

# Effect of Line-Tower Coupling on E1 Pulse Excitation on a Transmission Line

Luis San Martin  
Sandia National Laboratories  
EM Theory and Simulation  
Albuquerque, USA  
lsanmar@sandia.gov

Larry Warne  
Sandia National Laboratories  
EM Theory and Simulation  
Albuquerque, USA  
lkwarne@sandia.gov

Salvatore Campione  
Sandia National Laboratories  
EM Theory and Simulation  
Albuquerque, USA  
sncampi@sandia.gov

Matthew Halligan  
Sandia National Laboratories  
Electrical Sciences, Experiments  
Albuquerque, USA  
mhallig@sandia.gov

Ross Guttromson  
Sandia National Laboratories  
Energy and Earth Systems  
Albuquerque, USA  
rguttro@sandia.gov

**Abstract**—In a transmission line, we evaluate the coupling between a line and a tower above ground when the excitation is an E1 high-altitude electromagnetic pulse (HEMP). Our model focuses on capturing correctly the effect of the coupling on the peak of the HEMP induced current that propagates along the line. An assessment of this effect is necessary to accurately estimate the effect of the excitation on the systems and components of the power grid. This analysis is a step towards a quantitatively accurate evaluation of HEMP excitation on the power grid.

**Keywords**—Transmission line tower, E1 pulse, HEMP, EM pulse, attenuation.

## I. INTRODUCTION

The accurate estimation of the excitation induced by an E1 high-altitude electromagnetic pulse (HEMP) on a transmission line over ground is important to evaluate the effect of a HEMP on the systems and equipment of the power grid. The purpose of this paper is to estimate the effect of the coupling between a line and a supporting tower on the magnitude of a HEMP induced excitation on a transmission line over ground. To correctly bound this effect, we evaluate the current induced on an infinite line when the incident field angle corresponds to the angle of maximum coupling to the transmission line. We use a Norton equivalent circuit to inject this current to evaluate the line-tower coupling effect on the current of the line over ground.

Our frequency domain model uses the telegrapher's equation solver ATLOG – Analytic Transmission Line Over Ground, to assess the effect of the line-tower coupling. Initial contributions to the problem of a line over ground can be found in [1], [2]. The exact solution of a filament above a conductive ground was provided in [3, 4]. For additional information about ATLOG see [5, 6] and references therein, where ATLOG was developed and shown to provide results in good agreement with full-wave simulations.

The outline of the paper is as follows. Sec. II reviews the transmission line equations used in the analysis and the evaluation of the current induced on an infinite line by the military standard MIL-STD E1 HEMP excitation. In Sec. III the procedure to evaluate the attenuation along the transmission line caused by a tower is described. The analysis to estimate the

coupling between the conductor line and the metallic tower is given in Sec. IV. In Sec. V, the tower impedance and the footing impedance are given. The results for the attenuation caused by the tower are in Sec. VI.

## II. CURRENT INDUCED ON AN INFINITE LINE BY A MIL-STD E1 EXCITATION

The analysis of the excitation of a transmission line over ground by a HEMP was reported in detail in [9]. Here we include parts of the analysis relevant to this work. A solution to the transmission line equations of an infinite line, that includes the different contributions to the transmission line parameters that model ground losses is included. The transmission line equations are:

$$\frac{dV}{dz} = -ZI + E_z^{inc}, \quad \frac{dI}{dz} = -YV \quad (1)$$

In Eq. (1) the impedance is defined as  $Z = Z_0 + Z_2 + Z_4$ . Here we will focus on the case of the line height larger than the wire diameter  $b, h > b$  (we use  $\exp(j\omega t)$  time dependence)

$$Z_0 = \begin{cases} \frac{\omega \mu_0}{2\pi(1-j)\frac{a}{\delta}} \frac{1}{\delta} & \text{if } \frac{\delta}{a} < \frac{1}{10}, j\omega \frac{\mu_0}{8\pi} + \frac{1}{\sigma_0 \pi a^2} \text{ if } \frac{\delta}{a} > 10 \\ -j\omega \frac{\mu_0}{2\pi(1-j)\frac{a}{\delta}} \frac{J_0((1-j)\frac{a}{\delta})}{J_1((1-j)\frac{a}{\delta})} & \text{otherwise} \end{cases} \quad (2)$$

$$Z_2 = j\omega \frac{\mu_0}{2\pi} \ln\left(\frac{b}{a}\right) + j\omega \frac{\mu_0}{2\pi} \ln\left(\frac{h}{b} + \sqrt{\left(\frac{h}{b}\right)^2 - 1}\right) \quad (3)$$

$$Z_4 = j\omega \mu_0 \frac{H_0^{(2)}(k_4 h)}{2\pi k_4 b H_1^{(2)}(k_4 h)} \quad (4)$$

with  $k_4^2 = \omega \mu_0 (\alpha \epsilon_4 - j\sigma_4)$ ,  $\delta = \sqrt{2/(\omega \mu_0 \sigma_0)}$  and where  $\epsilon_4$  is the ground permittivity,  $\sigma_4$  the ground conductivity,  $\sigma_0$  the wire

conductivity,  $\sigma$  the wire radius,  $b$  the radius of the dielectric shell coating the wire, and  $h$  the height of the line from the conducting ground plane. The admittance is defined as

$$\frac{1}{Y} = \frac{1}{Y_e} + \frac{1}{Y_4} \text{ with } Y_e = j\omega C_e \text{ and}$$

$$\frac{1}{C_e} = \sqrt{\left(\frac{h/h_e}{C_0} + \frac{1}{C_2}\right)^2 - \left(\frac{b/h_e}{C_0} + \frac{A_2}{C_2}\right)^2} \quad (5)$$

where  $h_e = \sqrt{h^2 - b^2}$ ,  $C_2 = \frac{2\pi\epsilon_0}{\ln(b/a)}$ , and  $\epsilon_2$  the permittivity of a dielectric shell coating the wire,  $A_2$  given as

$$A_2 = 0.7(1-a/b)(1-h_e/h)\frac{(\epsilon_2 - \epsilon_0)}{(\epsilon_2 + \epsilon_0)}, \text{ with } C_0 \text{ given as}$$

$$C_0 = 2\pi\epsilon_0 / \ln\left(h/b + \sqrt{(h/b)^2 - 1}\right), \text{ and admittance } Y_4 \text{ as}$$

$$Y_4 = j\pi(\omega\epsilon_4 - j\sigma_4)k_4 h \frac{H_1^{(2)}(k_4 h)}{H_0^{(2)}(k_4 h)} \quad (6)$$

We analyze the case of an infinite transmission line. From Eq. (1) and with  $k_L = \sqrt{-ZY}$  we obtain

$$\left(\frac{d}{dz} + k_L^2\right)I = -YE_z^{inc} \quad (7)$$

We assume the incident wave is polarized in the plane containing the wire and perpendicular to the ground surface with incident angle  $\theta_0$  with the z-axis, as shown in Fig. 1

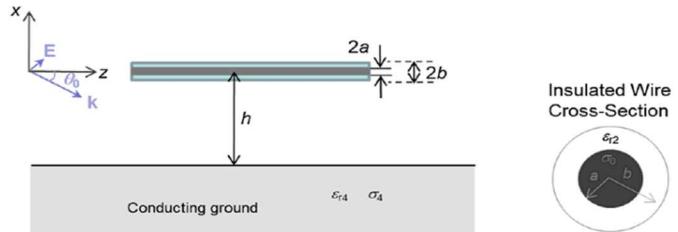


Fig. 1. Schematic description of the incident wave direction on the transmission line indicating parameters of the problem.

$$H_y^{inc} = \frac{1}{\eta_0} E_s(\omega) \left( e^{-jzk_0 \cos \theta_0 + jxk_0 \sin \theta_0} + R_H e^{-jzk_0 \cos \theta_0 - jxk_0 \sin \theta_0} \right), x > 0 \quad (8)$$

$T_H$  and  $R_H$  are the transmission and reflection coefficients and  $k_0$  the free space wave number. In the case above ground it is convenient to set the phase reference on the wire itself at  $z = 0$ , which can be done by setting  $E_s(\omega) = E(\omega)e^{-j\theta_0}$ , where  $E(\omega)$  represents the spectrum of the incident HEMP.

$$E_z^{inc} = \frac{dH_y^{inc}}{dx j\omega\epsilon_0} = E_s(\omega) \sin \theta_0 \left( e^{-jzk_0 \cos \theta_0 + jxk_0 \sin \theta_0} - R_H e^{-jzk_0 \cos \theta_0 - jxk_0 \sin \theta_0} \right) \quad (9)$$

$$\text{and } 1 + R_H = T_H, \quad 1 - R_H = \frac{\sqrt{(k_4/k_0) - \cos^2 \theta_0}}{(k_4/k_0)^2} T_H, \text{ from}$$

where we obtain

$$R_H = \frac{(k_4/k_0)^2 \sin \theta_0 - \sqrt{(k_4/k_0)^2 - \cos^2 \theta_0}}{(k_4/k_0) \sin \theta_0 + \sqrt{(k_4/k_0)^2 - \cos^2 \theta_0}} \quad (10)$$

From the above expressions the incident electric field along the wire can be evaluated as

$$E_z^{inc} = E_s(\omega) \sin \theta_0 \begin{pmatrix} e^{j\theta_0 \sin \theta_0} \\ -R_H e^{-j\theta_0 \sin \theta_0} \end{pmatrix} e^{-jzk_0 \cos \theta_0} = A_0 e^{-jzk_0 \cos \theta_0} \quad h > 0 \quad (11)$$

By combining Eq. (11) and Eq. (7) we obtain the differential equation for the transmission line

$$\left( \frac{d^2}{dz^2} + k_L^2 \right) I = -YA_0 e^{-jzk_0 \cos \theta_0} \quad (12)$$

whose general solution can be written as the sum the particular solution plus the homogeneous solution

$$I = C_1 e^{j\theta_0 \sin \theta_0} + C_2 e^{-j\theta_0 \sin \theta_0} - \frac{YA_0 e^{-jzk_0 \cos \theta_0}}{k_L^2 - k_0^2 \cos^2 \theta_0} \quad (13)$$

which for the infinite line, after dropping the homogeneous terms that blow up at  $\pm\infty$ , the solution is

$$I = -\frac{YA_0 e^{-jzk_0 \cos \theta_0}}{k_L^2 - k_0^2 \cos^2 \theta_0} \quad (14)$$

To estimate the attenuation of the HEMP induced EM-wave on a transmission line caused by the towers, we use the current induced on a line of infinite length by the E1 MIL-STD pulse given in Appendix. The line parameters used in the modeling are given as ground permittivity  $\epsilon_4 = 20\epsilon_0$ , ground conductivity  $\sigma_4 = 0.01 \text{ S/m}$ , wire radius  $a = 1 \text{ cm}$ , with no insulation, wire conductivity  $\sigma_0 = 2.9281 \times 10^7 \text{ S/m}$  and line height 10 m, 20 m, 30m and 40 m. The current induced on an infinite line at  $z=0$ , from Eq. (14), is given by  $I_i = -YA_0 / (\Gamma^2 - k_0^2 \cos^2 \theta_0)$ , which is evaluated at the angle of maximum coupling that produces the largest current on the line. We use a matched Norton equivalent circuit to inject this current into a semi-infinite line for which the solution that satisfy the radiation condition as  $z \rightarrow +\infty$  is

$$I = A_1 e^{i\Gamma z}, \quad V = -i\Gamma \frac{A_1}{Y} e^{i\Gamma z}, \quad (15)$$

with  $A_1 = 2I_i / (1 - i\Gamma Y_0 / Y)$  and  $Y_0 = \sqrt{Y/Z}$ . This is done to use a typical pulse induced by the E1 MIL-STD HEMP in our evaluation of attenuation caused by the towers. The injected waveforms are shown in Fig. 2 and the frequency content of the curves in **Error! Reference source not found.**3. The angle of maximum coupling depends on the height of the line and the conductivity of the ground. Table I contains the angles of

maximum coupling and the maximum currents at  $z = 0$ , on an infinite line for varying line heights.

TABLE I. ANGLES OF MAXIMUM COUPLING FOR VARYING LINE HEIGHTS

Line Height	Angle of Maximum Coupling	Maximum Current
[m]	[Rad]	[A]
10	$\theta = \pi / 33$	3546
20	$\theta = \pi / 49$	5350
30	$\theta = \pi / 57$	6676
40	$\theta = \pi / 64$	7760

### III. METHOD FOR THE EVALUATION OF THE LINE-TOWER COUPLING

To evaluate the attenuation caused by a tower, we define an elemental building block (EBB) composed of a length of line and one tower. The EBB is shown in Fig. 4. The entire transmission line can be generated by repetition of this EBB. The parameters are: the line segment length, and the heights of the tower. We evaluated the attenuation for the set of parameters given in Table II, which represent typical values [10]. The attenuation is obtained from the analysis of the circuit in Fig. 5. The circuit includes the Norton equivalent source. The line is represented in the circuit by its characteristic impedance. The ratio of the current with tower and without tower is given by the following formula

$$\frac{I_{w/Tower}}{I_{wo/Tower}} = \frac{1}{1 + \frac{Z_c}{2Z_{Total}}} \quad (16)$$

with  $Z_c = \sqrt{Z/Y}$  and  $Z_{TOTAL}$  the coupling impedance defined in Fig. 5. The attenuation on an EBB is the line attenuation on a segment of length  $L/2$ , then the attenuation due to the coupling in Eq. (16) and finally a second line attenuation corresponding to the second segment of length  $L/2$ , as shown in Fig. 4. The line attenuation without tower is obtained from Eq. (15) under the assumption that line attenuation is unperturbed by the presence of the tower.

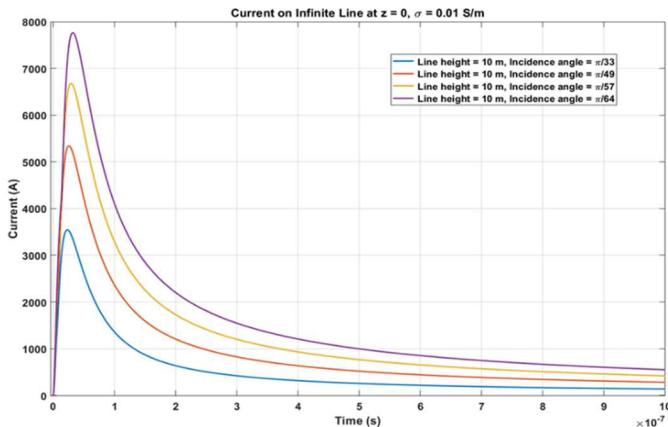


Fig. 2. Time domain waveform of the current at  $z = 0$  for an infinite length line with a height of 10 m, 20 m, 30 m and 40 m, over ground of 0.01 S/m for angle of maximum coupling.

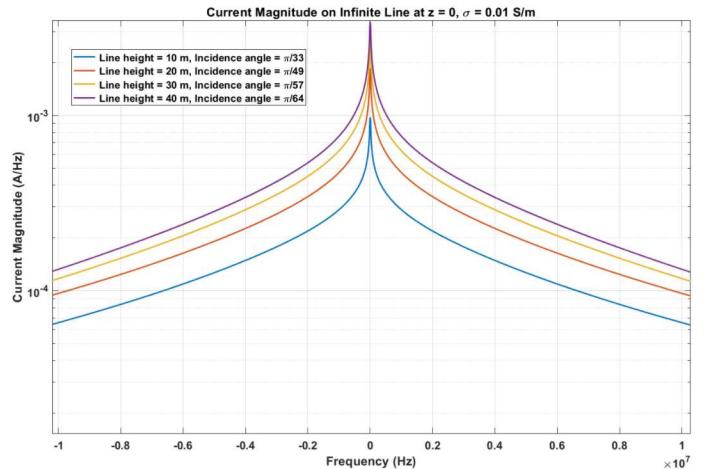


Fig. 3. Frequency domain plot of the current magnitude for conductivity of ground  $\sigma = 0.01$  S/m in logarithmic scale.

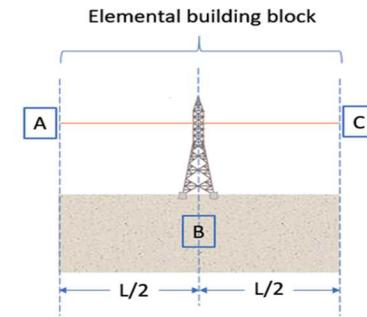


Fig. 4. The current is injected at point A. The attenuation from A to C has three parts. Attenuation from A to B, attenuation at B due to the coupling to ground through the tower and the attenuation from B to C.

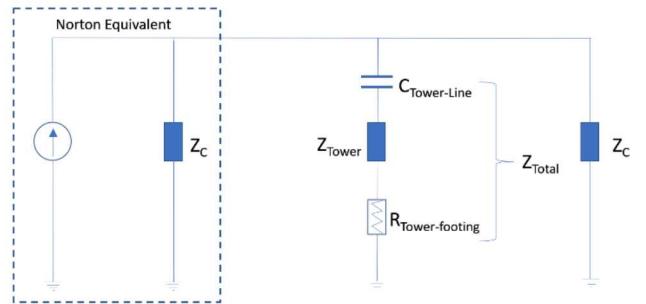


Fig. 5. Equivalent circuit.

TABLE II. PARAMETERS IN EVALUATION OF ATTENUATION ESTIMATIONS

Earth conductivity	[S/m]	0.01
Length of line	[m]	100, 200, 300, 400
Height of tower	$h_T$ [m]	20, 30, 40
Height of line	$h_L$ [m]	10, 20, 30, 40, 50

### IV. ESTIMATION OF THE CAPACITIVE COUPLING BETWEEN THE CONDUCTOR LINE AND THE METALLIC TOWER

In the following, we describe the main points of the estimation of capacitances between a line and a tower. The estimation is

made under the approximation of a PEC ground. We approximate the line and the tower by cylinders in proximity to one another at right angles ( $z$ -axis is taken to be directed along the horizontal line with  $z = 0$  at the position of the vertical cylinder, representing the tower, the  $y$ -axis directed vertical to the earth with  $y = 0$  at the surface with  $x = 0$  at the horizontal line and  $x = \Delta$  at the position of the vertical cylinder). We assume a line of length  $2L$  with radius  $a$ , with  $L \gg a$ , tower of height  $h_c$ , tower radius  $a_c$ , with  $h_c \gg a_c$ . In Fig. 6, the parameters used in the estimation of the capacitive coupling are shown. We estimate the electrostatic potential for the configuration of Fig. 6. This is a low frequency approximation appropriate to estimate the coupling of the EM wave, whose spectrum, shown in Figs. 3, is most significant under 10MHz. The potential for the situation shown in Fig. 6 can be written as

$$\phi(x, y, z) = \frac{1}{4\pi\epsilon_0} \lim_{L \rightarrow \infty} \left[ \int_{-L}^L \frac{q(z) dz}{\sqrt{x^2 + (y-h)^2 + (z-z')^2}} - \int_{-L}^L \frac{q(z) dz}{\sqrt{x^2 + (y+h)^2 + (z-z')^2}} \right] + \frac{1}{4\pi\epsilon_0} \int_{-h_c}^h \frac{q_c(y) dy}{\sqrt{(x-\Delta)^2 + (y-y')^2 + z^2}} \quad (17)$$

with  $q_c(-y) = -q_c(y)$  and  $h_c > h$ . The charge per unit length associated with the uniform line above earth is removed to focus on the charge associated to the presence of the tower.

$$\phi(x, y, z) = \frac{1}{4\pi\epsilon_0} \lim_{L \rightarrow \infty} \left[ \int_{-L}^L \frac{q_0 dz}{\sqrt{x^2 + (y-h)^2 + (z-z')^2}} - \int_{-L}^L \frac{q_0 dz}{\sqrt{x^2 + (y+h)^2 + (z-z')^2}} \right] + \frac{1}{4\pi\epsilon_0} \lim_{L \rightarrow \infty} \left[ \int_{-L}^L \frac{(q(z) - q_s) dz}{\sqrt{x^2 + (y-h)^2 + (z-z')^2}} - \int_{-L}^L \frac{(q(z) - q_s) dz}{\sqrt{x^2 + (y+h)^2 + (z-z')^2}} \right] + \frac{1}{4\pi\epsilon_0} \int_{-h_c}^h \frac{q_c(y) dy}{\sqrt{(x-\Delta)^2 + (y-y')^2 + z^2}} \quad (18)$$

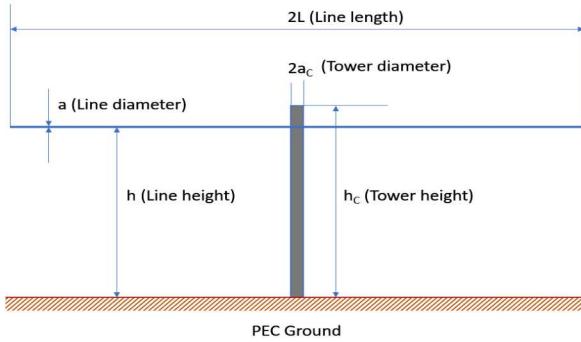


Fig. 6. Parameters used in capacitive coupling evaluation.

In this expression, the first two terms correspond to the potential of an infinite line and its image. The contribution of these two terms can be easily evaluated, by for example, evaluating the potential of a single infinite line and applying superposition. The

potential after this evaluation with  $(q(z') - q_s) = \Delta q(z')$  can be written as

$$\phi(x, y, z) = \frac{1}{4\pi\epsilon_0} \lim_{L \rightarrow \infty} \left[ \int_{-L}^L \frac{\Delta q(z') dz'}{\sqrt{x^2 + (y-h)^2 + (z-z')^2}} - \int_{-L}^L \frac{\Delta q(z') dz'}{\sqrt{x^2 + (y+h)^2 + (z-z')^2}} \right] + \frac{q_s}{2\pi\epsilon_0} \ln \sqrt{\frac{x^2 + (y+h)^2}{x^2 + (y-h)^2}} + \frac{1}{4\pi\epsilon_0} \int_{-h_c}^h \frac{q_c(y') dy'}{\sqrt{(x-\Delta)^2 + (y-y')^2 + z^2}} \quad (19)$$

To evaluate the capacitance, we assume a potential difference  $V$  between the horizontal and the vertical cylinder, with two boundary conditions  $\phi(x = 0, y = h + a, z) = V$ ,  $|z| < L \rightarrow \infty$ ,  $\phi(x = \Delta + a_c, y, z = 0) = 0$ , where we assume  $L \gg h, \Delta \gg a_c, a$ . The system of equations obtained is solved numerically to obtain  $q_c$  and  $\Delta q$ . The estimates of the capacitance for various values of parameters  $h$  and  $h_c$  with  $\Delta = 2.4$  m are given in Table 1. In the following, we use a lump circuit analysis to estimate the attenuation due to the line-tower coupling. The use of a lumped circuit is justified by the concentrated charge distribution near the point of closest distance between the tower and the line. The charge distribution of the line, due to the presence of the tower (i.e.  $\Delta q$  with respect to charge distribution in the absence of the tower) is shown in Fig. 7Fig., where normalized charge distributions on the conductor line are given for the different line and tower height. The charge distribution is symmetric but only positive distance is shown. As seen in Fig. 7, the charge is highly concentrated near  $z = 0$ . We also show in Fig. 8 the charge distribution on the tower for a representative example with  $h_c = 40$  m and  $h = 20$  m, with similar behavior.

TABLE 1. CAPACITANCE VALUES FOR DIFFERENT LINE AND TOWER HEIGHTS

$h$ [m]	$h_c$ [m]	Capacitance [pF]
10	20	19.2
10	30	20.4
10	40	20.8
20	30	30.7
20	40	32.8
30	40	38.5
40	50	44.5

## V. TOWER IMPEDANCE MODEL AND TOWER FOOTING RESISTANCE

As mentioned above the most significant contribution to the total impedance of the coupling is the capacitance. Due to this, we chose simple models to represent the tower impedance. Several models have been proposed to represent the surge impedance of a transmission line tower. In 1934 Jordan gave a first estimate [11] that was later improved by Takahashi [12].

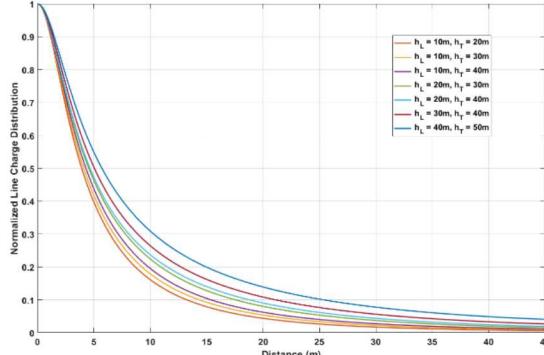


Fig.7. Normalized charge distribution on the conductor line. At zero is the tower and the charge distribution is symmetric with only positive distance is shown.

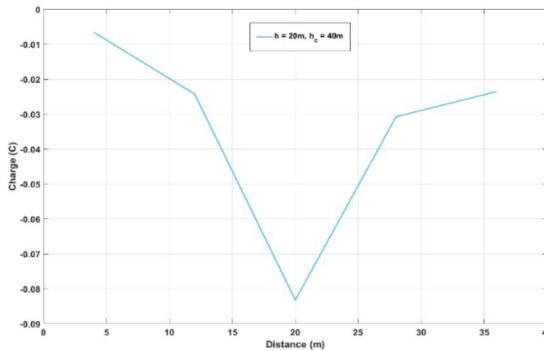


Fig. 8. Charge distribution on the tower.

The International Council on Large Electric Systems (CIGRE) has recommended formulas for tower surge impedances [13]. In the case of a cylindrical conductor the expressions proposed are:

$$Z_s = 60 \ln \left( \frac{h}{r_c} \right) \Omega \quad (20)$$

Recently, Gutierrez et al. proposed a new methodology to derive the tower impedance based on the use of transmission line segments [14]. For the case of a single vertical conductor over conductive ground, the field distribution of the conductor and its image can be approximated by that of a bi-conical antenna, which leads to the following expression for the surge impedance

$$Z_s = \frac{1}{2\pi} \sqrt{\frac{\mu}{\epsilon}} \ln \left[ \frac{h + \sqrt{h + r_c}}{r_c} \right] + \frac{1}{2\pi} \sqrt{\frac{\mu}{\epsilon}} \ln \left[ \frac{(h + p) + \sqrt{(h + p)^2 + r_c^2}}{h + \sqrt{h^2 + r_c^2}} \right] \quad (21)$$

where  $p$  is the complex skin depth  $p = 1/\sqrt{j\omega\mu_E\sigma_E}$  that accounts for the ground conductivity losses [15], with  $\mu_E$  and the soil magnetic permeability and electrical conductivity

$\sigma_E$ . In the above expressions  $h$  is the height of the cylinder that represents the tower and  $r_c$  its radius. Either of the expressions in Eqs. (20), (21), give similar results for the attenuations. In the results for the attenuation given below, we used the

expression in Eq. (21). This impedance corresponds to the characteristic impedance of the tower represented as a transmission line with the foot resistance as a termination load. The tower-foot resistance is represented in lightning strike studies by a non-linear resistance, where the non-linearity is caused by the ionization of the ground [16].

$$R_{\text{Tower-foot}} = \frac{R_0}{\sqrt{1 + I/I_g}} \quad (22)$$

In our case, we assume the current is not enough to trigger the non-linear effect  $R_T = R_0$ , with the value of  $R_0 = 12.5 \Omega$ , a high frequency approximation [17] for vertical ground rod divided by 4 to account for the 4 supports of a typical tower. Notice that the maximum current is not affected by the  $R_0$ , because for an E1 MIL-STD, the maximum current occurs at approximately 25 ns, shorter than the time required for the first reflection from the tower base to arrive. This is seen in Fig. 9, where the reflection from the tower footing, does not affect the value of the maximum current. The total impedance of the combined line-tower coupling is

$$Z_{\text{Total}} = \frac{1}{j\omega C_{\text{Line-Tower}}} + Z_{\text{Total}} \left( \frac{R_{\text{Tower-foot}} + jZ_{\text{Tower}} \tan(kL)}{Z_{\text{Tower}} + jR_{\text{Tower-foot}} \tan(kL)} \right) \quad (23)$$

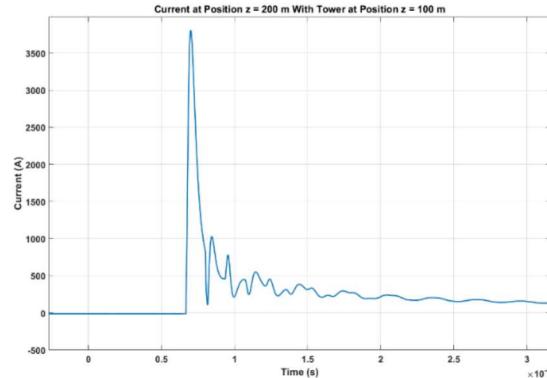


Fig. 9. Effect of reflection does not change the peak value.

## VI. ATTENUATION RESULTS FOR THE PARAMETERS IN TABLE 1

In Tables 3-9, we present the results for the attenuation at the end of the elemental building block, i.e. at position C in Fig. 4 for the case of  $\sigma = 0.01 \text{ S/m}$ . The angle of maximum coupling, is given in each table. In each case we include the values of attenuation without the tower (Wo/) and with tower (W/). The results less are all less than one because the attenuation is given as a per one reduction of the initial amplitude.

TABLE 3: TOWER HEIGHT = 20 M, LINE HEIGHT = 10 M,  $\Theta = \pi/33$

Tower distance	100 m	200 m	300 m	400 m
Wo/towers	0.9100	0.8297	0.7578	0.6940
W/towers	0.8473	0.7740	0.7080	0.6481

TABLE 4: TOWER HEIGHT = 30 M, LINE HEIGHT = 10 M,  $\Theta = \pi/33$ 

Tower distance	100 m	200 m	300 m	400 m
Wo/towers	0.9100	0.8297	0.7578	0.6940
W/towers	0.8390	0.7666	0.7014	0.6423

TABLE 5: TOWER HEIGHT = 40 M, LINE HEIGHT = 10 M,  $\Theta = \pi/33$ 

Tower distance	100 m	200 m	300 m	400 m
Wo/towers	0.9100	0.8297	0.7578	0.6940
W/towers	0.8346	0.7627	0.6979	0.6392

TABLE 6: TOWER HEIGHT = 30 M, LINE HEIGHT = 20 M,  $\Theta = \pi/49$ 

Tower distance	100 m	200 m	300 m	400 m
Wo/towers	0.9581	0.9182	0.8802	0.8443
W/towers	0.8327	0.8001	0.7689	0.7388

TABLE 7: TOWER HEIGHT = 40 M, LINE HEIGHT = 20 M,  $\Theta = \pi/49$ 

Tower distance	100 m	200 m	300 m	400 m
Wo/towers	0.9581	0.9182	0.8802	0.8443
W/towers	0.8210	0.7888	0.7580	0.7287

TABLE 8: TOWER HEIGHT = 40 M, LINE HEIGHT = 30 M,  $\Theta = \pi/57$ 

Tower distance	100 m	200 m	300 m	400 m
Wo/towers	0.9738	0.9483	0.9238	0.9000
W/towers	0.8205	0.8006	0.7812	0.7623

TABLE 9: TOWER HEIGHT = 50 M, LINE HEIGHT = 40 M,  $\Theta = \pi/64$ 

Tower distance	100 m	200 m	300 m	400 m
Wo/towers	0.9812	0.9629	0.9449	0.9274
W/towers	0.8095	0.7954	0.7818	0.7682

From this analysis, we conclude that the line-tower attenuation can be significant when the line height is 20 meters or more, because the characteristic impedance is higher in (16) and there is less loss along the line due to the earth.

## VII. ACKNOWLEDGMENT

Sandia National Laboratories is a multi-mission laboratory managed and operated by National Technology & Engineering Solutions of Sandia, LLC, a wholly owned subsidiary of Honeywell International Inc., for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-NA0003525. This paper describes objective technical results and analysis. Any subjective views or opinions that might be expressed in the paper do not necessarily represent the views of the U.S. Department of Energy or the United States Government.

## REFERENCES

- [1] Carson, J. R., "Wave propagation in overhead wires with ground return," The Bell System Technical Journal, Vol. 5, 539-554, 1926.
- [2] Sunde, E. D., Earth Conduction Effects in Transmission Systems, Dover, New York, 1967.
- [3] Wait, J. R., "Theory of wave propagation along a thin wire parallel to an interface," Radio Science, Vol. 7, 675-679, 1972.
- [4] Wait, J. R., "Tutorial note on the general transmission line theory for a thin wire above the ground," IEEE Transactions on Electromagnetic Compatibility, Vol. 33, 65-67, 1991.
- [5] Campione, S., L. K. Warne, L. I. Basilio, C. D. Turner, K. L. Cartwright, and K. C. Chen, "Electromagnetic pulse excitation of finite- and infinitely-long lossy conductors over a lossy ground plane," Journal of Electromagnetic Waves and Applications, Vol. 31, 209-224, 2017.
- [6] Warne, L. K. and K. C. Chen, "Long line coupling models," Sandia National Laboratories Report, Vol. SAND2004-0872, Albuquerque, NM, 2004.
- [7] Baum, C. F., "Effect of corona on the response of infinite-length transmission lines to incident plane waves," AFWL Interaction Note, Vol. 443, February 6, 1985.
- [8] Lee, K. S. H., "Electromagnetic coupling to transmission lines," Electromagnetics, Vol. 8, 107-124, January 1, 1988.
- [9] Campione, S., L. K. Warne, L. I. Basilio, C. D. Turner, K. L. Cartwright, and K. C. Chen, "Electromagnetic Pulse Excitation of Finite- and Infinitely-Long Lossy Conductors Over a Lossy Ground Plane," Journal of Electromagnetic Waves and Applications, Vol. 31, 209-224, 2017.
- [10] EPRI, "Transmission line reference book", Second edition (revised), 537.12 Tr69 1982.
- [11] C. A. Jordan "Lightning computations for transmission lines with overhead ground wires, Part II," Gen. Electr. Rev, vol.37, no pp.180-186,1934.
- [12] H. Takahashi, "Confirmation of the error of Jordan's formula on tower surge impedance," Trans. Inst. Elect. Eng. Jpn., vol. 114-B, pp. 112-113, 1994 (in Japanese).
- [13] S. Visacro and F. H. Silveira, "Guide to procedures for estimating lightning performance of transmission lines," CIGRE SC33-WG01, Tech. Brochure, Oct. 1991.
- [14] J. A. Gutierrez, R., J. L. Naredo, J. L. Bermudez, M. Paolone, C. A. Nucci, and F. Rachidi, "Nonuniform Transmission Tower Model for Lightning Transient Studies," IEEE Transactions on Power Delivery, vol 19, no. 2, April 2004.
- [15] A. Semlyen and A. Deri, "Time domain modeling of frequency dependent three-phase transmission line impedance," IEEE Trans. Power App. Syst., vol. PAS-104, pp. 1549-1555, June 1985.
- [16] Juan A. Martinez, and Ferley Castro-Aranda, "Tower Modeling for Lightning Analysis of Overhead Transmission Lines", IEEE Power Engineering Society General Meeting, San Francisco, Ca, USA, 2005.
- [17] L. Greev and M. Popov, "On high-frequency circuit equivalents of a vertical ground rod," IEEE Trans. Power Del., vol. 20, no. 2, pp. 1598-1603, Apr. 2005.

## VIII. APPENDIX

In the evaluation of the attenuation induced by the line-towers, we will use the E1 MIL-STD as a HEMP excitation. The E1 MIL-STD pulse is defined as

$$E^{inc} = E_0 K (e^{-\alpha t} - e^{-\beta t}) u(t), \quad (A.1)$$

with parameters  $\alpha = 4 \times 10^7 s^{-1}$ ,  $\beta = 6 \times 10^8 s^{-1}$ ,  $E_0 = 50 kV/m$

and  $K = 1/(e^{-\alpha_{max}} - e^{-\beta_{max}})$  with  $t_{max} = \log(\beta/\alpha)/(\beta - \alpha)$ , and  $u(t) = 1$  for  $t \geq 0$ . Its spectrum given by (with  $\exp(-i\omega t)$  time dependence)

$$E(\omega) = \int_{-\infty}^{\infty} E(t) e^{i\omega t} dt = \frac{E_0 K (\beta - \alpha)}{(\alpha - i\omega)(\beta - i\omega)} \quad (A.2)$$