# Non-Cooperative Optimization of Charging Scheduling of Electric Vehicle via Stackelberg Game

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**Abstract:** In this paper, we deal with the charging scheduling optimization problem of electric vehicle using Stackelberg game. Stackelberg game is one of game theory classified as hierarchical, repeating, and non-cooperative game. The charging station determines the price to maximize its own profit from selling energy and each EV determines the energy demand to maximize the charge benefit by competing with other EVs. At this time, we guarantee that Nash equilibrium exists within the EV group. Finally, using numerical simulation, we show that the game reaches an Stackelberg equilibrium.

**Keywords:** Electric vehicle, Charging scheduling, Stackelberg game

## 1. INTRODUCTION

In recent years, the development and practical use of EV (Electric Vehicle) is progressing due to environmental problems. Compared with conventional gasoline and diesel cars, EV has a remarkably short distance to travel without charge. Therefore, when traveling over long distances typified by expressway driving, it is necessary to plan the location and time of charging in advance. This problem is called charging scheduling problem of EV.

In the United States, carriers such as Car Charging provide CS (Charging Station). In addition, in some parts of China, the EV needs to charge by paying an additional fee other than electric fee, and the price can be freely decided by the CS operator [1]. In this way, as the number of CS increases, price competition between CSs is expected to intensify, and CS managers need to find a pricing method that maximizes their revenue. Furthermore, it is also necessary to consider that the number of chargers at the CS is finite. If many EVs concentrate on one charging station, the station manager will lose the profit, so it is desirable that the usage rates of all the CSs be uniform. In other words, the CS manager not only maximizes the profit by selling energy to the EV, but also aims to place the EV at each station desirably by controlling the price.

On the other hand, EV selects the CS according to own purpose. Cooperative algorithm has already been proposed for EVs to cooperate with each other and to achieve scheduling [2]. However, it is not necessarily realistic that the EVs cooperate with each other. Therefore, it is necessary to consider situations where EVs behave selfishly and compete against each other. In addition, in many previous studies, the charge amount and service time of EVs are assumed to follow the exponential distribution, and the utility of each EV and the decision making mechanism are not mentioned, and assumption that all EVs are uniformly fully charged. In reality, it is unnatural that all the EVs are fully charged, and EV should consider that the charge amount can be freely determined in the scheduling problem. Even in this point, it can be said that considering the macro situation where EVs compete with each other is useful.

The main contribution of this paper is a proposition of non-cooperative scheduling algorithm using the Stackelberg game [3] classified as a hierarchical, repetitive, and noncooperative game in which EVs and CSs are regarded as players, respectively. The advantage of the Stackelberg game is that it can capture the behavior of the player microscopically and the order of action is determined by the hierarchy. That is, we can express the competition mechanism between EVs and the order of behavior decision between CS and EV, and we can propose more realistic algorithm [4]. However, the equilibrium in the Stackelberg game is generally difficult to theoretically analyze, and many of them are limited to numerical solutions [5].

In this paper, we model the flow of EV, the transition of the number of vehicles in CS, and battery of EV. After that, formulate the one-leader multi-follower Stackelberg game with EVs as followers and CS as a leader. Furthermore, we discuss the solution of Stackelberg game, and indicate the global Nash equilibrium necessary to show the existence of Stackelberg equilibrium, and propose noncooperative optimal charge scheduling algorithm. Finally, confirm that the proposed algorithm achieves equilibrium using numerical simulation.

## 2. PROBLEM STATEMENT

An overview of the network used in this research is shown in Fig.1. Each CS belongs to the same administrator and exchanges information with each other. When CS interval is assumed to be $T$, each station presents the charging price to each vehicle at the scheduling time $k = nT (n = 1, 2, \dots)$. This exchange is done through V2I and I2V. In addition, each EV communicates with other surrounding EVs to determine their desired charging demand while maintaining a non-cooperative attitude towards other EVs. Setting up such a network is reasonable considering that most CSs are connected to the network for paying credit cards, and car navigation systems are installed in EVs.

† Miyu Yoshihara is the presenter of this paper.

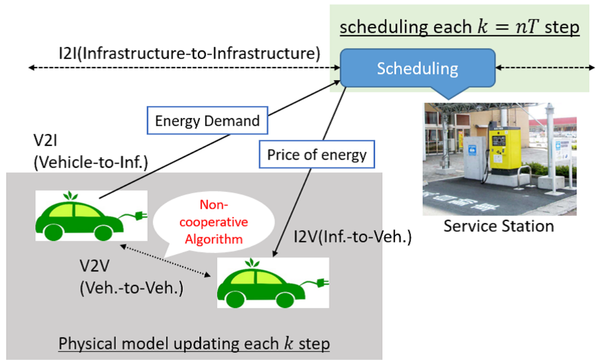


Fig. 1 Network condition

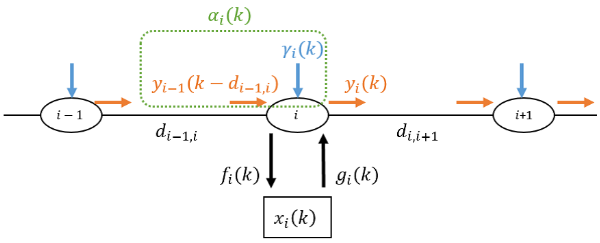


Fig. 2 Traffic flow model

### 2.1. EV flow and CS model

The first component of the model describes the flow of EVs on the highway. Entrances/exits or CS to each of which we associate one node of the network. Let the total number of nodes be $N$. We consider a chain topology in which node $i$ is connected only with the nodes of $i-1$, $i+1$. Thus edges represente visual links between two successive entrances, exits or CS. A generic portion of the chain involving node $i$ and its predecessor and successor is depicted in Fig.2.
Let $\alpha(k)$ be the average EV flow arriving at a node $i$, which is given by

$$\alpha_i(k) = \begin{cases} \gamma_i(k), & \text{if } i = 1 \\ \gamma_i(k) + y_{i-1}(k - d_{i-1,i}), & \text{if } i \neq 1 \end{cases} \tag{1}$$

where $\gamma_i(k)$ is the exogenous flow entering the chain network at node $i$ at time $k$, $y_{i-1}(k)$ is average EV flow coming from node$(i-1)$ to node $i$, $d_{i-1,i}$ is the time required for an EV to traverse the edge from node $i-1$ to node $i$. The number of EVs departing from node $i$, $y_i(k)$, can be written as follows.

$$y_i(k) = \alpha_i(k) + g_i(k) - f_i(k), \tag{2}$$

where $g_i(k)$ represents EV flow comming out from service station $i$, and $f_i(k)$ represents EV flow entering to service station $i$. When node $i$ is not a CS, $g_i(k) = f_i(k) = 0$.
In addition, we present a queue model at each SS. Let $x_i(k) \geq 0$ be the number of EV in CS $i$.

$$x_i(k+1) = x_i(k) + f_i(k) - g_i(k). \tag{3}$$

### 2.2. EV energy model

This subsection we model the power comsumption of the EVs, which is gives as

$$e^-_{v,i+1} = e^+_{v,i} - d_{i,i+1} r^-_v, \tag{4}$$

$$e^+_{v,i} = e^-_{v,i} + \frac{E_{v,i}}{\mu_v}, \tag{5}$$

where $e^-_{v,i+1}$ is the SOC (State of Charge) when thr generic EV $v$ arrives at node $i+1$, $e^+_{v,i}$ indicates the SOC when EV $v$ leaves the node $i$. Since the time required to move from the node $i$ to node $i+1$ is $d_{i,i+1}$, $r^-_v$ is the SOC required for unit time running. The second term in the right-hand-side of (4) represents the amount of charge consumed to move between nodes $i-1, i$.
The equation (5) shows the SOC of the EV departing node $i$. $E_{v,i}$ is the amount of energy that EV $v$ charges at node $i$, and $\mu_v$ is the battery capacity. Each EV bids the desired amount of charge to the CS, whereas the CS individually sets a price that maximizes its own profit.
The charging strategy of each EV satisfies the following two constraints;

$$E^{min}_{v,i} \leq E_{v,i} \leq E^{max}_{v,i}, \tag{6}$$

$$e^-_{v,i} + \frac{E_{v,i}}{\mu_v} \leq 1. \tag{7}$$

Inequalities (6) are constraints that the upper and lower bounds of the amount of energy that an EV can charge at CS, and inequality (7) describes the fact that the SOC caanot exceed 100 %.

## 3. STACKELBERG GAME

At the time $k$, EV that toward the CS $i$ which has $c_i$ charger express $\mathcal{K}_i(k) := \{1, \dots K(k)\}$. The CS, as EV can achieve the energy demand, reasonable price set $\boldsymbol{p}(k) = [p_1(k), \dots, p_{K(k)}(k)]$, and eventually charging some EVs among them. Here, $p_v(k)$ is the energy price per unit for some EV $v$. The charging energy demand of sets of the target EV $\boldsymbol{E}(k) = [E_{1,i}(k), \dots, E_{K(k),i}(k)]$. Here, $E_{v,i}(k)$ is charging energy amount EV $v$ at the CS $i$. There is the case that EV is not charge,$E_{v,i}(t) = 0$.
In this paper, EV once after conveyed the desired amount of energy to the CS, the CS presents the charging price so as to maximize its own profits to the EV, EV is based on the asking price. It will repeat a series of flow, such as declaring the amount of energy that maximizes the utility of my own again. Therefore, this problem is a repetitive non-cooperative game with a hierarchical structure, CS has the decision privilege to EV. Such a problem is formulated as one leader/multiple follower game of Stackelberg game.

### 3.1. Follower model

$U^k_{v,i}$ ,represents the utility function of EV $v$, is written as folows.

$$\begin{aligned} U^k_v =& \mu_v E_{v,i}(k) - \frac{1}{2}\theta_{v,i}(k)\left(E_{v,i}(k)\right)^2 \\ & - p_{v,i}(k) E_{v,i}(k) - p_{v,i}(k)\left(E_{v,i}(k) - \bar{E}_i(k)\right), \end{aligned} \tag{8}$$

where, $\bar{E}_i(k)$ is the average energy demand of all EVs. In addition, $\theta_{v,i}(k)$ is

$$\theta_v(k) = \frac{\frac{1}{\left(\mu_v - e_v^-(k)\right)}}{\sum_k \left(\frac{1}{\left(\mu_v - e_v^-(k)\right)}\right)}. \quad (9)$$

The equation (9) shows a satisfaction parameter indicating the measure of satisfaction of SEV $v$ obtained by charging an unit of energy with SS $i$. For example, if SEV $v$ has a higher need for energy demand than SEV $v+1$ (eg, it is going to travel farther, has a larger battery, etc.) then SEV $v$ is the same to achieve satisfaction, more energy is required than SEV $v+1$. Therefore, it becomes $\theta_v(k) \leq \theta_{v+1}(k)$.
Thus, the optimization problem solving for each EV can be written as follows.

$$\mathbf{P1}: \max_{E_{v,i}(k)} U_{v,i}^k \left(E_{v,i}(k), \boldsymbol{E}_{-v,i}(k), \mu_v, p_{v,i}(k)\right),$$
$$\text{s.t. } (6), (7).$$

### 3.2. Leader model

The CS has two objectives. One is to uniformize the utilization of each CS as shown in [2]. This problem is written as follows.

$$\mathbf{P2.1}: \min_{F_i(k)} \left[\max_i \left(\frac{x_i(k)}{c_i}\right) - \min_j \left(\frac{x_j(k)}{c_j}\right)\right],$$
$$\text{s.t. } 0 \leq F_i(k) \leq \sum_{k=nT}^{(n+1)T} \alpha_i(k),$$

where, $F_i(k) = \sum_{j=n}^{n+1} f_i(jT)$. Since $\frac{x_i(k)}{c_i}$ indicates the utilization rate of CS $i$, it is written as a problem to minimize the difference between the maximum utilization rate and the minimum utilization rate.

The second objective is to maximize own profit by selling energy to EV. The optimization solution of the profit function is the price pair $\boldsymbol{p}(k)$ to each EV and the shared energy demand pair $\boldsymbol{E}(k)$. Thus, the utility function of the CS,

$$Q_i^k = \sum_v \left(p_{v,i}(k)E_{v,i}(k) + p_{v,i}(k)\left(E_{v,i}(k) - \bar{E}_i(k)\right)\right) - \frac{a}{2}\left(\sum_v E_{v,i}(k)\right)^2. \quad (10)$$

The second optimization problem solved by CS can be written as follows.

$$\mathbf{P2.2}: \max_{p_i(k)} Q_i^k \left(\boldsymbol{E}_i(k), \boldsymbol{p}_i(k)\right),$$
$$\text{s.t. } p_i^{min} \leq p_i(k) \leq p_i^{max}.$$

## 4. THE SOLUTION OF THE STACKELBERG GAME

The purpose of the Stackelberg game is to realize a point with no incentive for both players to deviate, called Stackelberg equilibrium. In game theory, the entire set of optimal strategies that players should take in each information set is called partial game complete equilibrium. The Stackelberg equilibrium is consistent with the actual strategic route which can actually occur out of the partial game complete equilibrium. Therefore, when there is an optimal solution uniquely in each of the follower game and the leader game and the partial game complete equilibrium exists, the Stackelberg game reaches equilibrium. In this paper, after showing the existence and uniqueness of the Nash equilibrium in non-cooperative follower game, we show that the optimal solution exists for the follower game and the leader game.

### 4.1. The solution of follower game

For the price $\mathbf{p}(k)$ in $\mathbf{P1}$ indicated by equation (10), the utility function of each EV is expressed by the concave function of $E_v(k)$, and the constraint is linear. Therefore, the Lagrangian dual decomposition method can be applied. At the time $k$, EV $v$, the Lagrangian function is as follows.

$$\begin{aligned}\mathcal{L}_v^k =& \mu_v E_{v,i}(k) - \frac{1}{2}\theta_v(k)E_{v,i}^2(k) - p_{v,i}(k)E_{v,i}(k) \\ & - p_{v,i}(k)\left(E_{v,i}(k) - \frac{\sum_v E_{v,i}(k)}{K_i(k)}\right) \\ & - \kappa_v(k)\left(E_{v,i}^{min} - E_{v,i}(k)\right) \\ & - \lambda_v(k)\left(E_{v,i}(k) - E_{v,i}^{max}\right) \\ & - \nu_v(k)\left(e_{v,i}^- + \frac{E_{v,i}(k)}{\mu_v} - 1\right), \end{aligned} \quad (11)$$

where, $\kappa_v(k), \lambda_v(k), \nu_v(k)$ are the Lagrangian multipliers for the constraints (6) and (7) respectively. The optimal charging energy demand can be written as follows.

$$\begin{aligned}E_{v,i}^*(k) =& \frac{1}{\theta_v}\Big(\mu_v - \frac{2K_i(k) - 1}{K_i(k)}p_{v,i} \\ & + \kappa_v(k) - \lambda_v(k) - \frac{\nu_v(k)}{\mu_v}\Big). \end{aligned} \quad (12)$$

In non-cooperative game, the existence and uniqueness of equilibrium points can not be guaranteed because of competition among multiple players. The following theorem is shown.

**Theorem 1:** When a price vector $\boldsymbol{p}(k)$ is presented, there is a unique global Nash equilibrium solution in multiple EV non-cooperative game where each EV behaves according to an optimal solution.

**Proof:** In a market where prices are adjusted by CS, each EV chooses a strategy to maximize its utility. At this time, for all EVs connected to CS $i$, a new utility function is defined as follow.

$$U_{sum}^k\left(\boldsymbol{E}_i(k), \boldsymbol{p}(k)\right) = \sum_{v \in \mathcal{K}}^{K} U_{v,i}^k. \quad (13)$$

Since it is written as a linear combination of the utility of each EV, consider the existence and uniqueness of equilibrium when maximizing the equation (13). Therefore, this problem is a jointly convex generalized nash equilibrium problem (GNEP). According to theorem[6], when $\boldsymbol{Z}$ is chosen as the existence space of the solution and

$$\boldsymbol{F}(\boldsymbol{E}) = -\left(\nabla_E U^k_{v,i}(\boldsymbol{E}_v)\right)^K_{v=1}, \tag{14}$$

then solutions of the variational inequality VI$(\boldsymbol{Z}, \boldsymbol{F})$ are equal to GNEP solution. Then we check if the solutions of VI$(\boldsymbol{Z}, \boldsymbol{F})$ are maximize the equation (15). Consider the following optimization problem.

$$\min_{\boldsymbol{E}\in\boldsymbol{Z}} f(x) = \min_{\boldsymbol{E}\in\boldsymbol{Z}} -\left(U^k_{v,i}(\boldsymbol{E}_v)\right)^K_{v=1}, \tag{15}$$

where, $f(x)$ is the potential function of the vector field $\boldsymbol{F}$ of VI $(\boldsymbol{Z}, \boldsymbol{F})$. The requirement for the local optimal solution of the constrained minimization problem (15) is

$$-\nabla f(\boldsymbol{E}) \in NC_Z(\boldsymbol{E}). \tag{16}$$

By $\nabla f(\boldsymbol{E}) = \boldsymbol{F}$, the equation (16) means $-\boldsymbol{F}(\boldsymbol{E}) \in NC_{\boldsymbol{Z}}$ and it is equal to the variational inequality problem. Therefore, the solutions to the variational inequality problem maximize (13). The existence of the potential function of the vector field $\boldsymbol{F}$ is discussed below.

Furthermore, by considering the energy demand when a price vector is given, we show the existence and uniqueness of the variational inequality solution. The KKT condition of VI $(\boldsymbol{Z}, \boldsymbol{F})$ is [6],

$$\begin{aligned}
&\boldsymbol{F}(\boldsymbol{E}) + \kappa_v(k)\nabla_{\mathbf{E}^k}\left(E^{min}_{v,i} - E_{v,i}(k)\right)\\
&\quad + \lambda_v(k)\nabla_{\mathbf{E}^k}\left(E_{v,i}(k) - E^{max}_{v,i}\right)\\
&\quad + \nu_v(k)\nabla_{\mathbf{E}^k}\left(e^-_{v,i} + \frac{E_{v,i}(k)}{\mu_v} - 1\right)\\
&\quad \times \kappa_v(k)\left(E^{min}_{v,i} - E_{v,i}(k)\right) \longrightarrow 0\\
&\quad \times \lambda_v(k)\left(E_{v,i}(k) - E^{max}_{v,i}\right) \longrightarrow 0\\
&\quad \times \nu_v(k)\left(e^-_{v,i} + \frac{E_{v,i}(k)}{\mu_v} - 1\right) \longrightarrow 0.
\end{aligned} \tag{17}$$

From the definition given in [7],

$$\boldsymbol{F} = -\begin{bmatrix} \mu_1 - \theta_1 E_{1,i}(k) - \dfrac{(2K-1)p_1, i(k)}{K} \\ \vdots \\ \mu_v - \theta_v E_{v,i}(k) - \dfrac{(2K-1)p_v, i(k)}{K} \end{bmatrix}. \tag{18}$$

Therefore, Jacobian matrix is

$$\mathbf{J} = \begin{bmatrix} \theta_1 & 0 & \cdots & 0 \\ 0 & \theta_2 & \cdots & 0 \\ \vdots & & & \\ 0 & 0 & \cdots & \theta_v \end{bmatrix}. \tag{19}$$

Since the Jacobian matrix is a diagonal matrix in the separable vector field, it can be confirmed that the potential function satisfies the symmetry. Due to the positive definite of Jacobian matrix, $\boldsymbol{F}$ is monotonically increases on solution set $\boldsymbol{Z}$. Therefore, the variational inequality solution uniquely exists. □

The CS solves P2.1 in order to equalize the usage rate of each SS. It is obvious that the utilization rate should be higher. The CS chooses

$$F_i(k) = F^*_i(k) \to x_i(k) = c_i. \tag{20}$$

For each CS the result is naturally as follows.

$$F_i(k) = \begin{cases} c_i, & \text{if } c_i \le \sum_{k=nT}^{(n+1)T} \alpha_i(k) \\ \sum_{k=nT}^{(n+1)T} \alpha_i(k). & \text{otherwise} \end{cases} \tag{21}$$

Each CS separately calculates $F_i(k)$ and exchanges $F_i(k)/c_i$ with the neighboring CS so that it can finally realize a unique utilization rate, $F_i(k)/c_i$ to be agreed. That is, the initial value is set to $F_i(k)$ by the expression (21) and updating is performed according to the following update rule.

$$\begin{aligned}
\frac{\dot{F}_1(k)}{c_1} &= \zeta\left(-\frac{F^*_1(k)}{c_1} + \left[\frac{2F^*_1(k)}{3c_1} + \frac{F^*_2(k)}{3c_2}\right]\right),\\
\frac{\dot{F}_i(k)}{c_i} &= \zeta\left(-\frac{F^*_i(k)}{c_i} + \left[\frac{F^*_{i-1}}{3c_{i-1}} + \frac{F^*_i(k)}{3c_i} + \frac{F^*_{i+1}}{3c_{i+1}}\right]\right),\\
\frac{\dot{F}_N(k)}{c_N} &= \zeta\left(-\frac{-F^*_N(k)}{c_N} + \left[\frac{F^*_{N-1}}{3c_{N-1}} + \frac{2F^*_N(k)}{3c_N}\right]\right).
\end{aligned} \tag{22}$$

The above update rule can be written as follows,

$$\dot{\mathbf{F}}^*(k) = \zeta(-I + P)\mathbf{F}^*(k), \tag{23}$$

$$P = \begin{bmatrix} \frac{2}{3} & \frac{1}{3} & 0 & \cdots & \cdots & 0 \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 & \cdots & 0 \\ & \ddots & \ddots & \ddots & & \\ 0 & \cdots & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ 0 & \cdots & \cdots & 0 & \frac{1}{3} & \frac{2}{3} \end{bmatrix}, \tag{24}$$

where,$\mathbf{F}^*(k) = [F^*_1(k)/c_i, \dots, F^*_K(k)/c_N]$. The following proposition 1 holds.

**Proposition 1:** Let the second small eigenvalue of matrix $(I - P)$ be $\lambda_2$, the smpling time $T$, and $0 < \delta < 1$. At this time, if $\zeta \ge \dfrac{4}{\lambda_2 \delta T}$ is chosen, the optimal inflow vehicle number by the consensus algorithm converges to the follwing value.

$$\frac{F_i(kT + \delta T)}{c_i} \to \frac{\sum_{j=1}^N \min[c_j, \sum_{k=nT}^{(n+1)T} \alpha_j(k)]}{\sum_{j=1}^N c_j}. \tag{25}$$

**Proof:** The exponential convergence rate of the equation (23) is given by $\zeta\lambda_2$ and it is also given by the settling time $\delta T = \dfrac{4}{\zeta\lambda_2}$. Since $P$ is row - stochastic, column - stochastic, if we choose gain as $\zeta \ge \frac{4}{\lambda_2 \delta T}$ for

$0 < \delta < 1$, the equation (15) is guaranteed to converge asymptotically [2]. It is obvious that the result of the agreement problem will be the average of the initial values, $\dfrac{\sum_{j=1}^{N} \min[c_j, \sum_{k=nT}^{(n+1)T} \alpha_j(k)]}{\sum_{j=1}^{N} c_j}$. □

After the target $F_i^*(k)$ has been determined, the administor of CS calculates the demand response of all EVs $\mathbf{E}^*(k)$ to maximize its utility It is necessary to present the price $\mathbf{p}^*(k)$ to the EV. The optimal price vector is defined as $\mathbf{p}^*(k) = [p_1^*(k), \ldots, p_v^*(k), \ldots, p_{K(k)}^*(k)]$, the optimization problem $\mathbf{P2.2}$ can be rewritten as follows.

$$\mathbf{p}^*(k) = \arg\max Q_i^k \left(\mathbf{E}^*(k)\right), \tag{26}$$

$$\text{s.t. } p_i^{min} E_{v,i}(k) \leq p_i(k) E_{v,i}(k) \leq p_i^{max} E_{v,i}(k). \tag{27}$$

Similarly, we obtain optimal price vector $\mathbf{p}^*(k)$.

$$\begin{aligned}\mathcal{M}_v^k = &\sum_v \left(p_{v,i}(k) E_{v,i}^*(k) + p_{v,i}(k)\left(E_{v,i}^*(k) - \bar{E}^*(k)\right)\right) \\ &- \frac{a}{2}\left(\sum_v E_{v,i}(k)\right)^2 \\ &- \sum_v \left(\psi_v(k)\left(p_i^{min} E_{v,i}(k) - p_i(k) E_{v,i}(k)\right)\right) \\ &- \sum_v \left(\omega_v(k)\left(p_i(k) E_{v,i}(k) - p_i^{max} E_{v,i}(k)\right)\right),\end{aligned} \tag{28}$$

where, $\psi_v(k), \omega_v(k)$ are Lagrangian multipliers for the constraint. Then, the optimal price is

$$\begin{aligned}p_{v,i}^*(k) = &\frac{1}{\left(2 + \psi_v(k) - \omega_v(k)\right) K(k) - 1} \\ &\left(a E_{v,i}(k) + \psi_v(k) p_i^{min} - \omega_v(k) p_i^{max}\right).\end{aligned} \tag{29}$$

### 4.2. Algorithm

Algprithm 1 is a summary of the above. The gradient method is used for updating the Lagrangian multiplier.

## 5. SIMULATION

### 5.1. Simulation conditions

Simulation conditions are shown in Table 1. The simulation area is the new Tomei Expressway, among which 9 nodes between the Tokyo IC - Nakai service area. In Fig.3, the circle indicates the IC, the service station with the square charging spot, the black number indicates the time required between nodes, and the red letter indicates the number of chargers existing in the station. The optimal inflow rate $F^*(k)$ agreement algorithm iteration number is 30 [step], and the Stackelberg game iteration number is 300 [step]. The number of vehicles flowing into the model from the outside $\gamma(k)$ is Fig.4, and the battery capacity and initial charge of each EV are given by Fig.5.

**Algorithm 1** Energy charging scheduling algorithm at the service station $i$

**Require:** $F_i^*(k)$, $\alpha_i(k)$ ,and a strongly connected communication topology between EVs.

1: **if** $k = nT, n \in \{1, 2, \ldots, n_{final}\}$ **then**
2: **Initialization:** Set the iteration $q = 0$, set initial $E_{v,i}(k)$,$\mathbf{p}(k)$,$\kappa_{v,q}(k)$,$\lambda_{v,q}(k)$,$\nu_{v,q}(k)$,$\psi_{v,q}(k)$ and $\omega_q(t)$, $k \in \mathcal{K}(t)$
3: compute $F_i^*(k)$ according to (30)(31).
4: set $\mathcal{S}_i = \{\}$
5: **for** $k = 1, 2, \ldots, K(t)$ **do**
6: compute battery status $e_{v,i}^-$
7: **if** $e_{v,i}^- < d_{i,i+1} r_v^-$, $v \in \mathcal{K}_i$ **then**
8: $\mathcal{S}_i \leftarrow v$
9: $E_{v,i}^{min} = d_{i,i+1} r_v^-$
10: **end if**
11: Given $\mathbf{p}_{v,i}(k)$, each EV $k$ compute $E_{v,i,q+1}(k)$ according to (19), and $Y_v(k)$ based on (20).
12: **if** $p_{v,i}(k) \leq Y_v(k)$ **then**
13: submit $E_{v,i,q+1}(k)$ to the maneger of service station
14: **end if**
15: Updating the dual variables by using GDM as follow:

$$\kappa_{v,q+1}(k) = \left(\kappa_{v,q}(k) + x\left(E_{v,i}^{min} - E_{v,i}(k)\right)\right)^+$$

$$\lambda_{v,q+1}(k) = \left(\lambda_{v,q}(k) + y\left(E_{v,i}(k) - E_{v,i}^{max}\right)\right)^+$$

$$\upsilon_{v,q+1}(k) = \left(\upsilon_{v,q}(k) + z\left(e_{v,i}^- + \frac{E_{v,i}(k)}{\mu_v} - 1\right)\right)^+$$

where $x, y, z > 0$ are sufficiently small positive step sizes.
16: **end for**
17: Given $\mathbf{E}_{i,q+1}(t)$, the service station maneger updates the price vectors $\mathbf{p}_{q+1}(t)$ according to (36).
18: Updating the dual variable by using GDM as follow:

$$\begin{aligned}\psi_{v,q+1}(k) = &\big(\psi_{v,q}(k) \\ &+ v\left(p_i^{min} E_{v,i}(k) - p_i(k) E_{v,i}(k)\right)\big)^+\end{aligned}$$

$$\begin{aligned}\omega_{v,q+1}(k) = &\big(\omega_{v,q}(k) \\ &+ w\left(p_i(k) E_{v,i}(k) - p_i^{max} E_{v,i}(k)\right)\big)^+\end{aligned}$$

where $v, w > 0$ are sufficiently small positive step sizes.
19: Set $q = q + 1$ and repeat step 11 to step 18 until convergence.
20: **if** $E_{v,i}(k)$, $p_{v,i}(k)$ converge **then**
21: compute $\left(Q_i^k\right)_v = p_{v,i}(k) E_{v,i}(k) + p_{v,i}(k)\left(E_{v,i}(k) - \bar{E}_i(k)\right) - \frac{a}{2} E_{v,i}^2(k)$
22: rank $\left(Q_i^k\right)_v$ in an ascending order for $v \notin \mathcal{S}_i$
23: choose EVs for $F_i^*(k) - |\mathcal{S}_i|$th largest $\left(Q_i^k\right)_v$
24: **end if**
25: **end if**

Table 1 simulation parameter

| Parameter | Symbol | setting |
|---|---|---|
| Number of step | $K$ | 60 [min] |
| Sampling time | $k$ | 1 [min] |
| Scheduling interval | $T$ | 10 [min] |
| Battry usage | $r_v^-$ | 0.05 [%/min] |
| Minimal energy demand | $E^{min}$ | 0 [kWh] |
| Maximal energy demand per unit time | $E^{max}$ | 2.5[kWh/min] |
| Minimal price | $p^{min}$ | 0 [dol./kWh] |
| Maximal price | $p^{max}$ | 1 [dol./kWh] |
| Initial Lagrangian multipliers | $\kappa(0)$ | 4 |
| | $\lambda(0),\ \nu(0)$ | 30 |
| | $\psi(0)\ \omega(0)$ | 10 |
| Updating step size | $x'$ | 0.001 |
| | $y, z, v, w$ | 0.01 |

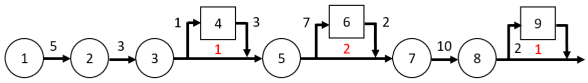

Fig. 3 Simulation map

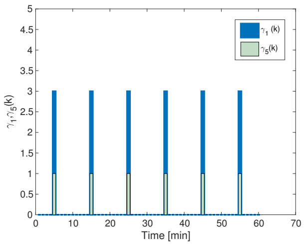


Fig. 4 Inflow

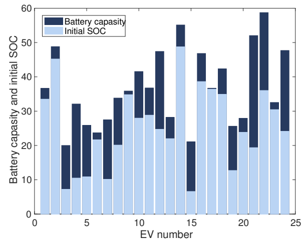


Fig. 5 Initial SOC

### 5.2. Simulation result

The simulation results are shown in Fig.6-9. From Fig.6, it can be confirmed that the agreement algorithm (Proposition 1) of the optimal incoming vehicles number at each CS is operating correctly. On the other hand, it was decided by Fig.6 that the optimum inflowing vehicle number at each CS at step = 11 was determined to be 0.5 with respect to the number of chargers, but it was used for Fig.7, the result shows that the rate is not 0.5 uniformly. This is because the number of chargers $c_4 = 1$ in node 4, so utilization rate 0.5 can not be realized in this CS. Since the vehicle does not arrive at node 9, it is obviously impossible to let the vehicle flow in and the utilization rate becomes 0 between step = 10 and 20. From Fig 8,9 we can confirm that the optimul energy demand and price by Stackelberg games converge. In Fig.8, it was confirmed that the constraints are kept in all EVs. Fig.9 also seems to finally converge the price in constrained conditions.

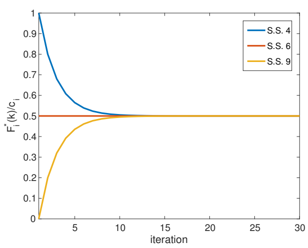


Fig. 6 Optimal inflow

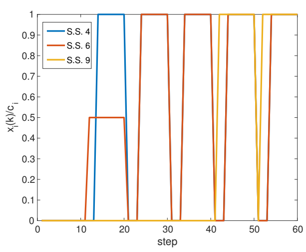


Fig. 7 Utilization ratio

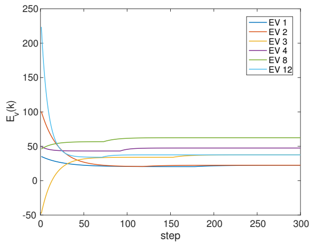


Fig. 8 Energy demand

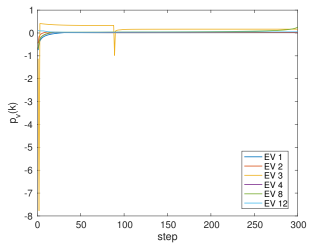


Fig. 9 Price of enegy

## 6. CONCLUTION

In this paper, after modeling the EV flow, the number of vehicles in the CS, and SOC of EV, we formalize the Stackelberg game. Furthermore, we discussed the solution of the Stackelberg game and showed that there exists an optimal solution and Nash equilibrium in follower game. Finally, we proposed a non-cooperative optimal charging scheduling algorithm and confirmed that it achieves equilibrium using numerical simulation. For future work, introduction of queue and considering different energy prices is presented for each EV. This problem is called the Hoteling problem.

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