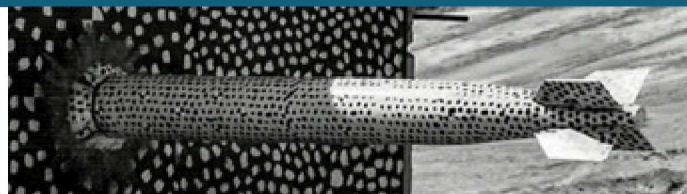


WEC Co-design September 2020 webinar



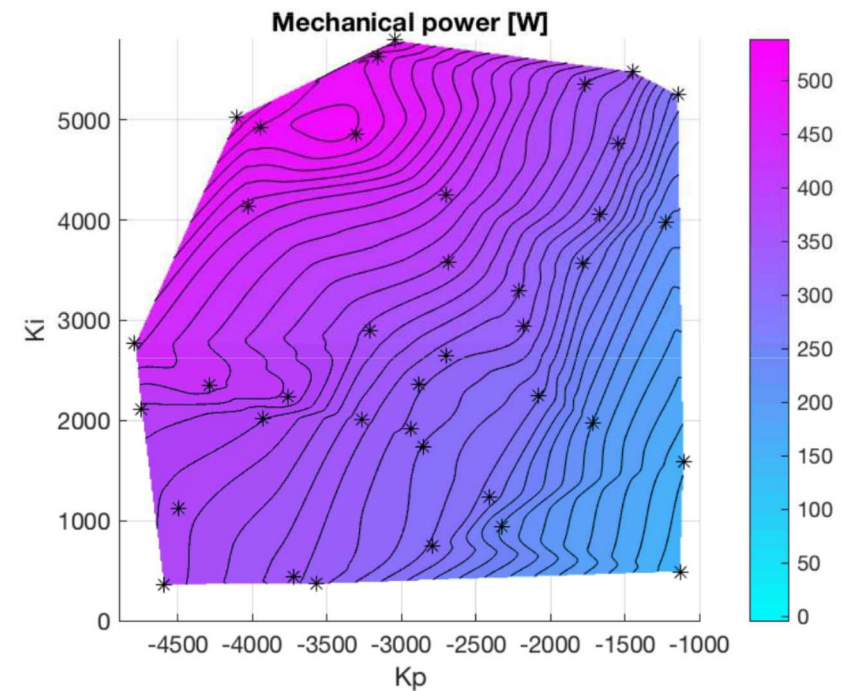
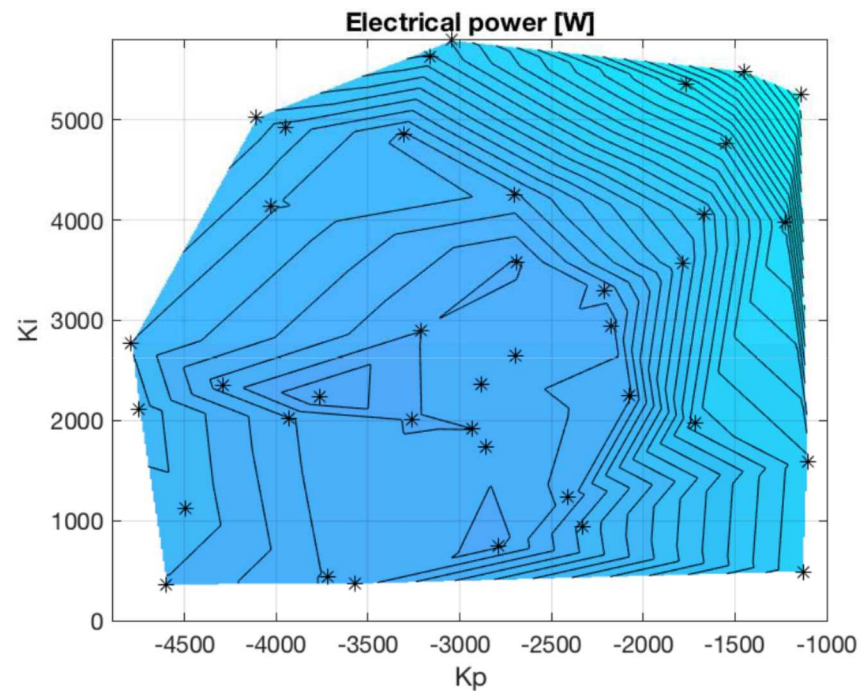
Presented by

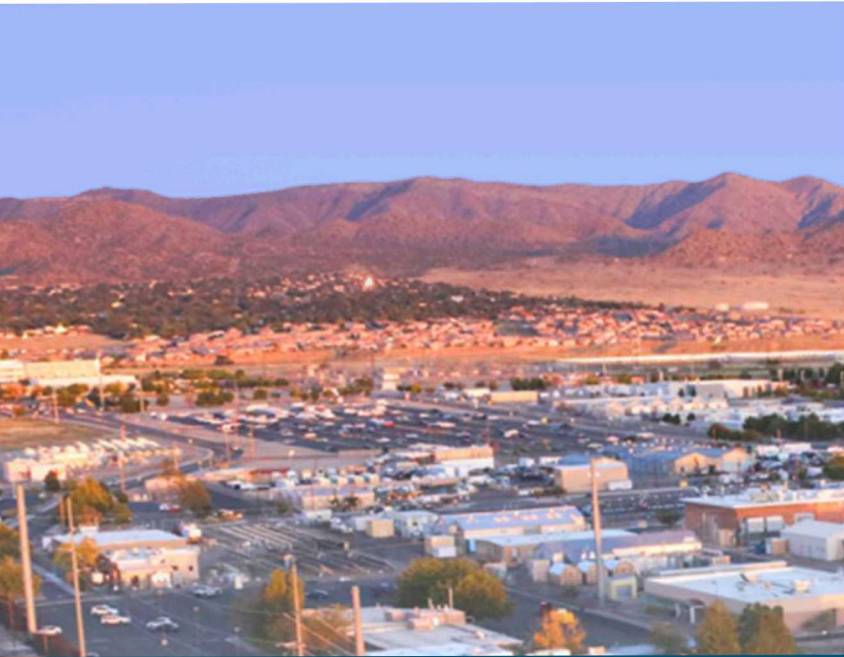
Ryan Coe, Giorgio Bacelli, Dominic Forbush



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- I. Background
- II. Problem statement and approach
- III. Examples in MATLAB





Background



Causal impedance matching approach

Why work in the frequency domain?

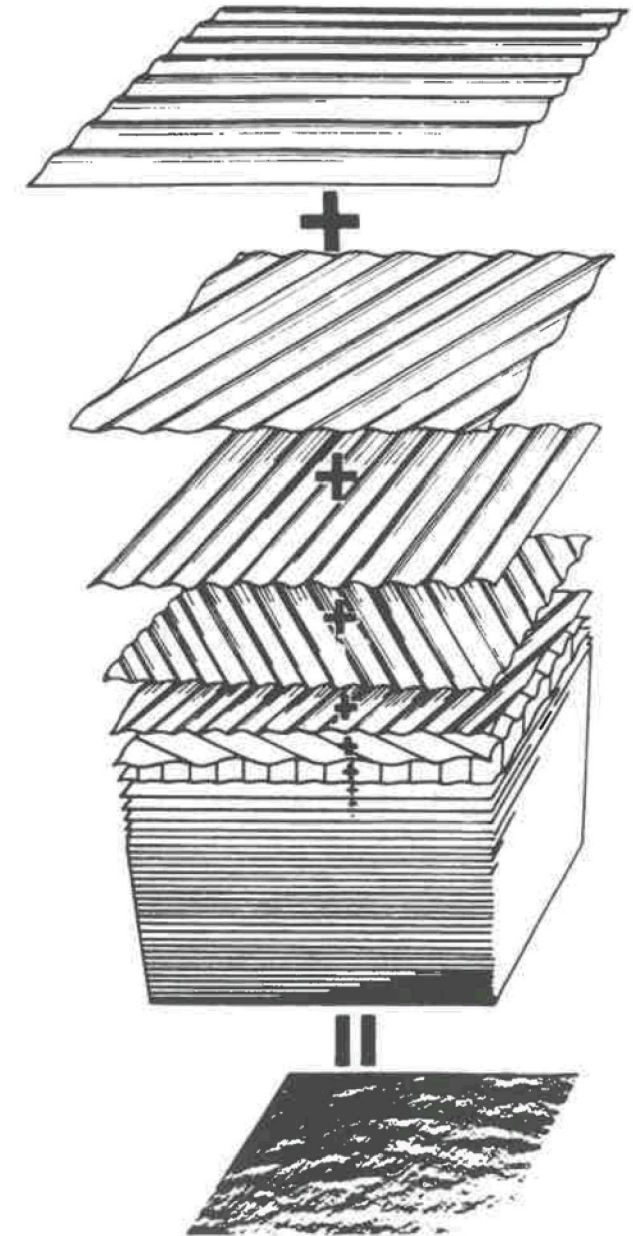
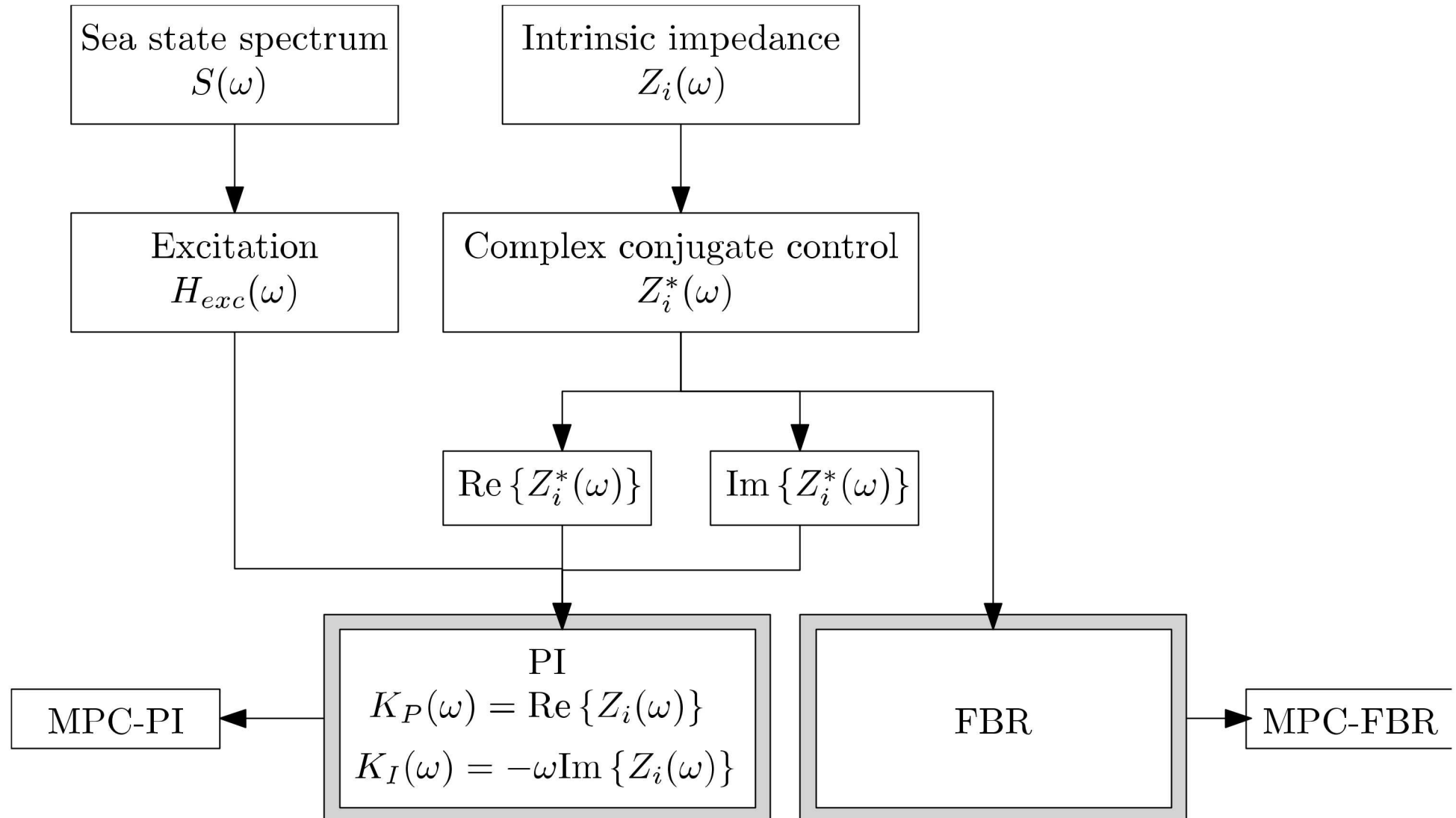


Fig. 1.3. Structure of random sea (Pierson *et al.*, 1958).

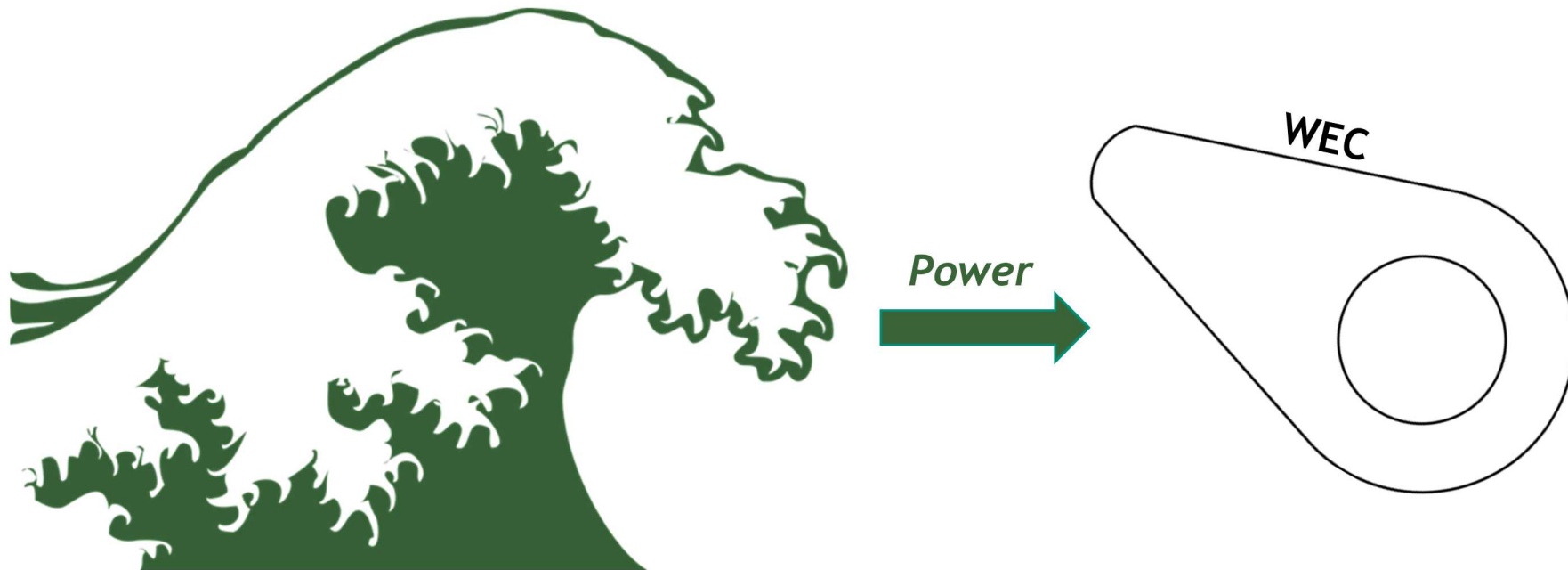
WEC control hierarchy



6 Impedance matching

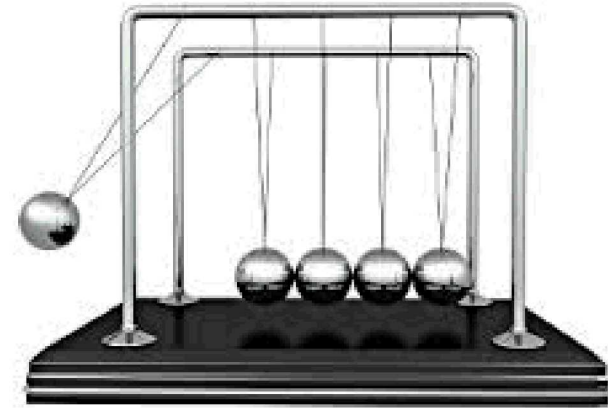
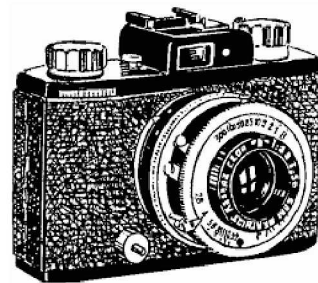
Maximum power transfer (from waves to PTO)

$$Z_{source} = Z_{load}^*$$



7 Impedance matching

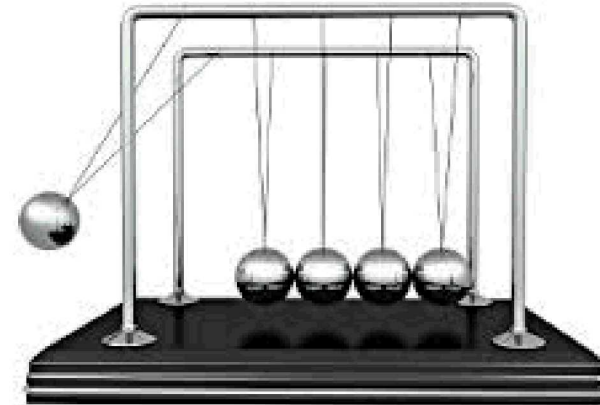
Where else is this used?



8 Impedance matching

●	●●
●	●●
●	●●
●	●●
●	●●
●	●●
●	●●
●	●●

<http://www.lockhaven.edu/~dsimanek>



$$Z_i(\omega) = i\omega \underbrace{(M + m(\omega))}_{\text{mass}} + \underbrace{B_v + R(\omega)}_{\text{damping}} + \underbrace{\frac{S}{i\omega}}_{\text{stiffness}}$$

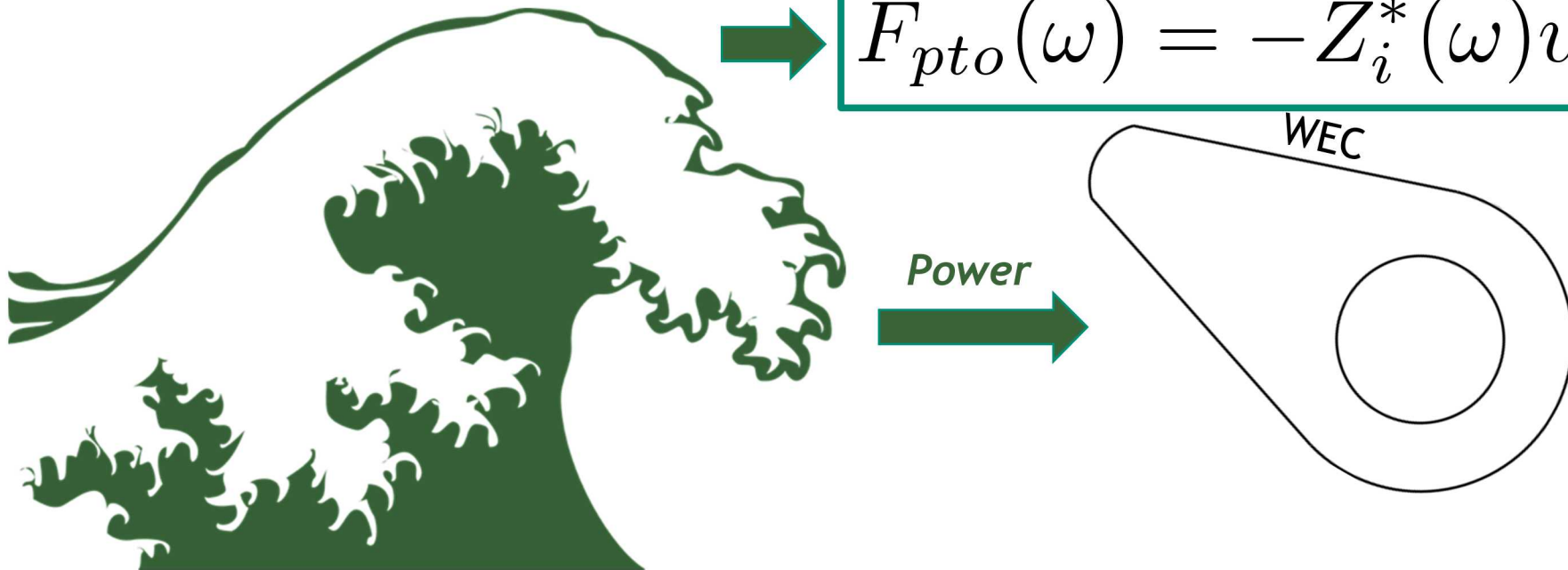
$$\begin{aligned} F_u(\omega) &= f(v(\omega)) \\ &= -Z_u(\omega)v(\omega) \end{aligned}$$

Maximum power transfer (from waves to PTO)

$$Z_{source} = Z_{load}^*$$

$$Z_i = Z_u^* \rightarrow (Z_i^* = Z_u)$$

$$F_{pto}(\omega) = -Z_i^*(\omega)v(\omega)$$

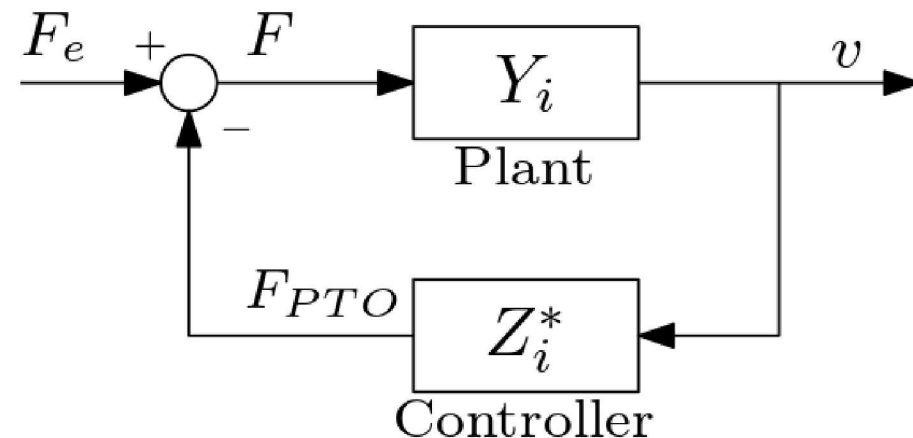


Maximum power transfer (from waves to PTO)

$$Z_{source} = Z_{load}^*$$

$$Z_i = Z_u^* \rightarrow (Z_i^* = Z_u)$$

$$\Rightarrow F_{pto}(\omega) = -Z_i^*(\omega)v(\omega)$$



Acausality - Let's get things straight

- Two components of noncausality
 - **Excitation:** *matter of perspective*
 - **Complex conjugate control:** *A common problem seen in other fields*

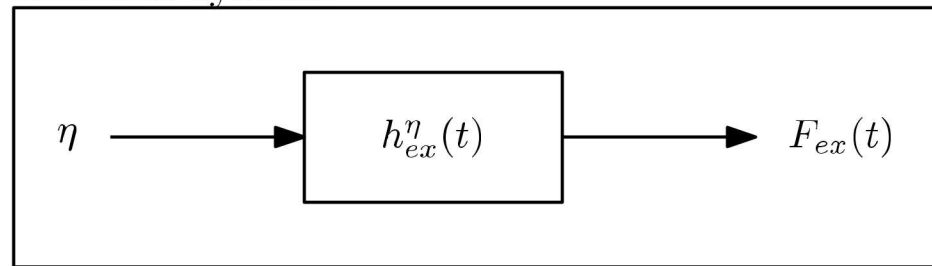
Causal: $y_{\text{now}} = f(t \leq \text{now})$

Acausal: $y_{\text{now}} = f(0 \leq t \leq \text{tomorrow})$

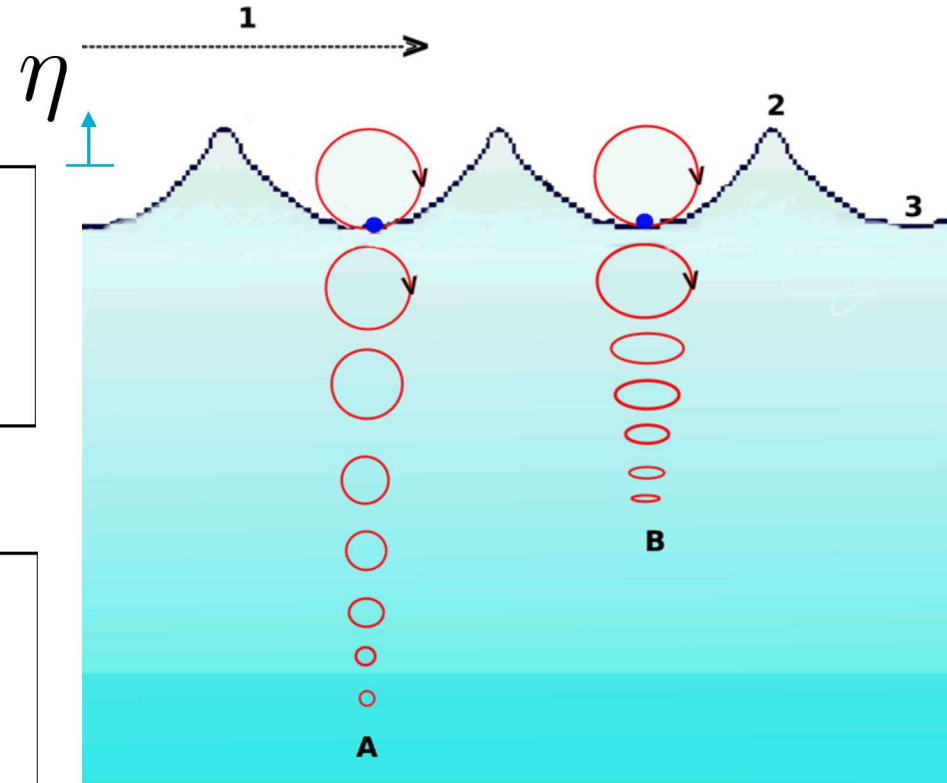
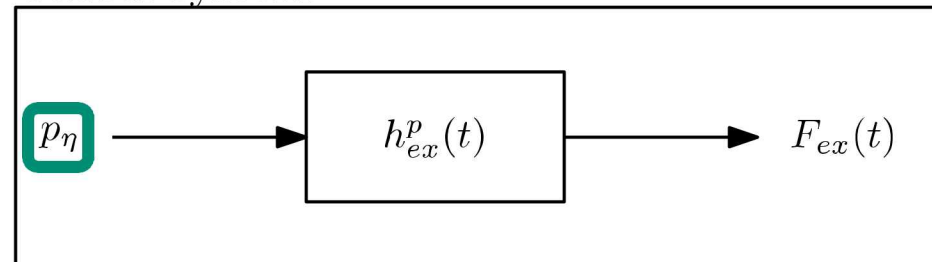
Acausality – excitation (cont.)

- Two components of noncausality
 - Excitation:** *matter of perspective*
 - Complex conjugate control:** *A common problem seen in other fields*

Acausal system



Causal system



Acausality – impedance matching

- Two components of noncausality
 - Excitation: *matter of perspective*
 - Complex conjugate control: *A common problem seen in other fields*

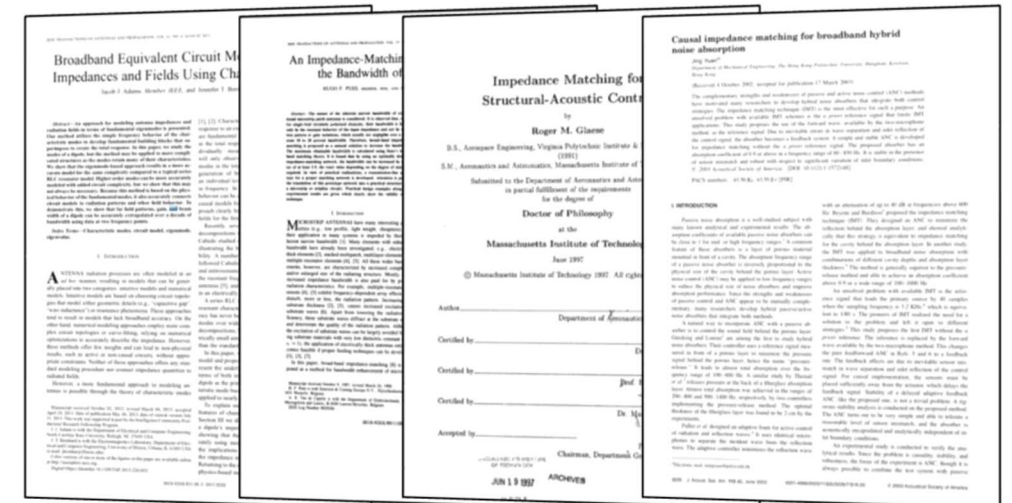
The complex conjugate of any causal system must be acausal

(proof left for your free time)



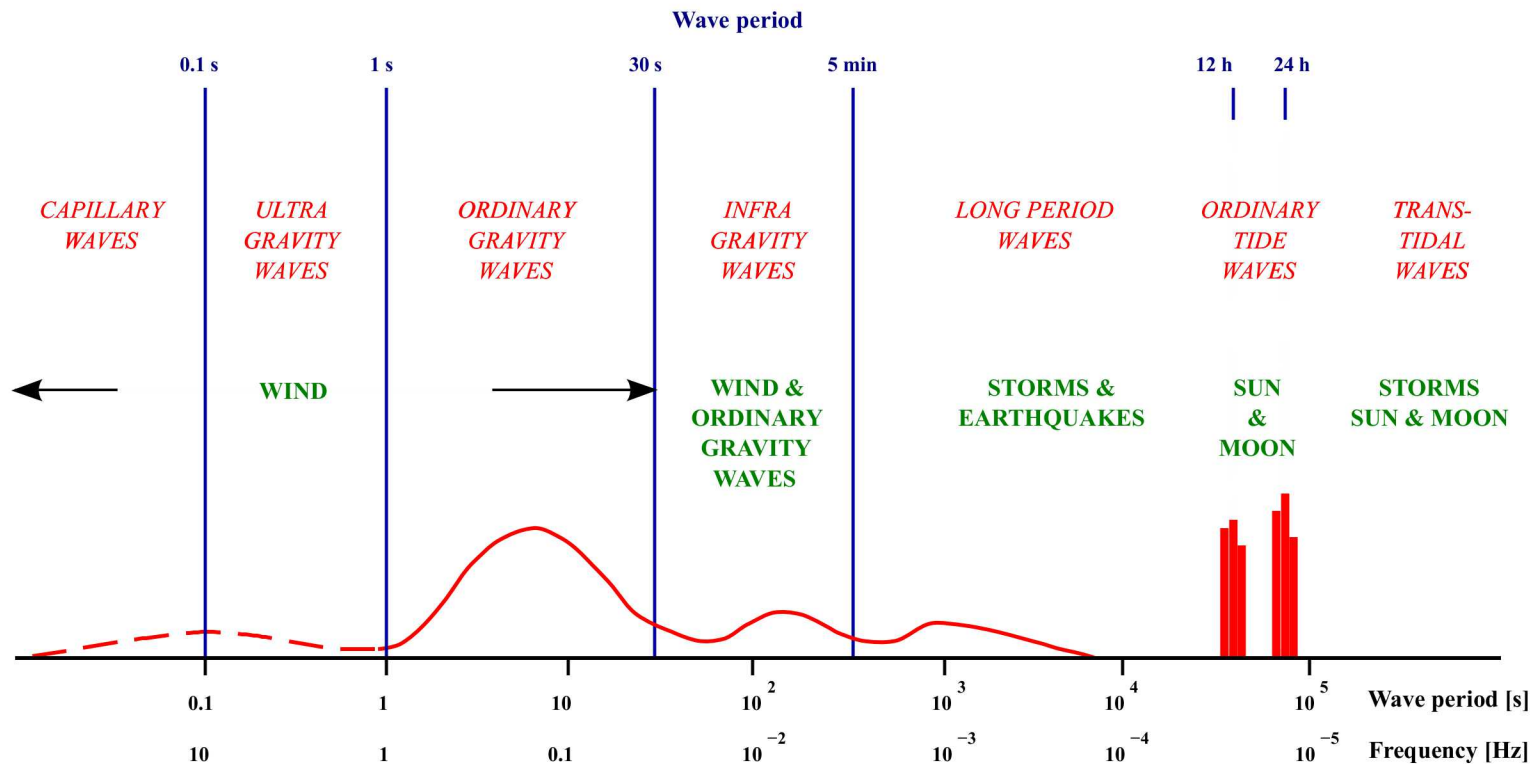
Need an engineering solution

Acausality in impedance matching is a problem seen in many other fields



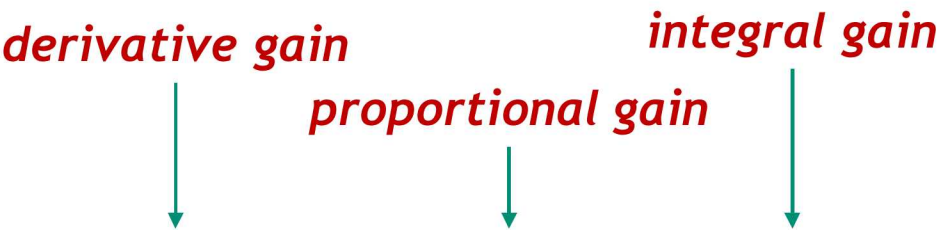
Band-limited input

System	Frequency range
US broadcast television	54-700 MHz
AM radio	0.6-1.6 MHz
Visible light	430-750 THz
Audible sound	20-20,000 Hz
Gravity water waves	0.05-1 Hz
Individual sea state (example)	0.1-0.2 Hz



Causal realization

While perfectly implementing impedance matching in a causal controllers is not possible, we can come quite close

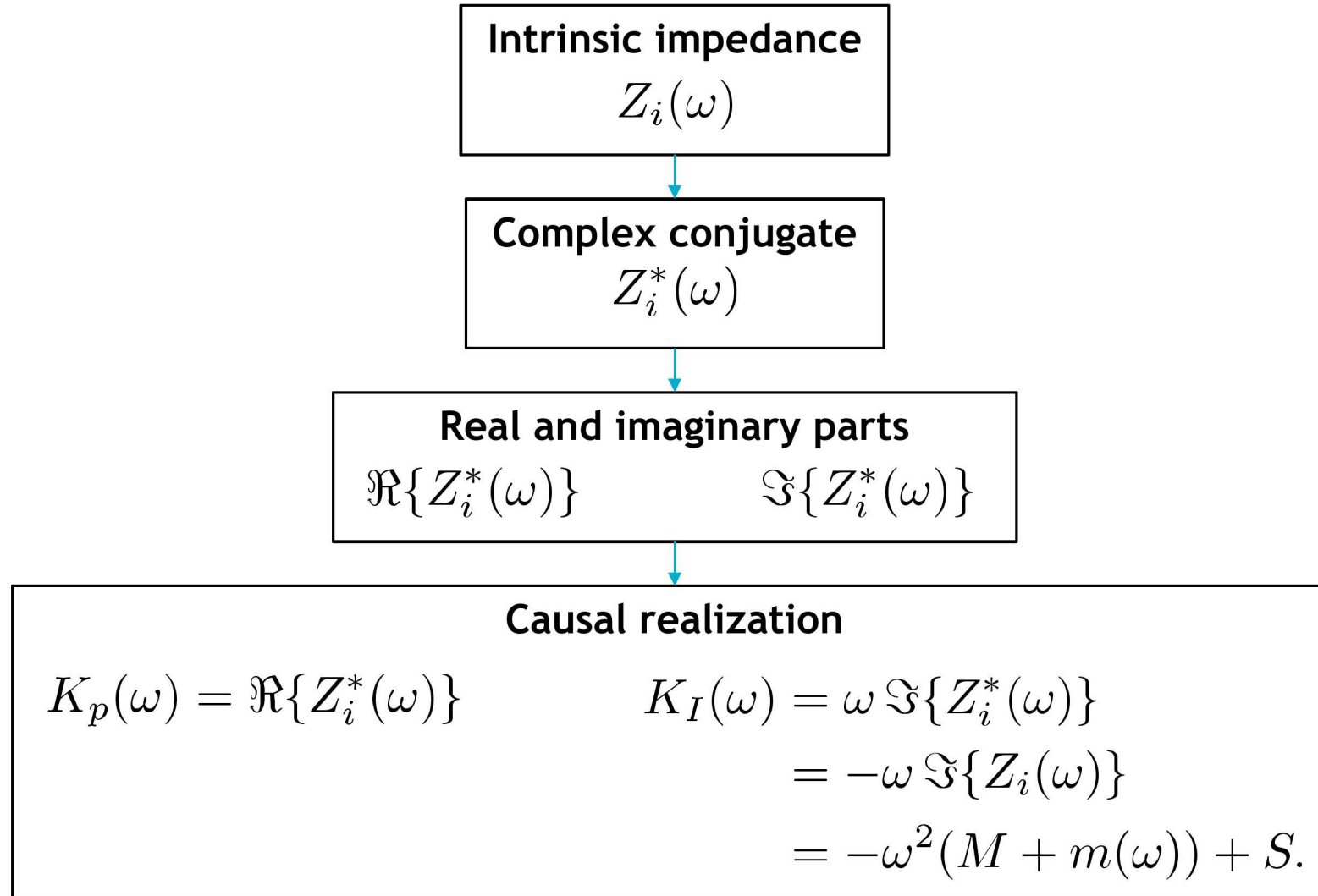


The diagram shows three red labels with arrows pointing to the terms in the numerator of the transfer function: **derivative gain** points to $K_D s^2$, **proportional gain** points to $K_P s$, and **integral gain** points to K_I .

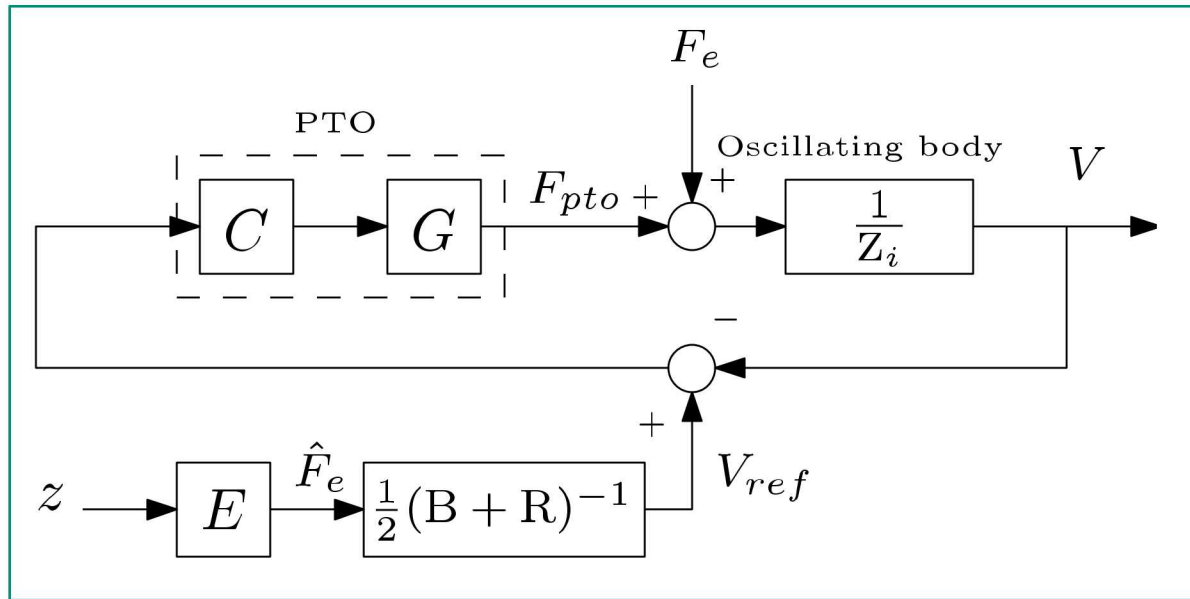
$$C_{PID}(s) = \frac{K_D s^2 + K_P s + K_I}{s}$$

*most of us learn about “PID” in the context of error minimization, this is not really the intent here

“WEC control hierarchy”



Two paths - a WEC reformation

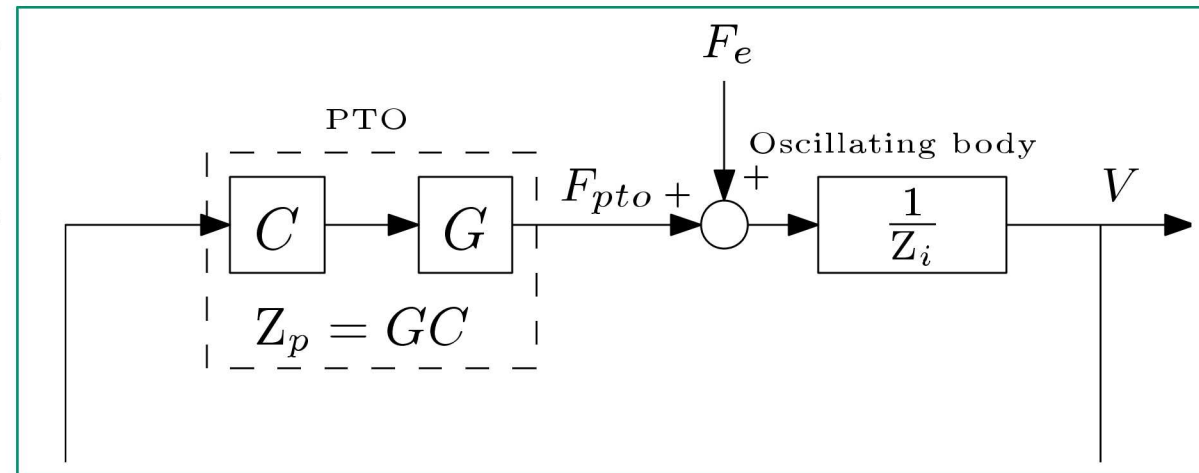


**Velocity tracking
controller
structure**

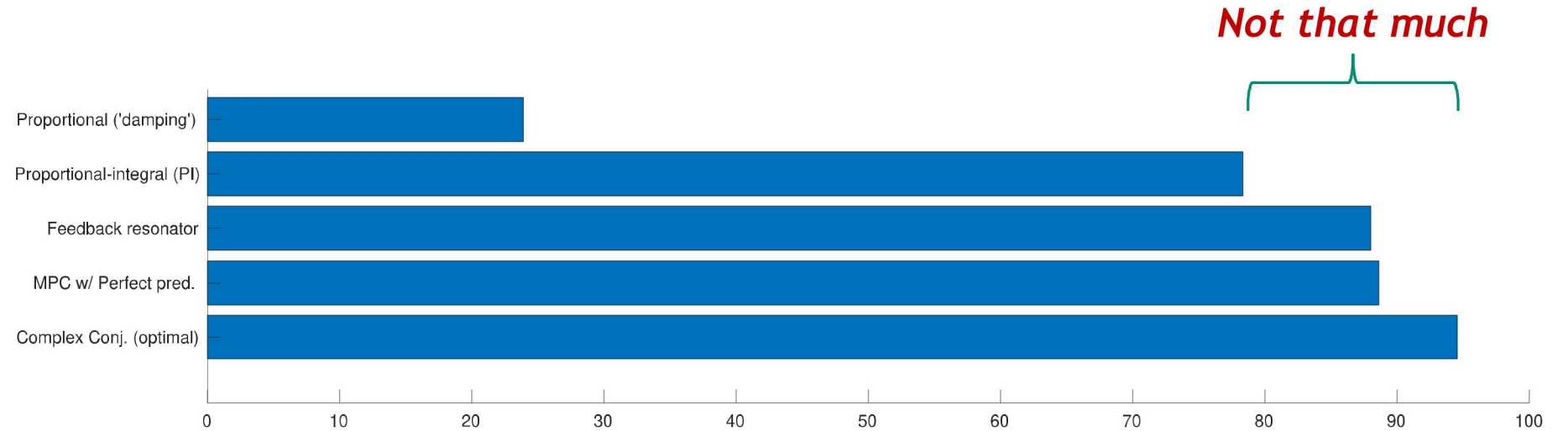
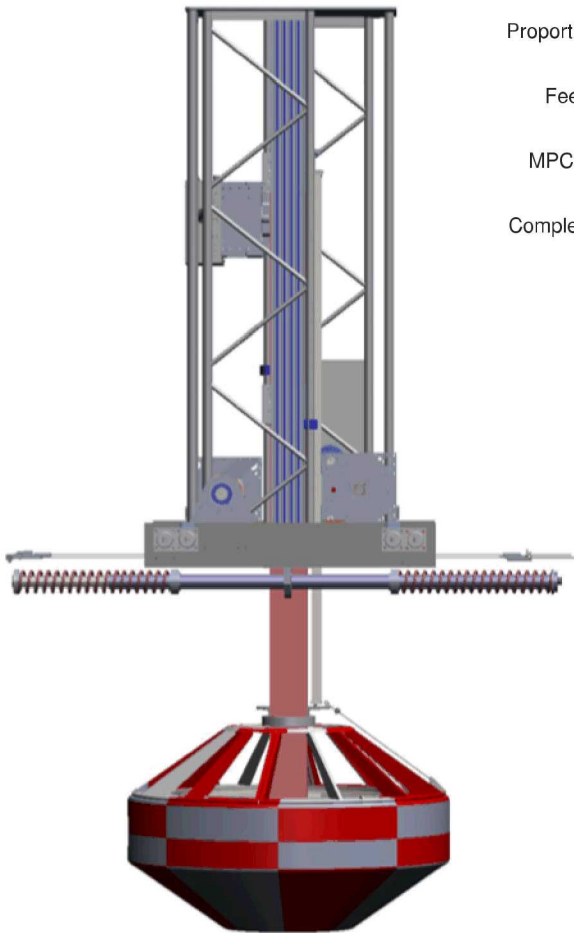
$$V_{ref} = \frac{F_e}{2\Re\{Z_i\}}$$

**Impedance
matching
controller
structure**

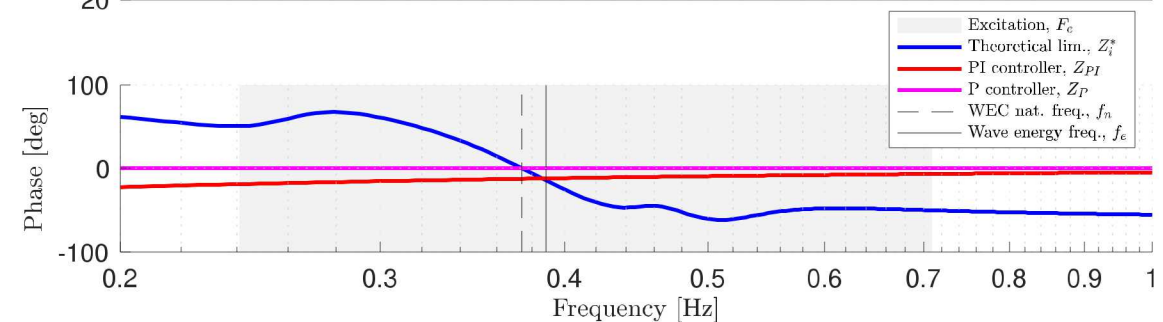
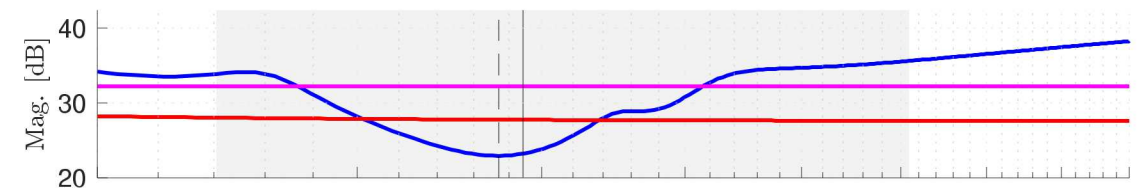
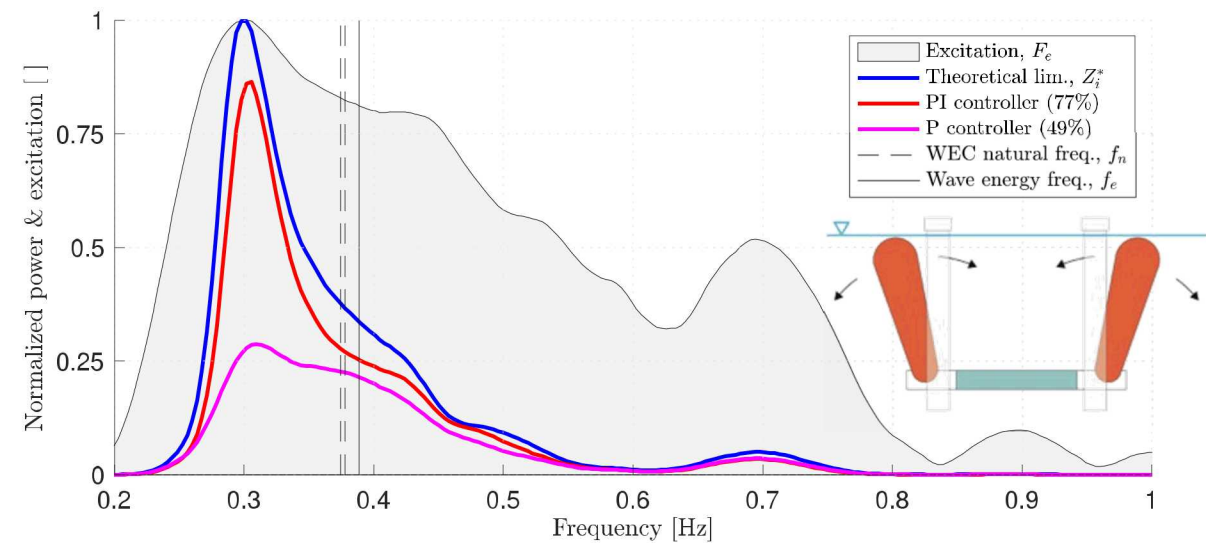
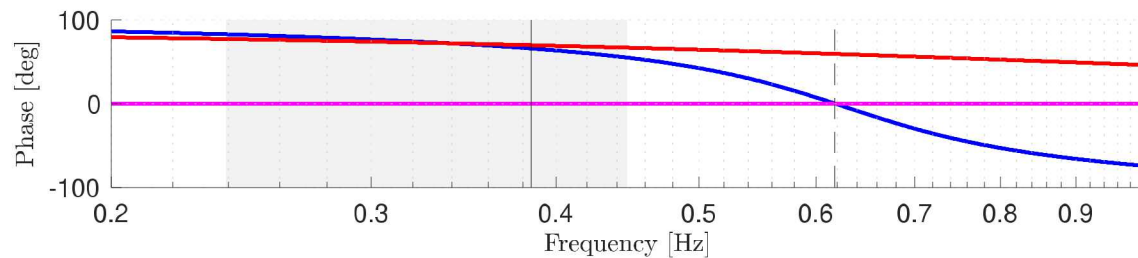
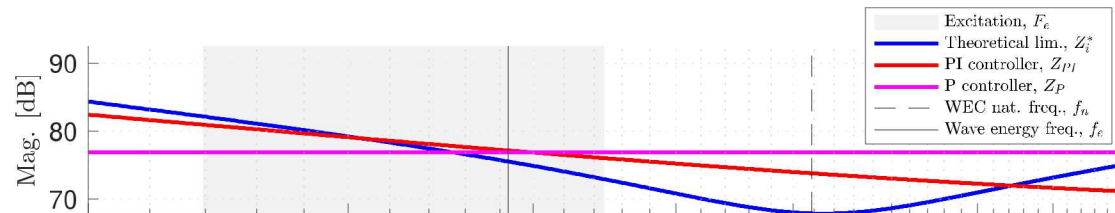
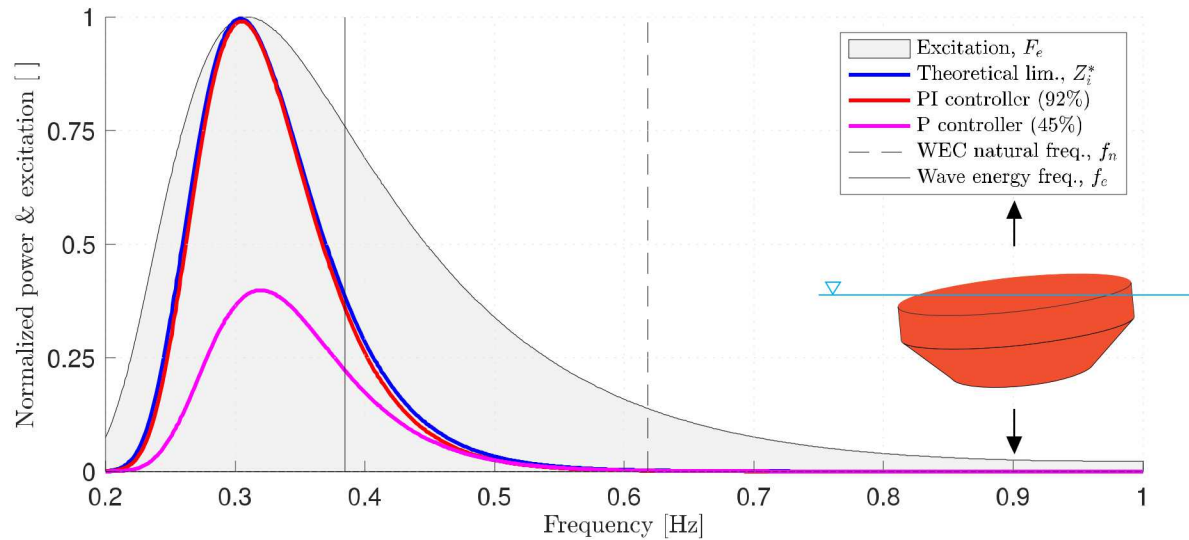
$$Z_p = Z_i^*$$



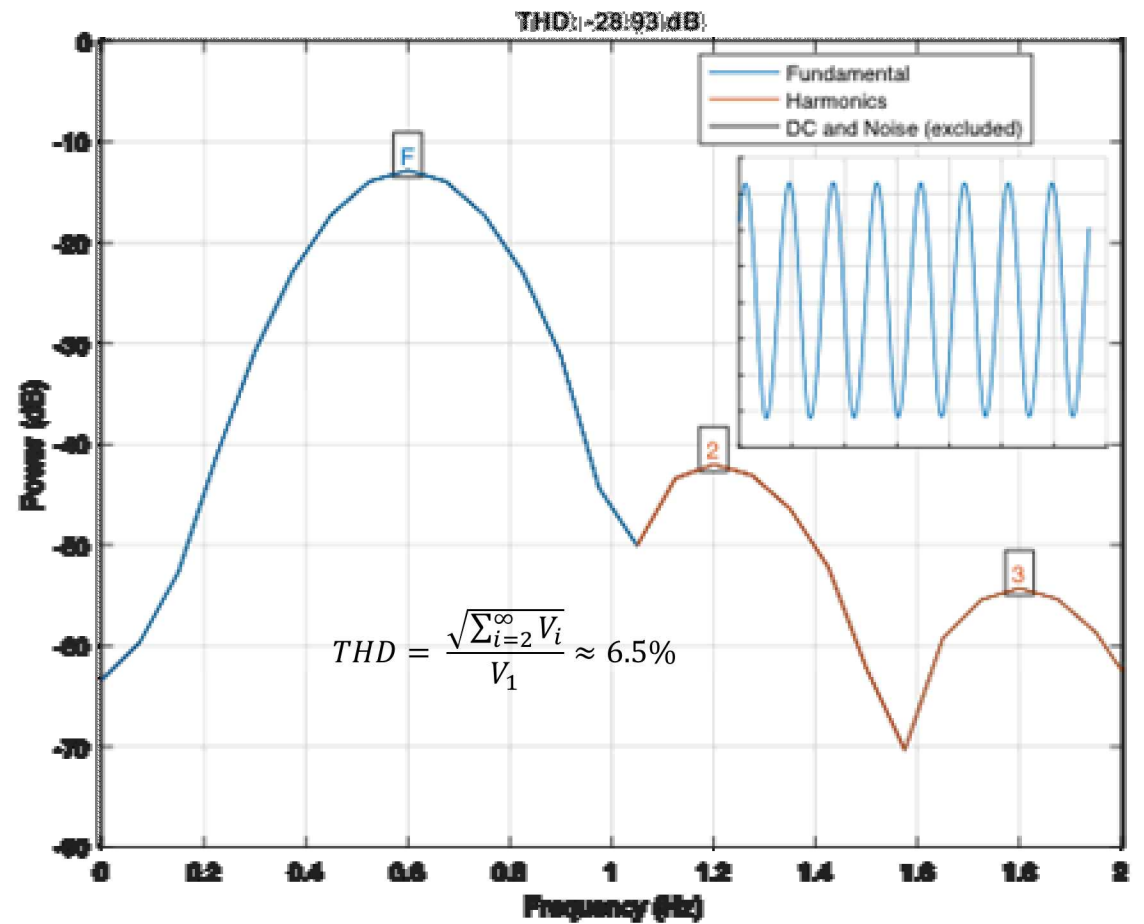
What am I missing out on?



What am I missing out on (cont.)?

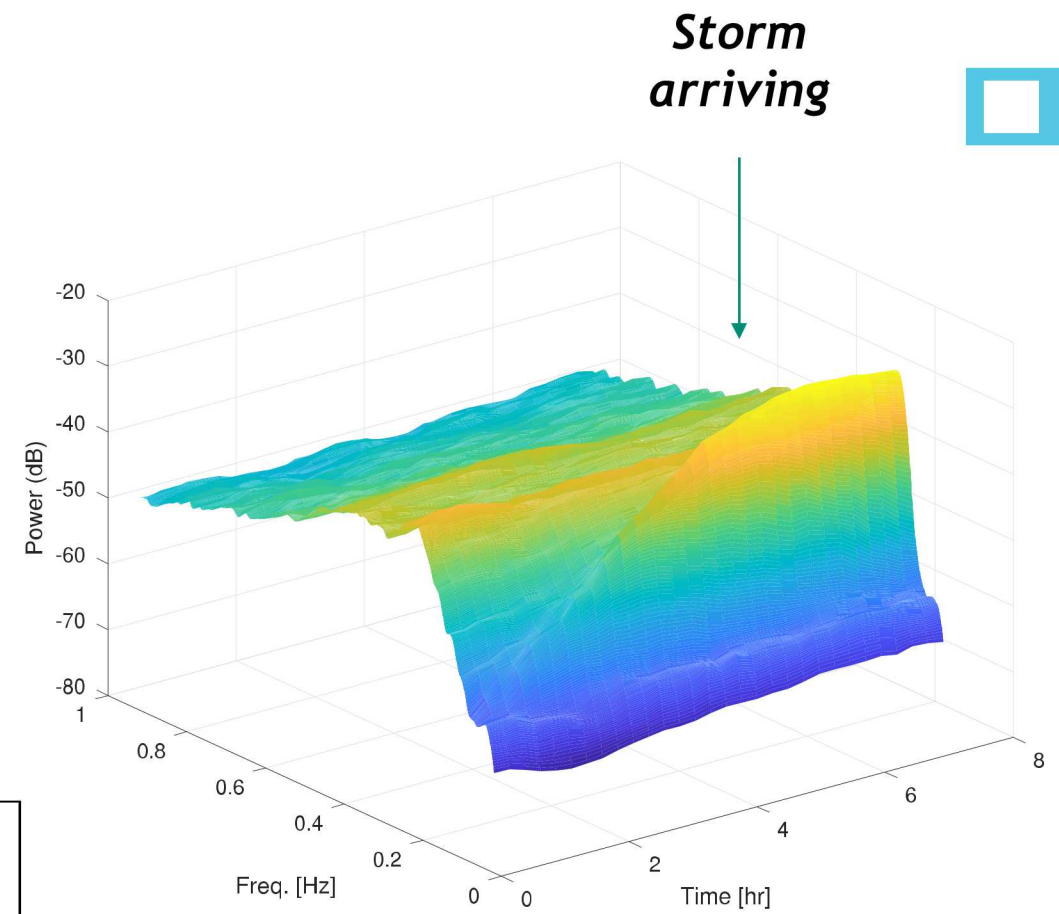
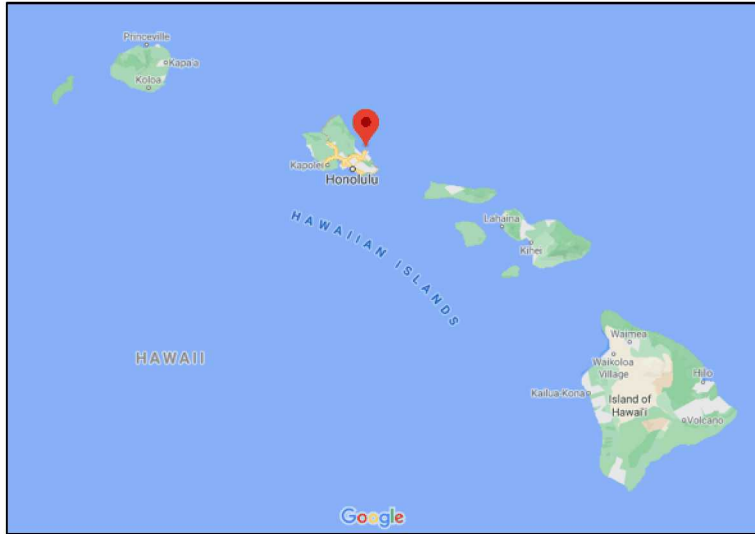


What about nonlinearities?

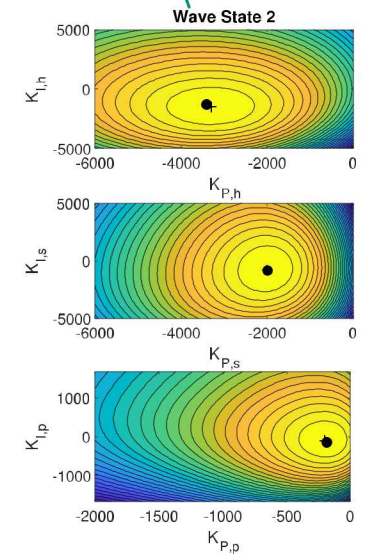
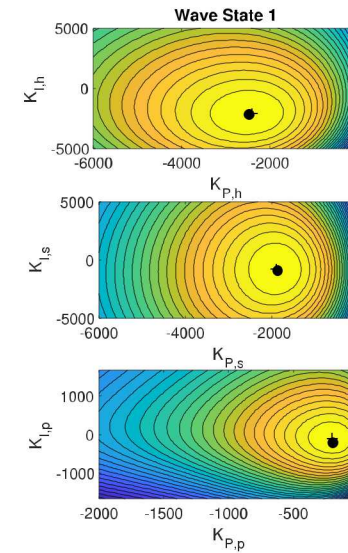
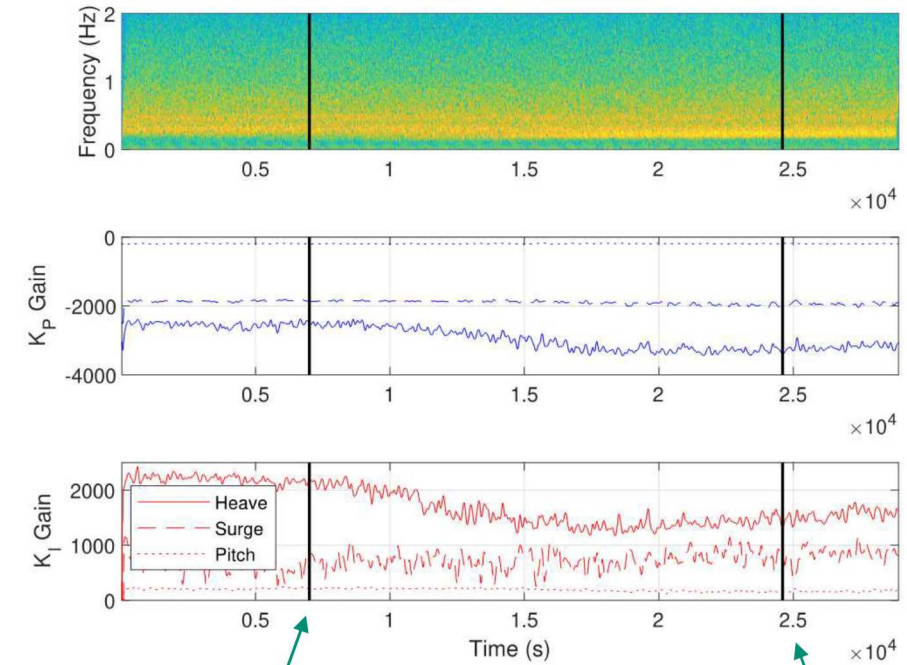
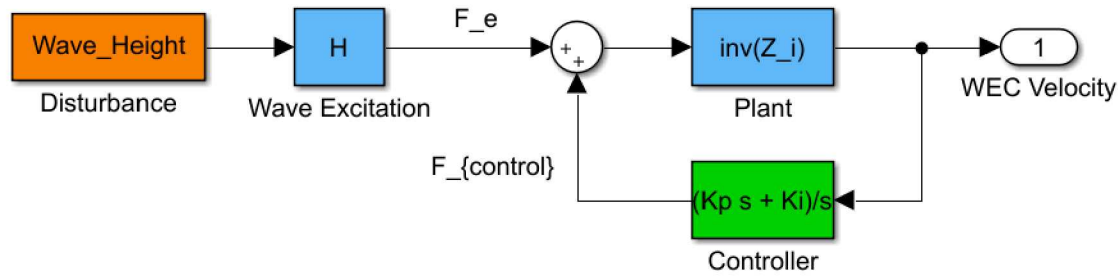


Isn't PI too narrow-banded?

CDIP225, KANEHOHE BAY, WETS, HI



Isn't PI too narrow-banded (cont.)?



Isn't PI too narrow-banded (cont.)?

Bode-Fano limit: there will always be a tradeoff between bandwidth and accuracy (reflection coefficient)

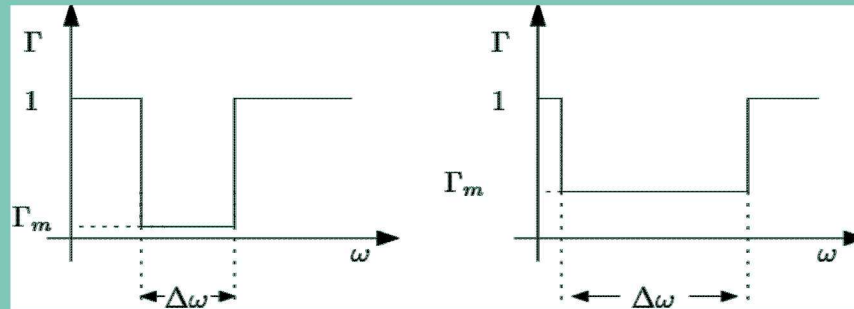
$$\int_0^{\infty} \ln \frac{1}{|\Gamma(\omega)|} d\omega \leq M < \infty$$

Γ Reflection coefficient

$\Gamma = 1 \rightarrow$ NO power transferred to the load
 $\Gamma = 0 \rightarrow$ All power transferred to the load (perfect match)

Example

$$\Delta\omega \ln \frac{1}{\Gamma_m} \leq M$$



Hendrik Bode



https://en.wikipedia.org/wiki/Hendrik_Wade_Bode

Robert Fano



https://en.wikipedia.org/wiki/Robert_Fano

- Narrow banded devices (High Quality factor Q) have lower M
- Narrow banded WECs are not the best choice (e.g. spar buoy)



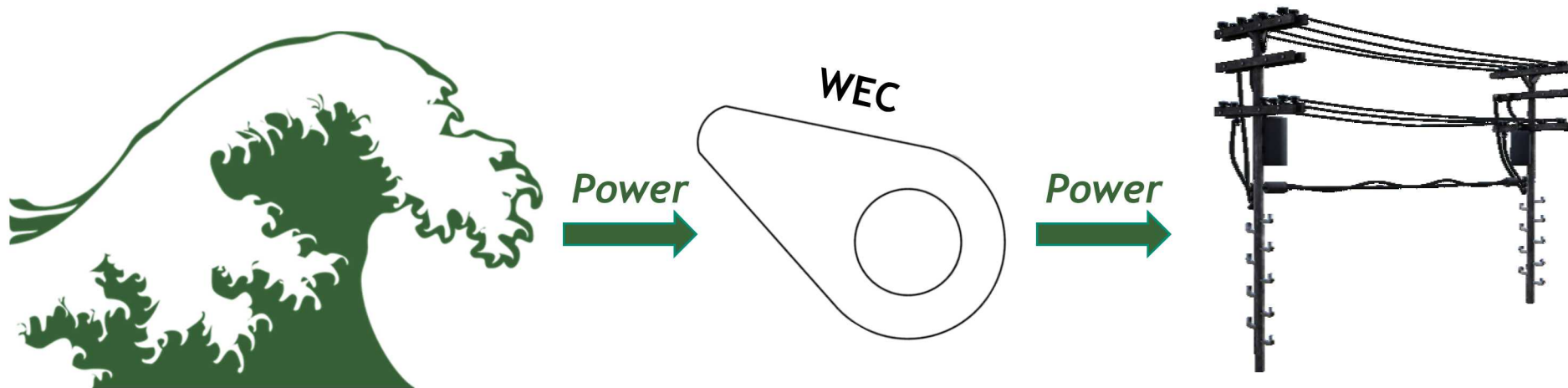
Another good reason not to design narrow-banded WECs

Impedance matching

Maximum power transfer (from waves to PTO)

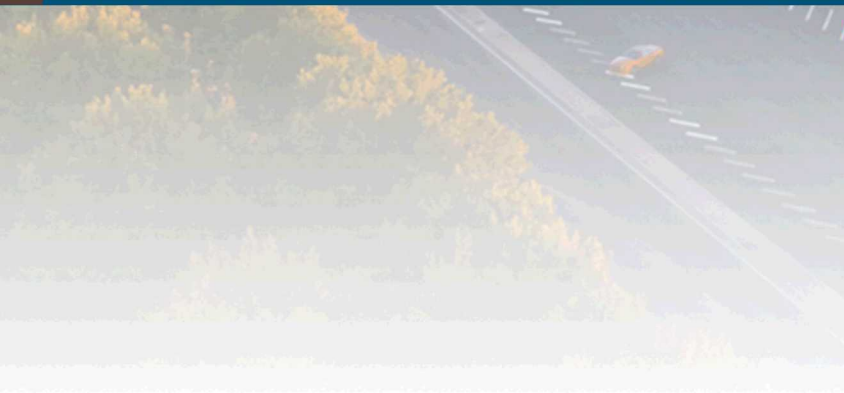
$$Z_{source} = Z_{load}^*$$

...actually, the real problem is closer to

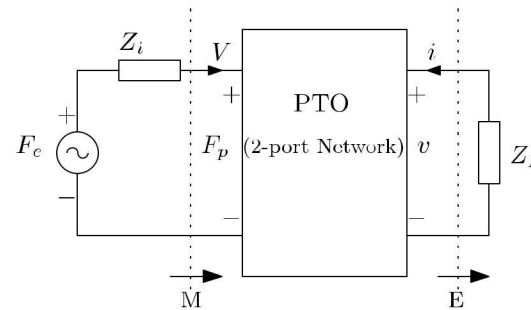
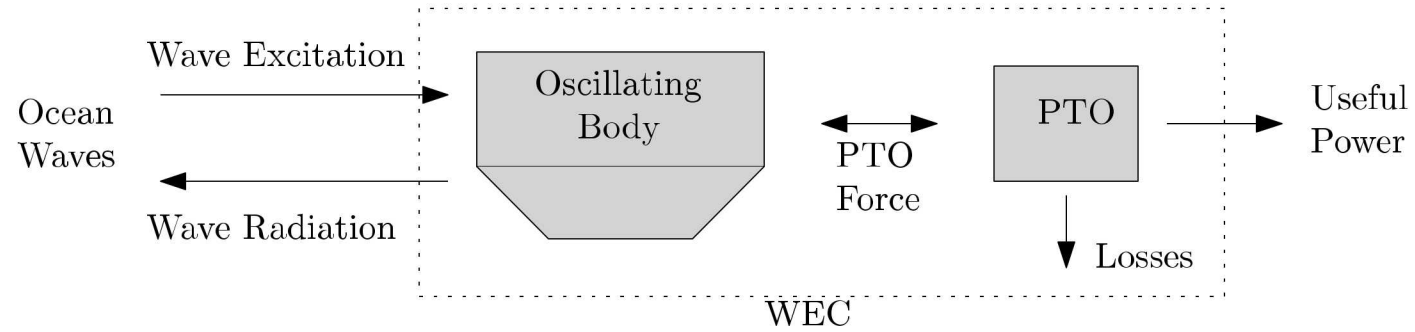




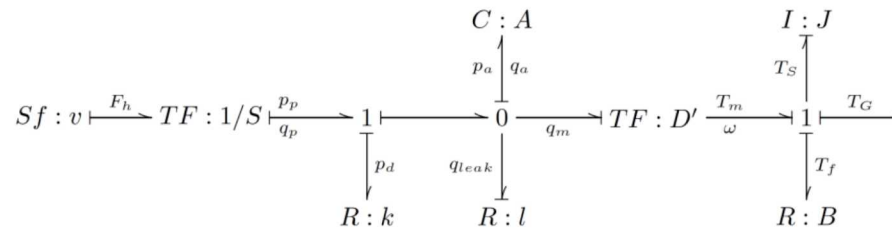
Co-design problem and approach



Description of power flow



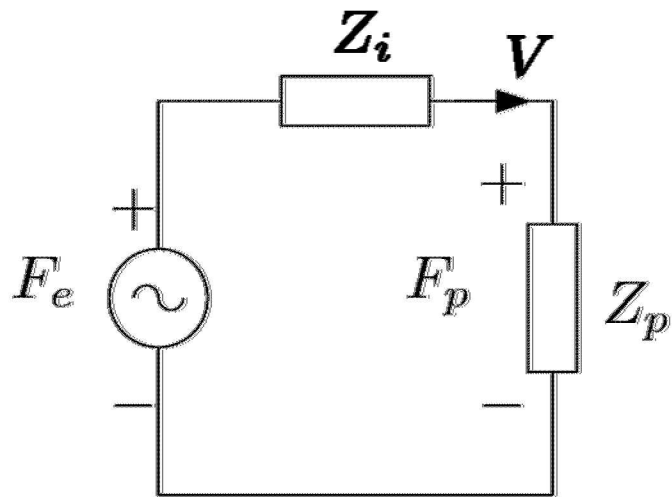
Multiport



Bond-graph

Why use Multiport theory?

It gives condition for maximum power transfer:



Impedance matching

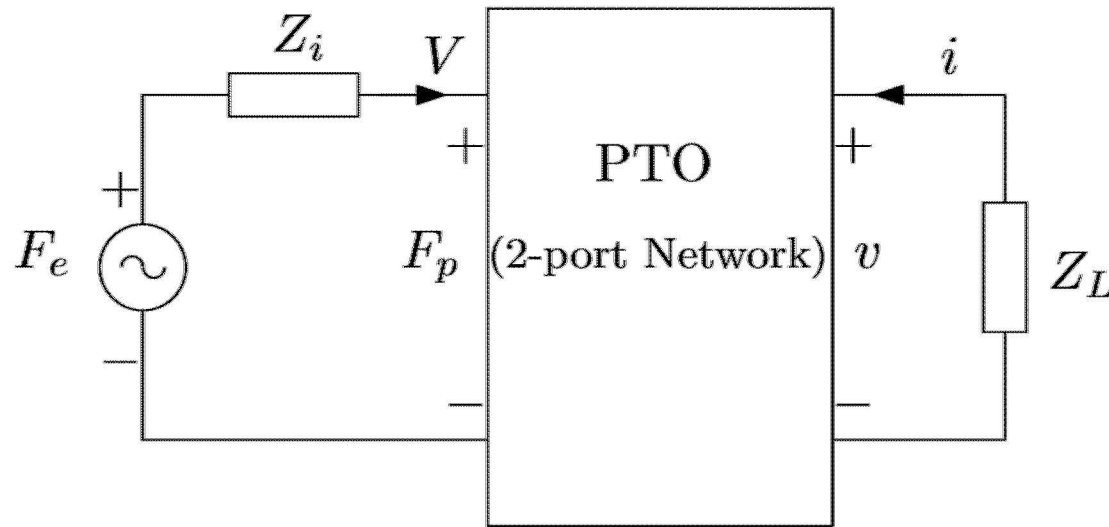
$$Z_p = Z_i^*$$

Maximum power transferred from F_e to Z_p

How to account for the dynamics and efficiency of the PTO?

Objective:
Transfer max power to load Z_L

Thevenin
equivalent transformation



Impedance matching condition for Max power transfer to Z_L

$$Z_L = Z_{th}^*$$

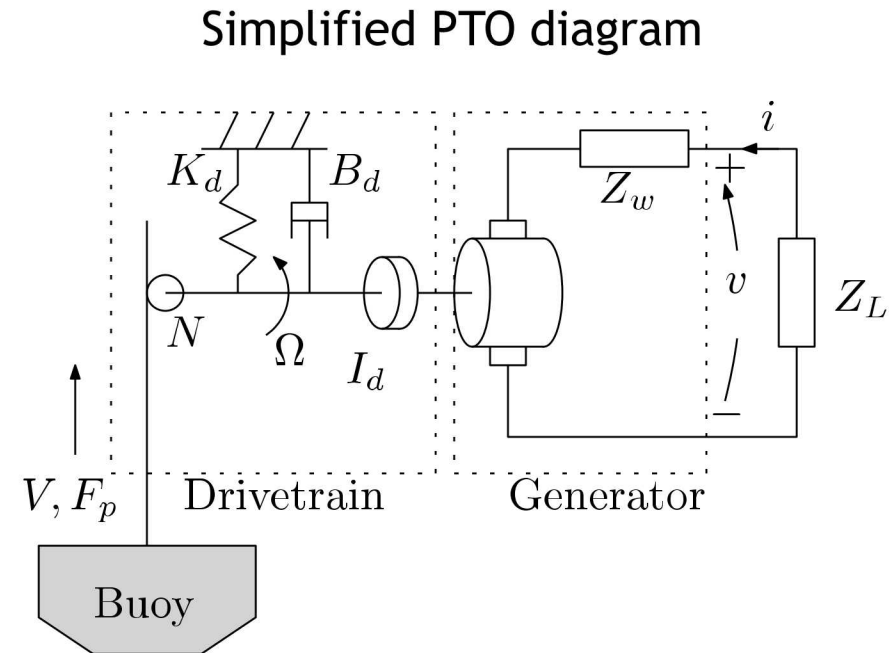
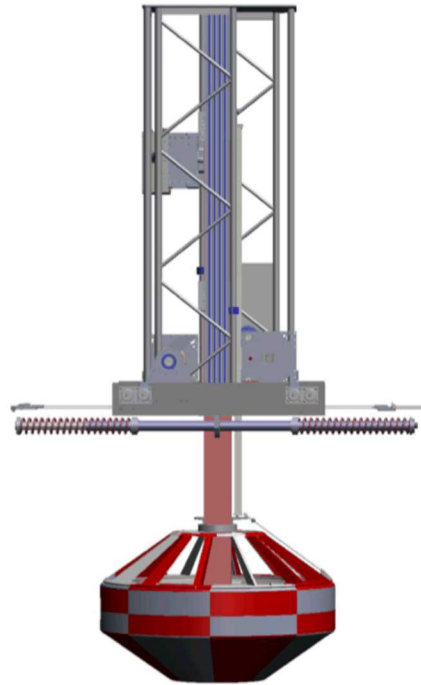
Z_{th} depends
on the PTO



Control system can
be designed to
account for PTO

How to account for the dynamics and efficiency of the PTO?

Example:



$$Z_{th} = Z_w + \frac{\frac{2}{3} K_t^2 N^2}{N^2 Z_d + Z_i}$$



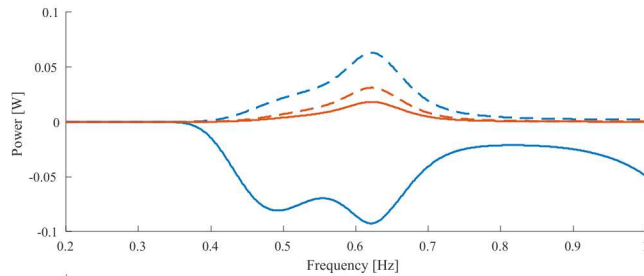
$$C = Z_L = Z_{th}^*$$

Control systems designed to account for PTO dynamics and efficiency

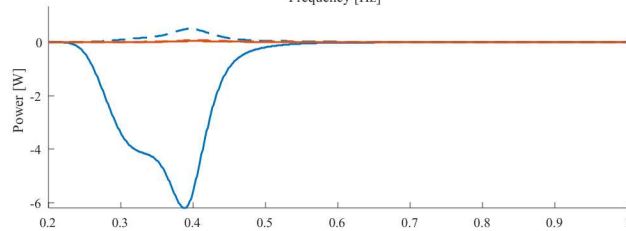
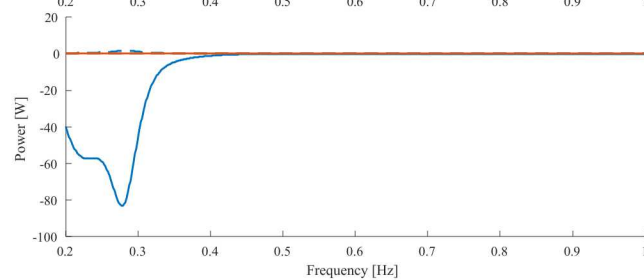
Electrical vs. Mechanical power

 $T_p = 1.58 \text{ s}$
 $T_p = 2.5 \text{ s}$
 $T_p = 3.5 \text{ s}$

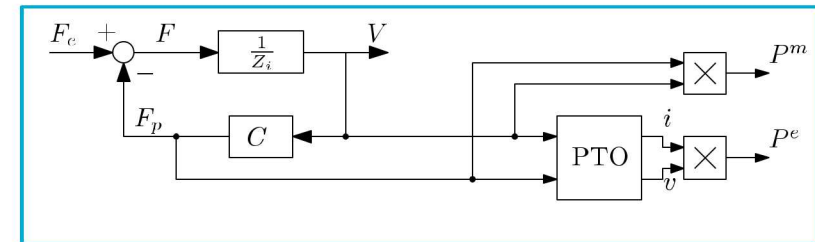
	P_m [W]	η_m	P_e [W]	η_e		P_m [W]	η_m	P_e [W]	η_e		P_m [W]	η_m	P_e [W]	η_e
CC on mech	1.06E+1	1.00	-2.91E+1	-2.74		5.21E+1	1.00	-7.47E+2	-14.34		1.29E+2	1.00	-7.85E+3	-60.69
PI on mech	9.72E+0	0.92	-1.8E+1	-1.70		4.96E+1	0.95	-6.29E+2	-12.08		1.21E+2	0.94	-5.85E+3	-45.26
CC on elec	4.41E+0	0.42	2.52E+0	0.24		8.26E+0	0.16	4.39E+0	0.08		6.8E+0	0.05	3.5E+0	0.03
PI on elec	4.38E+0	0.41	2.51E+0	0.24		8.22E+0	0.16	4.37E+0	0.08		6.77E+0	0.05	3.5E+0	0.03


 $T_p = 1.58 \text{ s}$

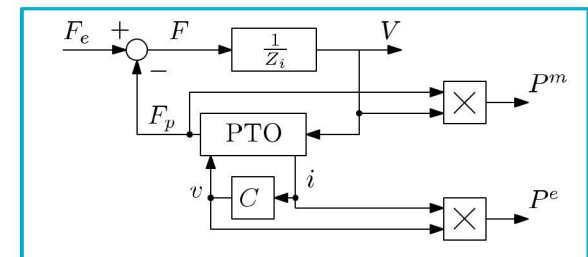
- Mechanical efficiency - CC control for max P_m
- Electrical efficiency - CC control for max P_m
- Mechanical efficiency - CC control for max P_e
- Electrical efficiency - CC control for max P_e


 $T_p = 2.50 \text{ s}$

 $T_p = 3.50 \text{ s}$

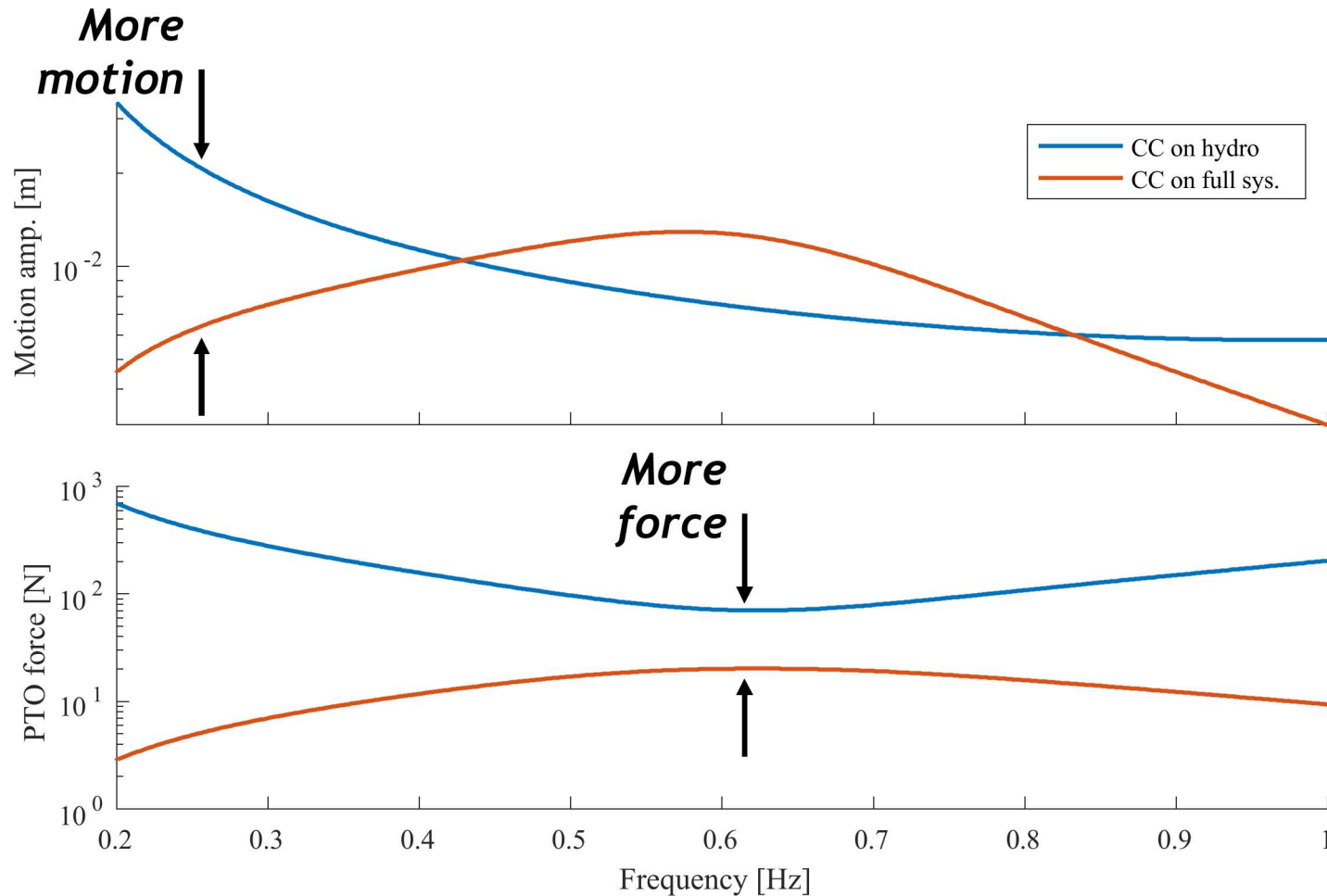
Mechanical power



Electrical power

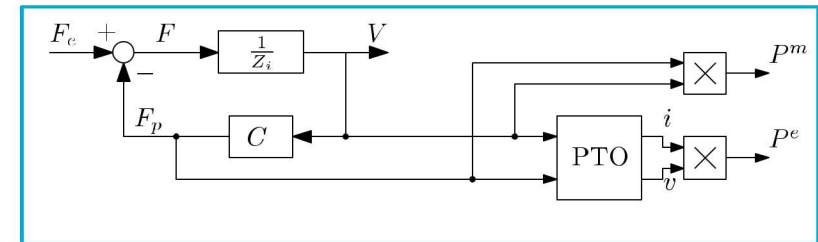


Electrical vs. Mechanical power (cont.)

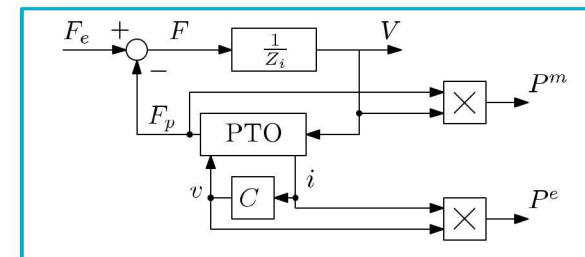


Making other design decisions without a good approximation of the control and PTO is problematic

Mechanical power

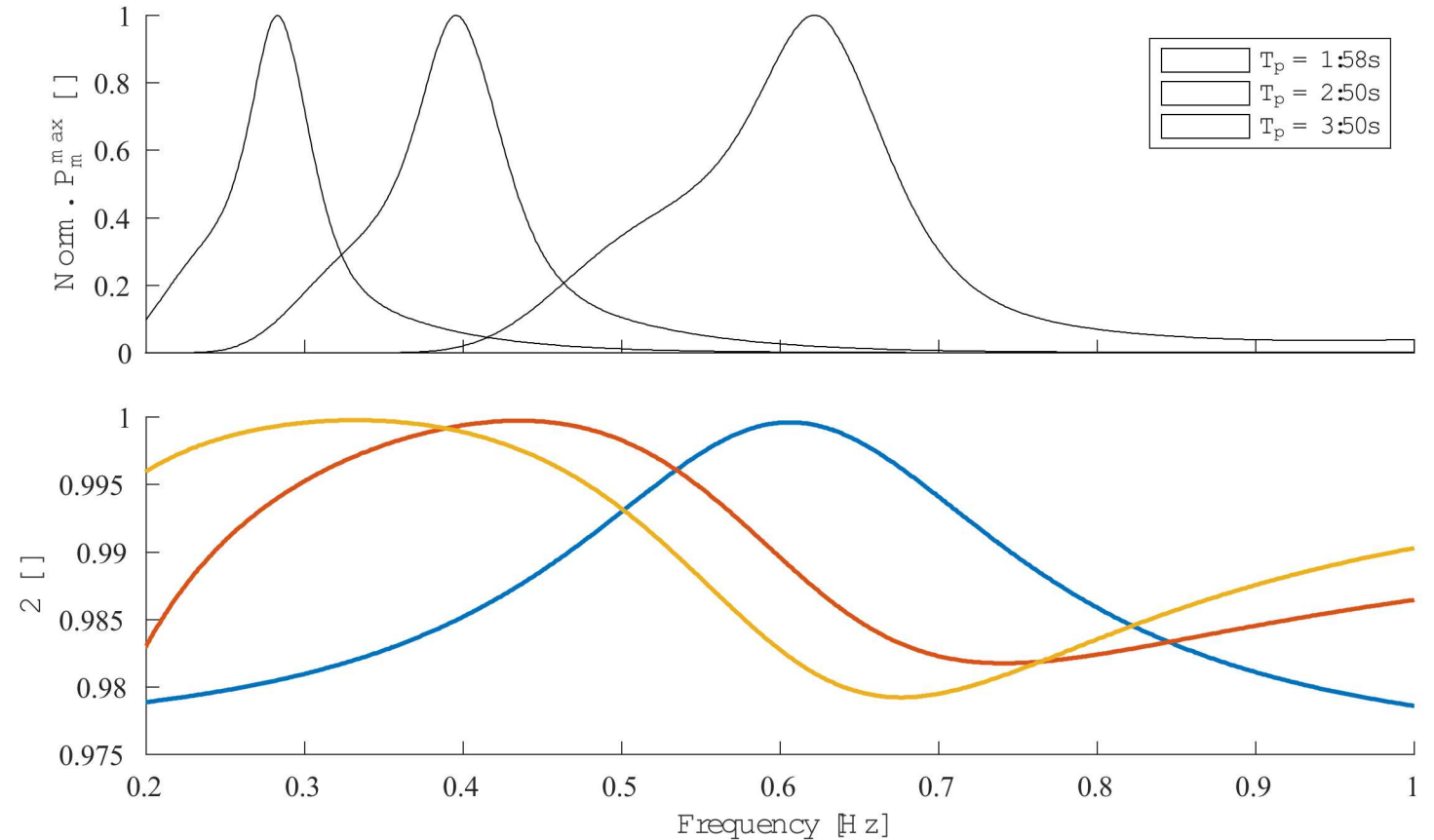


Electrical power



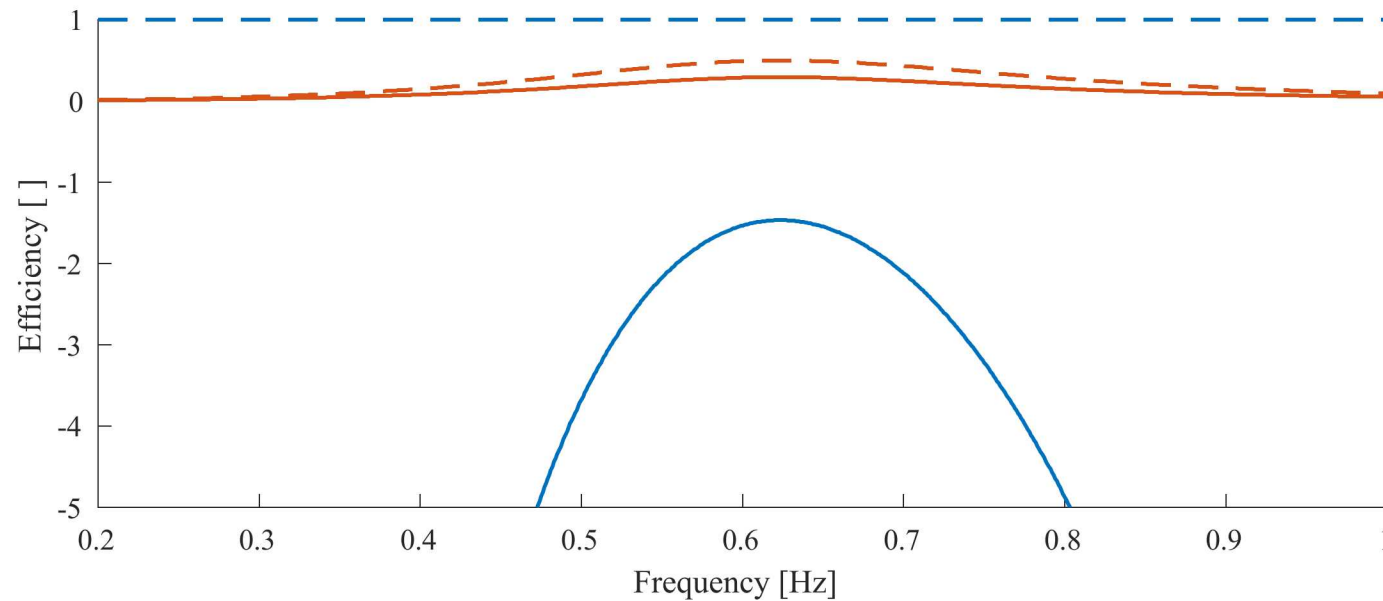
PTO design is the important problem

$$\eta = \frac{P_L}{P_m^{max}}$$



A perfect controller is not that much better than a good controller, especially when the PTO is not optimized

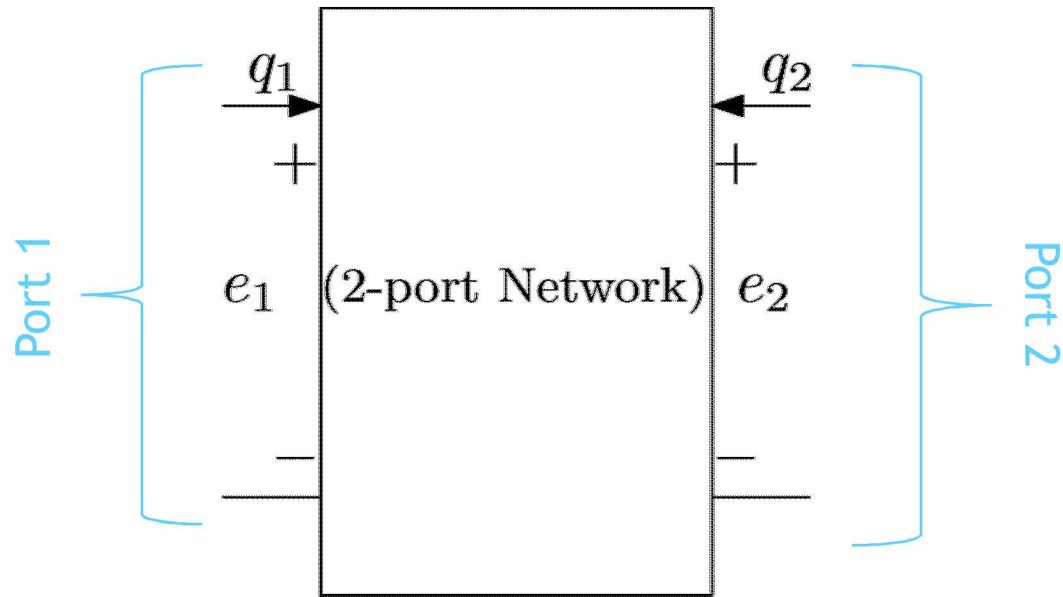
Efficiency is highly frequency dependent



- Mechanical efficiency - CC control for max P_m
- Electrical efficiency - CC control for max P_m
- Mechanical efficiency - CC control for max P_e
- Electrical efficiency - CC control for max P_e

Multiport framework

Example: Generic 2-port



e_1, e_2 : effort (across) variables

q_1, q_2 : flow (through) variables

2-port Impedance:

$$\begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}$$

$$Z_{11} = \left. \frac{e_1}{q_1} \right|_{q_2=0} \quad Z_{12} = \left. \frac{e_1}{q_2} \right|_{q_1=0}$$

$$Z_{21} = \left. \frac{e_2}{q_1} \right|_{q_2=0} \quad Z_{22} = \left. \frac{e_2}{q_2} \right|_{q_1=0}$$

Multiport framework (cont.)

Example: Electric generator

$$e_1 = \tau_m$$

$$e_2 = v_q$$

$$q_1 = \Omega$$

$$q_2 = i_q$$

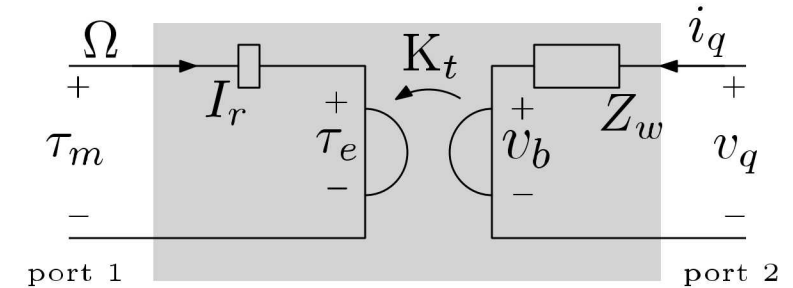
$$\tau_m = I_r j\omega\Omega + \tau_e$$

$$v_q = v_b + Z_w i_q$$

$$\tau_e = -\sqrt{3/2} K_t i_q$$

$$v_b = \sqrt{3/2} K_t \Omega$$

$$\begin{aligned} Z_{11} &= \left. \frac{e_1}{q_1} \right|_{q_2=0} & Z_{12} &= \left. \frac{e_1}{q_2} \right|_{q_1=0} \\ Z_{21} &= \left. \frac{e_2}{q_1} \right|_{q_2=0} & Z_{22} &= \left. \frac{e_2}{q_2} \right|_{q_1=0} \end{aligned}$$



Electrical generator
(2-port network)

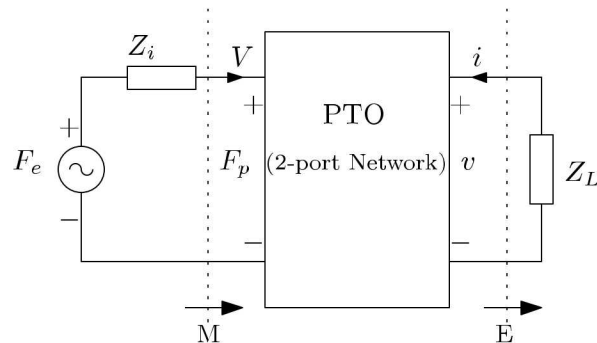
$$Z_{gen} = \begin{bmatrix} j\omega I_r & -\sqrt{\frac{3}{2}} K_t \\ \sqrt{\frac{3}{2}} K_t & Z_w \end{bmatrix}$$

How do we optimize PTO?

How can we tune the controller to account for the PTO?

How can we design a good PTO simultaneously with a controller?

Consider the PTO as an impedance matching network and design it to maximize the power delivered to the load.



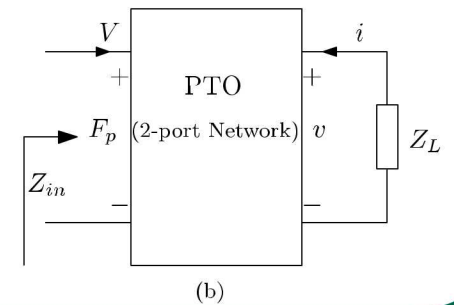
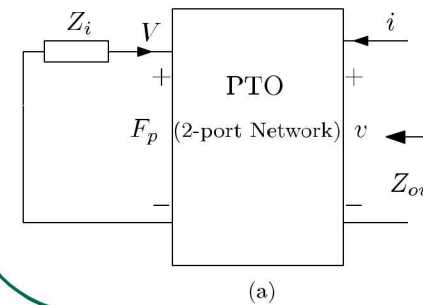
2 matching conditions for maximization of electrical power

$$Z_L = Z_{out}^*$$

Output matching

$$Z_i = Z_{in}^*$$

Input matching



Theoretical upper bound for mechanical power

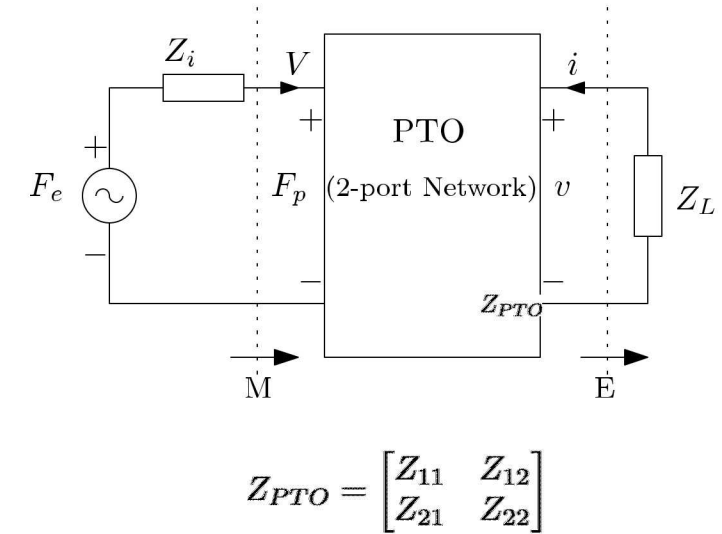
$$P_m^{max} = \frac{|F_{exc}|^2}{8 R_i}$$

Power on the load as function of the PTO and Controller ($C = Z_L$)

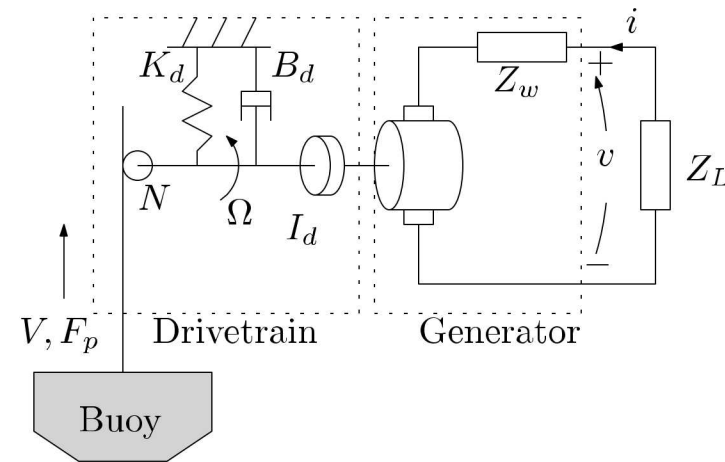
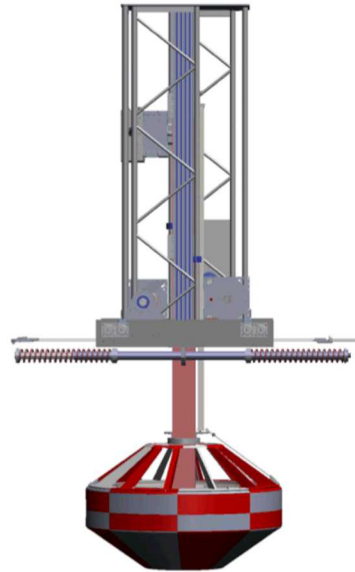
$$P_L = \left| \frac{Z_{21}}{(Z_{11} + Z_i)(Z_{22} + Z_L) - Z_{12} Z_{21}} \right|^2 \frac{|F_e|^2}{2} R_L$$

Electrical conversion efficiency

$$\eta = \frac{P_L}{P_m^{max}} = \left| \frac{Z_{21}}{(Z_{11} + Z_i)(Z_{22} + Z_L) - Z_{12} Z_{21}} \right|^2 4 R_i R_L$$



Power and efficiency: WaveBot



$$P_L = \frac{\frac{3}{2} K_t^2 N^2 |F_e|^2 R_L}{2 |(N^2 Z_d + Z_i) (Z_w + Z_L) + \frac{3}{2} K_t^2 N^2|^2}$$

$$\eta = \frac{6 K_t^2 N^2 R_i R_L}{2 |(N^2 Z_d + Z_i) (Z_w + Z_L) + \frac{3}{2} K_t^2 N^2|^2}$$

Simultaneous optimization of PTO and control

$$\arg \max_{C, \Theta} P_L(F_{\text{exc}}, Z_i, Z_{PTO}(C, \Theta), Z_L(C, \Theta))$$

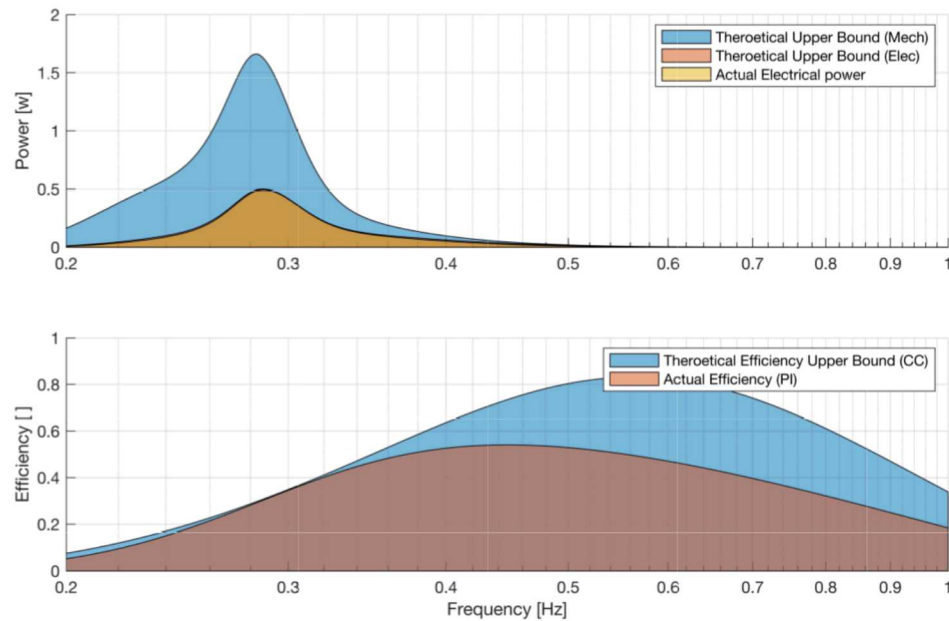
$$Z_{PTO} = Z_{PTO}(C, \Theta) \qquad Z_L = Z_L(C, \Theta)$$

$$\Theta = \{N, I_d, K_d, \dots\} \quad \text{PTO parameters}$$

$$C = \{K_p, K_i\} \quad \text{Control parameters (e.g. PI)}$$

Example I: Co-optimization of PTO and control

Nominal PTO



Optimized PTO

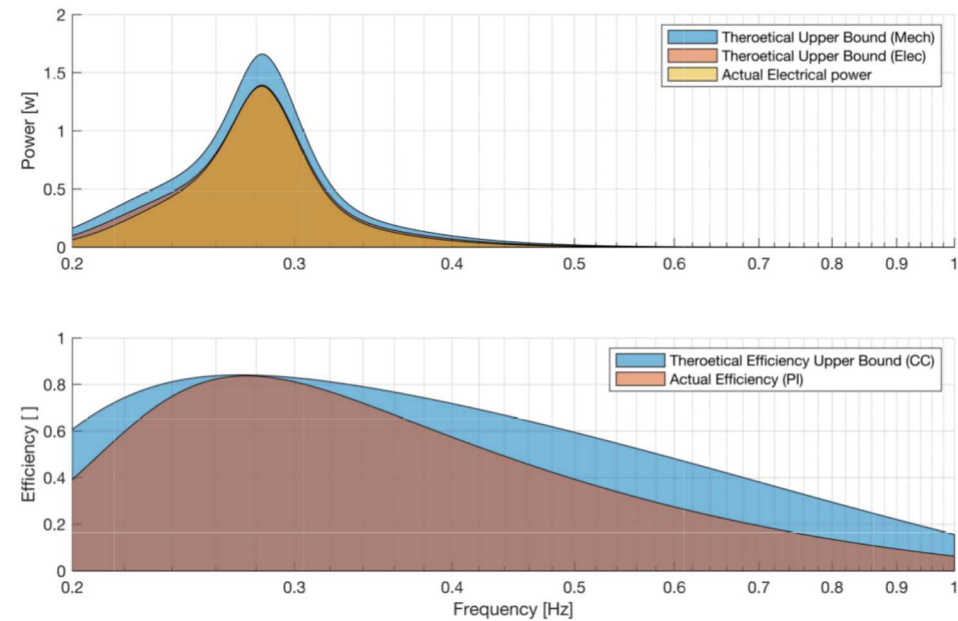


Table 1: Performance improvements with a co-optimized PTO

	Non co-optimized PTO		Co-optimized PTO	
	P[W]	η	P[W]	η
Mechanical (CC)	129.3	1	129.3	1
Electrical (CC)	41.2	0.32	103.7	0.80
Electrical (PI)	38.2	0.30	97.4	0.75

PTO parameters to be optimized:

$$\Theta = \{N, I_d, K_d\}$$

Example 2: Co-optimization of PTO and control with non-colocated controller

$$i = C \Omega$$

Measure angular velocity of the shaft Ω
Control the current on the generator i



$$Z_L = Z_w + \frac{3 K_t}{2 C}$$

Non-colocated controller

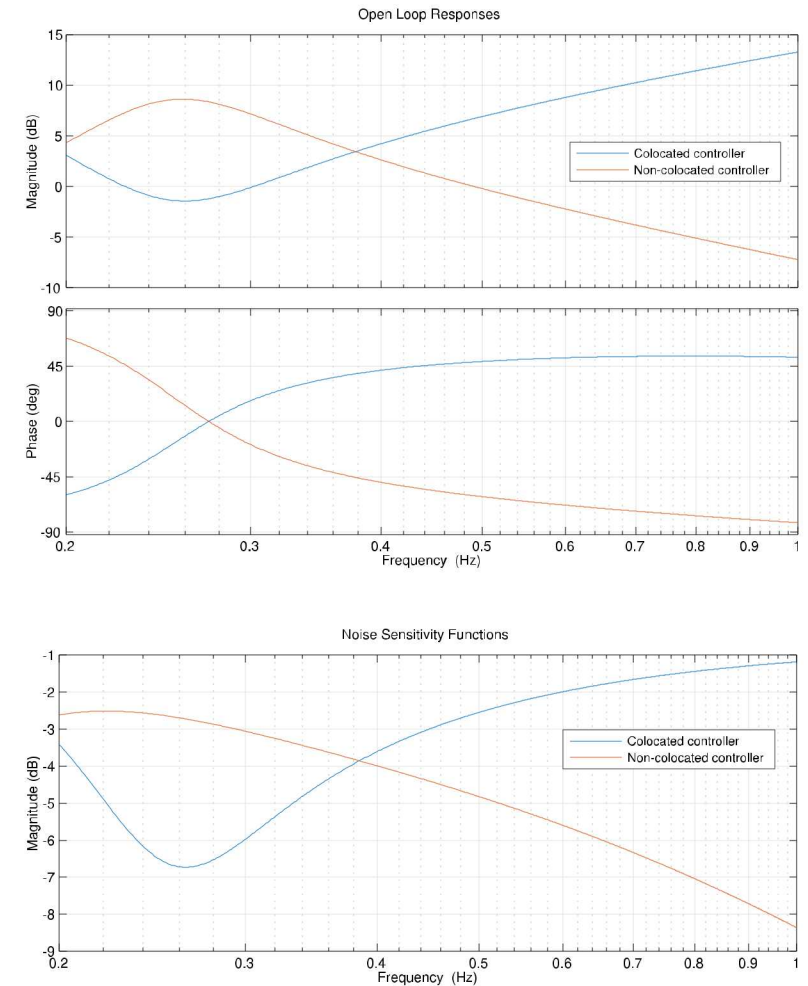
$$C \neq Z_L$$

Table 1: Performance improvements with a co-optimized PTO and non-colocated controller

	Non co-optimized PTO		Co-optimized PTO	
	P[W]	η	P[W]	η
Mechanical (CC)	129.3	1	129.3	1
Electrical (CC)	41.2	0.32	103.7	0.80
Electrical (PI)	40.3	0.31	97.4	0.75

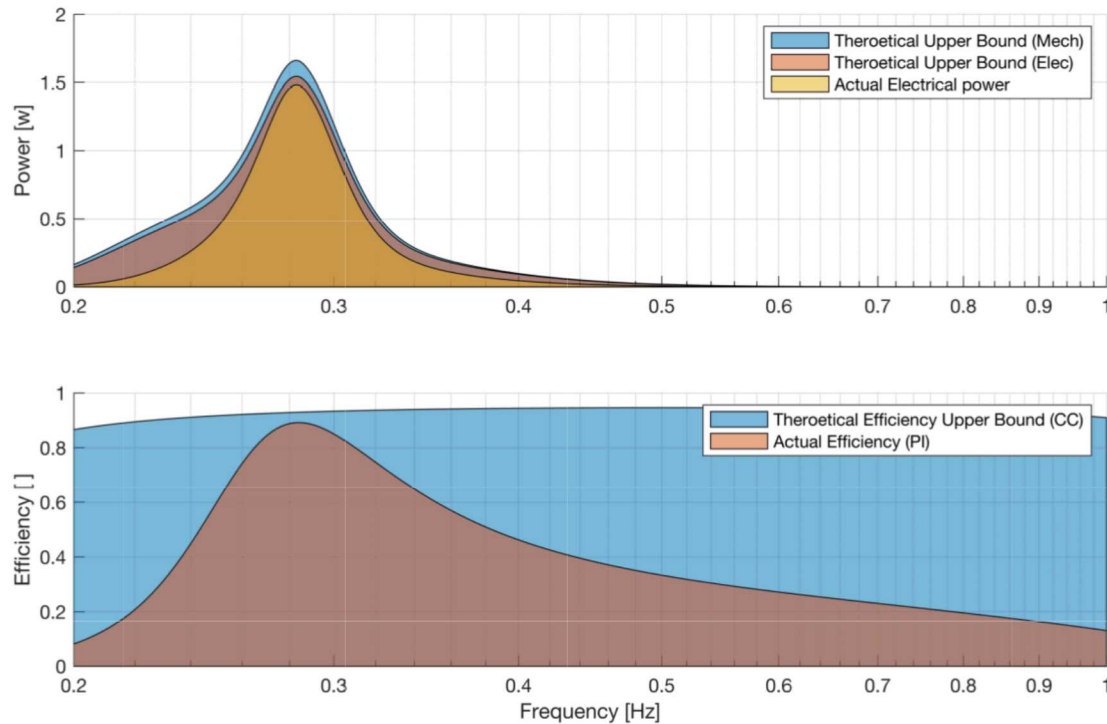
Example 2: Co-optimization of PTO and control with non-colocated controller (cont.)

- NC controller has same power performance as colocated
- NC controllers have to be implemented when cannot measure and control directly quantities related to power
- Structure of the feedback loop depends on the availability and quality of measurements (sensors) and actuators.
- Very different dynamic behavior



Example 3: Reducing winding resistance vs co-optimization

Nominal PTO with very small ($1 \text{ m}\Omega$) R_w



Optimized PTO with larger ($100 \text{ m}\Omega$) R_w
 $\Theta = \{N, I_d, K_d\}$

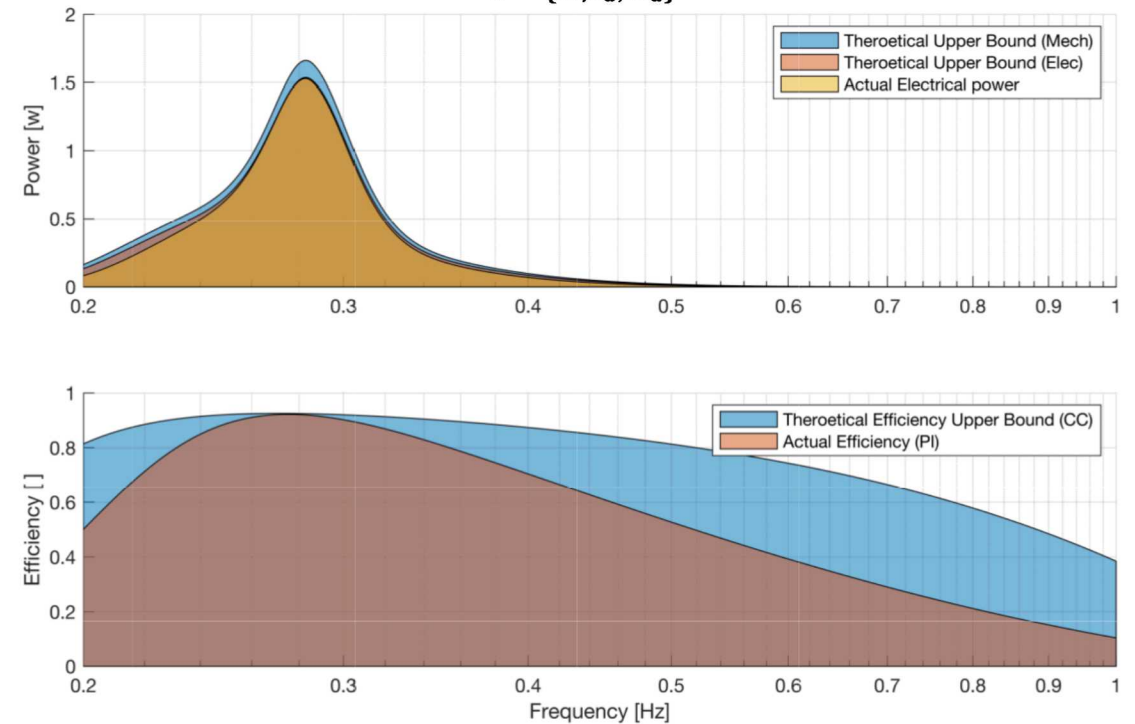


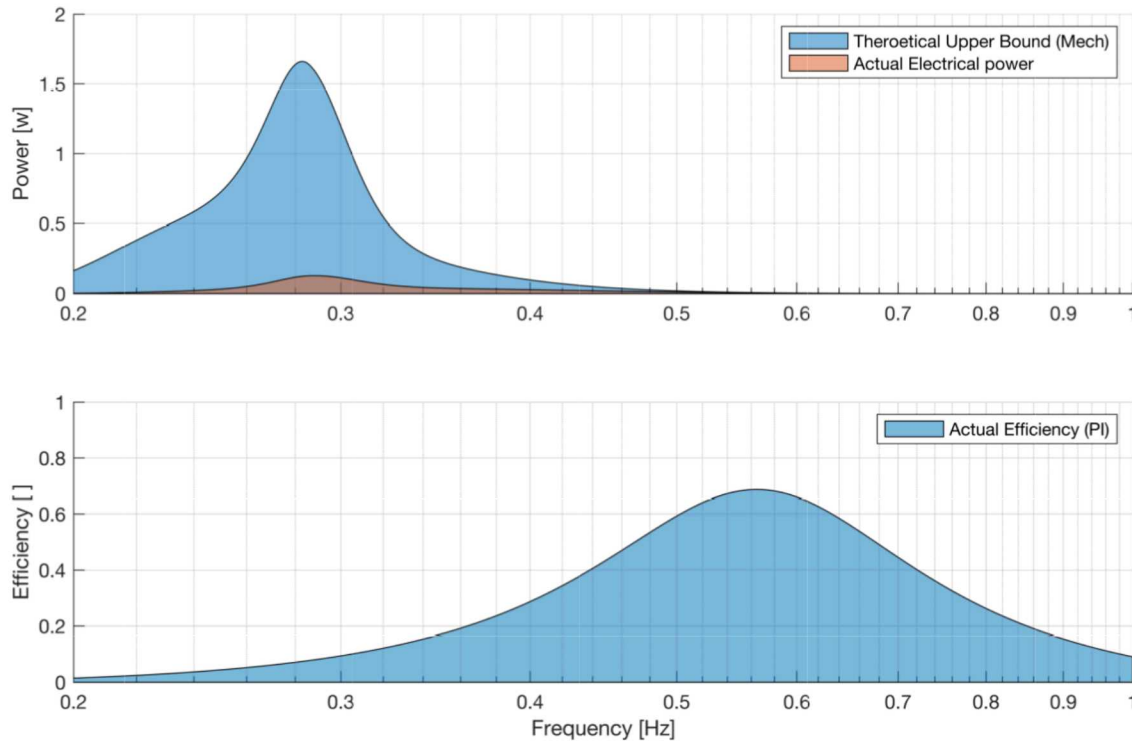
Table 1: Performance improvements with a co-optimized PTO and non-colocated controller

	Non optimized PTO $R_w = 1\text{m}\Omega$		Optimized PTO $R_w = 100\text{m}\Omega$	
	P[W]	η	P[W]	η
Mechanical (CC)	129.3	1	129.3	1
Electrical (CC)	119.7	0.93	117.5	0.91
Electrical (PI)	89.0	0.69	110.0	0.85

Similar overall efficiency with much less efficient (cheaper) generator

Example 4: Fixed load (damping) with tunable PTO

Fixed load ($Z_L = 10\Omega$), Nominal PTO



Fixed load ($Z_L = 10\Omega$), Tuned PTO

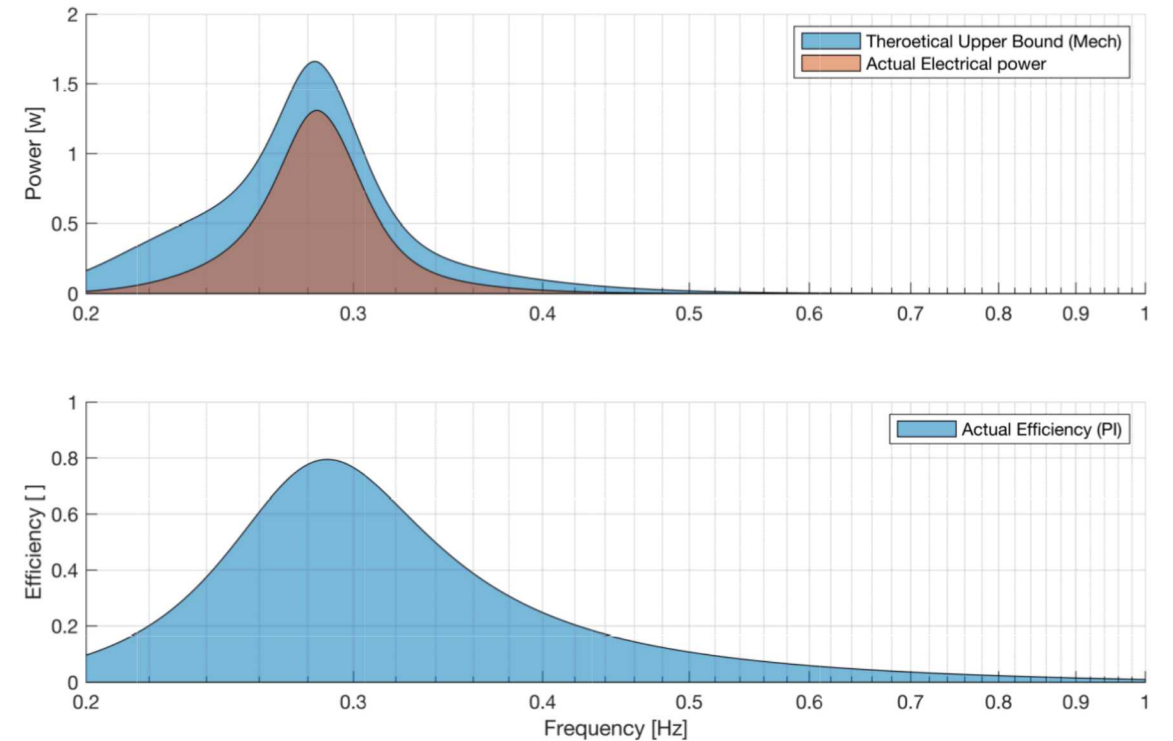


Table 1: Performance improvements of a tuned PTO with a fixed load

	Non Tuned PTO		Tuned PTO	
	P[W]	η	P[W]	η
Theoretical limit ($\bar{P}_m$)	129.3	1	129.3	1
Electrical (P_e)	12.9	0.1	88.0	0.68

$$\Theta = \{I_d, K_d\}$$

Let's see some examples

<https://github.com/SNL-WaterPower/fbWecCntrl/tree/codesign>

The screenshot shows the GitHub repository page for `SNL-WaterPower/fbWecCntrl` at the `codesign` branch. The repository is private and has 5 stars, 1 fork, and 0 issues. It contains 2 branches and 0 tags. The repository description is "Feedback controller design for wave energy converters (WECs)". The repository is licensed under GPL-3.0. The repository contains the following files and folders:

- `codesign`: do three sea states (3 hours ago, 46 commits)
- `data`: FOSWEC impedance - fix consistency (#1) (5 months ago)
- `src`: move WecPower to stand-alone function (2 months ago)
- `.gitignore`: initial commit (5 months ago)
- `COPYING`: initial commit (5 months ago)
- `README.md`: use line breaks for better tracking of edits (4 months ago)
- `demo_foswec.m`: FOSWEC impedance - fix consistency (#1) (5 months ago)
- `demo_waveBot.m`: improve inline documentation (5 months ago)
- `fbWecCntrlTest.m`: add unit tests for mimoPi function (2 months ago)

The README.md file contains the following text:

fbWecCntrl

NOTE: this code is fully-functional, but not supported and users should not expect responses to issues

fbWecCntrl is a set of MATLAB functions and scripts demonstrating a causal impedance matching approach to wave energy converter (WEC) control design. The methods applied in this code are detailed in the following paper, and is a fork of code originally published on [MHK-DR](#).

@Article{Coe2020a,
author = {Ryan G. Coe and Giorgio Bacelli and Dominic Forbush},
title = {A practical approach to wave energy modeling and control},
date = {2020-06-11}}

