Z. Bentz, D. Lin, and K. J. Webb, *Phys. Rev. Appl.* 10(3), 034021 (2018)