

## Large Bubbles in Vibrated Liquid Are Levitated by Wall Motions

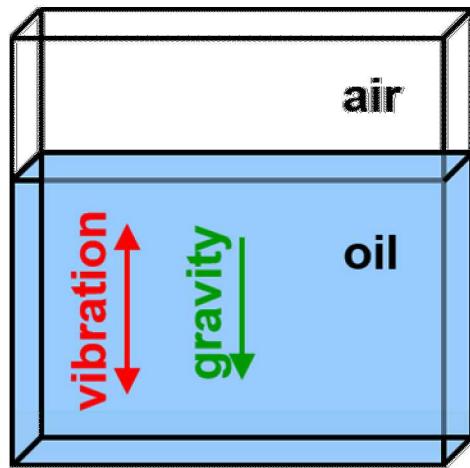
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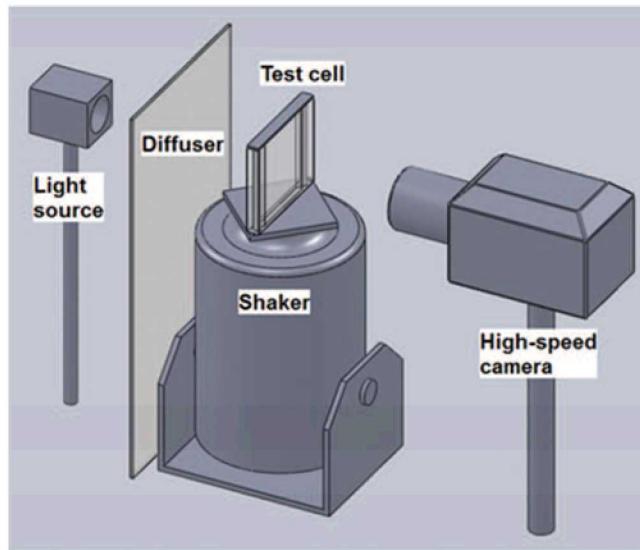
American Physical Society Division of Fluid Dynamics 73<sup>rd</sup> Annual Meeting  
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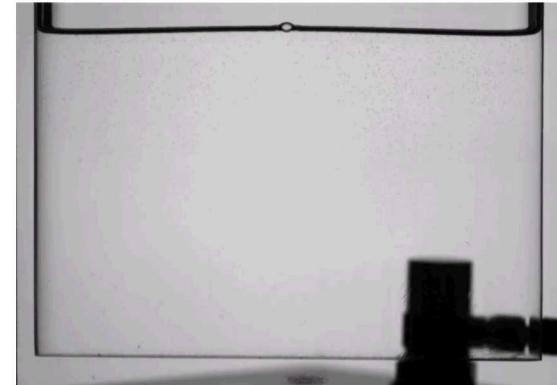
# O'Hern Experiment: Bubbles Sink Easily



Lab Experiment



215 Hz, 25 G: bubbles sink, form gas cavity



O'Hern PoF (2012) experiment: bubbles sink easily and form stable cavities

- Chamber: acrylic, 3" x 3" x 0.115" side walls
- Liquid: 20-cSt PDMS oil, 3" x 2" x 0.25" slab
- Gas: ambient air, 83 kPa and 25 C
- Vibration: 100-300 Hz, amplitude 10-30 G

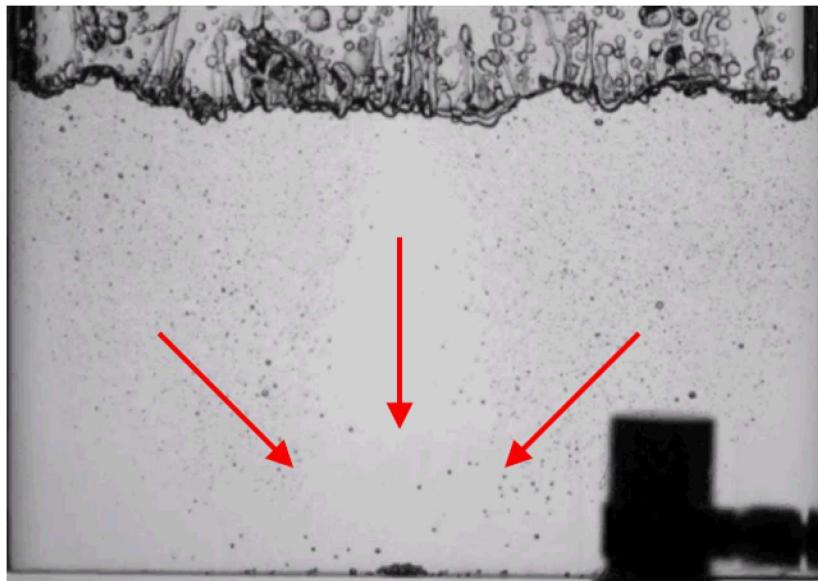
Bjerknes (1906) theory: much stronger vibration needed to make bubbles sink

Why do bubbles sink much more easily in experiment than theory suggests?



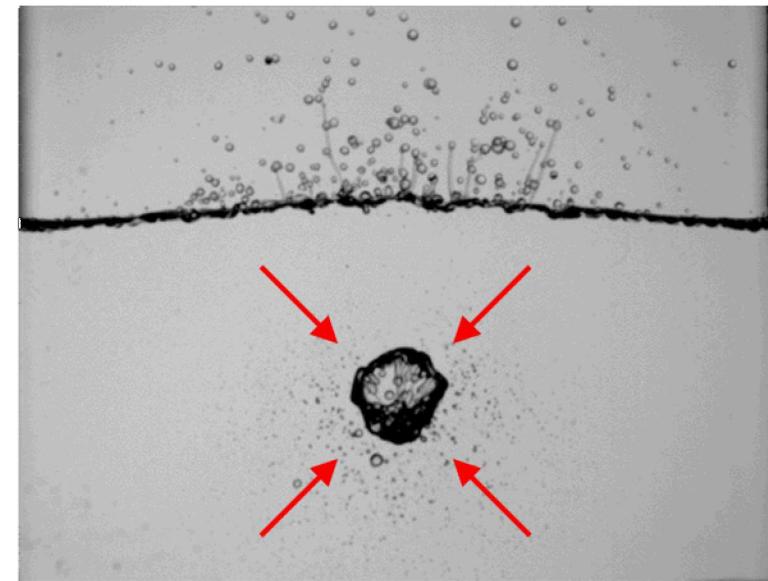
# Two Locations for Submerged Gas Cavities

Case 1: 215 Hz, 25 G



Gas cavity is on bottom

Case 2: 280 Hz, 15 G



Gas cavity is levitated

Stable submerged gas cavities in O'Hern experiment appear in two locations

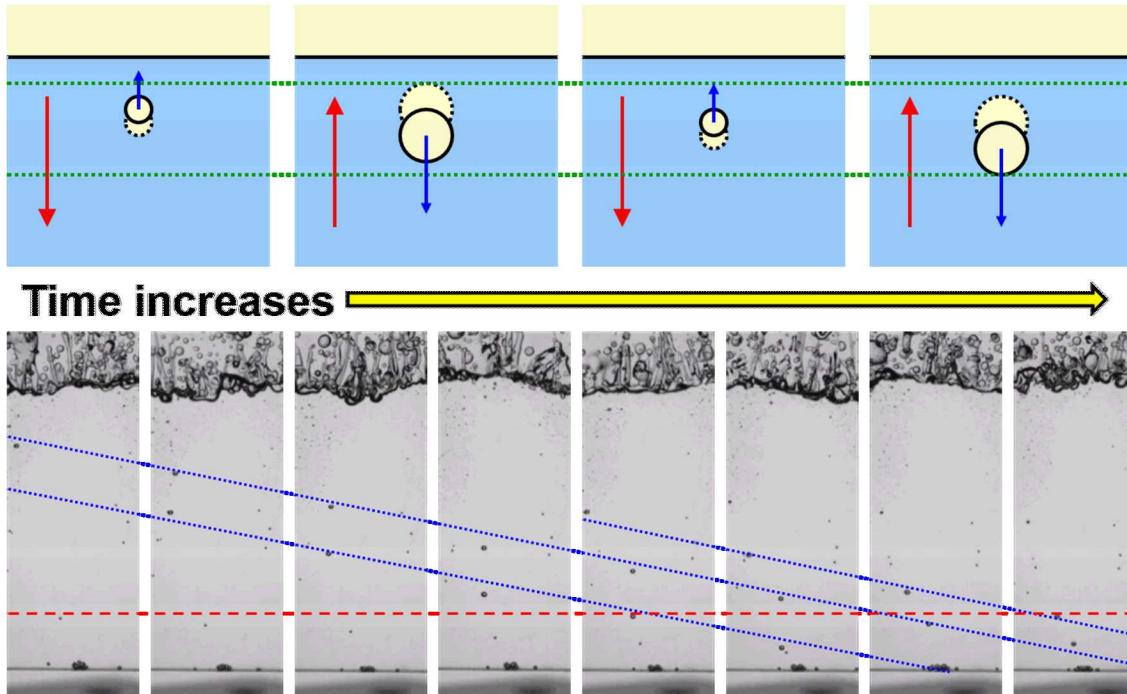
1. Smaller cavity is pressed against bottom and centered in chamber
2. Large cavity is levitated above bottom and centered in chamber

Experimental conditions are similar, so why different gas-cavity locations?

- On bottom versus levitated (laterally centered in both cases)



# Where Bubbles Go Is Determined By Force



Total force on bubble determines the direction in which bubbles try to move

- **Buoyancy** force pushes bubbles upward toward **free surface** (Archimedes)
- **Bjerknes** force pushes bubbles toward **oscillating-pressure maximum**
  - Net force from bubble volume & pressure gradient oscillating in phase

Neutral depth from force balance for uniform oscillating-pressure gradient

- Bubbles below neutral depth sink, but bubbles above neutral depth rise

$$\mathbf{F} = -\langle V_B \nabla p \rangle$$

$$\mathbf{F} = \mathbf{F}_{\text{Buoyancy}} + \mathbf{F}_{\text{Bjerknes}}$$

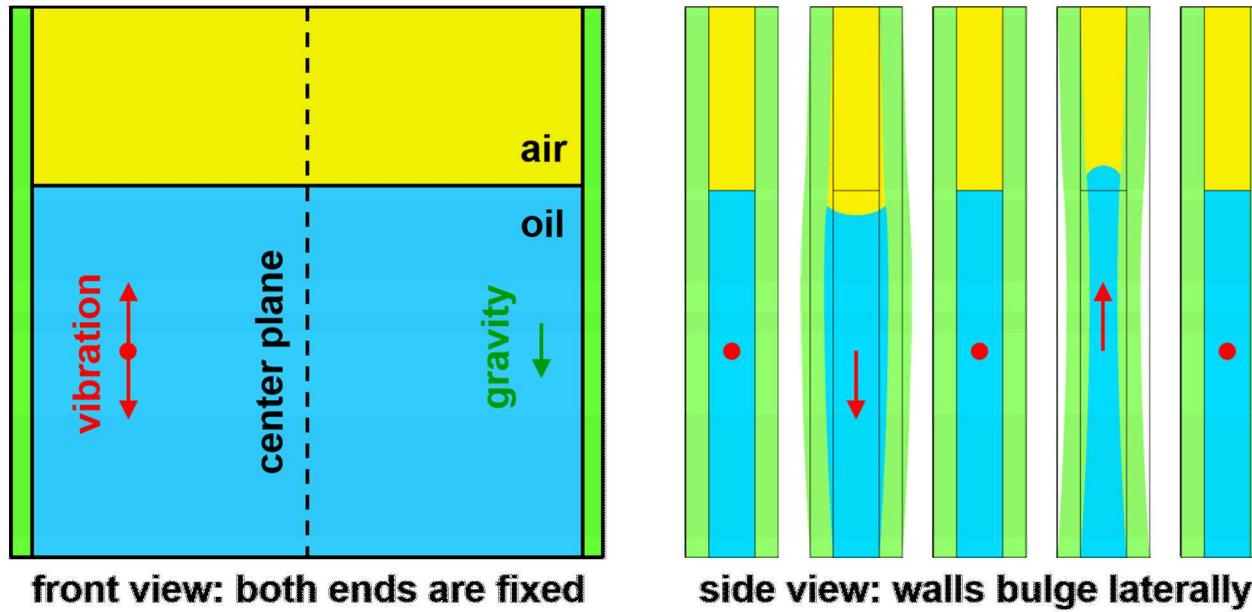
$$\mathbf{F}_{\text{Buoyancy}} = -\rho_L V_B \mathbf{g}$$

$$\mathbf{F}_{\text{Bjerknes}} = \frac{V_B \tilde{\mathbf{p}}}{2\gamma p} \nabla \tilde{\mathbf{p}}$$

$$Z_{\text{Neutral}} = \frac{2\gamma p}{\rho_L g \hat{a}^2}$$



# Flexible Walls Increase Bjerknes Force



Oscillating acceleration causes flexible side walls to bulge laterally

- Bulging is small for end planes (left) but is large for center plane (right)

Bulging walls produce much larger pressure oscillation than rigid walls

- Side walls and gas combine to form a spring with a strong resonance

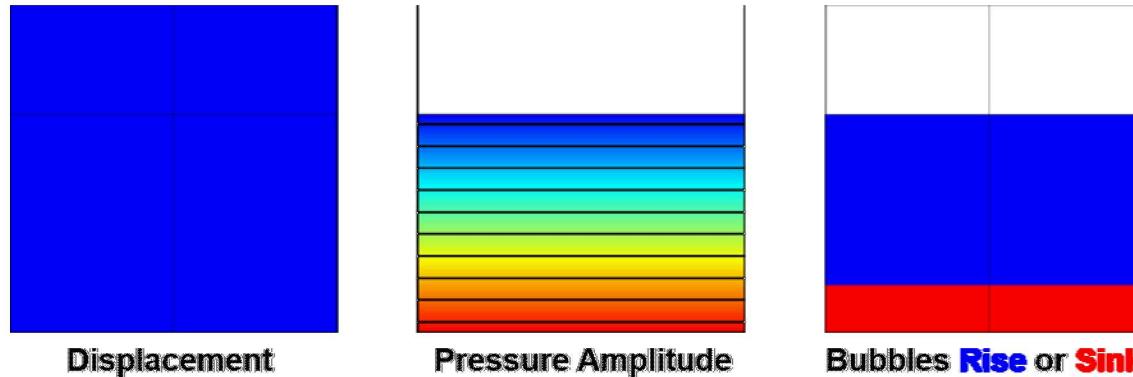
Much larger pressure oscillation yields much larger Bjerknes force

- Bubble model must include both liquid acoustics and side-wall elasticity
- COMSOL Multiphysics is used to implement this acoustic-elastic model

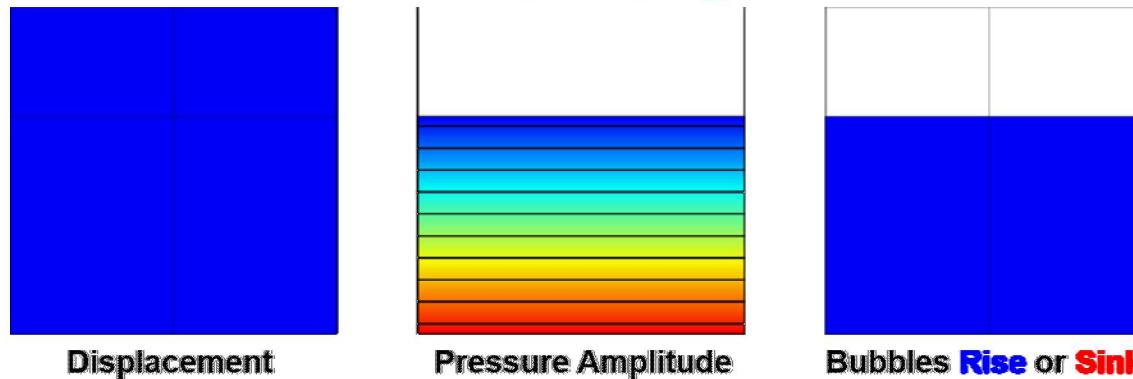


# Rigid Walls: Bubbles Rise

Case 1: 215 Hz, 25 G; **Rigid Walls**



Case 2: 280 Hz, 15 G; **Rigid Walls**



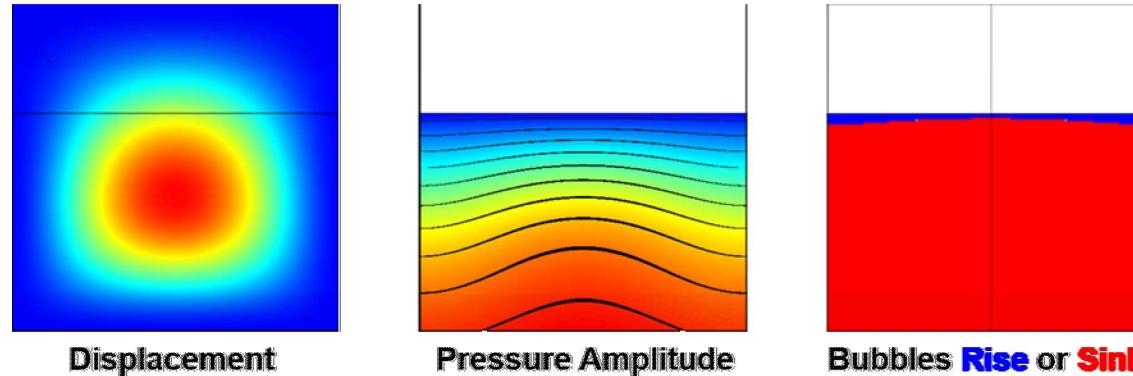
Acoustic-elastic model with **rigid walls** shows that bubbles mostly **rise**

- Buoyancy force exceeds Bjerknes force almost everywhere
- Case 1: neutral depth agrees with theory for uniform pressure gradient

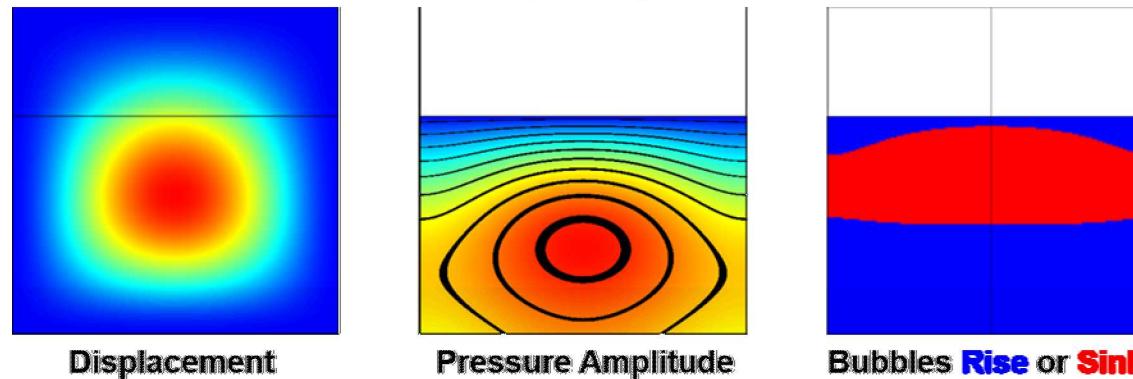


# Flexible Walls: Bubbles Sink

Case 1: 215 Hz, 25 G; **Flexible** Walls



Case 2: 280 Hz, 15 G; **Flexible** Walls

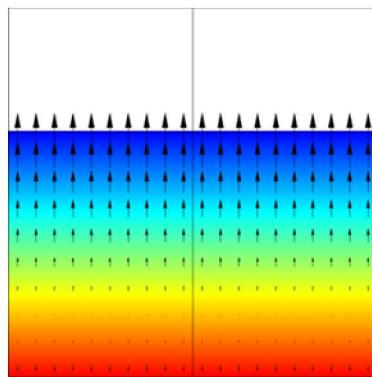


Acoustic-elastic model with **flexible** walls shows that bubbles mostly **sink**

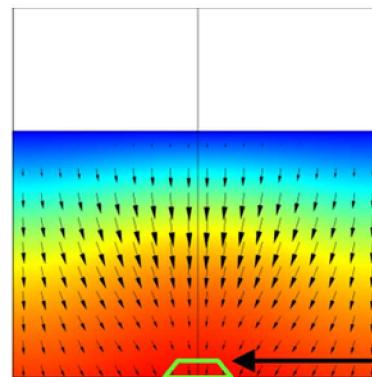
- Bjerknes force pushes bubbles to maximum of oscillating pressure
- Centered at bottom for Case 1 but centered above bottom for Case 2



# Model Matches Experiment

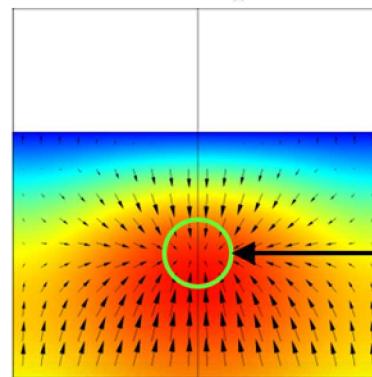
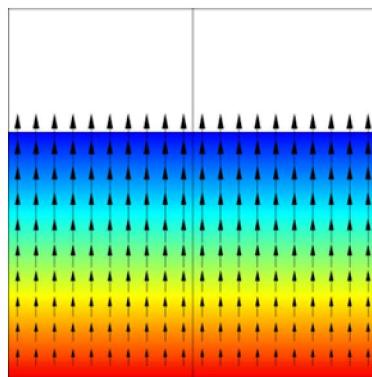
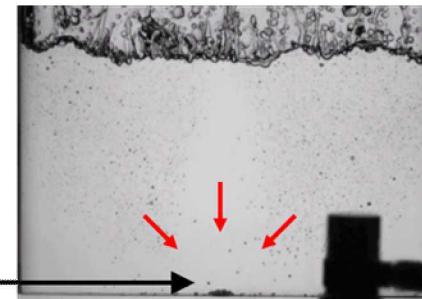


**Rigid walls**  
Bubbles **rise** to free surface

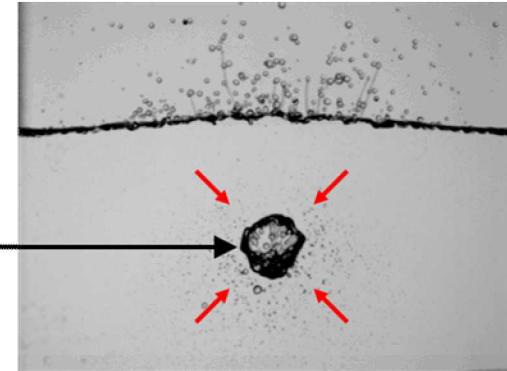


**Flexible walls**  
Bubbles **sink** to observed place

**Case 1: 215 Hz, 25 G**  
**Below resonance**



**Case 2: 280 Hz, 15 G**  
**Above resonance**



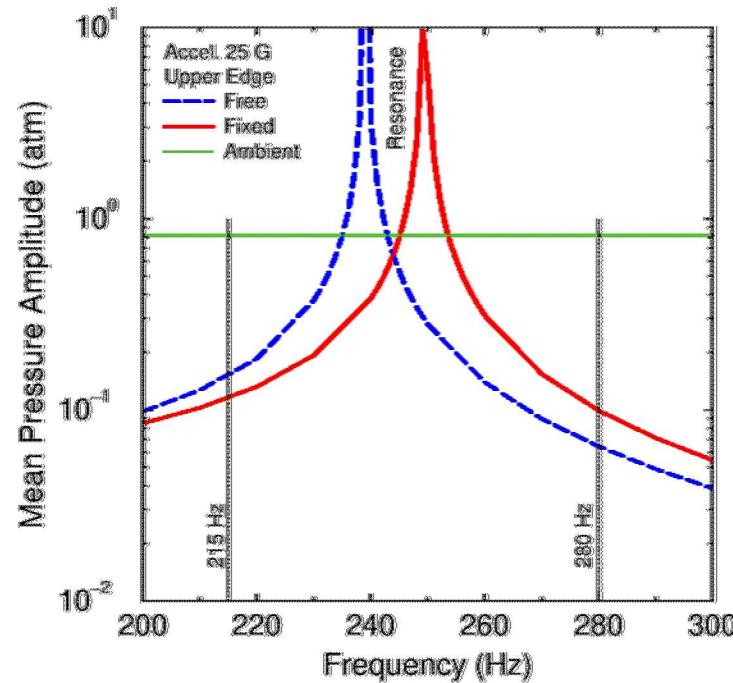
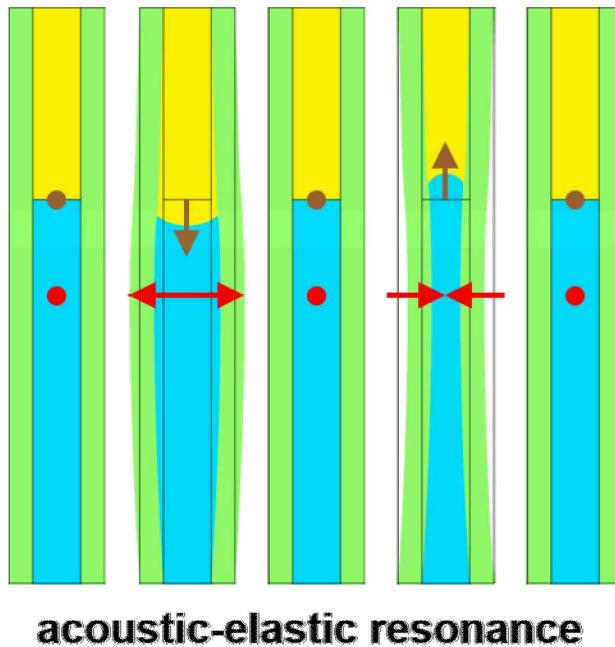
Acoustic-elastic model reproduces results of O'Hern (2012) experiment

- Arrows are total force (buoyancy + Bjerke's), color is oscillating pressure

Total force vectors push bubbles toward gas cavities seen in experiment



# Model Explains Different Gas Locations



Acoustic-elastic resonance is responsible for different locations of gas regions

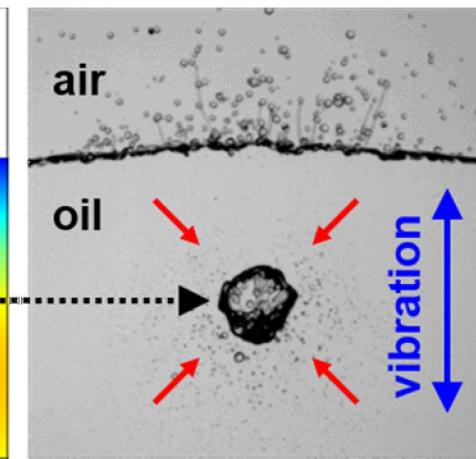
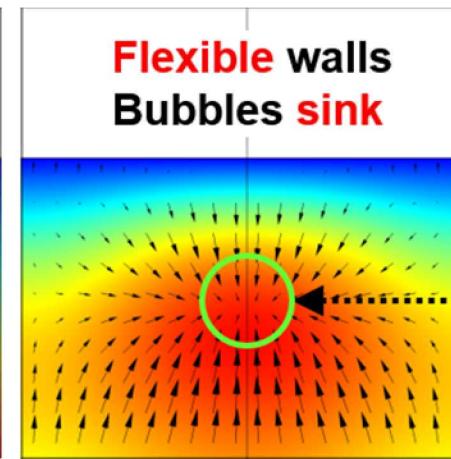
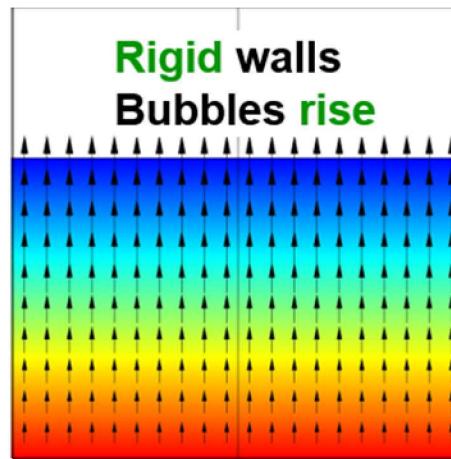
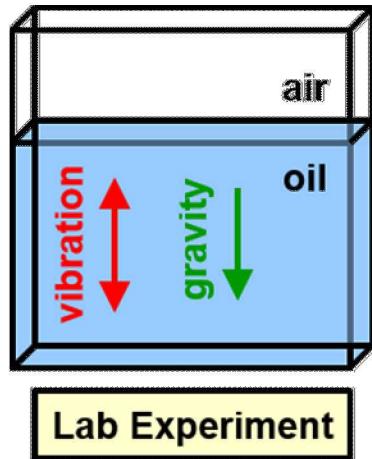
- Liquid and side walls form a spring-mass system, which has a resonance
  - Spring is composed of upper ambient gas region and flexible side walls
  - Mass is based on liquid and side-wall masses

Vibration frequency determines location of oscillating pressure maximum

- On bottom if below resonance, but above bottom if above resonance



# Conclusions



Flexible walls can greatly increase Bjerknes force on bubbles during vibration

- Walls and gas create a spring against which liquid/wall mass oscillates
- When frequency is near resonance, oscillating pressure becomes large
- When oscillating pressure becomes large, Bjerknes force becomes large

Acoustic-elastic model explains large bubble motion in O'Hern experiment

- For rigid walls, Bjerknes force is smaller than buoyancy, so bubbles rise
- For flexible walls, Bjerknes force is larger than buoyancy, so bubbles sink
- Gas-cavity location is governed by being below or above resonance

