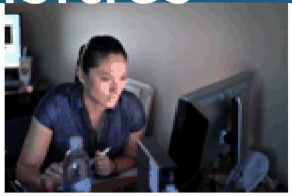
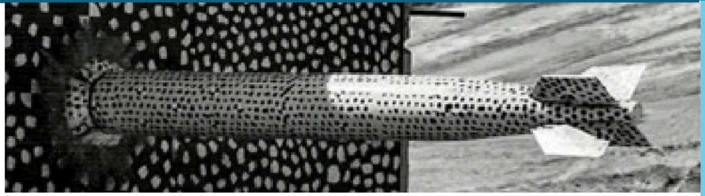
SAND2020-10013C

An Isotropic Large Deformation Viscoplastic Damage Model for Flexible Foams Across a Range of Relative Densities



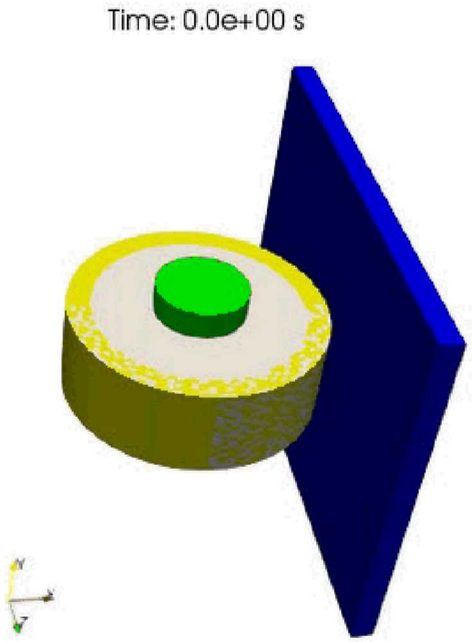


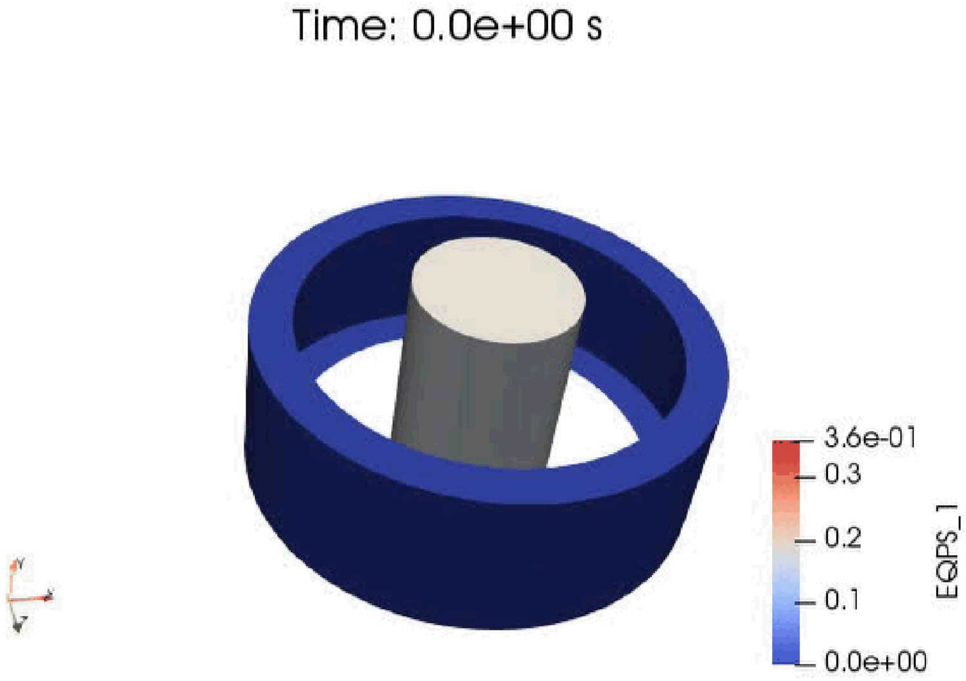
Presented By

Craig M. Hamel, Kevin N. Long, Robert Waymel, Sharlotte Kramer, Mike Neilsen

2020 ASME International Mechanical Engineering Congress and Exposition

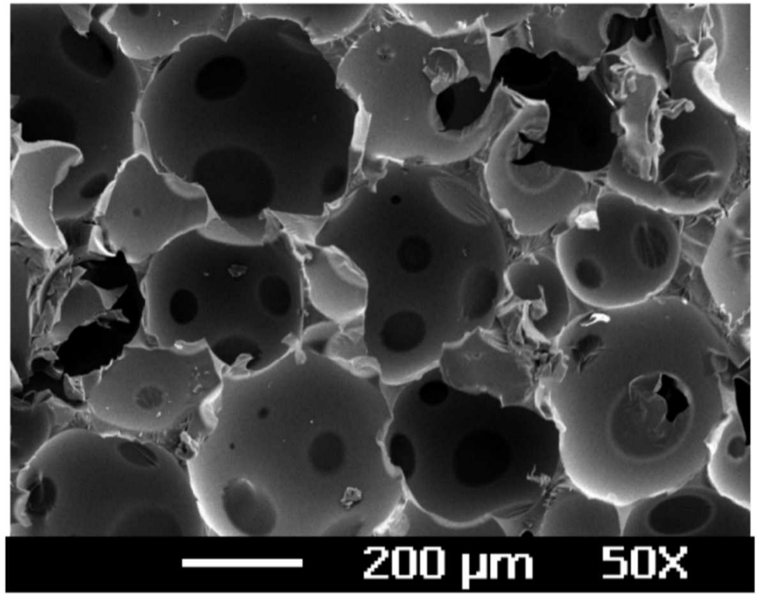
Motivation

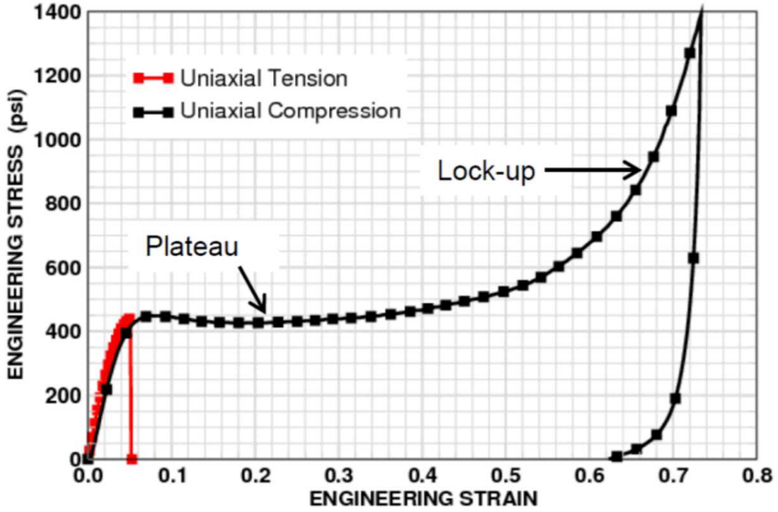




- ❑ Flexible foams are often utilized to alleviate mechanical shock and vibrations
- ❑ They are used in shipping containers to protect assets during transportation
- ❑ Mechanically, their main role is to absorb energy in an impact or crash scenario

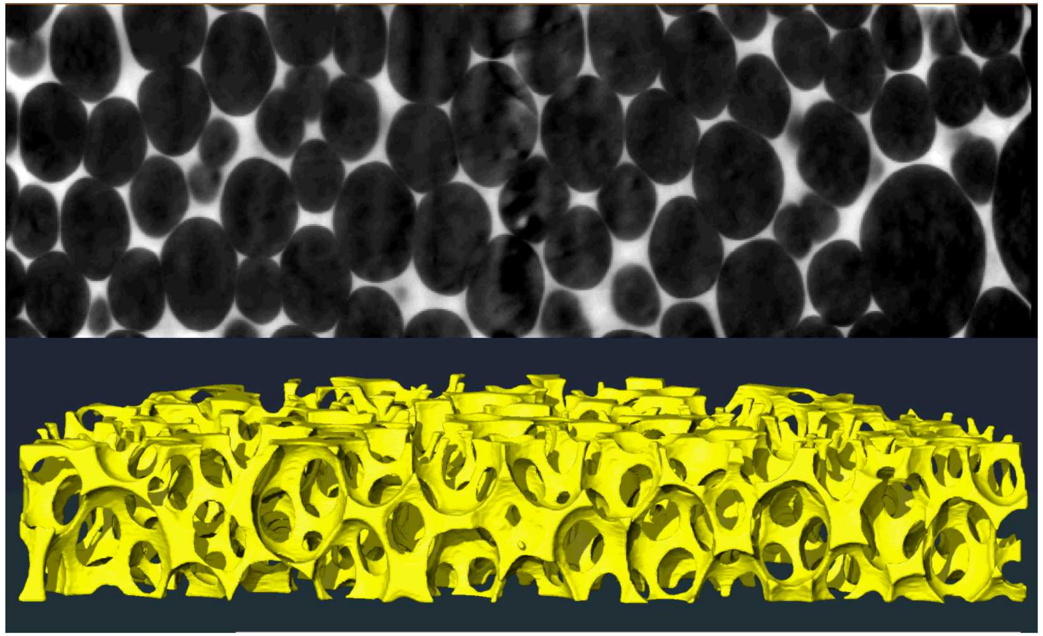
Background and Introduction



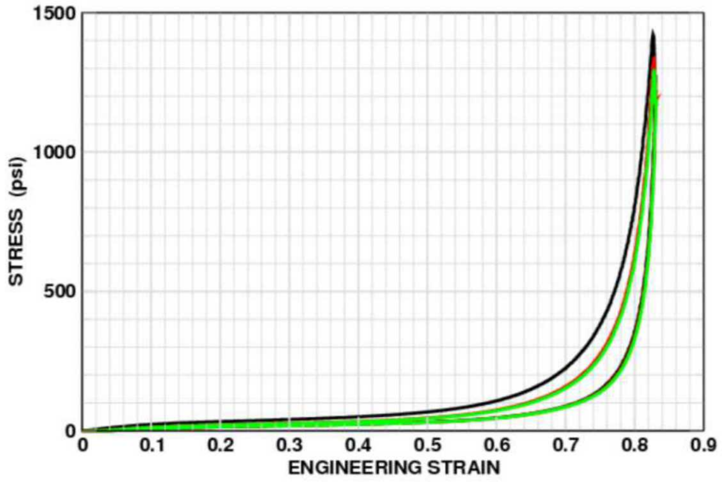


Rigid Foams

Rise Direction



- ❑ The main differences between the two types of foam are recovery vs. non-recovery and the stress scales at different parts of the stress-cycle



Flexible Foams

Constitutive Model Theory – Flexible Response

Lagrangian Mass Balance

$$\varphi = \varphi_0 \frac{V_0}{V}$$

Pseudo strain measure

$$\varepsilon = \int_{s=0}^{s=t} \left(\mathbf{R}^{\mathrm{T}}\mathbf{D}\mathbf{R}\right) \mathbf{ds} = \int_{s=0}^{s=t} (\mathbf{d})\, \mathrm{ds}$$

$$E_f(\theta, \varphi) = E_{f0} E_f(\theta) E_f(\varphi) \left(1 + d_{\mathrm{mult}} \|\varepsilon_{\mathrm{dev}}\|\right)$$

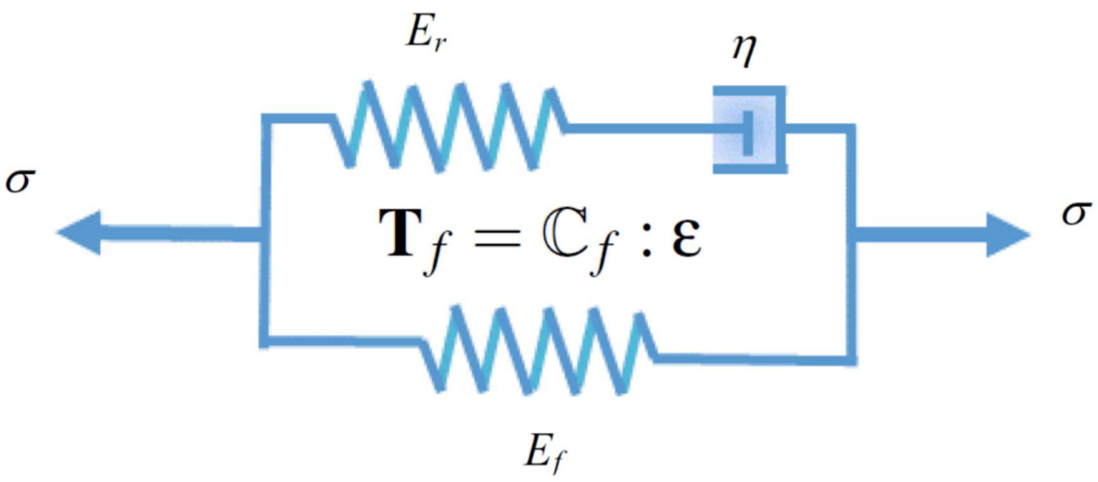
$$\nu_f(\theta, \varphi) = \nu_{f0} \nu_f(\theta) \nu_f(\varphi)$$

$$G_f = \frac{E_f}{2(1+\nu_f)} \qquad K_f = \frac{E_f}{3(1-2\nu_f)}$$

Additive decomposition of Cauchy stress into a flexible and rigid branch

$$\dot{\mathbf{T}} = \dot{\mathbf{T}}_f + \dot{\mathbf{T}}_r$$

$$\dot{\mathbf{T}}_r = \dot{\mathbb{C}}_r : \left(\varepsilon - \varepsilon^{\mathrm{inelastic}}\right) + \mathbb{C}_r : \left(\dot{\varepsilon} - \dot{\varepsilon}^{\mathrm{inelastic}}\right)$$



$$\mathbf{T}_f = 2G_f \,\mathrm{dev}(\varepsilon) + K_f \,\mathrm{tr}(\varepsilon)\,\mathbf{I}$$

Constitutive Model Theory – Flexible Damage Formulation

Flexible Damage Formulation

$$\mathcal{F} = \mathcal{H} - \mathcal{D} \leq 0,$$
$$\mathcal{D} = \text{MIN}\left[\text{MAX}\left(\mathcal{H}[s]|_{s\in[0,t]}\right), \mathcal{D}_{max}\right],$$
$$\mathcal{H} = \frac{C_1 (\text{tr}(\boldsymbol{E}))^2}{2(1-\log\varphi_0)^2} + \frac{C_2 \mathbf{E}_{\text{dev}}^2}{2},$$

- ❑ C1 and C2 are fitting parameters
- ❑ Damage is split into a volumetric and shear response

"Damaged" Young's Modulus

$$E_f(\theta, \varphi, ||\mathbf{E}_{\text{dev}}||, \mathcal{D}) = (1-\mathcal{D}) E_{f0} E_f(\theta) E_f(\varphi) (1 + d_{\text{mult}}||\mathbf{E}_{\text{dev}}||)$$

Constitutive Model Theory – Rigid Response

Stress update

$$\dot{\mathbf{T}}_r = \dot{\mathbb{C}}_r : \left(\varepsilon - \varepsilon^{\text{inelastic}}\right) + \mathbb{C}_r : \left(\dot{\varepsilon} - \dot{\varepsilon}^{\text{inelastic}}\right)$$

Yield surface

$$f = T^* - a$$

$$T^* = \sqrt{\bar{T}_r^2 + \frac{a^2}{b^2} p_r^2}, \quad \bar{T}_r = \sqrt{\frac{3}{2} \text{dev}(\mathbf{T}_r) : \text{dev}(\mathbf{T}_r)}, \quad p_r = \frac{\text{tr}\,\mathbf{T}_r}{3}$$

Flow direction

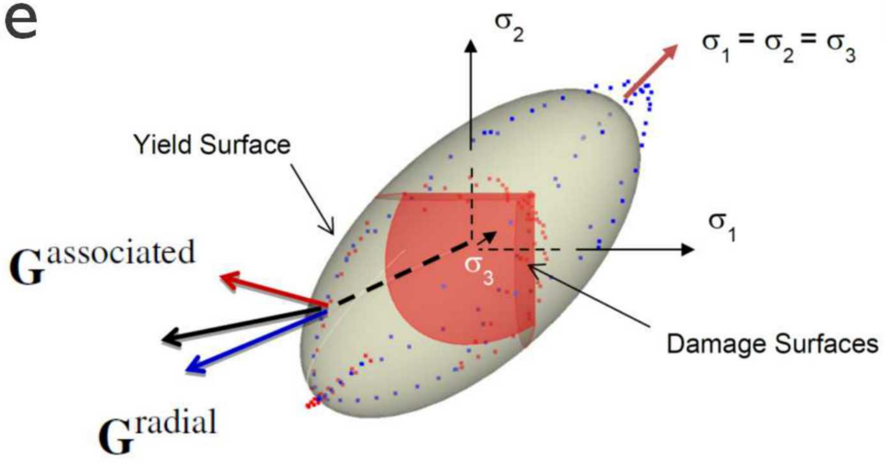
$$\dot{\varepsilon}^{\text{inelastic}} = \dot{\gamma}\mathbf{G}$$

$$\dot{\gamma} = \exp(h_r) \left(\frac{T^*}{a}\right)^{p_r}$$

$$\mathbf{G} = \frac{(1-\beta)\mathbf{G}^{\text{associated}} + \beta\mathbf{G}^{\text{radial}}}{\|(1-\beta)\mathbf{G}^{\text{associated}} + \beta\mathbf{G}^{\text{radial}}\|}$$

$$\mathbf{G}^{\text{associated}} = \frac{\frac{\partial f}{\partial \mathbf{T}_r}}{\left\|\frac{\partial f}{\partial \mathbf{T}_r}\right\|}$$

$$\mathbf{G}^{\text{radial}} = \frac{\mathbf{T}_r}{\|\mathbf{T}_r\|}$$



M.K. Neilsen et. al 1995
V.S. Deshpande et. al 2000
V.S. Deshpande et. al 2001

Shear and hydrostatic strengths a function of relative density

$$a = a[\varphi], \quad b = b[\varphi]$$

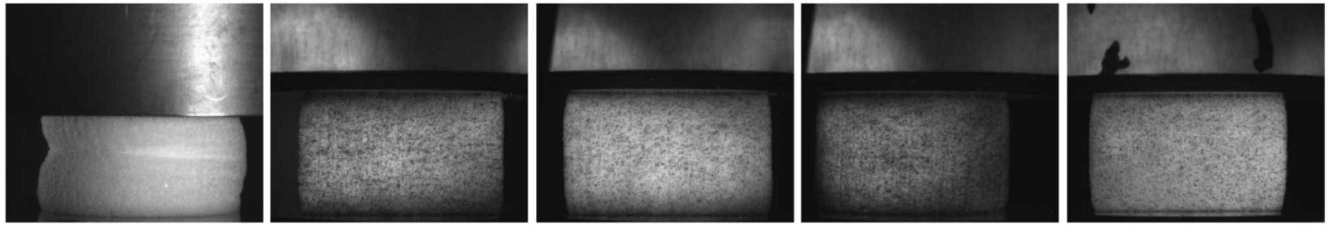
Flow direction a function of relative density

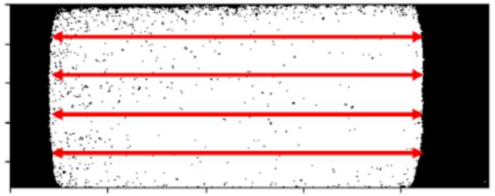
$$\beta = \beta[\varphi]$$

Overview of how the model is calibrated

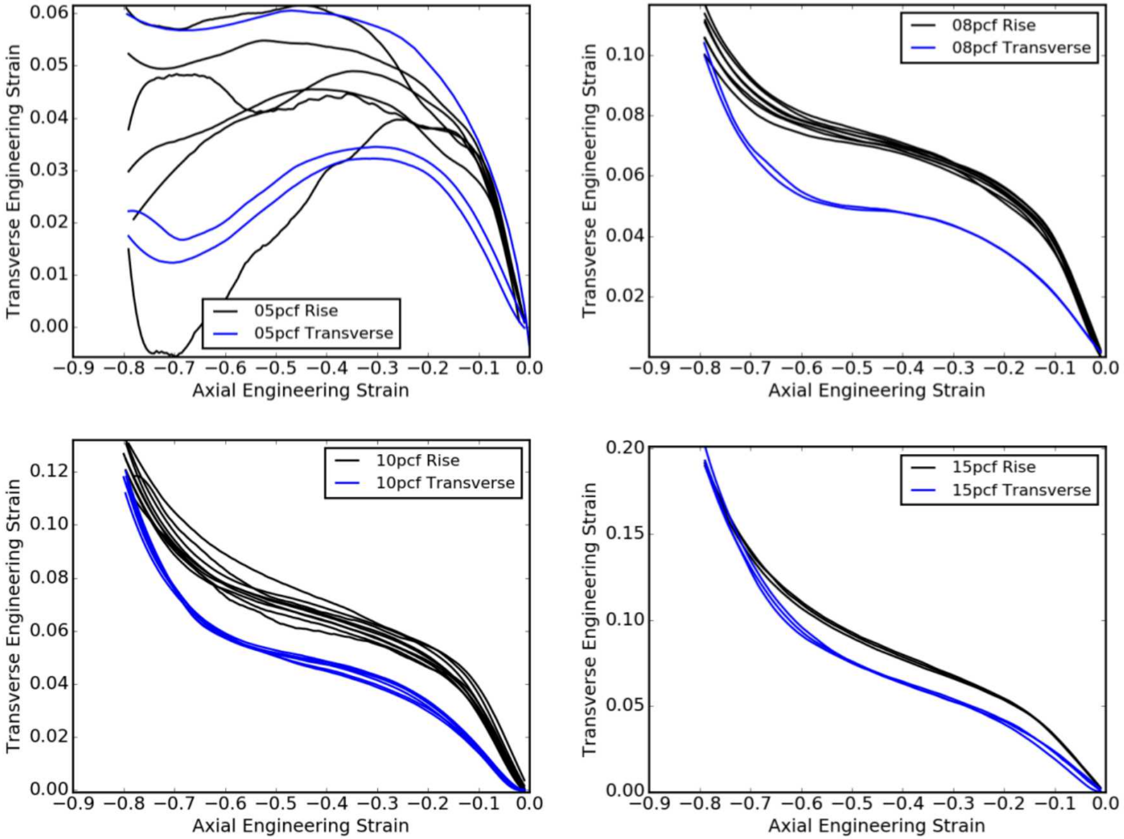
1. Calibrate rate independent flexible branch well above Tg of the matrix material
   a) Uniaxial compression experiments are conducted to extract the solid volume fraction dependent Poisson's ratio
   b) The same experiments are used to determine the solid volume fraction dependent Young's modulus of the rubbery state
2. Damage in the flexible branch is determined by considering the ratio of the stress-strain response between two subsequent cycles
3. Rate dependence model parameters are then tuned using the above material properties as ground truth
4. Triaxial tests are also necessary at cold temperatures to determine the changes of the yield surface, hardening behavior, and flow direction as a function of relative density
5. In this work we focused on the room temperature response which is well above Tg of the matrix material

Uniaxial Compression and Lateral Strain Response





- The solid volume fraction dependent Poisson's ratio was extracted experimentally using edge tracking from DIC experiments under the assumption of homogenous motion
- This part is key so that given a state of compression the, the solid volume fraction can be accurately predicted and for the calibration of other material properties



$$\lambda_{\text{axial}} = 1 + \varepsilon_{\text{axial}}$$

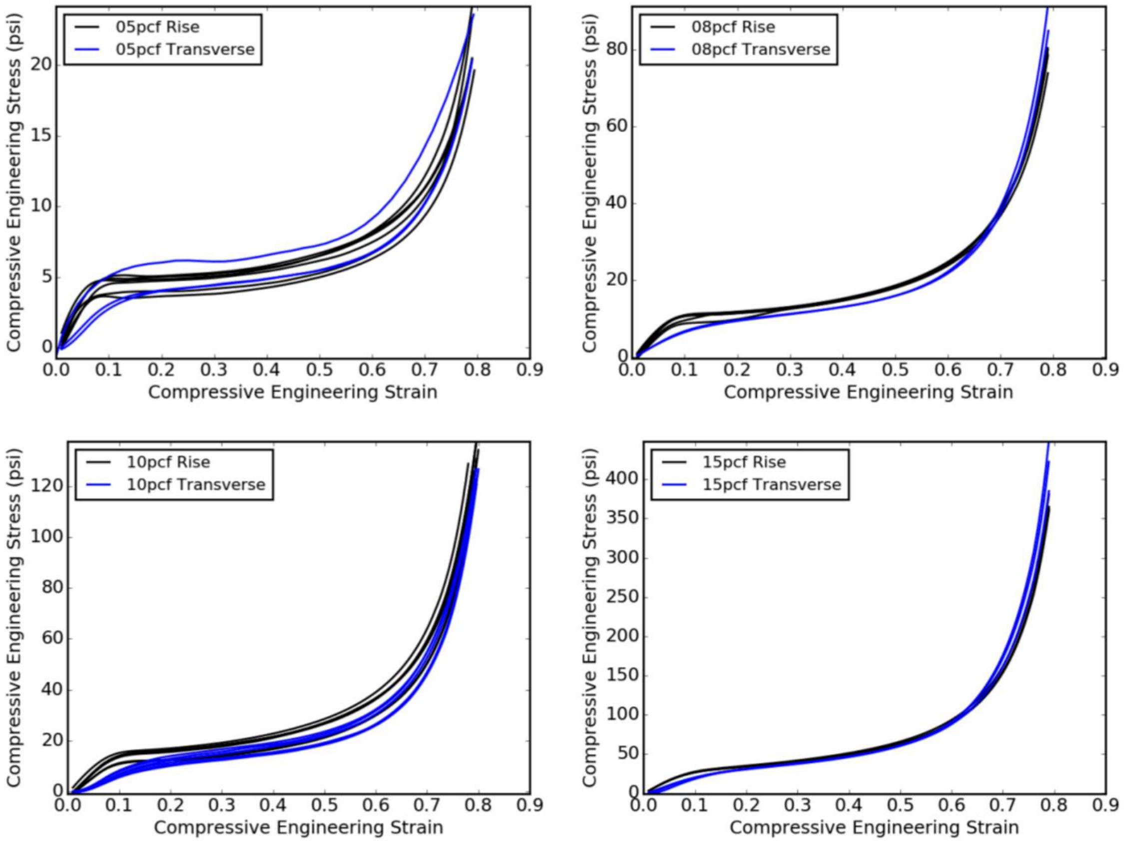
$$\lambda_{\text{transverse}} = 1 + \varepsilon_{\text{transverse}}$$

$$\nu_s = -\frac{\ln \lambda_{\text{transverse}}}{\ln \lambda_{\text{axial}}}$$

$$J = \lambda_{\text{axial}} \lambda_{\text{transverse}} \lambda_{\text{transverse}}$$

$$\varphi = \varphi_0 J^{-1}$$

Uniaxial Compression and Stress-strain response



- Now that the evolution of the solid volume fraction is known as a function of compressive strain the secant Young's modulus can now be calculated
- It should be noted that the current model is isotropic in nature, yet the materials themselves show anisotropy depending on what direction they are compressed relative to the rise direction
- Due to this, different model calibrations are determined and appropriately used depending upon the application the state of loading in application

$$\lambda_{\text{axial}} = 1 + \varepsilon_{\text{axial}}$$

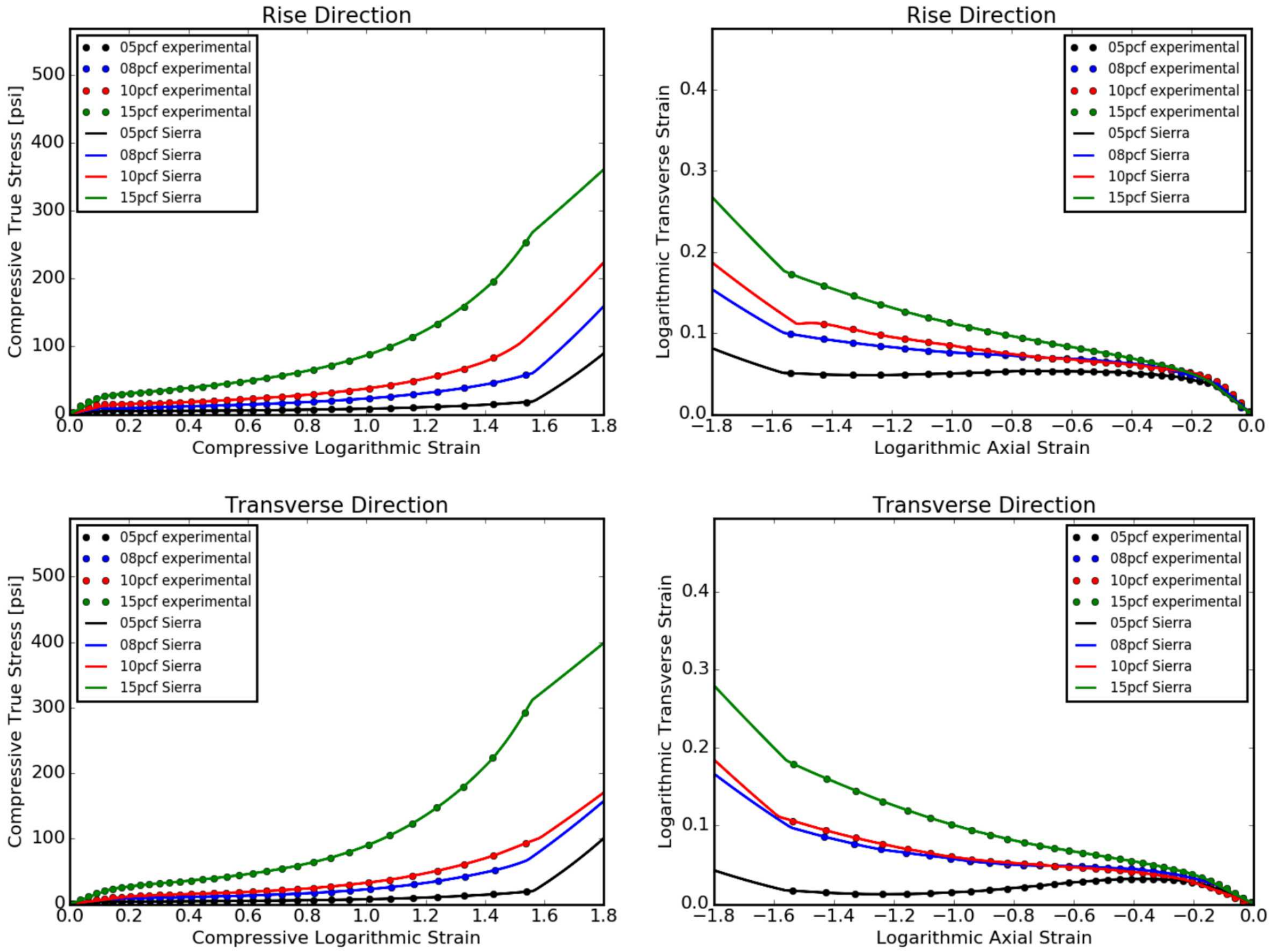
$$\lambda_{\text{transverse}} = 1 + \varepsilon_{\text{transverse}}$$

$$J = \lambda_{\text{axial}} \lambda_{\text{transverse}} \lambda_{\text{transverse}}$$

$$\varphi = \varphi_0 J^{-1}$$

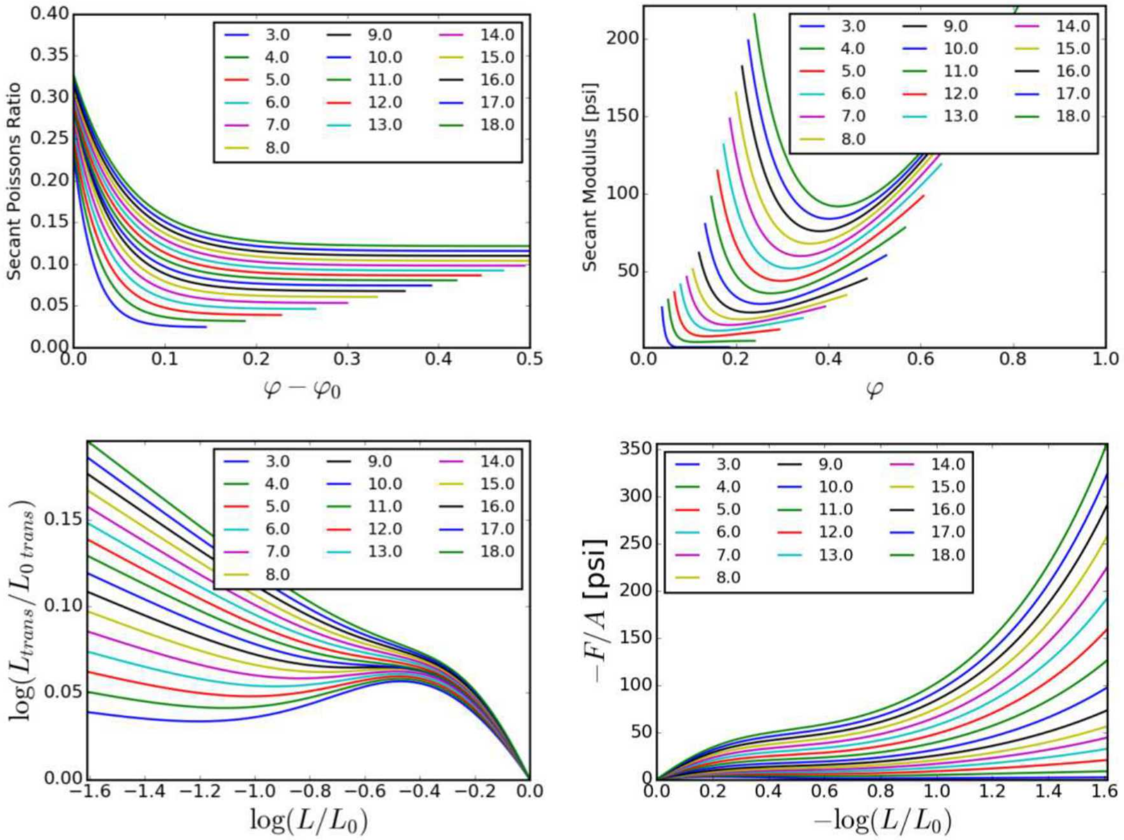
$$E_s = \frac{\sigma_{\text{axial}}}{\ln \lambda_{\text{axial}}}$$

Model Calibration – Individual Density Calibrations



- ❑ Using the above experimental data, model calibrations for four densities of foams loaded in two different directions were produced
- ❑ The experimental data was extrapolated to solid polymer behavior at a relative density of 1
- ❑ Agreement between experiments and the model were achieved for both the stress-strain response and the lateral strain response
- ❑ However, if a model calibration for a new density of foam is needed this process must be repeated
- ❑ Furthermore, per the manufacturers specifications the actual foam density can vary by ±10% for these materials

Model Calibration – Density as an Input

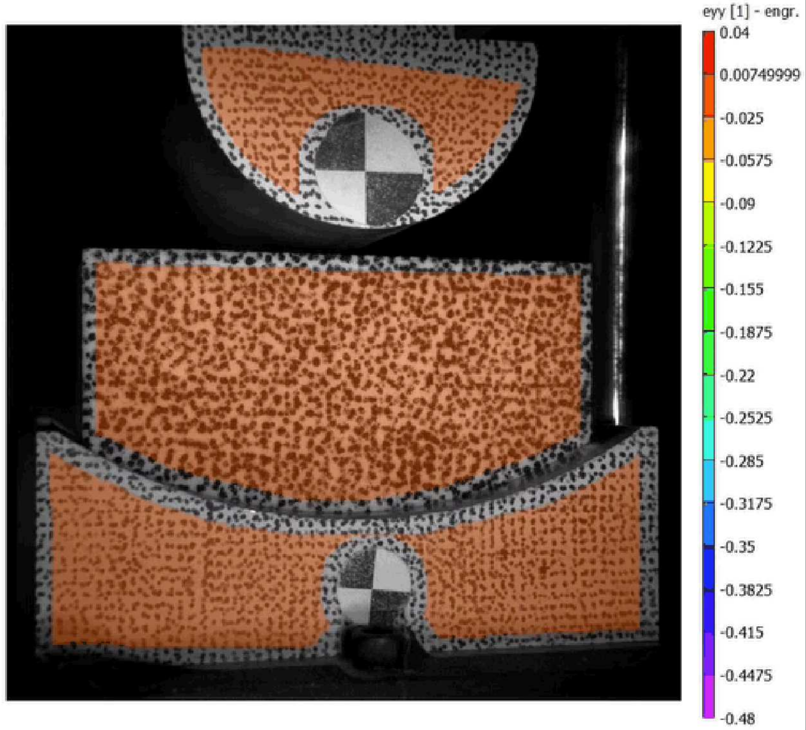


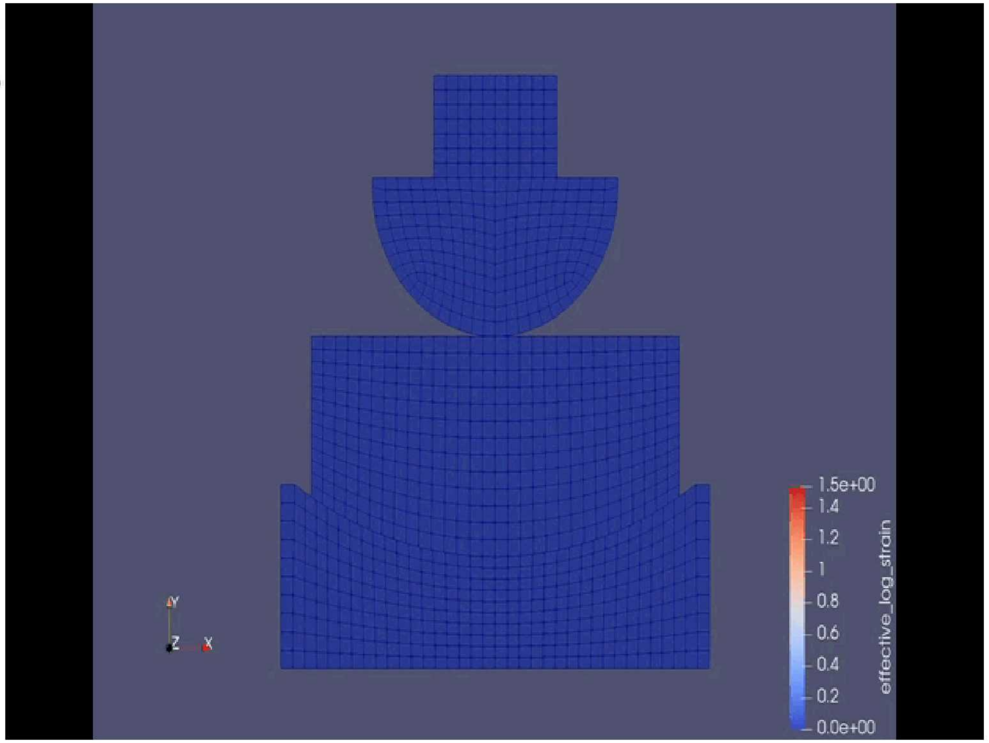
- Since foam density has variance
- Foam density also has a large impact on the material response of the foams
- We have therefore extended the model to take density as a model input
- This is achieved through using the individual density calibrations
- Empirical relations are determined for the solid volume fraction dependent Young's modulus and Poisson's ratio
- These relations are fit to the properties of individual densities
- The fits are then interpolated/extrapolated for other densities

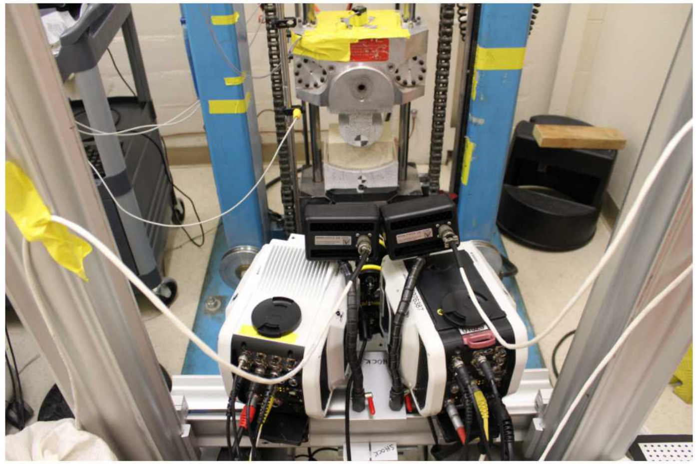
$$E_s(\varphi_{\text{rel}}) \approx d(\varphi_0)\,\varphi^{-e(\varphi_0)} + f(\varphi_0)\,\varphi^{g(\varphi_0)}$$

$$\nu_s(\varphi_{\text{rel}}) \approx a(\varphi_0)\left[1 - b(\varphi_0)\exp\left(-\frac{\varphi_{\text{rel}}}{c(\varphi_0)}\right)\right]$$

Cylinder-Saddle Validation Problem



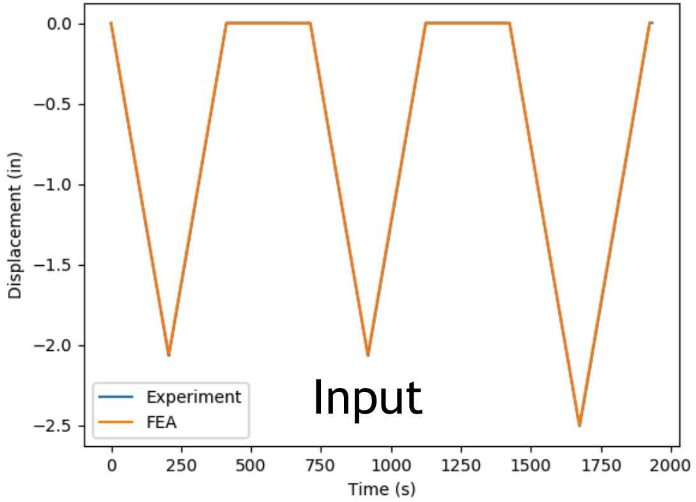


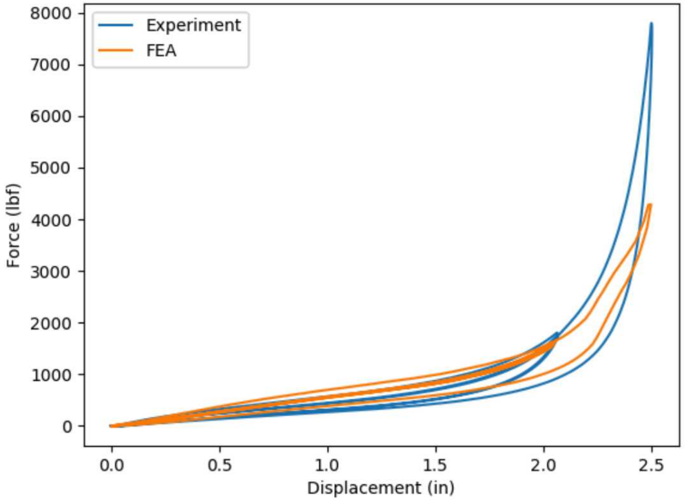


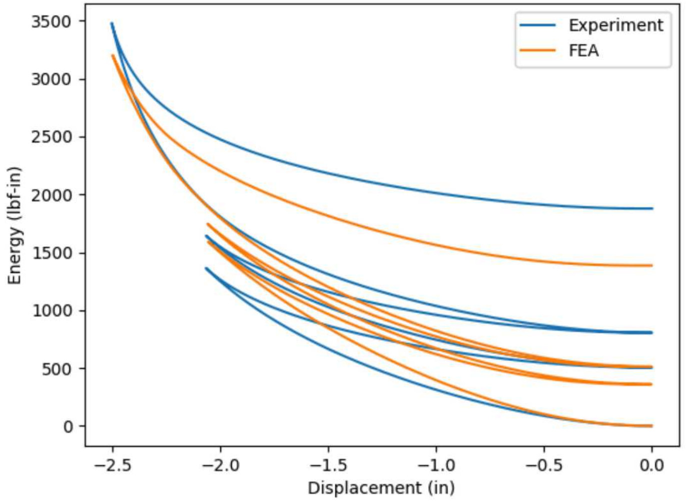
Test Setup

- ❑ A foam indentation problem is used as an initial validation problem for the constitutive model calibrations
- ❑ A slow displacement rate is used to mimic quasi-static conditions
- ❑ Full-field digital image correlation and global mechanical quantities of interest were determined experimentally

Cylinder-Saddle Validation Problems







- The results shown are for a foam specimen of nominal density 15pcf loaded in the Rise direction
- The global quantities in the plots shown above are determined from the force and displacement of the indenter for both the experiments and simulation
- Reasonable agreement is achieved between experiments and simulations for these global quantities
- We plan to extend this validation effort with the full field DIC data in the future

Summary

- A constitutive model for flexible foams has been developed which accounts for large deformation, rate dependence, and damage
- Model calibrations were developed for foams of different densities individually
- The individual density model calibrations were then utilized to interpolate material responses between densities that were tested and extrapolated outside of those densities
- Validation boundary value problems of complex non-homogenous motions were conducted and global quantities of interest correlate with experimental findings