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Reduced-physics model with symbolic regression using Bingo for fatigue life prediction of welds in shell structures.

Keven Carlson, Dr. Jacob Hochhalter, Dr. John Emery

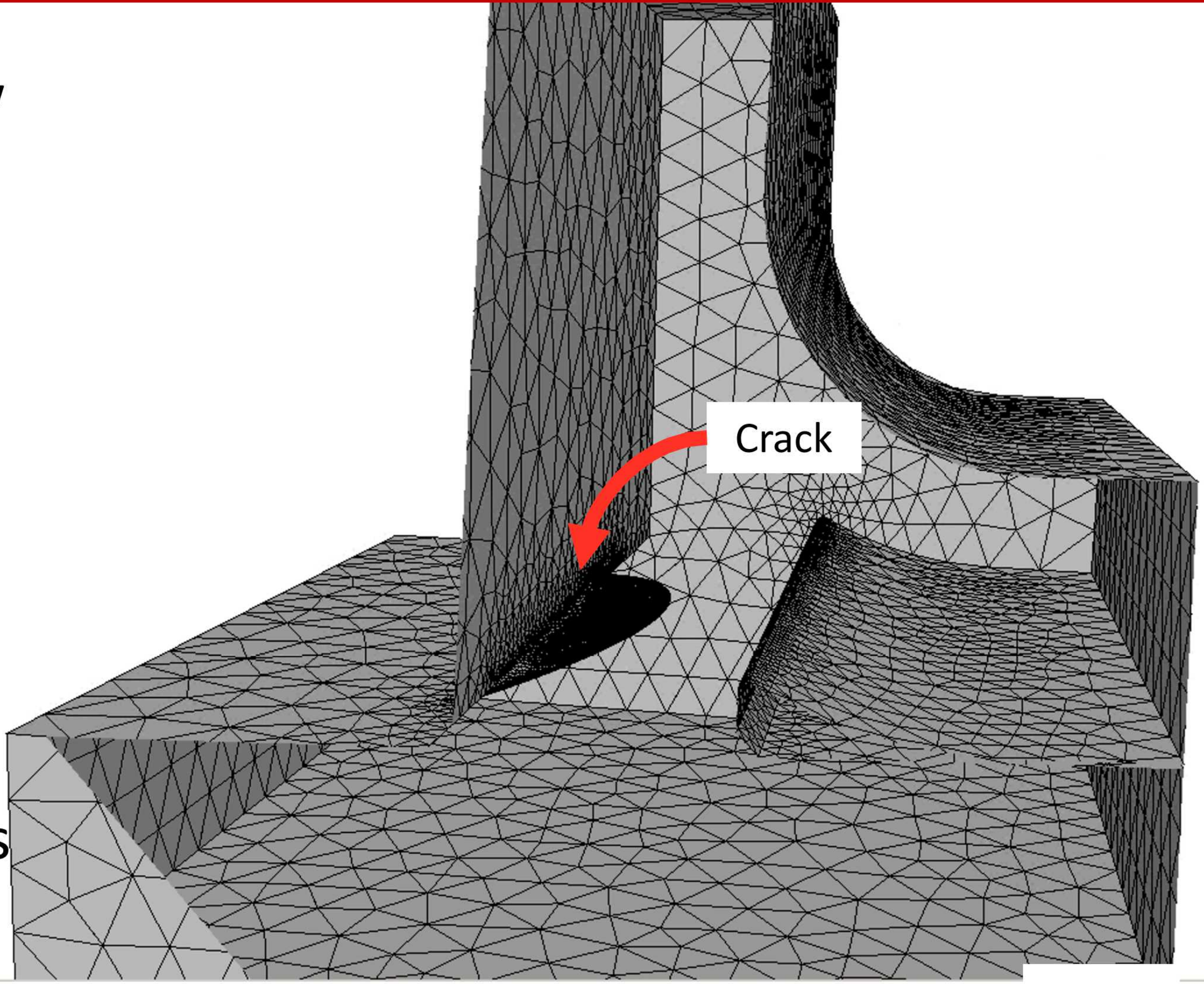
- Problem Definition and Overview
- Finite Element Model
- Stress Intensity Factor Computation
- Symbolic Regression
- Results
- Conclusion

Problem Definition

- Stress Intensity Factors (SIFs) drive crack growth.
- SIFs are accurately calculated in 3-D continuum element models
  - Computationally expensive
- Shell element models are computationally cheaper, but cannot model 3-D crack growth.
- "Line-weld" elements are used in shell element models to represent welds.
- We seek a low-fidelity crack growth model that maps stresses in shell elements to SIFs using Symbolic Regression (SR).

FRANC3D

1. FRANC3D used to insert and grow cracks into model mesh.
2. Reads displacements and material properties from Abaqus
3. Uses M-integral to compute SIFs
4. SIF results along crack front determine crack kink angle
5. Cracks are propagated in kink angle direction
6. Repeat steps 2-5 until crack grows through 80% of thickness



Inside of local model into which a crack has been inserted and gro

Department of MECHANICAL ENGINEERING
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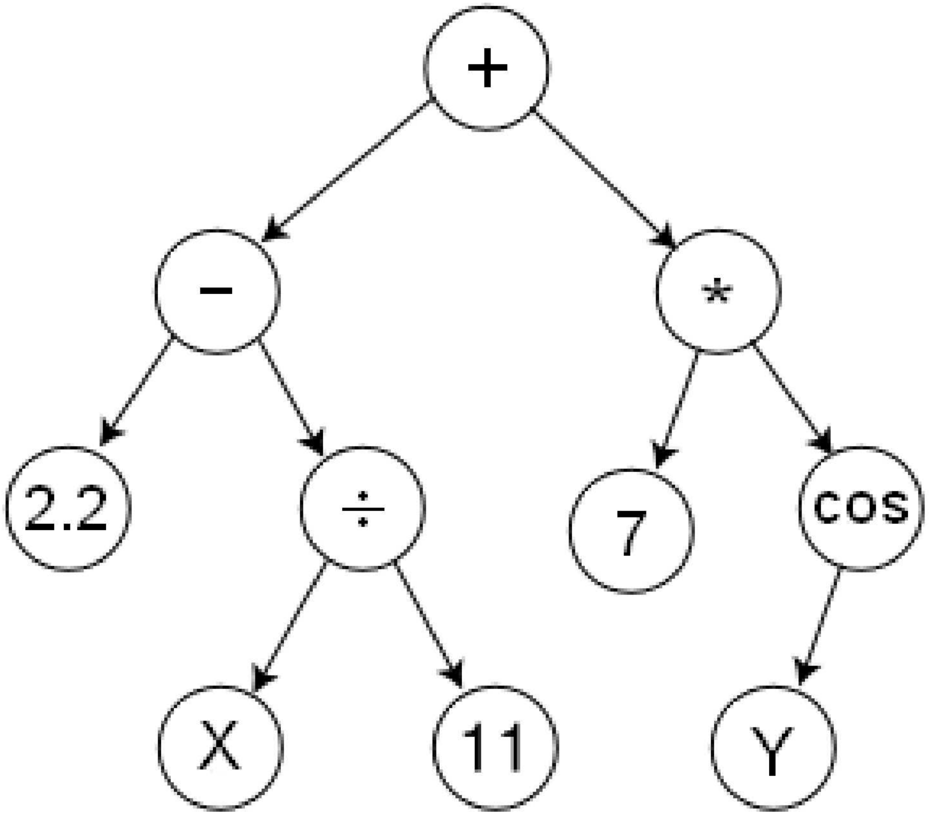
Symbolic Regression

- **Machine Learning:** Computers using data to create predictive models
- **Genetic Programming:** Evolution of computer programs based on fitness
- **Symbolic Regression:** Searches space of mathematical functions to fit equation to inputted data

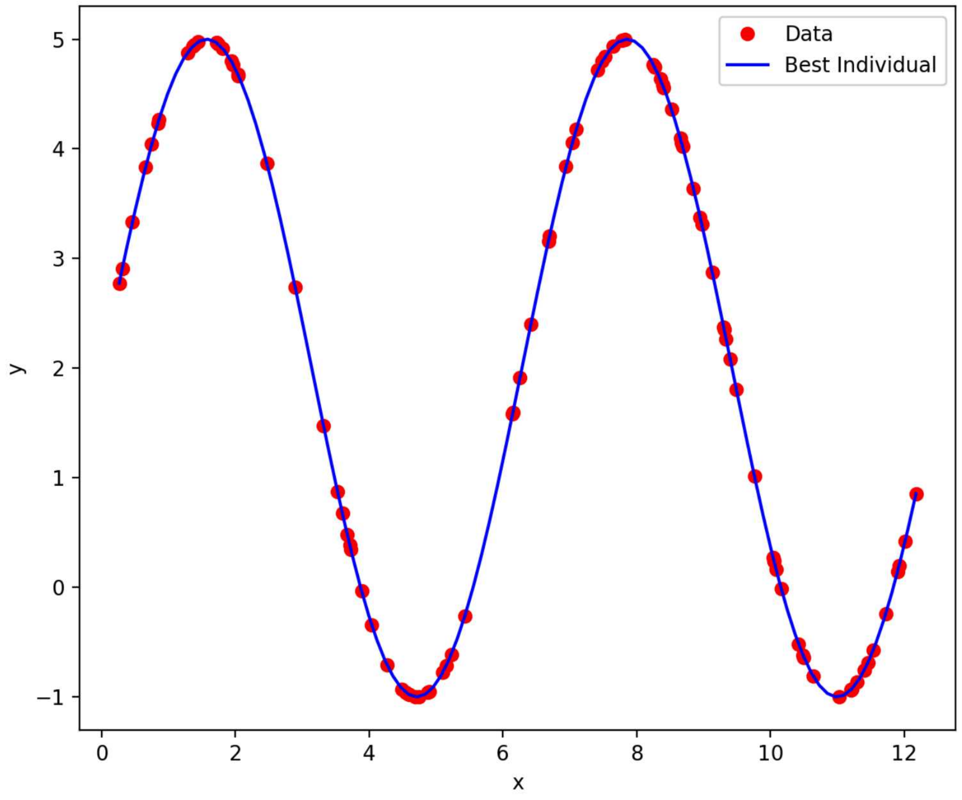


$$\left(2.2 - \left(\frac{X}{11}\right)\right) + \left(7 * \cos(Y)\right)$$

Simple Example

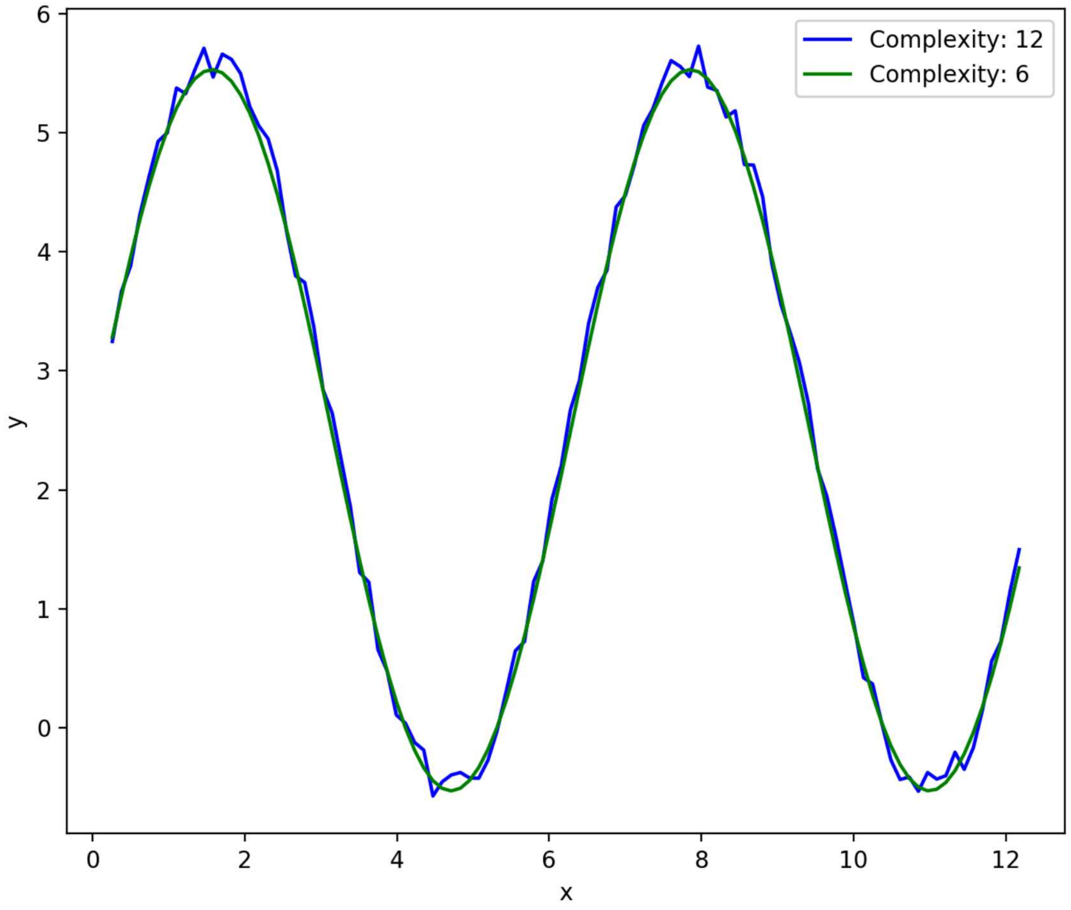
- Data artificially generated from equation $f(x) = 3 * \sin(x) + 2$
- Allowed equation operators: +, –, x, ÷, sin, cos
- Trained bingo until error tolerance < 1e-4, stack size = 40, population size = 100, max generations = 50
- Output equation:

$$f(x) = 3 * \sin(x) + 2$$



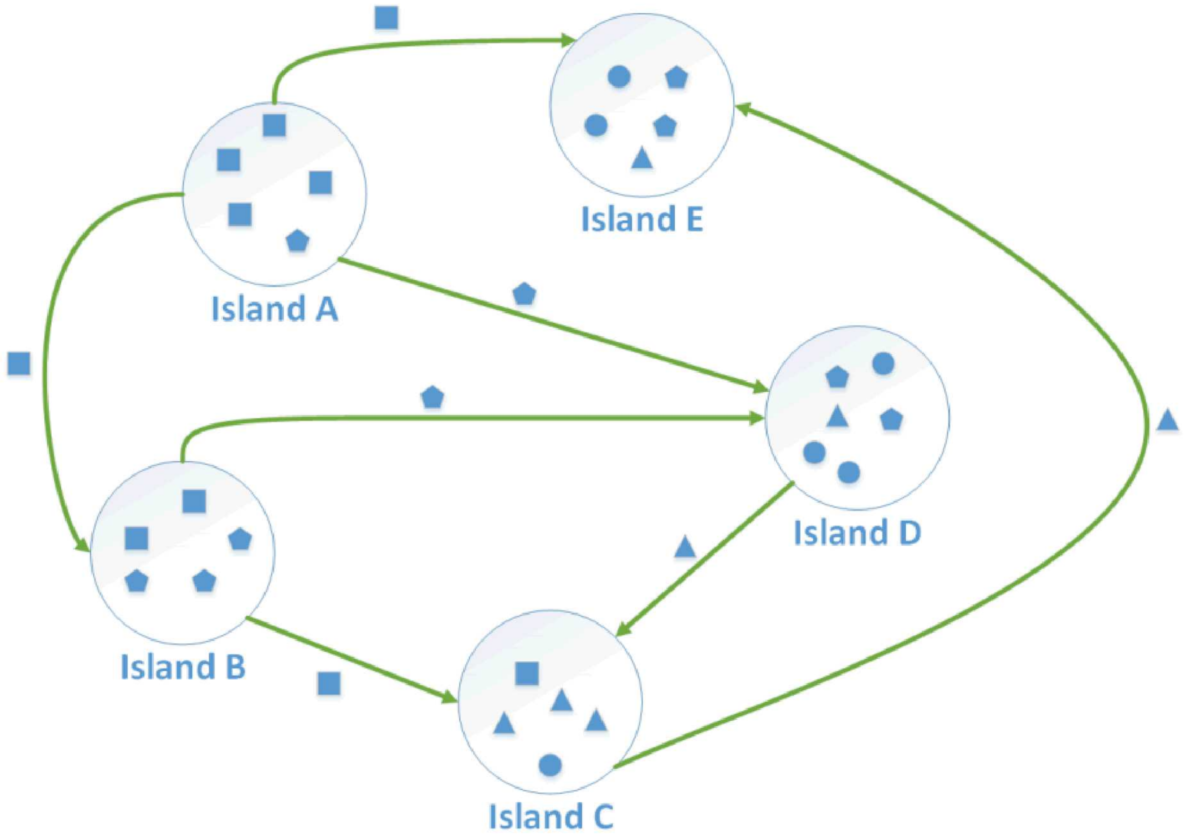
Pareto Front

- Experimental data
  - Possible noise or small error
- Overfitting
  - Model fits equation to noise
  - Not generalization
  - Performs poorly on test data
- Illustrated when noise is randomly added to data
- Tradeoff between complexity and interpretability



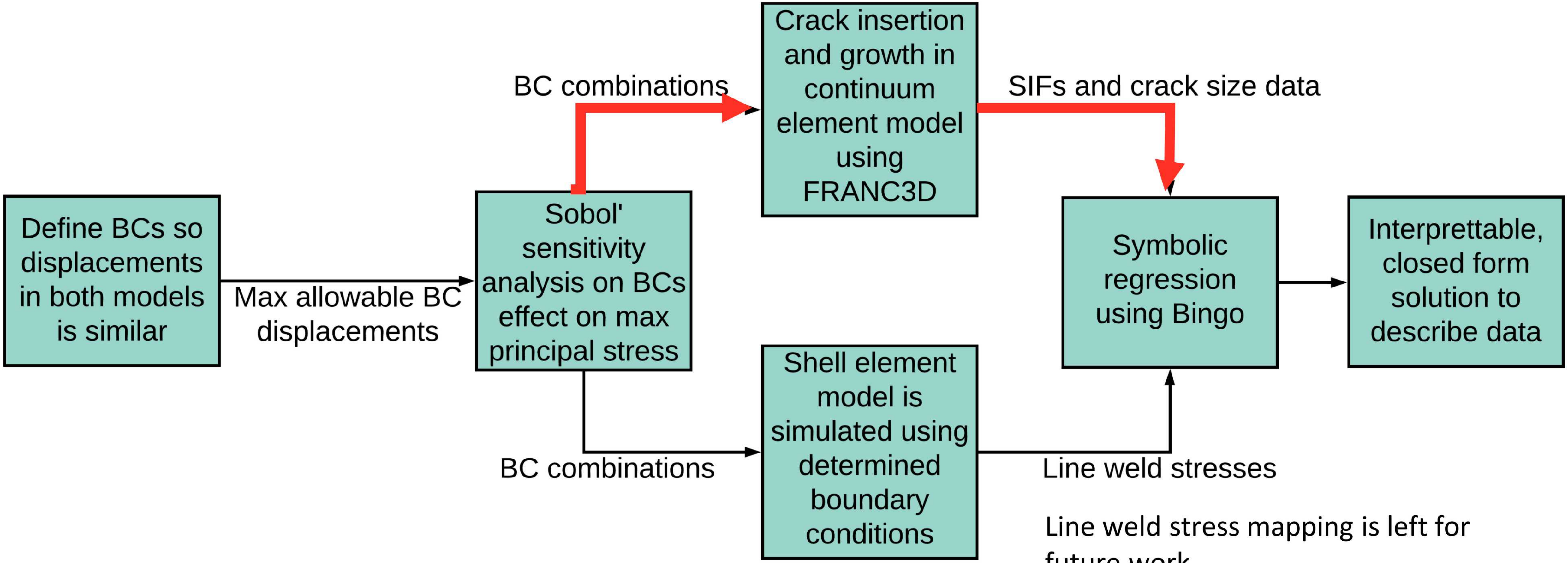
Bingo

- NASA-produced, open-source symbolic regression software using evolutionary algorithms
- Parallel island evolution strategy can be used with MPI
- Island evolution strategy allows for periodic migration of models between islands



Dynamic Island Model Base on Spectral Clustering in Genetic Algorithm
(Qinxue Meng et al. 2018)

Overview



Line weld stress mapping is left for future work

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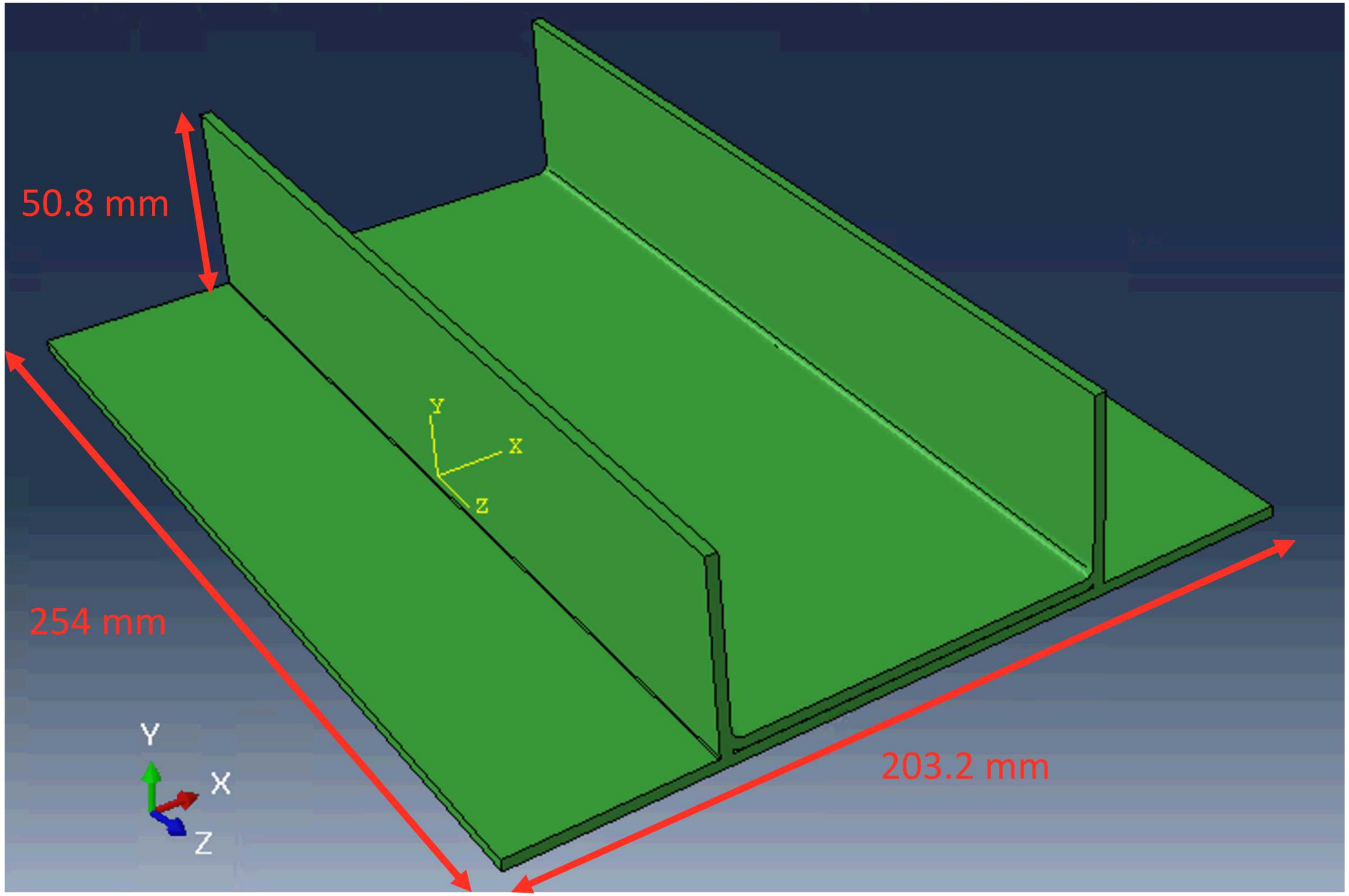
Finite Element Model Geometry



- C-Channel:
  - Welded to a plate
  - Flange length: 50.8 mm in y-direction
  - Constant thickness of 3.048 mm
- Plate:
  - 254 mm in z-direction
  - 203.2 mm in x-direction
  - 3.048mm in y-direction
- Weld Thickness: 3.048mm
- Idealized as one solid material with no heat-affected zone from weld

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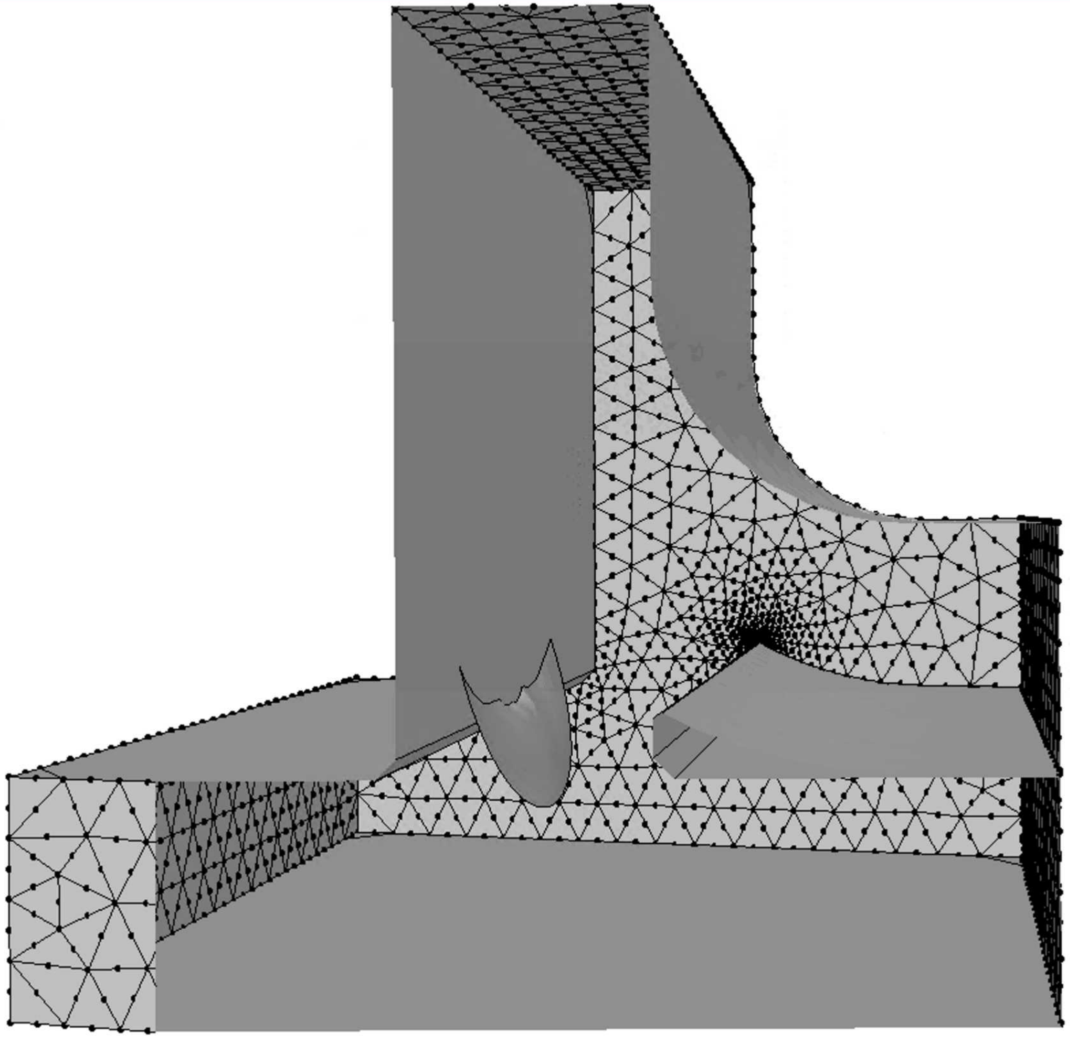
Training Data Generation

- 180 different BC combinations
  - Number of discrete displacements applied to surfaces dependent on sensitivity
- 0.05 mm initial crack size
  - Much lower than what non destructive evaluation would detect
- Crack inserted perpendicular to MPS at location on weld surface with greatest MPS
- Crack grown to 80% thickness of weld or about 2.4 mm at crack steps of 0.24 mm
  - Smaller crack step sizes show minimal difference on output
- Important values used in symbolic regression model: SIF, MPS, crack geometry, initial crack orientation, crack growth path

Note:

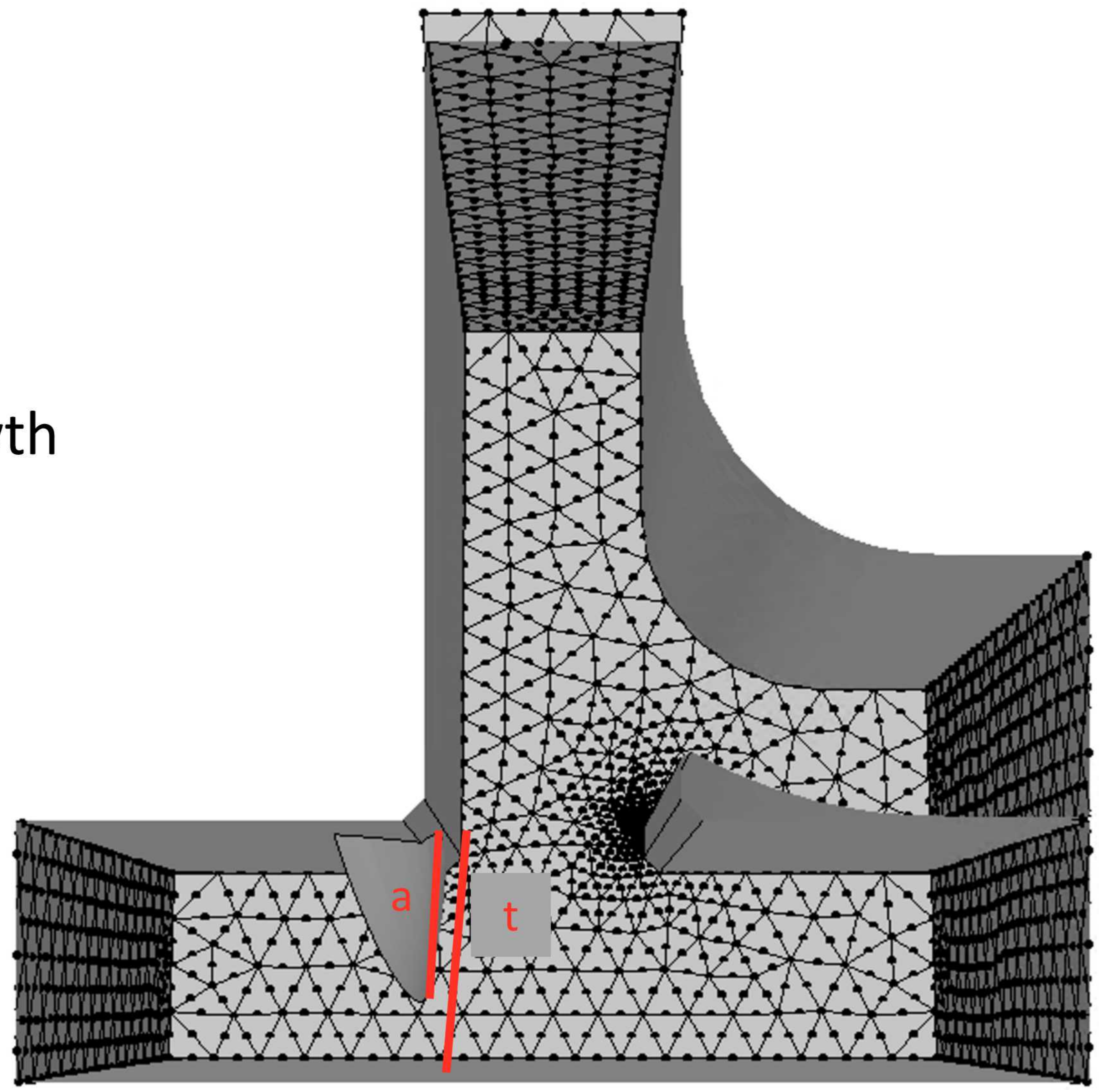
- Some cracks grow downward into the plate
  - Not likely behavior in practice because of the weld boundary
- The crack growth path
  - Plane fit to points of all crack fronts after initial crack insertion
- Thickness
  - Defined as the distance from crack insertion to where the crack would break through the surface



Downward growing crack.

Different Crack Growth Directions

- Horizontal Crack Growth
- Downward Horizontal Crack Growth



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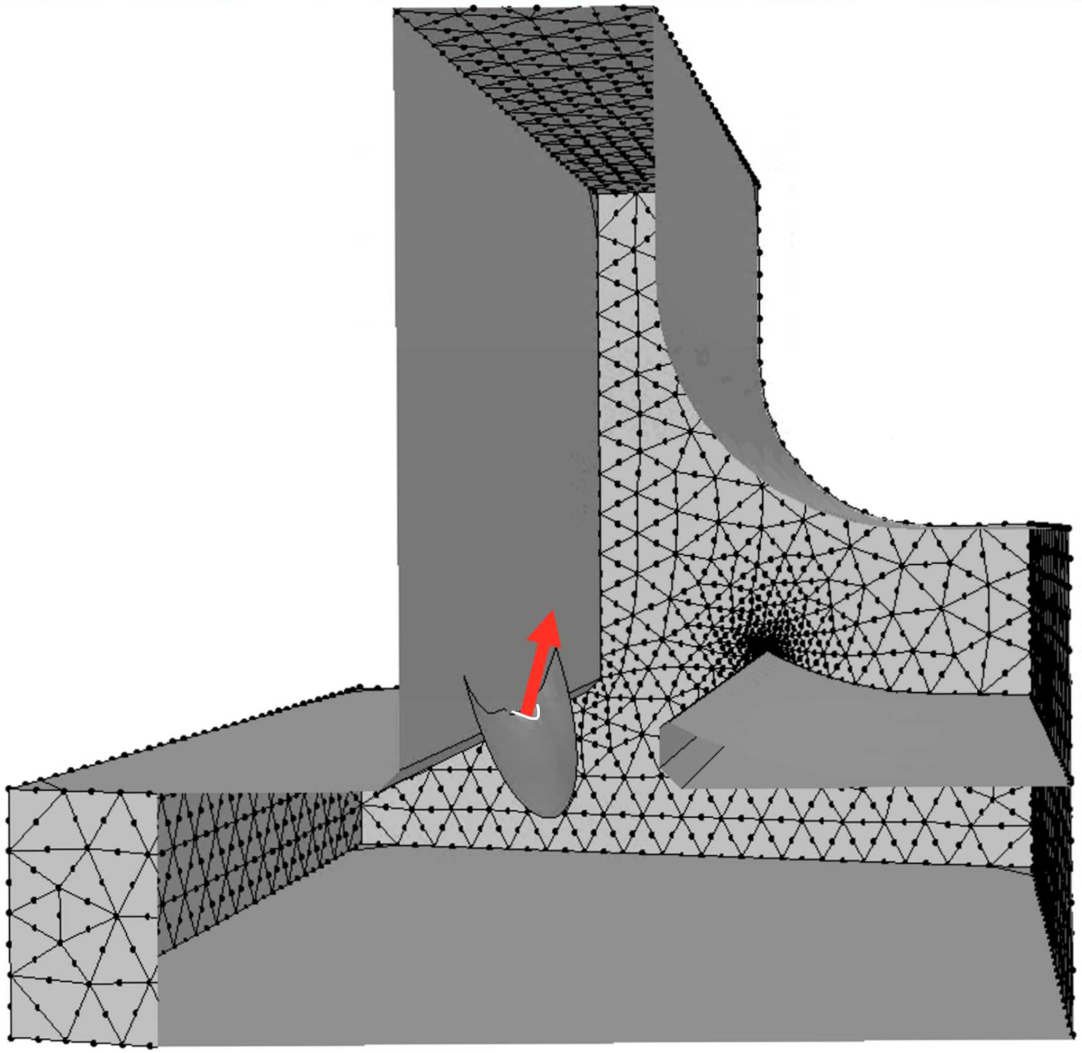
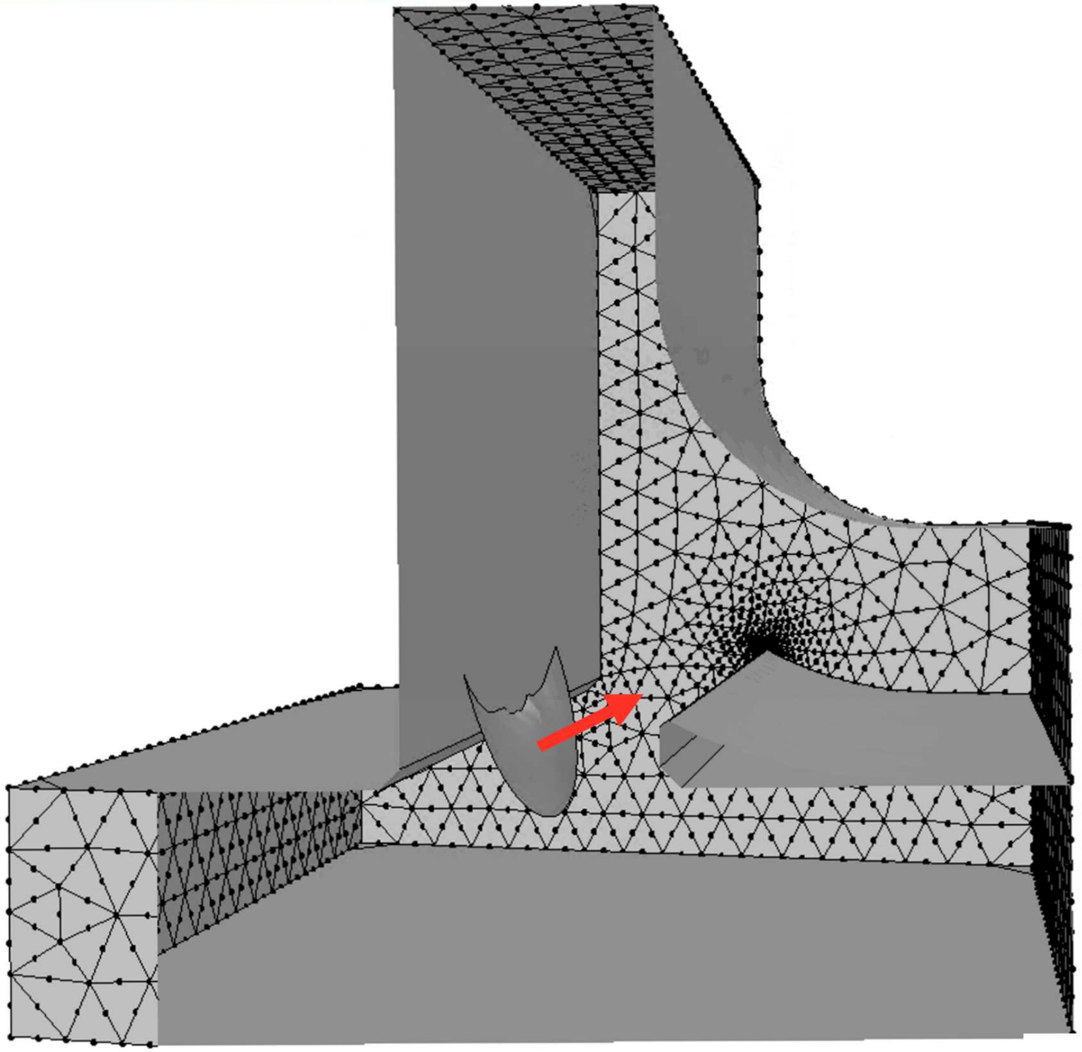
Bingo Model For SIFs

- $\frac{K_I}{\sigma\sqrt{\pi * a}} = f\left(\frac{a}{c}, \frac{a}{t}, \hat{n}_1, \hat{n}_2\right)$
- a: crack depth
- c: half crack width
- t: thickness relative to crack growth path
- σ: maximum principal stress at point of crack insertion
- $\hat{n}_1 = \langle x, y, z \rangle$
- $\hat{n}_2 = \langle x_2, y_2, z_2 \rangle$
- $\hat{n}_1$: normal unit vector of crack at insertion
- $\hat{n}_2$: normal unit vector of plane fit through all crack front points

Normal vectors



$\hat{n}_1$



$\hat{n}_2$

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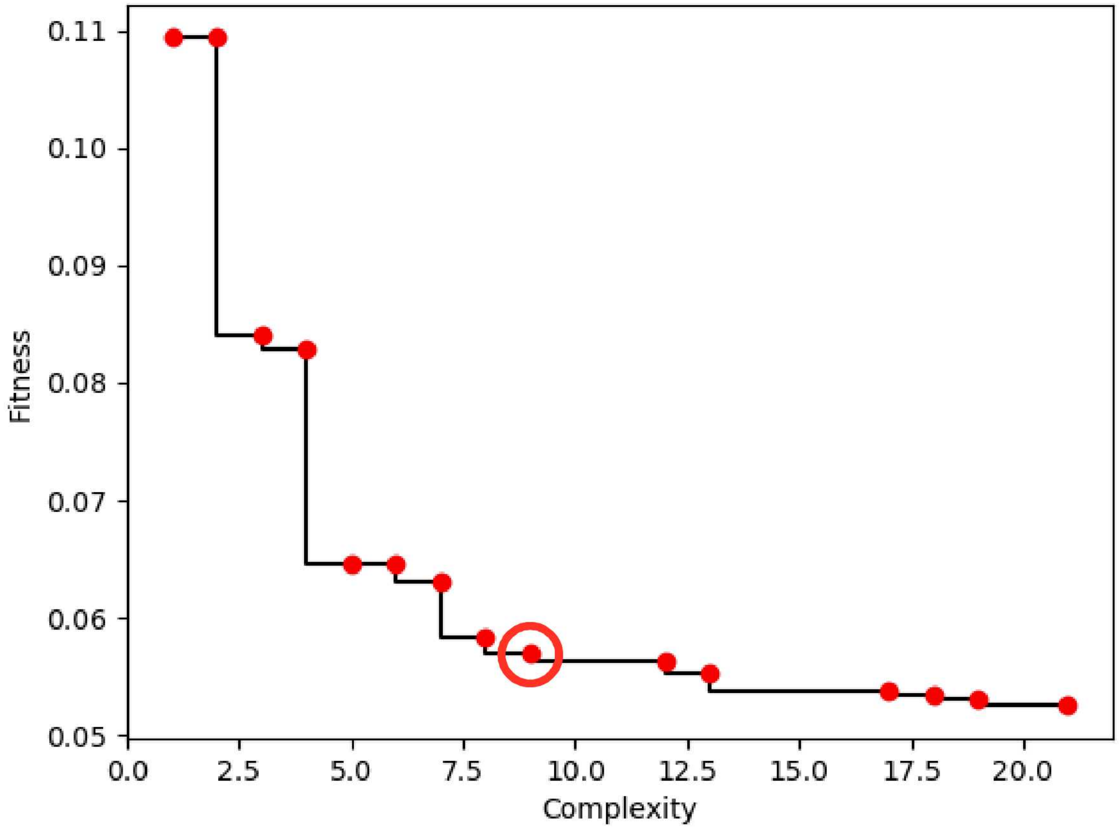
Symbolic Regression

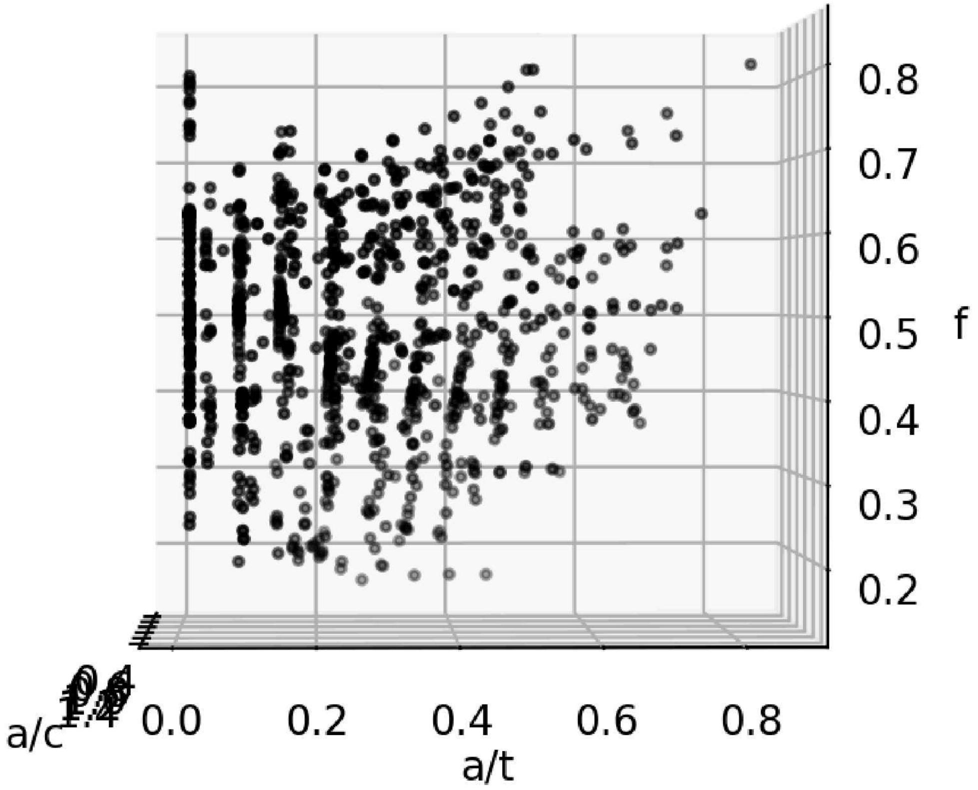
- Model from Pareto front with operations that made sense
  - i.e. some models had equations with components of the form x^x
- Of the lower complexity models, performed best on test dataset
- Mean absolute error: 0.0543

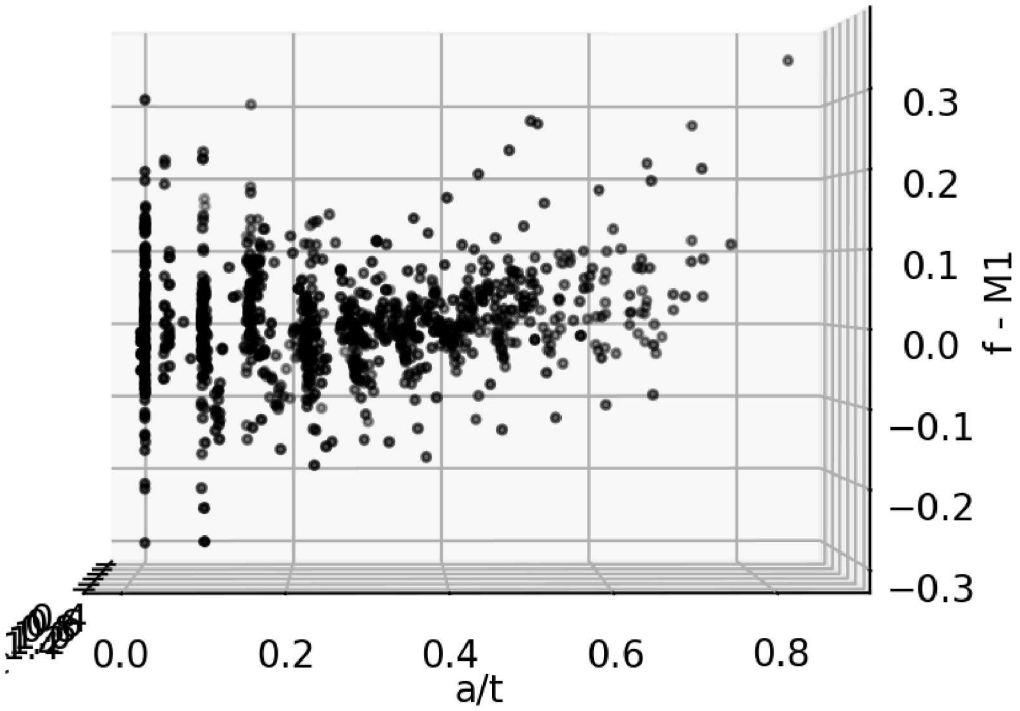


Gradient Boosting

- Boosting
  - Using an ensemble of ”weak” learners that together create a strong one.
- Gradient Boosting
  - Each ”weak” learner is found from the error of the previous learner
  - Prediction $\square$ is found with: $\square = \sum_{m=1}^{M} \square_m(\square)$ where $\square_m(\square)$ is each function found through symbolic regression using the results from $\square_m(\square) = \square - \sum_{m=1}^{m-1} \square_m(\square)$.

Subtract First Model





- Becomes more correlated with a/t

Second and Third Models

- Second Model
  - Trained on residual error of f_1 on train dataset.
  - Similar selection method as first model
  - Fitness: 0.0482

$$\rho = |y|^{\frac{a}{t}}$$

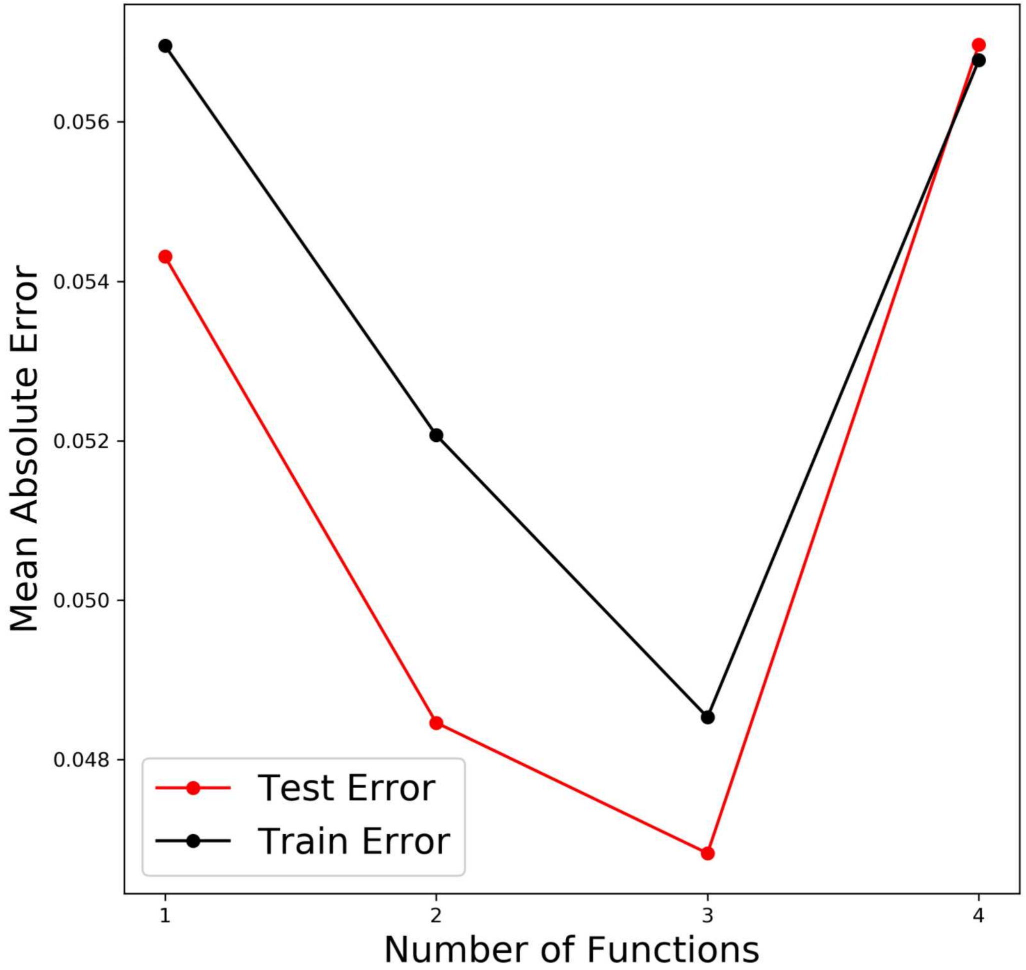
$$f_2 = -0.1657\,\rho\sqrt{|z|} - 0.1657(\rho(\rho * \rho^{\rho})^{y})^{0.25} + 0.176$$

- Third Model
  - Trained on residual error of $f_1 + f_2$
  - Fitness: 0.0466

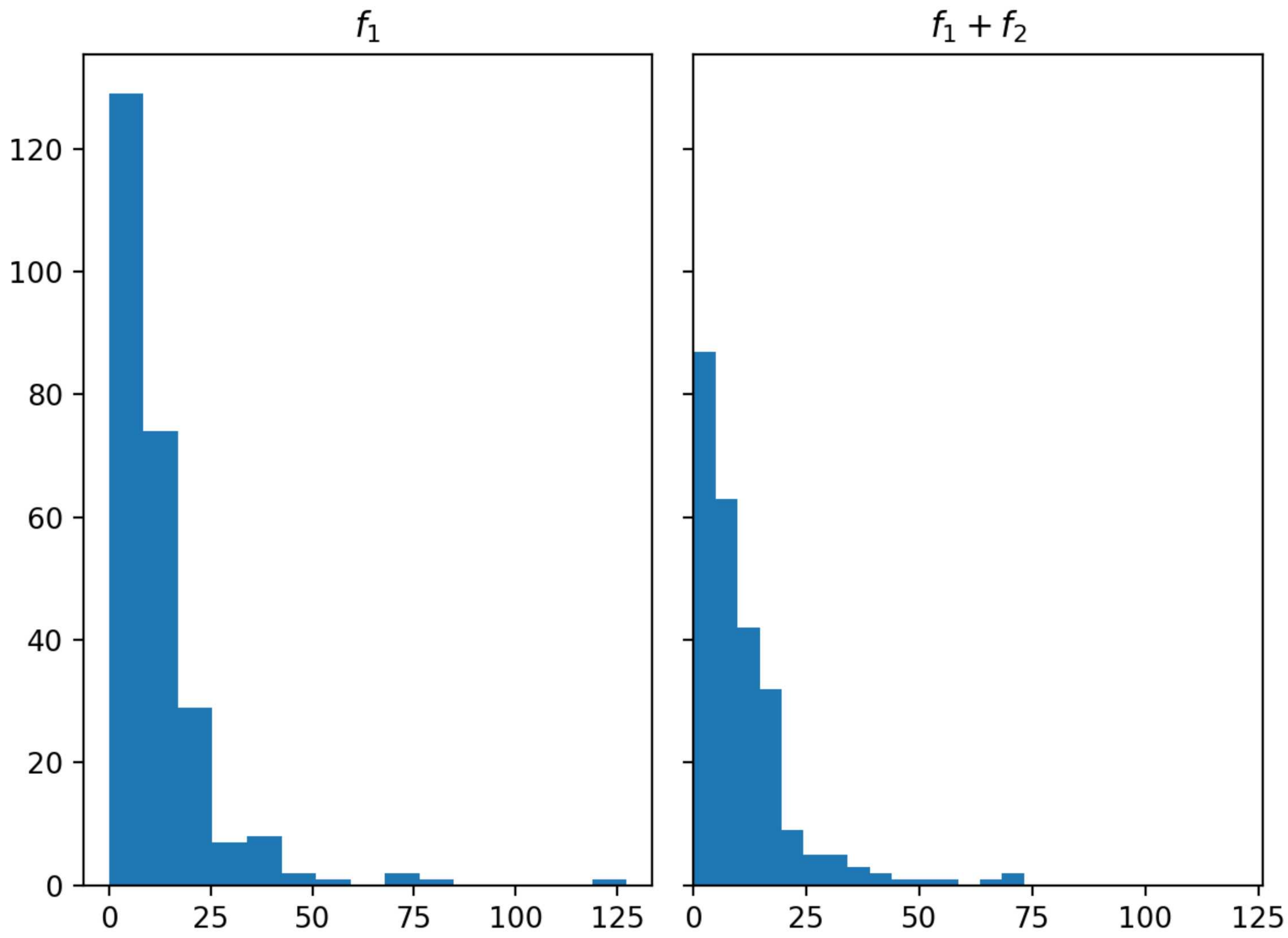
$$f_3 = \frac{z\sqrt{|y|}(z_2 - 0.933)}{z(z-1)+1}$$

Check For Convergence

- Fourth model did not improve overall model.
- Test error is below training error, may be a result of fewer datapoints and thus fewer chances of large outliers.
- 80% train, 20% test randomly split



Removing Outliers



$f_1 = \frac{a}{c}$ … $+ 0.141) + 0.159$

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Future Implications

- Raju-Newman Semi-Elliptical Surface Problem
- Gradient Boosting with SR can automate this process

$$F_s = \left[M_1 + M_2\left(\frac{a}{t}\right)^2 + M_3\left(\frac{a}{t}\right)^4\right] g f_\phi f_w$$

For $a/c \leq 1$

$$M_1 = 1.13 - 0.09\left(\frac{a}{c}\right)$$

$$M_2 = -0.54 + \frac{0.89}{0.2 + \left(\frac{a}{c}\right)}$$

$$M_3 = 0.5 - \frac{1}{0.65 + \frac{a}{c}} + 14\left(1 - \frac{a}{c}\right)^{24}$$

$$g = 1 + \left[0.1 + 0.35\left(\frac{a}{t}\right)^2\right](1 - \sin \phi)^2$$

Conclusion

- Symbolic regression can give interpretable results of how each input affects the output
- Models for SIFs given MPS and crack geometry found to be much easier for SR than line-weld stresses to MPS
- Gradient boosting can be used with SR to improve the overall model when more complex equations are expected
- Use gradient boosting with SR to revisit classical fracture mechanics SIF solutions
- Simpler geometry could be beneficial for finding physically interpretable results

Thank You

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