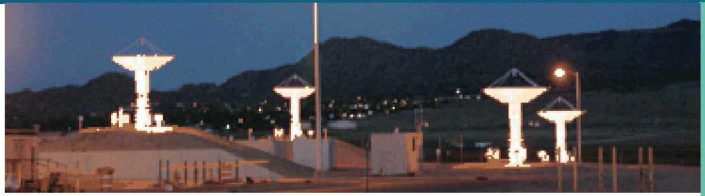
SAND2020-9667C

Variation of the Restoring Force Surface Method to Estimate Nonlinear Stiffness and Damping Parameters



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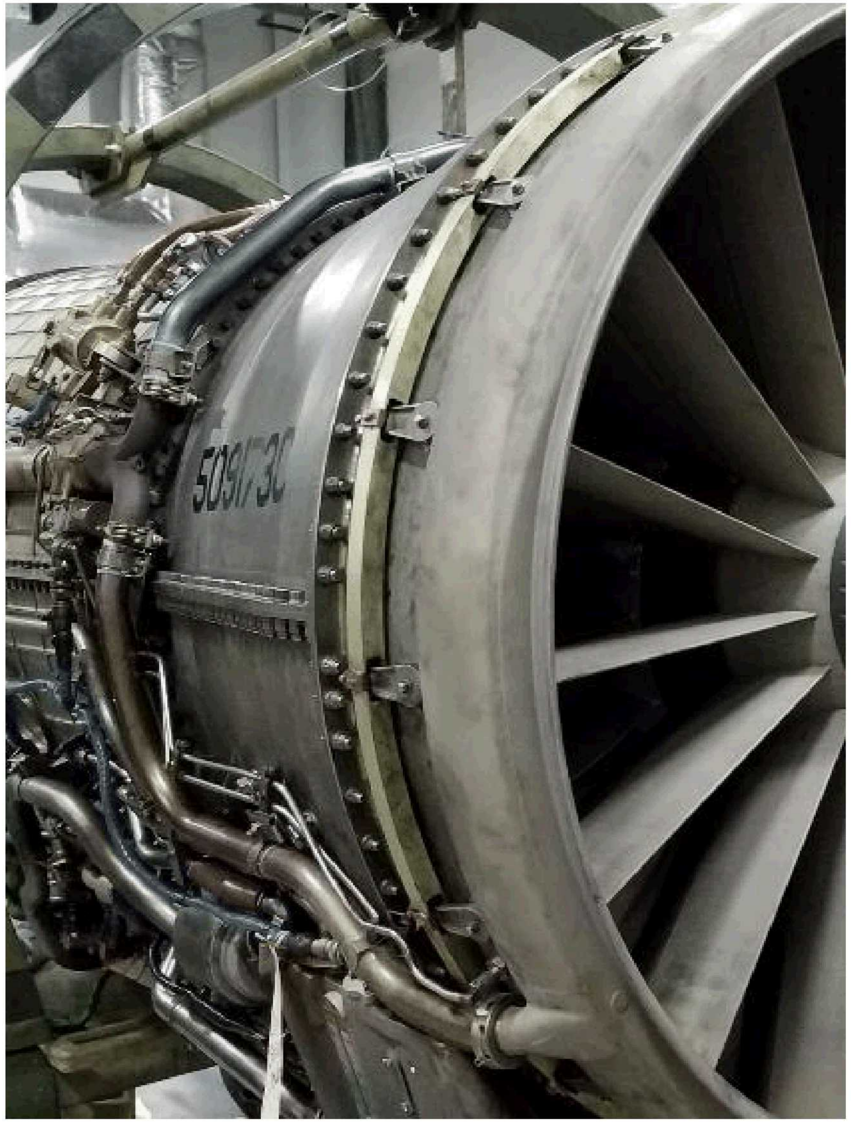
Nonlinear Analysis is Critical to Designing Efficient Structures that can Survive in Challenging Environments

- Understanding and taking advantage of nonlinear structural behavior will be crucial in next generation designs.
- Structures with bolted joints exhibit drastic variations in stiffness and energy dissipation.
- Nonlinear system identification techniques are required to accurately characterize these effects.
- To reach high response amplitudes that sufficiently excite the nonlinearity, the structure must be actively forced.
- Most nonlinear system ID techniques either cannot process forced response, or are limited to specific types of forcing
- This work presents a general nonlinear system ID technique for determining amplitude dependent natural frequency and damping ratio curves for structures under arbitrary forcing.



Theoretical Framework

Fit c and k with a Variation of the RFS Method

- Suppose a nonlinear system can be represented with a typical EOM by taking c and k as functions of time.

$$m\ddot{x} + c(t)\dot{x} + k(t)x = f$$

- Convenient forms for $c(t)$ and $k(t)$ are general polynomials of time.

$$c(t) = c_0 + c_1 t + \cdots \quad \wedge \quad k(t) = k_0 + k_1 t + \cdots$$

- Substitute these into the EOM and move the acceleration term to the right side.

$$(c_0 + c_1 t + \cdots)\dot{x} + (k_0 + k_1 t + \cdots)x = f - m\ddot{x}$$

- Put this into matrix form as below, where the responses and time are assumed to be column vectors.

$$[\dot{x} \quad t\dot{x} \quad \cdots \quad x \quad tx \quad \cdots]\begin{bmatrix} c_0 \\ c_1 \\ \cdots \\ k_0 \\ k_1 \\ \cdots \end{bmatrix} = f - m\ddot{x}$$

Fit c and k with a Variation of the RFS Method

- The polynomial coefficients are now determined in a least squares sense.

$$\begin{bmatrix} c_0 \\ c_1 \\ \cdots \\ k_0 \\ k_1 \\ \cdots \end{bmatrix} = [\dot{x} \quad t\dot{x} \quad \cdots \quad x \quad tx \quad \cdots]\backslash[f - m\ddot{x}]$$

- The natural frequency and damping ratio may then be determined as:

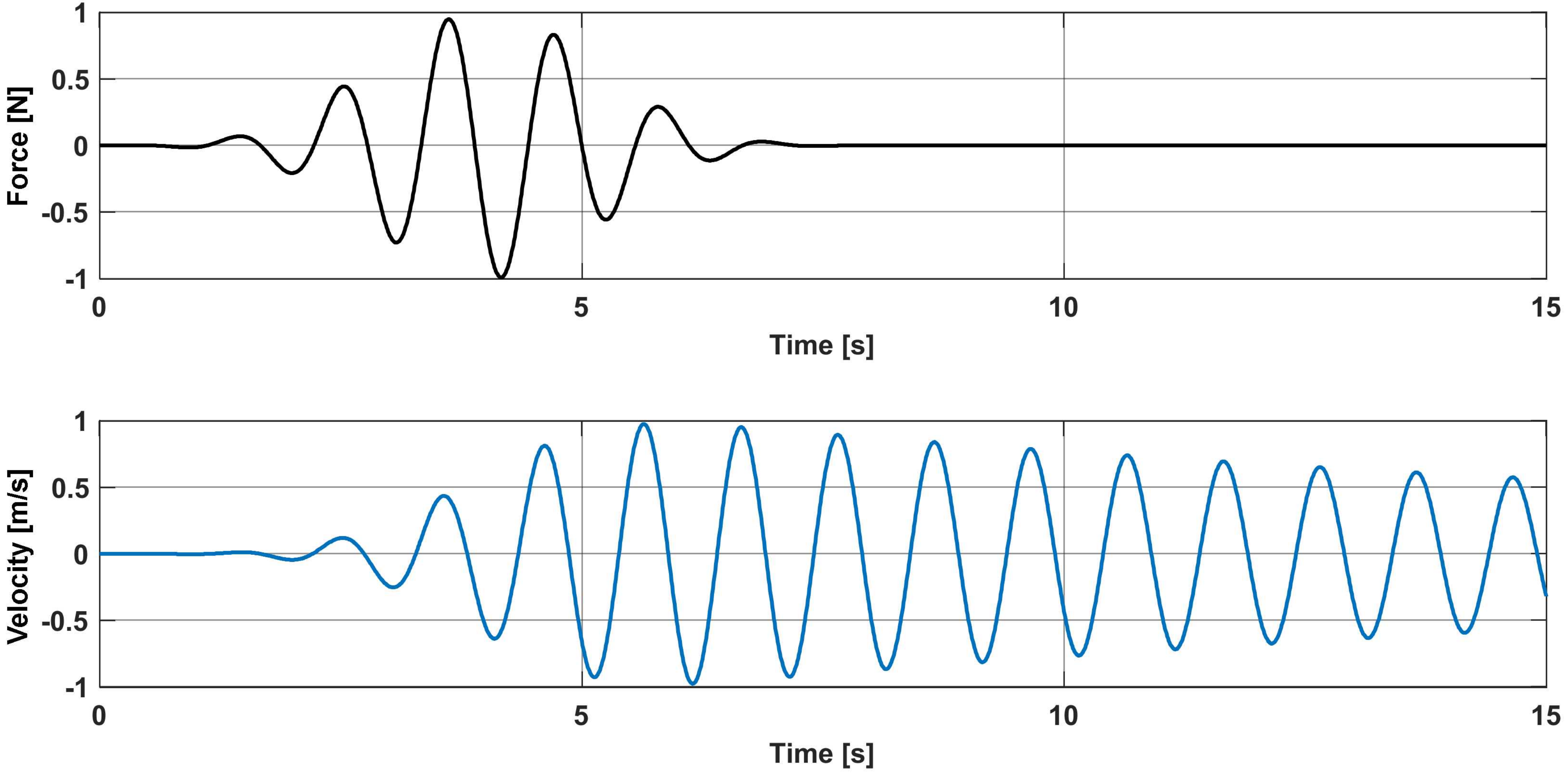
$$\omega_n(t) = \sqrt{k_0 + k_1 t + \cdots} \quad \wedge \quad \zeta(t) = \frac{c_0 + c_1 t + \cdots}{2\omega_n(t)}$$

- After determining the amplitude at each point in time, the time dependence can be swapped for the associated amplitude dependence

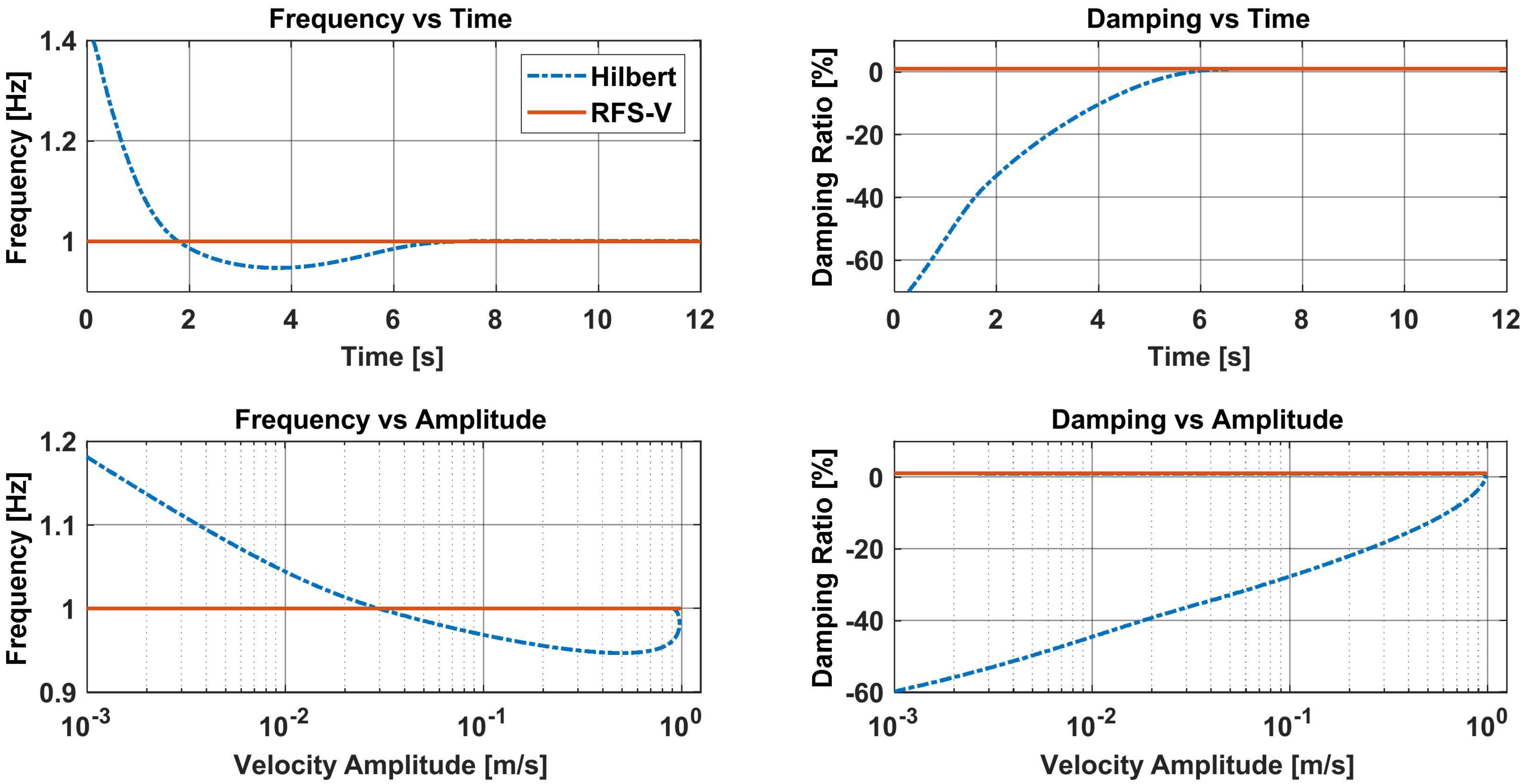
$$\omega_n(t) \rightarrow \omega_n(A) \quad \wedge \quad \zeta(t) \rightarrow \zeta(A)$$

Numerical Case Study - SDOF Linear System

Sine Beat Applied to the Linear Model and the Subsequent Response

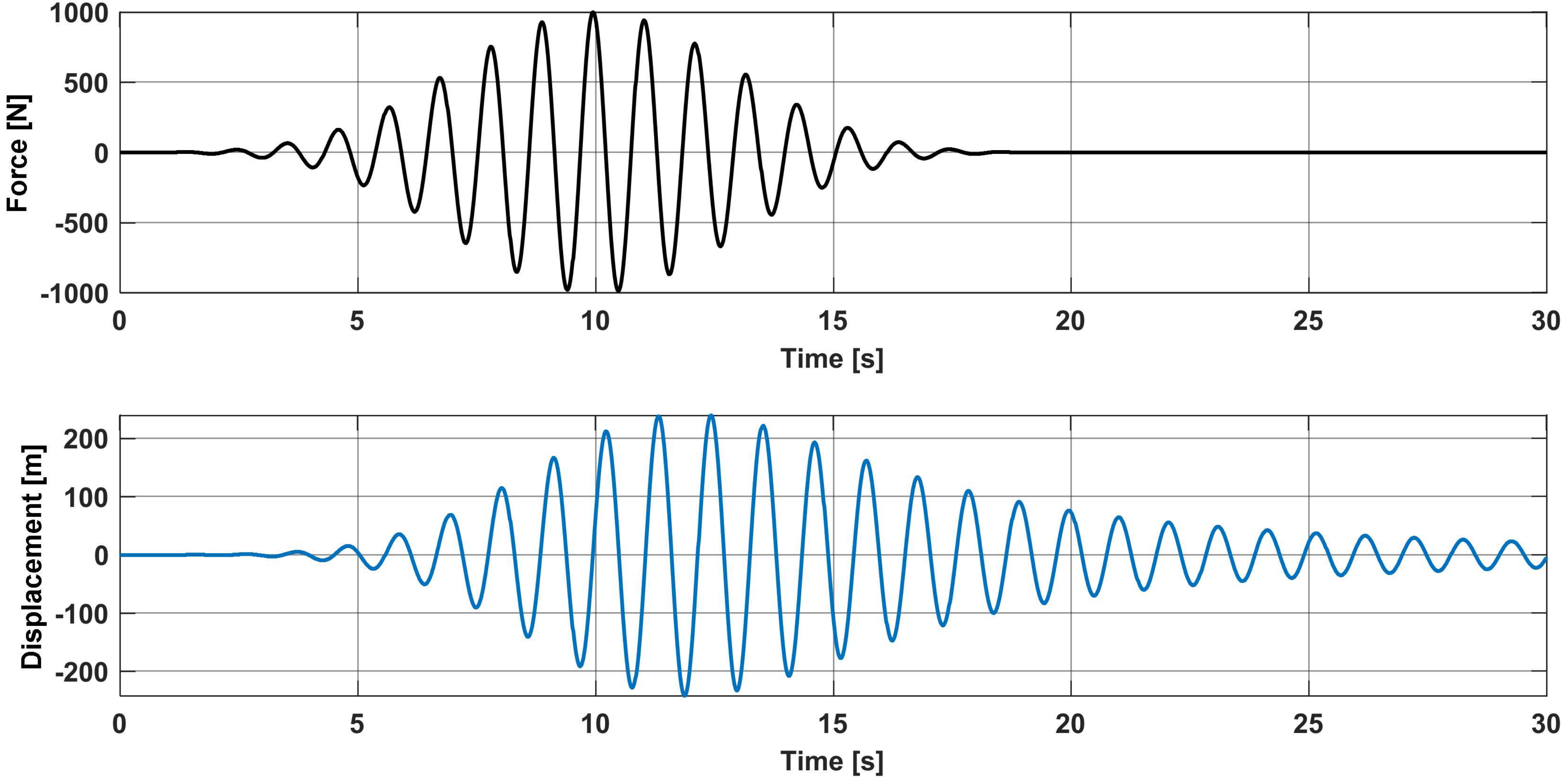


Frequency and Damping as Functions of Time and Amplitude for the Simulated Linear System

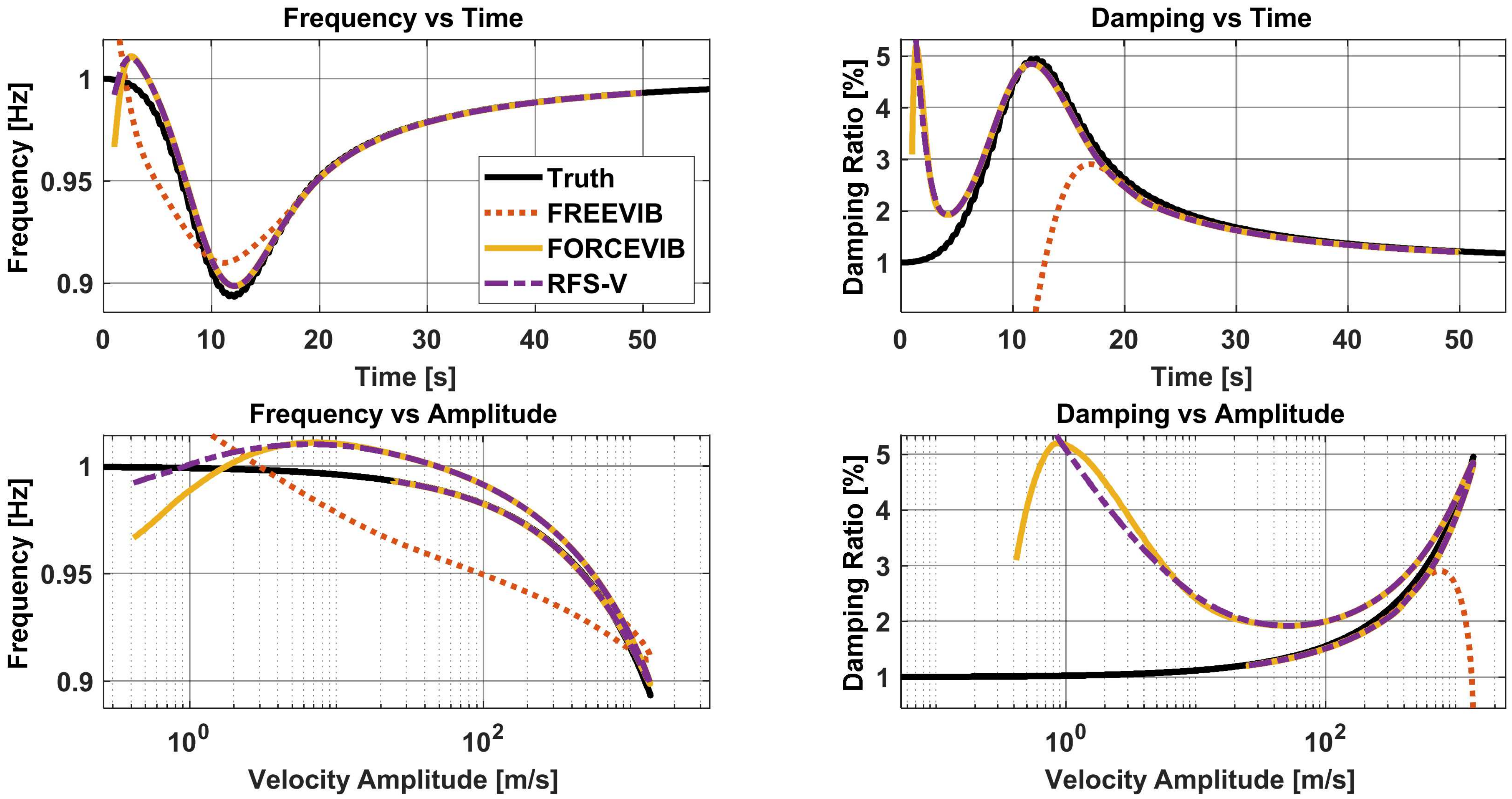


Numerical Case Study - Modal Iwan Model

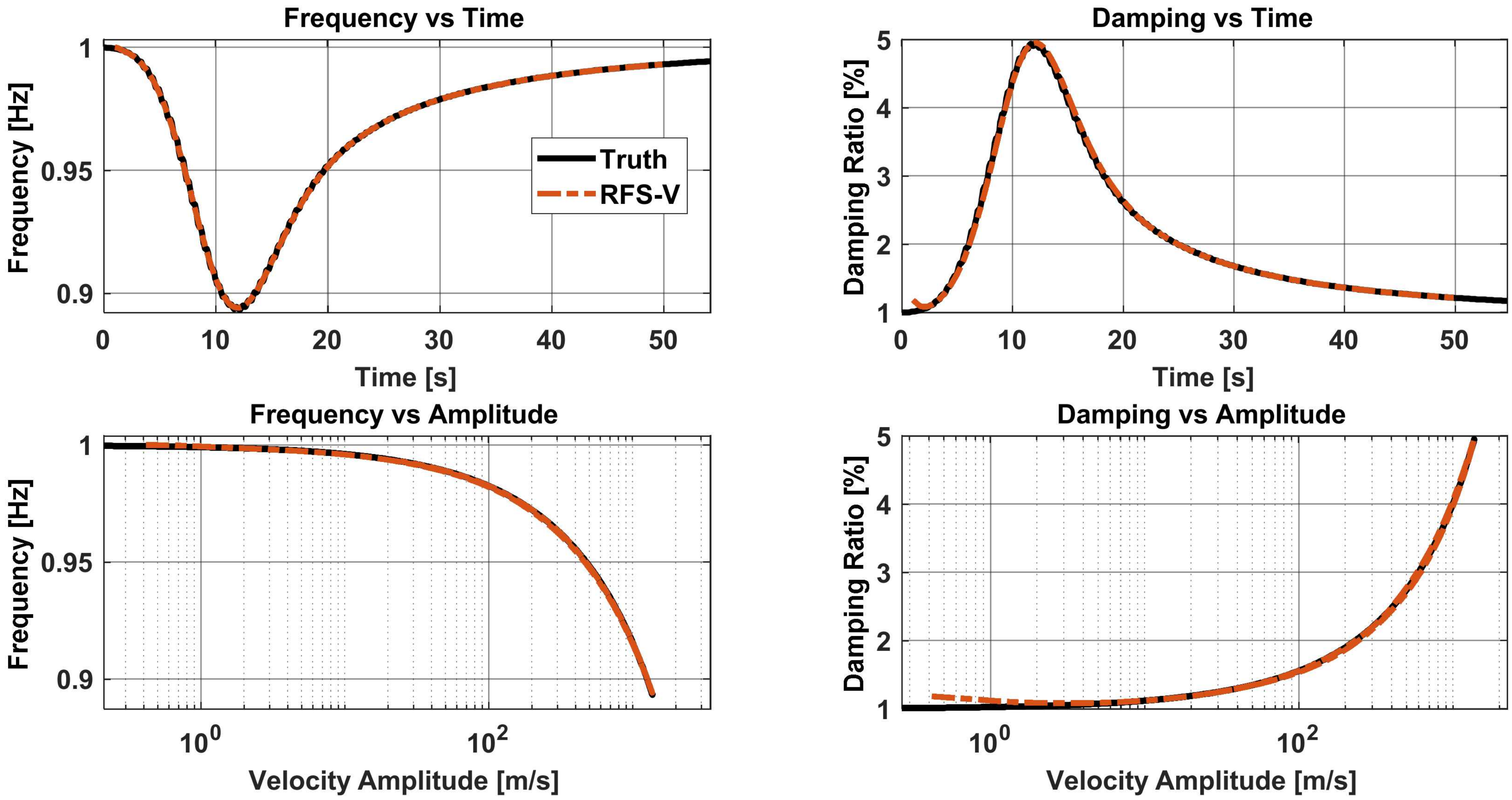
Sine Beat Applied to the Iwan Model and the Subsequent Response



Frequency and Damping as Functions of Time and Amplitude for the Iwan Model

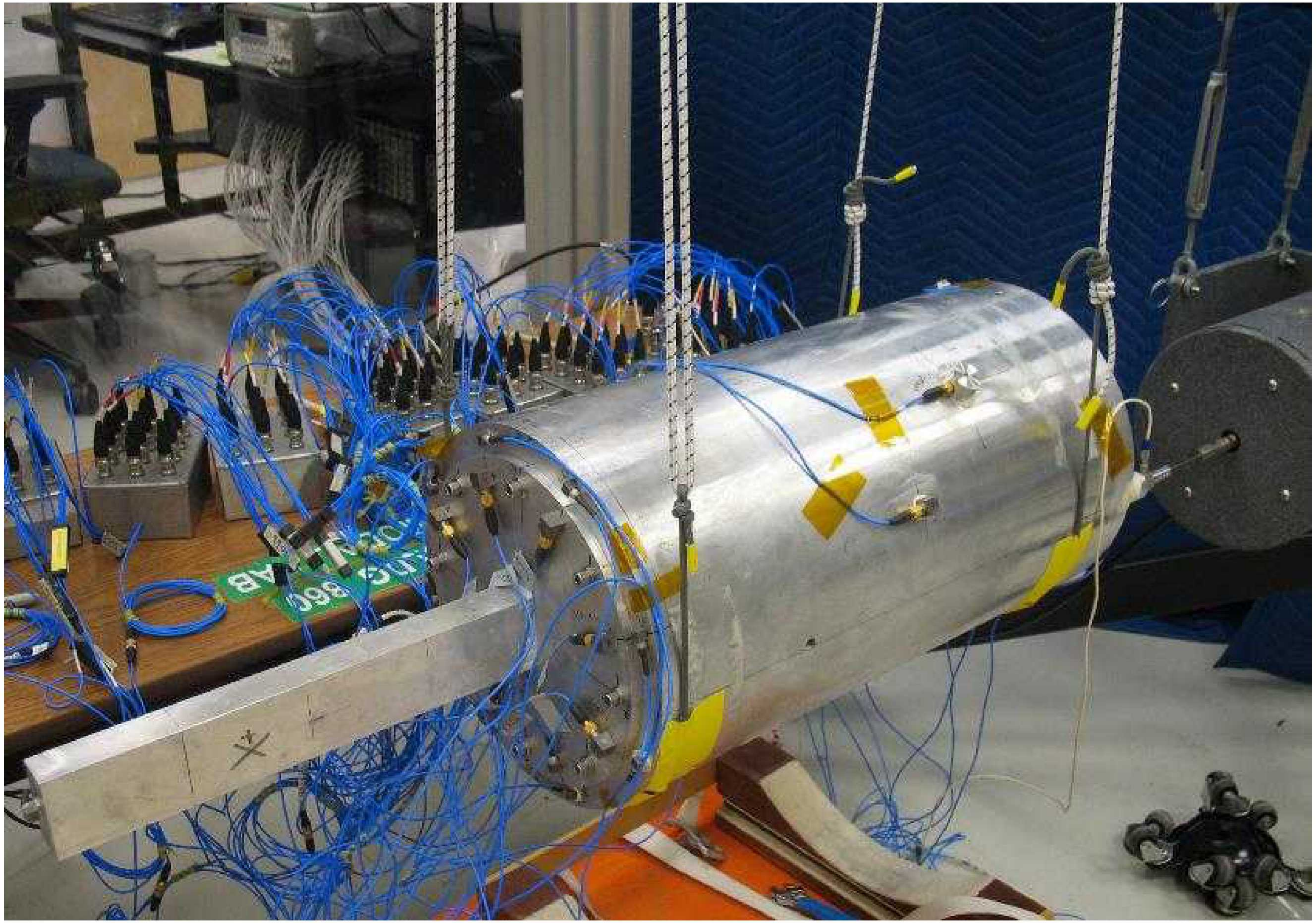


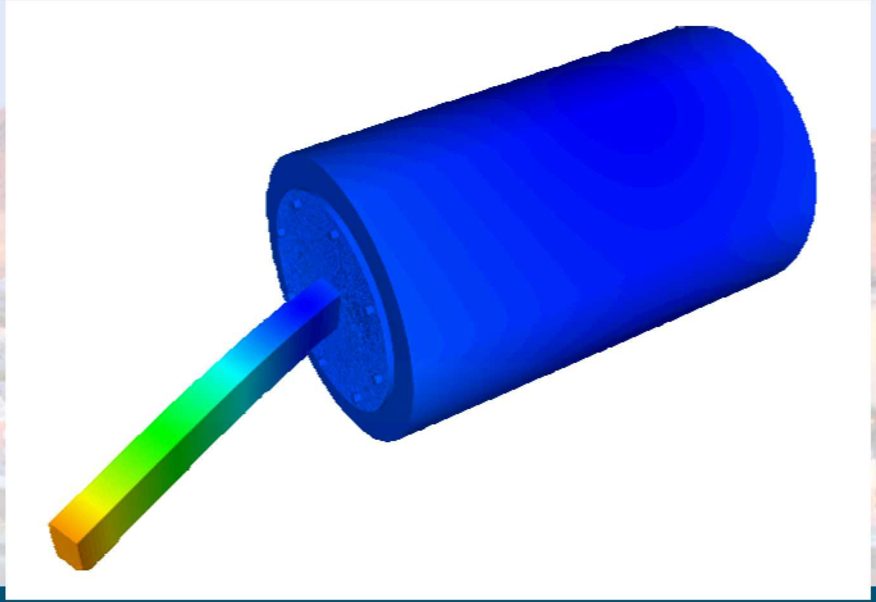
Frequency and Damping as Functions of Time and Amplitude for the Iwan Model



Experimental Demonstration - Cylinder-Plate-Beam Structure

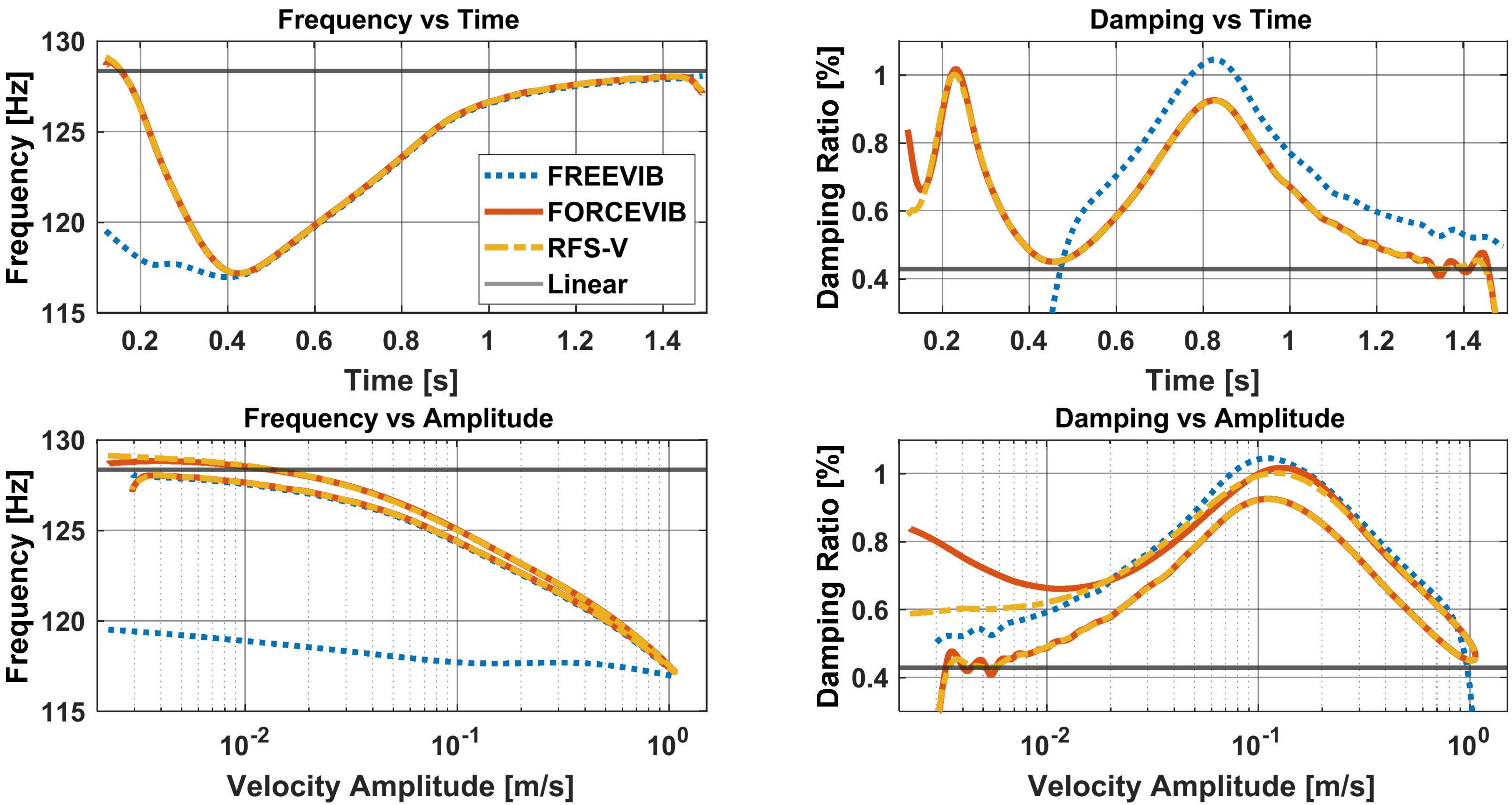
Cylinder-Plate-Beam Experimental Structure



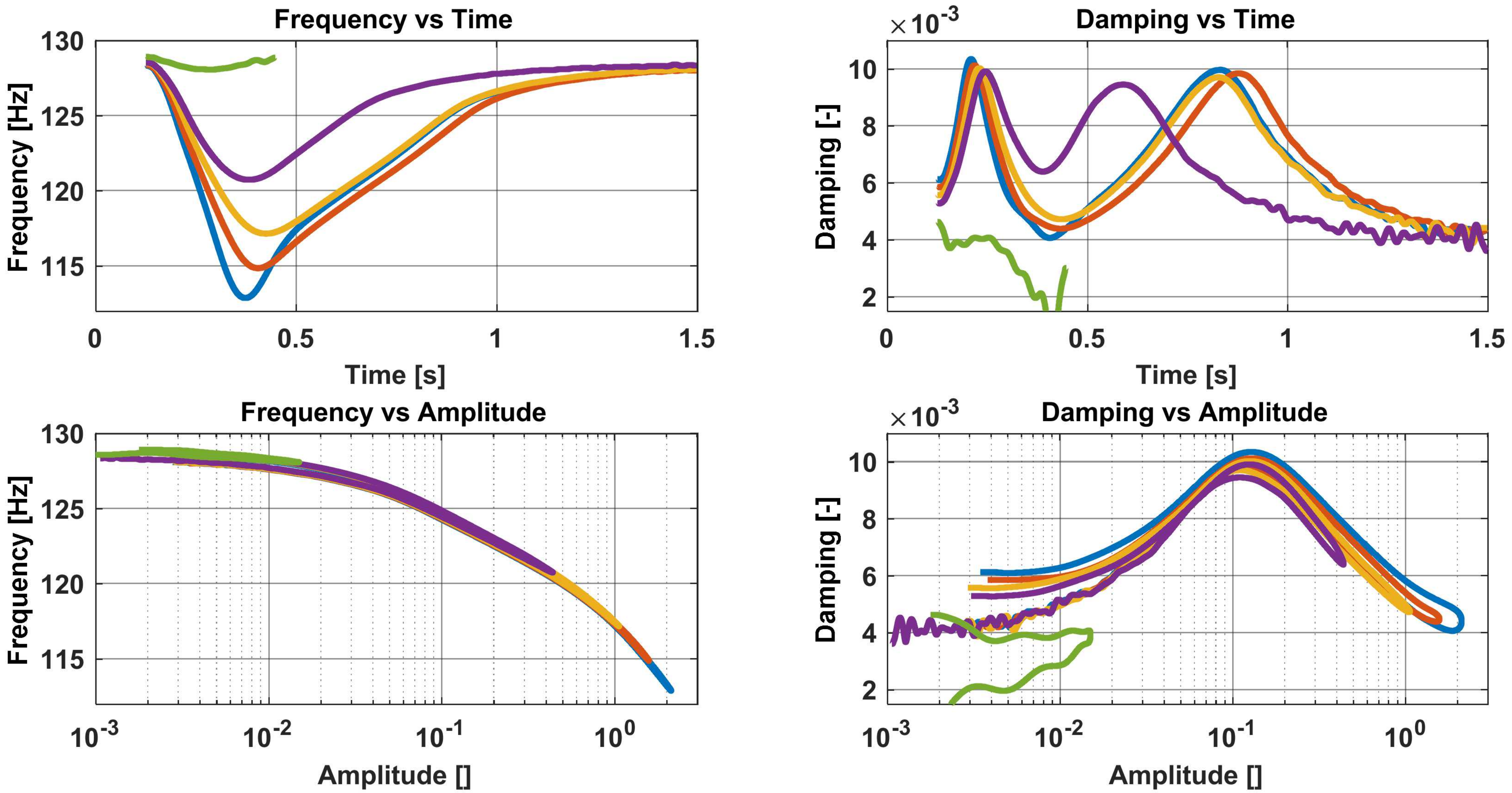


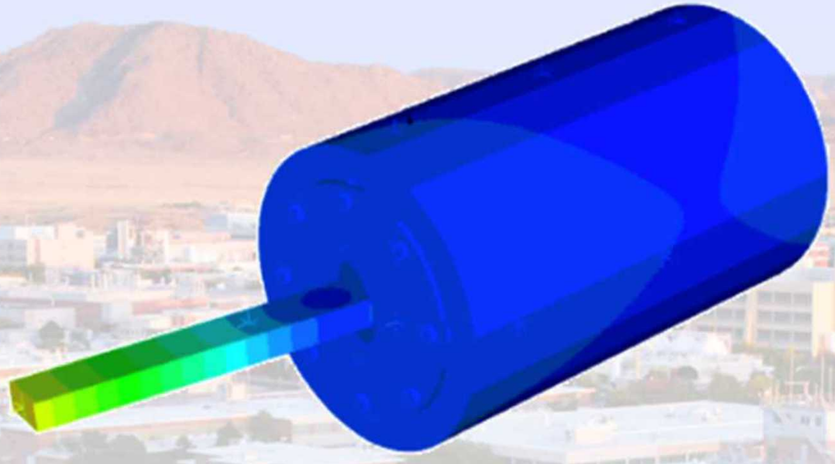
Fitting Experimental Modes - 1st CPB Mode: Beam Cantilever 1

Frequency and Damping as Functions of Time and Amplitude for the First Elastic Mode of the CPB



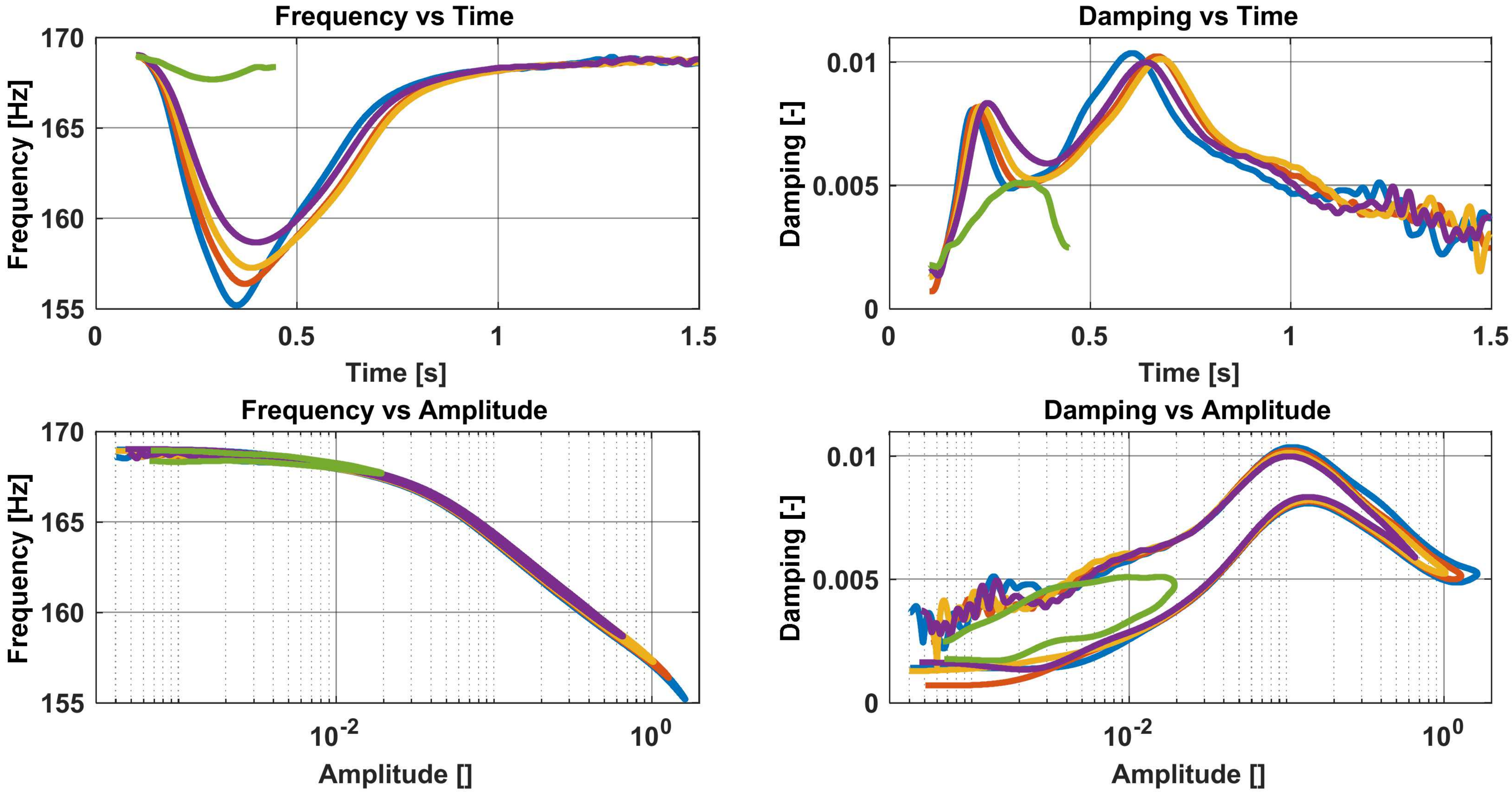
Frequency and Damping as Functions of Time and Amplitude for the First Elastic Mode of the CPB at Various Forcing Amplitudes

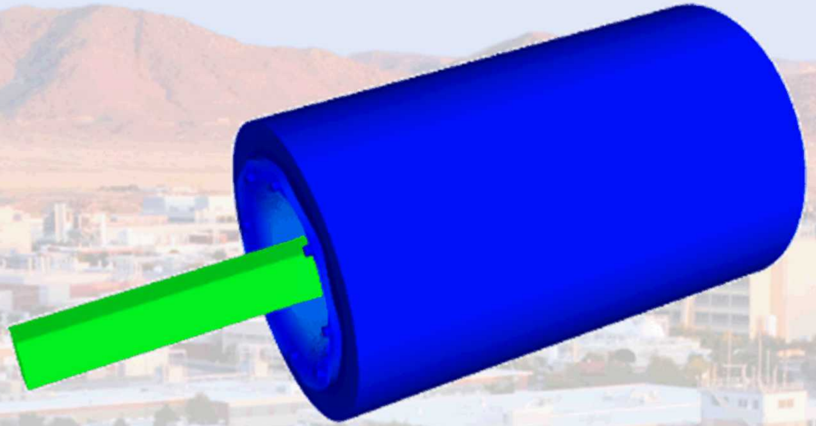




Fitting Experimental Modes - 2nd CPB Mode: Beam Cantilever 2

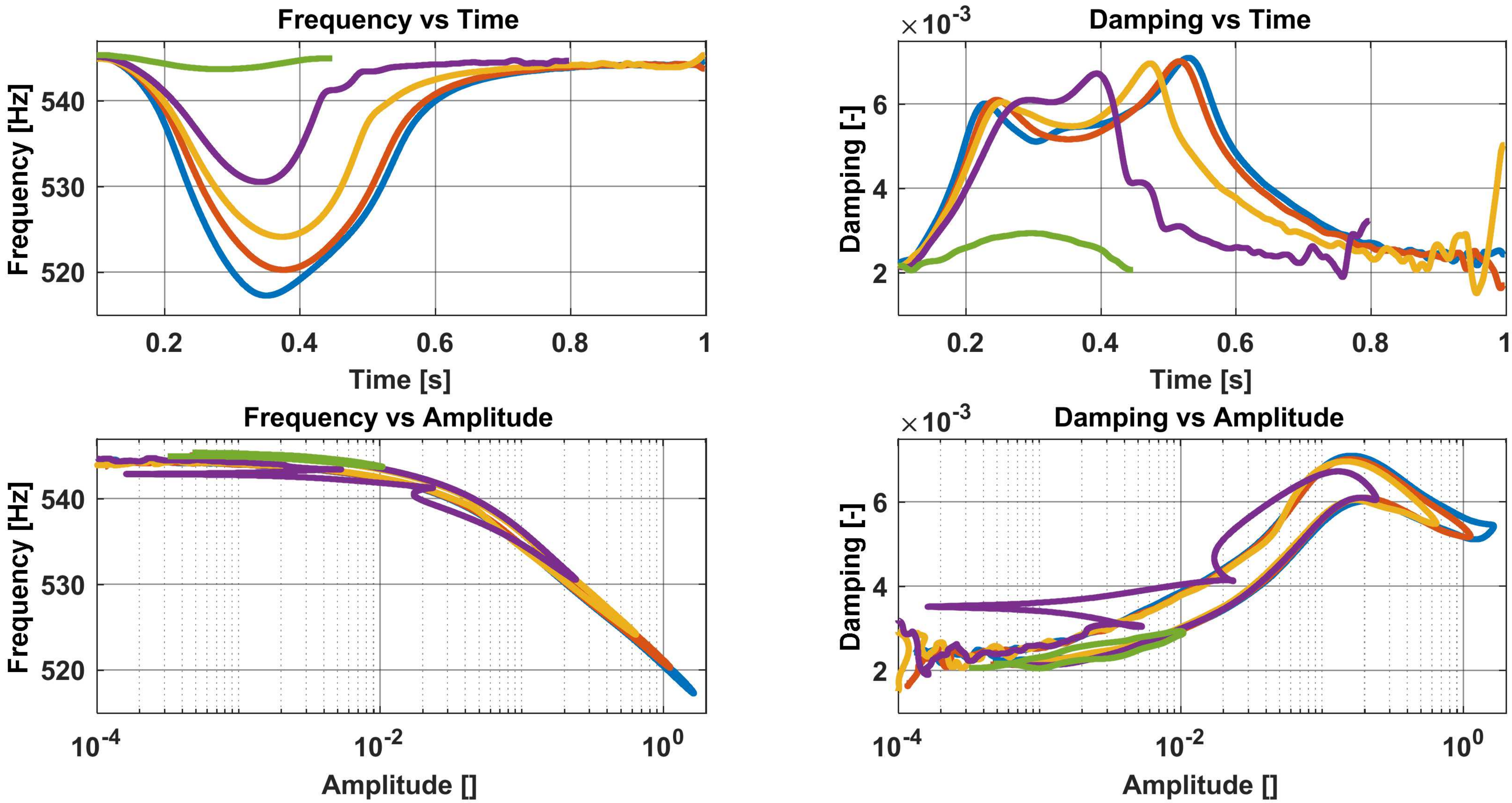
Frequency and Damping as Functions of Time and Amplitude for the Second Elastic Mode of the CPB at Various Forcing Amplitudes





Fitting Experimental Modes - 3rd CPB Mode: Plate Drum

Frequency and Damping as Functions of Time and Amplitude for the Third Elastic Mode of the CPB at Various Forcing Amplitudes



Conclusions

- A novel nonlinear system ID technique, RFS-V, has been mathematically described and demonstrated on several numerical and experimental case studies.
- RFS-V produces natural frequency and damping ratio curves that are comparable to those from the FORCEVIB method.
  - RFS-V appears to be less susceptible to fitting end effects as FORCEVIB. This is likely due to FORCEVIB being a function of first and second order derivatives of a spline fit, while RFS-V produces a solution directly from the original polynomial fits.
  - Since FORCEVIB is derived from the complex magnitude-phase space, it has independent equations for the stiffness and damping parameters. RFS-V is based on real valued quantities, so it results in one equation with two unknowns that is solved in a least squares sense.
- The numerical case studies show that RFS-V gives equivalent or better results when compared to FORCEVIB.
- The experimental case showed that RFS-V produced consistent backbone curves for frequency and damping.
  - The difference in the result from raising vs falling amplitude is likely due to inaccuracies in the modal force from errors in the modeshapes used to modal filter or calibration of the force sensor.
- While demonstrated on sine beats here, RFS-V could be utilized to identify structures under any forcing, as long as it is accurately measured. This method could be particularly useful speeding up phase-quadrature testing by estimating the quadrature point from a stable distance away.

Extra Slides

The Standard RFS Method

- In standard RFS, a response is assumed to fit the form below, where m, c and k are their respective linear values and f_{NL} is the nonlinear Restoring Force.

$$m\ddot{x} + c\dot{x} + kx + f_{NL} = f$$

- The nonlinear Restoring Force can be determined by bring the other terms on the left-hand side to the right.

$$f_{NL} = f - m\ddot{x} - c\dot{x} - kx$$

- A set of polynomials formed from the responses are then fit via least squares to the Restoring Force.

$$[\sigma \quad \lambda]\begin{bmatrix} x^3 \\ \dot{x}^2 \end{bmatrix} = f_{NL}$$

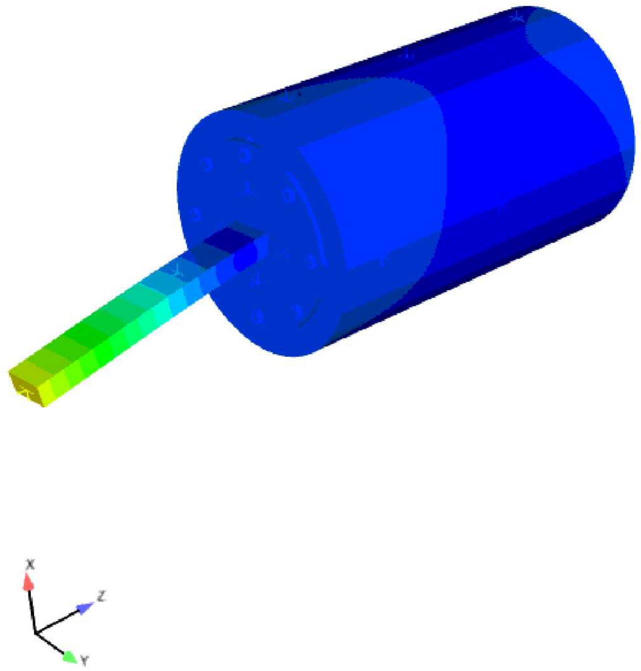
- This yields terms to replace the Restoring Force in the EOM.

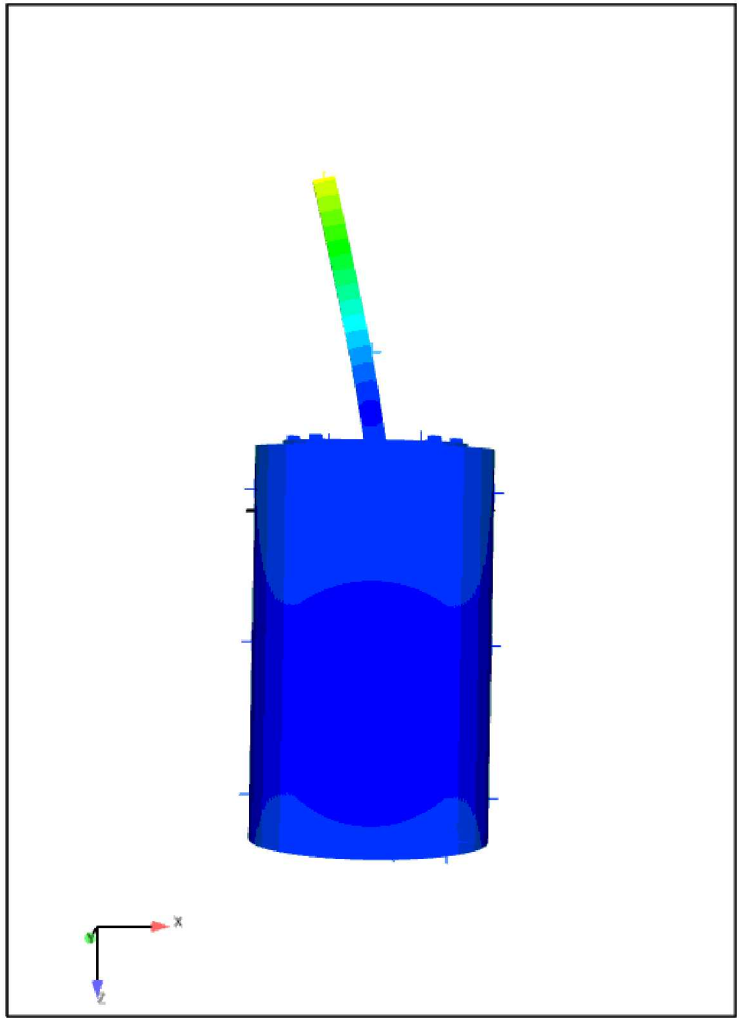
$$m\ddot{x} + c\dot{x} + kx + \sigma x^3 + \lambda \dot{x}^2 = f$$

Iwan Model Parameters

Table 1: Parameters used in the SDOF Modal Iwan Model Numerical Case Study

Parameter	F_S	K_T	χ	β	K_∞
Value	400 [N]	25.27 [N/m]	-.35 [-]	10 [-]	14.21 [N/m]


X
Z
Y


X
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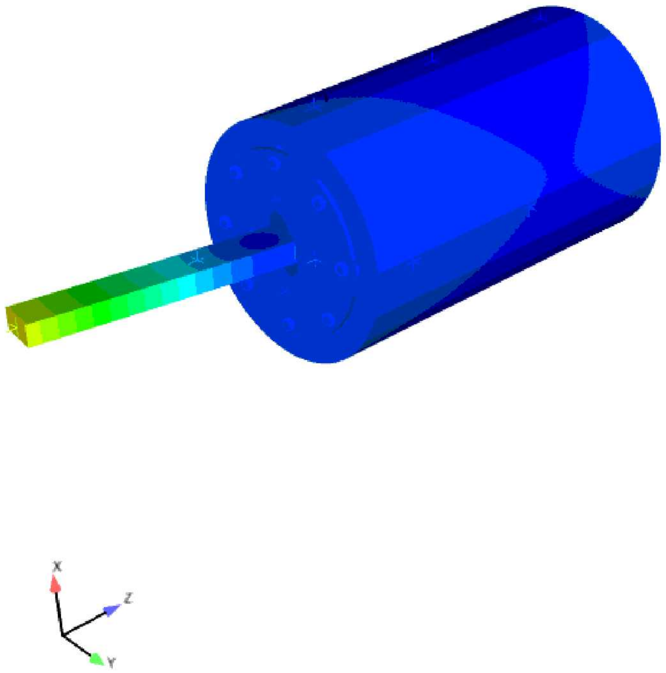
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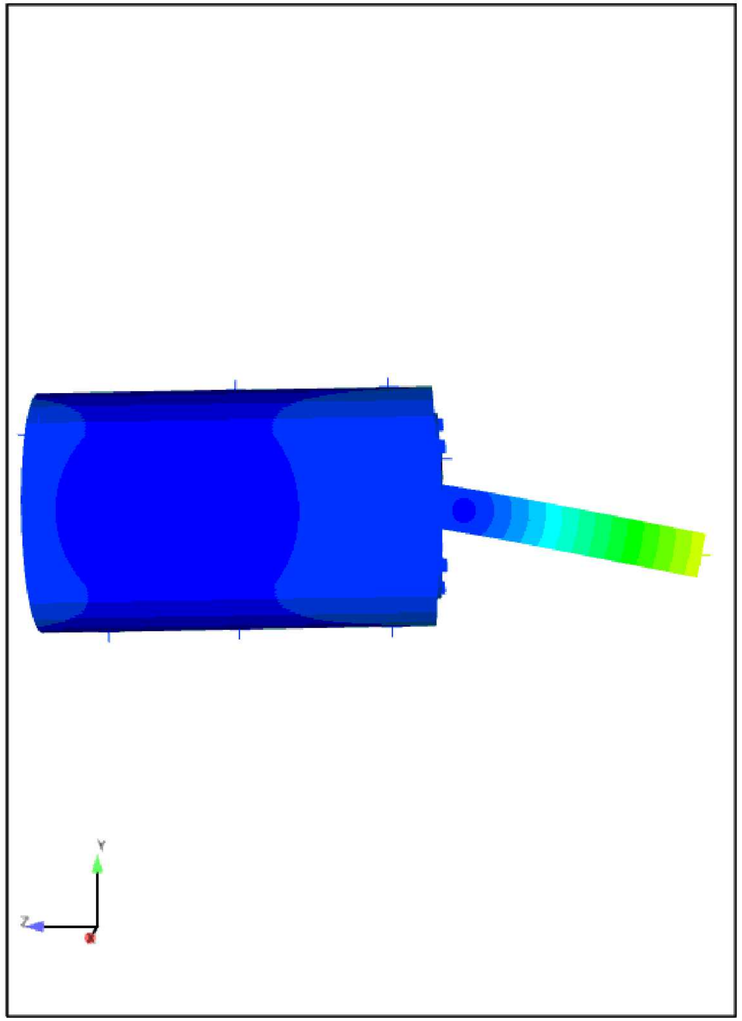
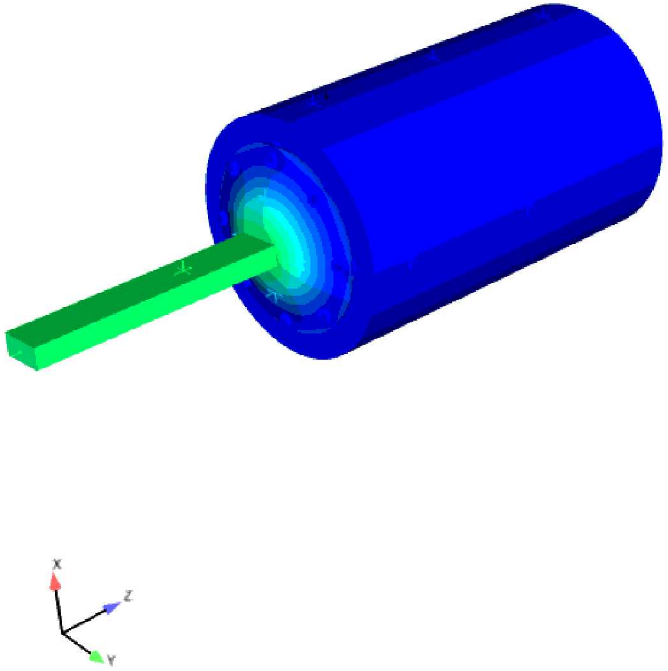
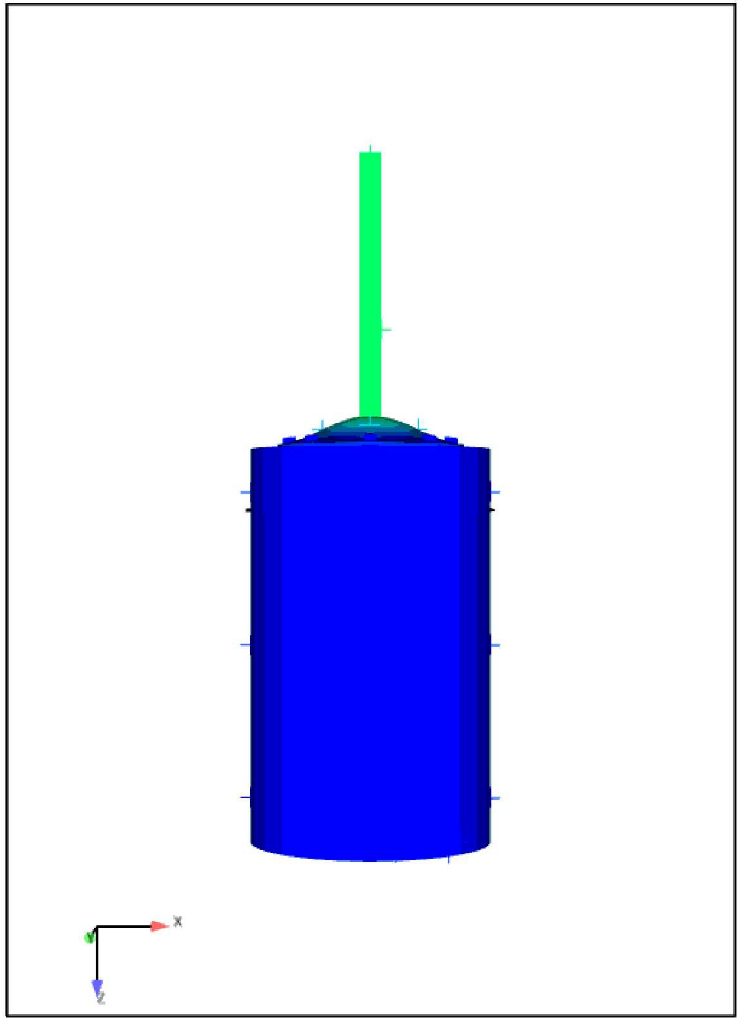
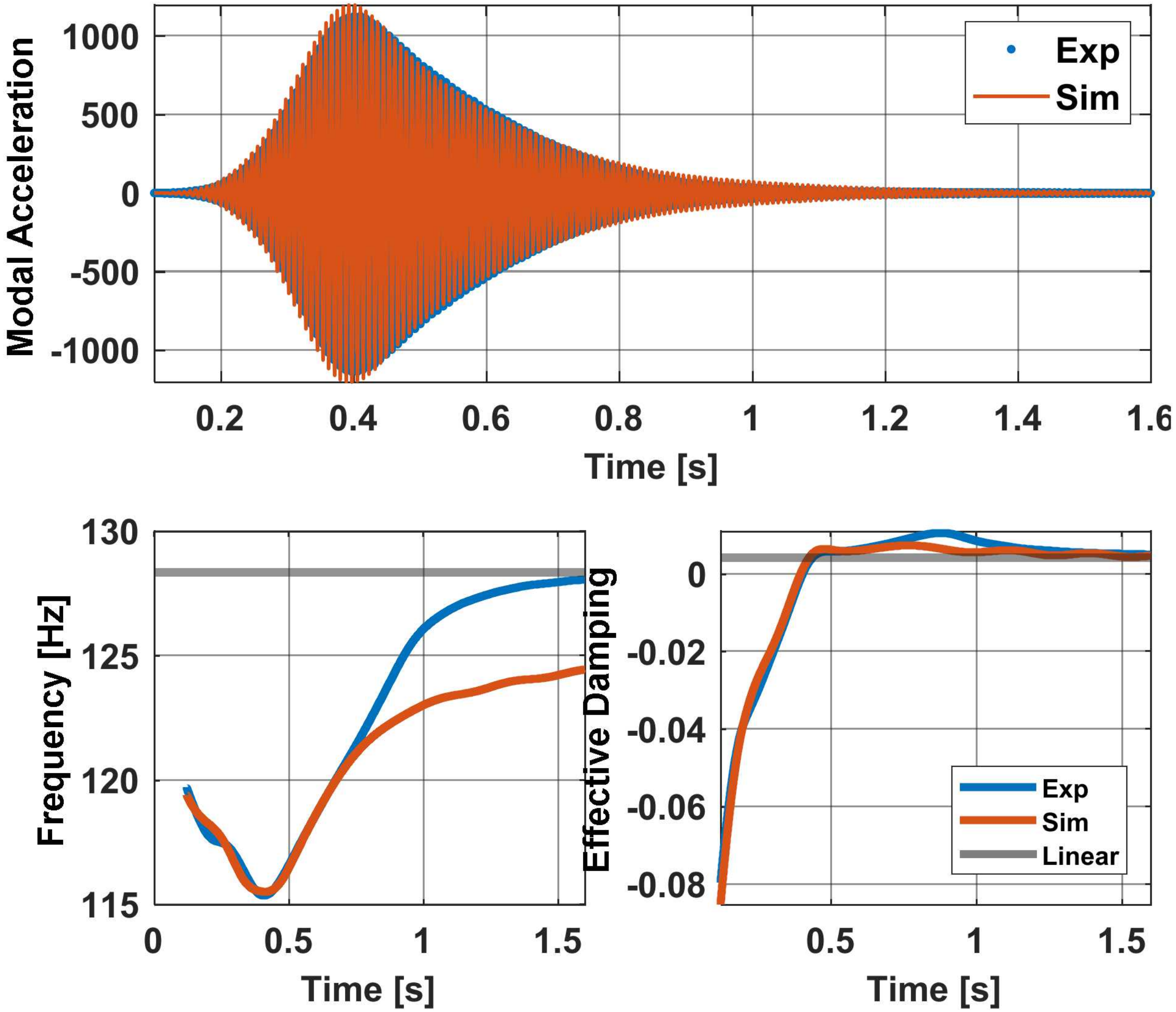

Y
Z


X
Z
Y


X
Z

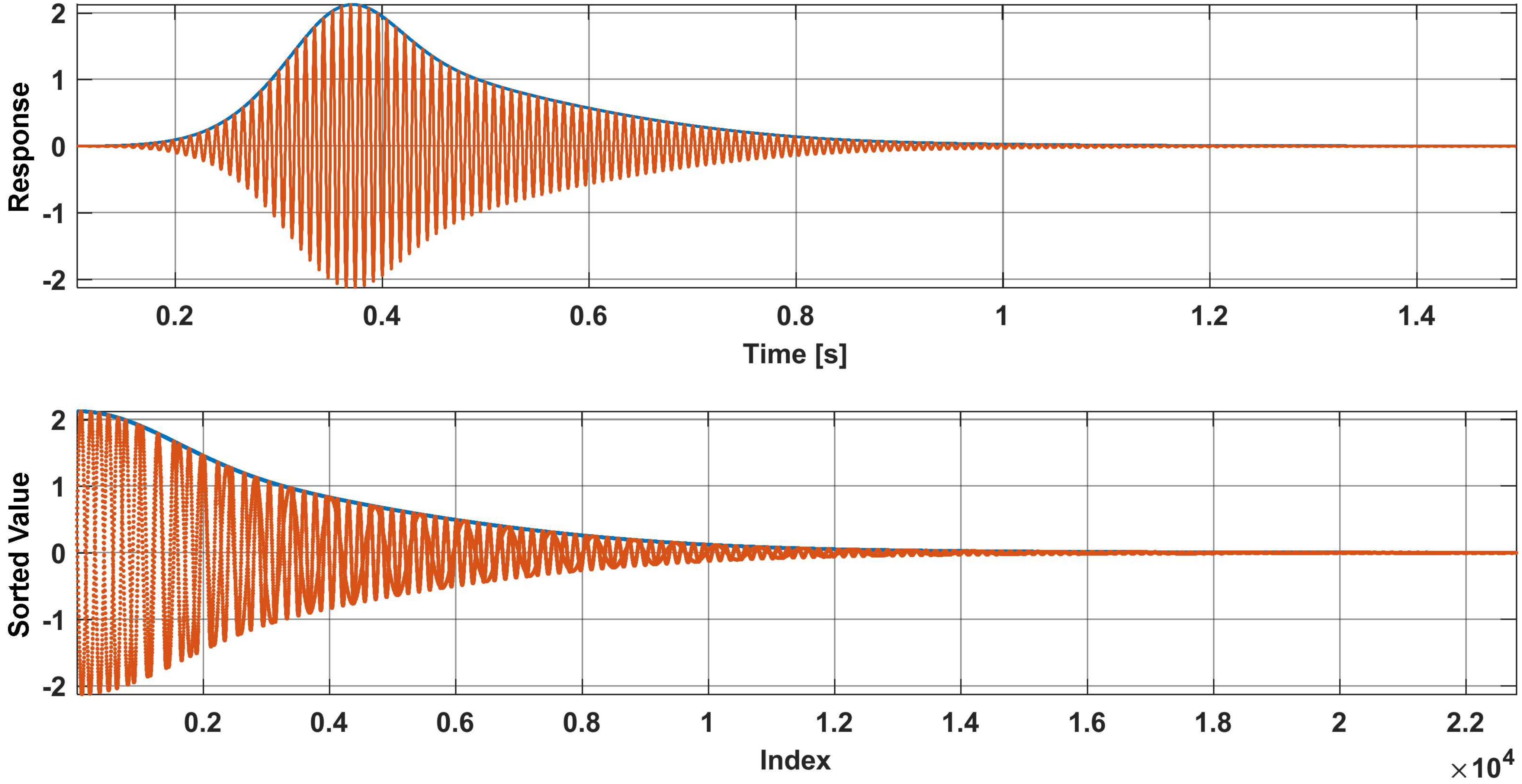
Previously had Trouble Implementing RFS

- Used RFS to form models of nonlinear CPB modal response
- Gauged accuracy by simulating response and comparing freq and damping curves to those from the exp data
- RFS process did not produce an accurate model and seemed too open-ended to be reliable

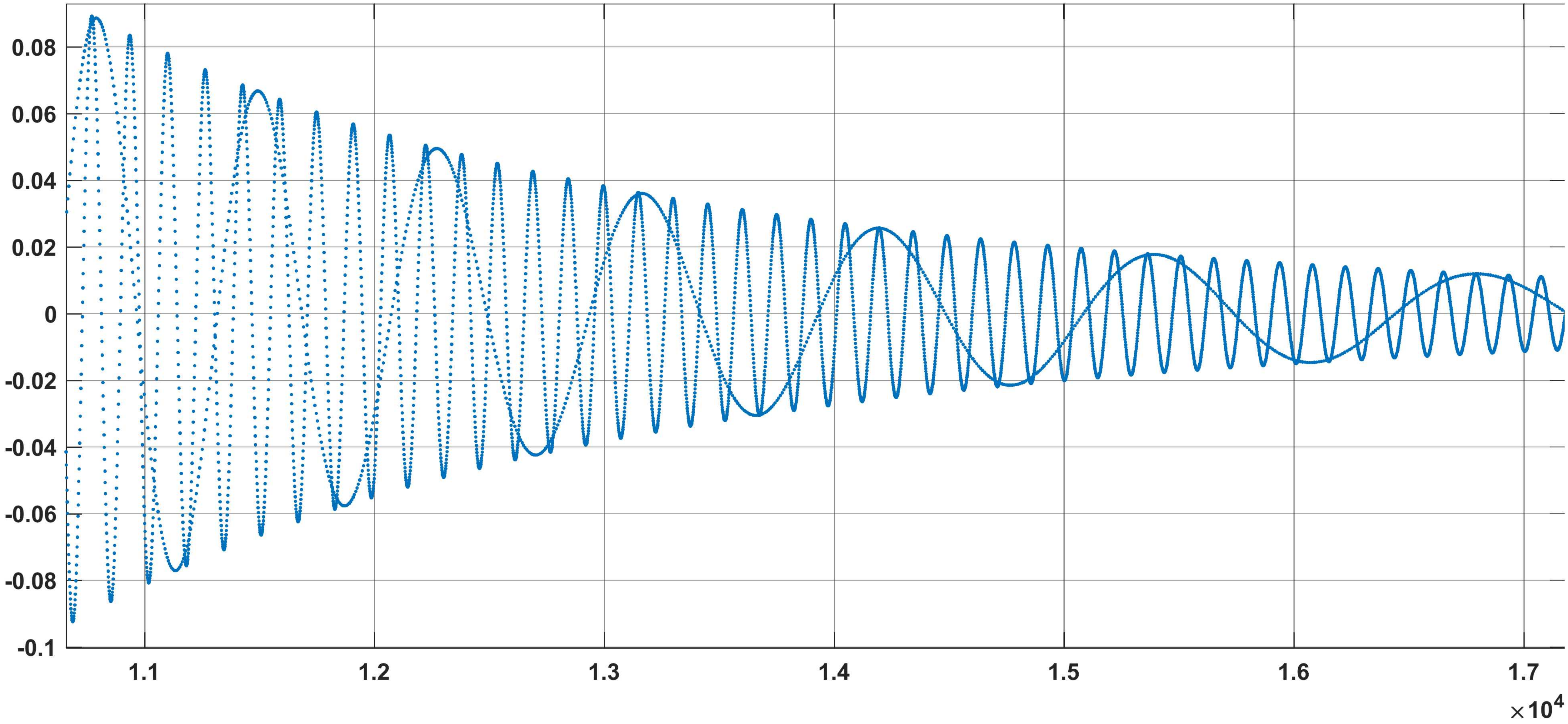


Reorder Response with Respect to Amplitude

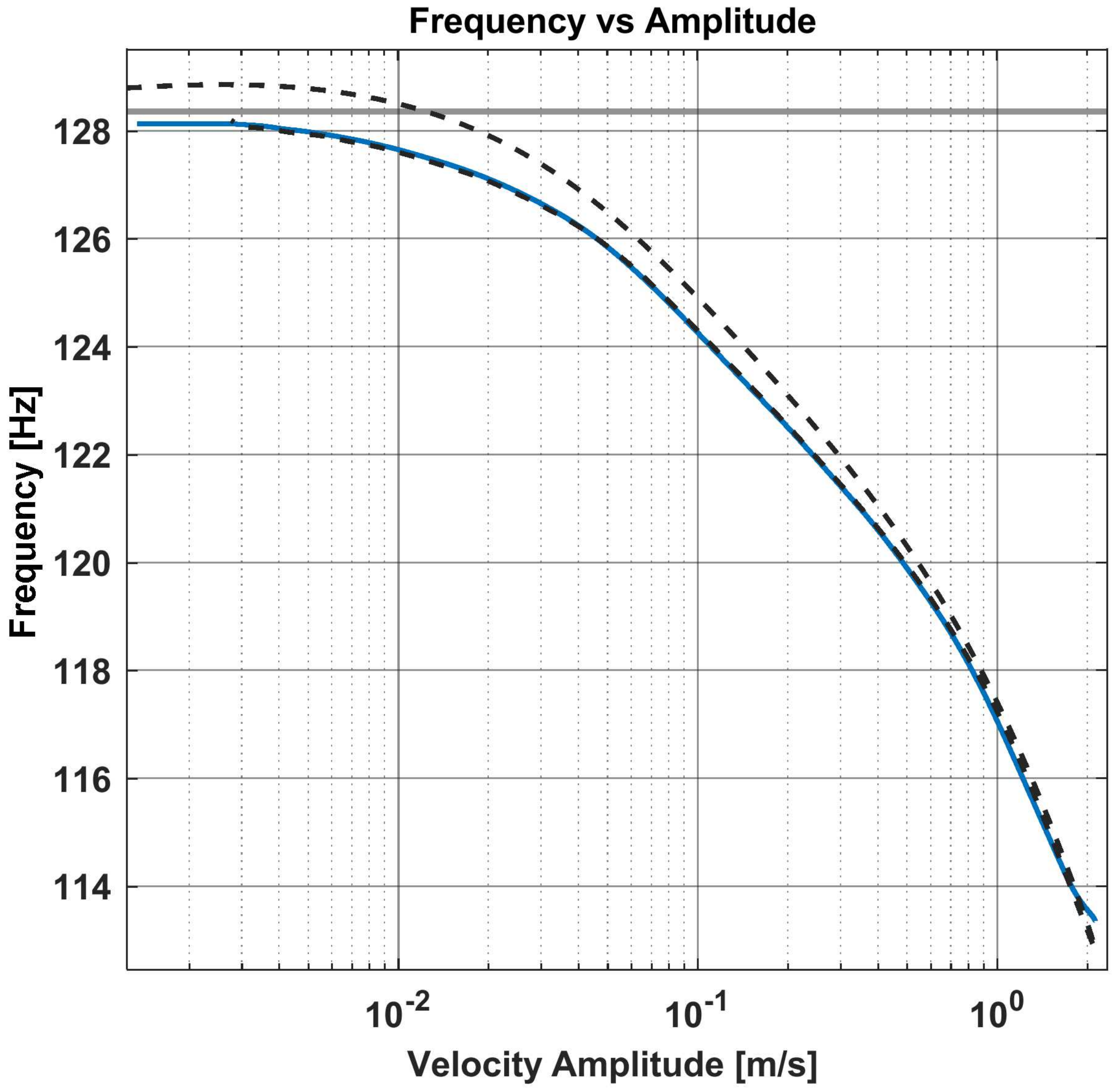
Sort by Response Amplitude

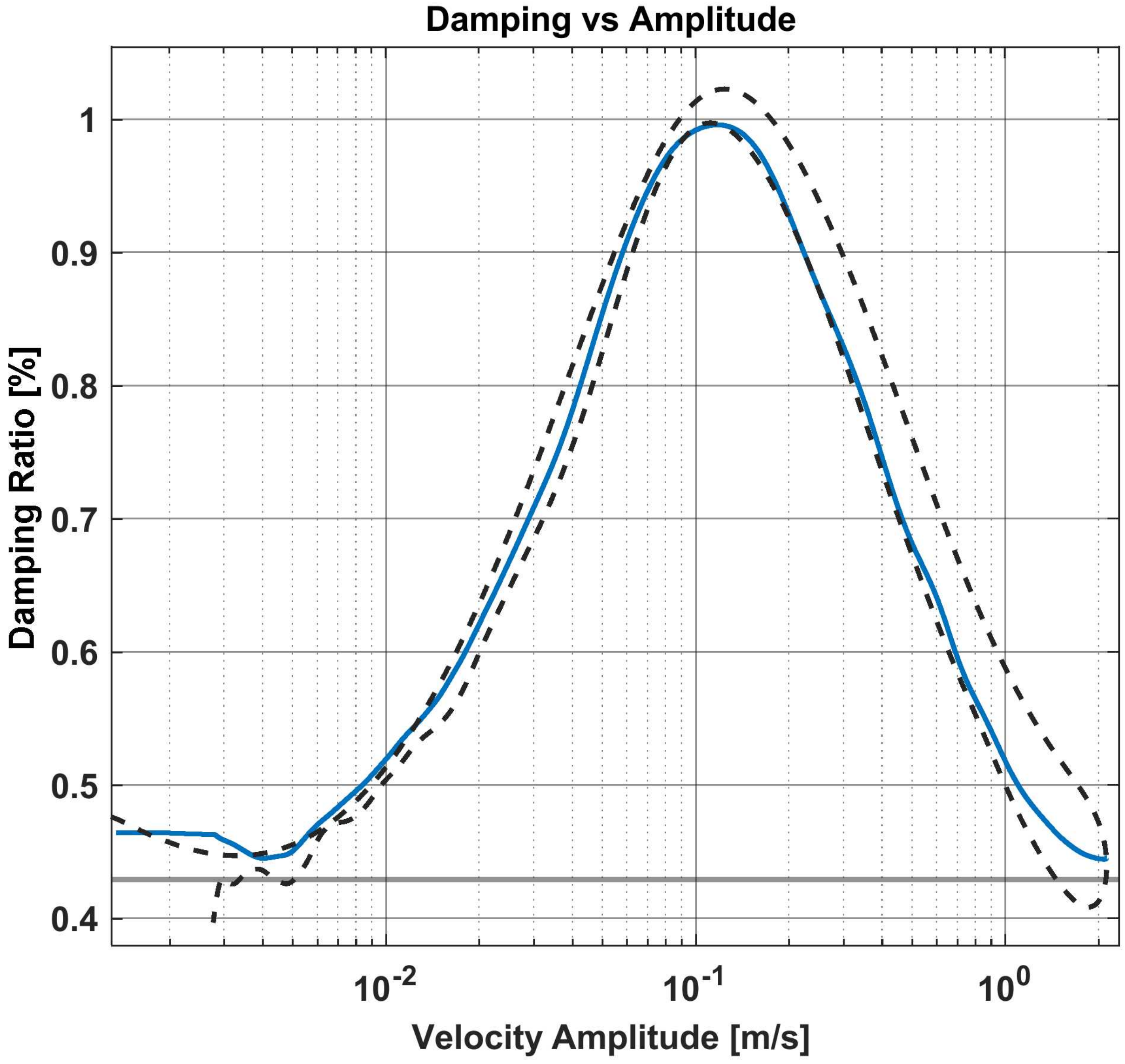


Sort by Response Amplitude

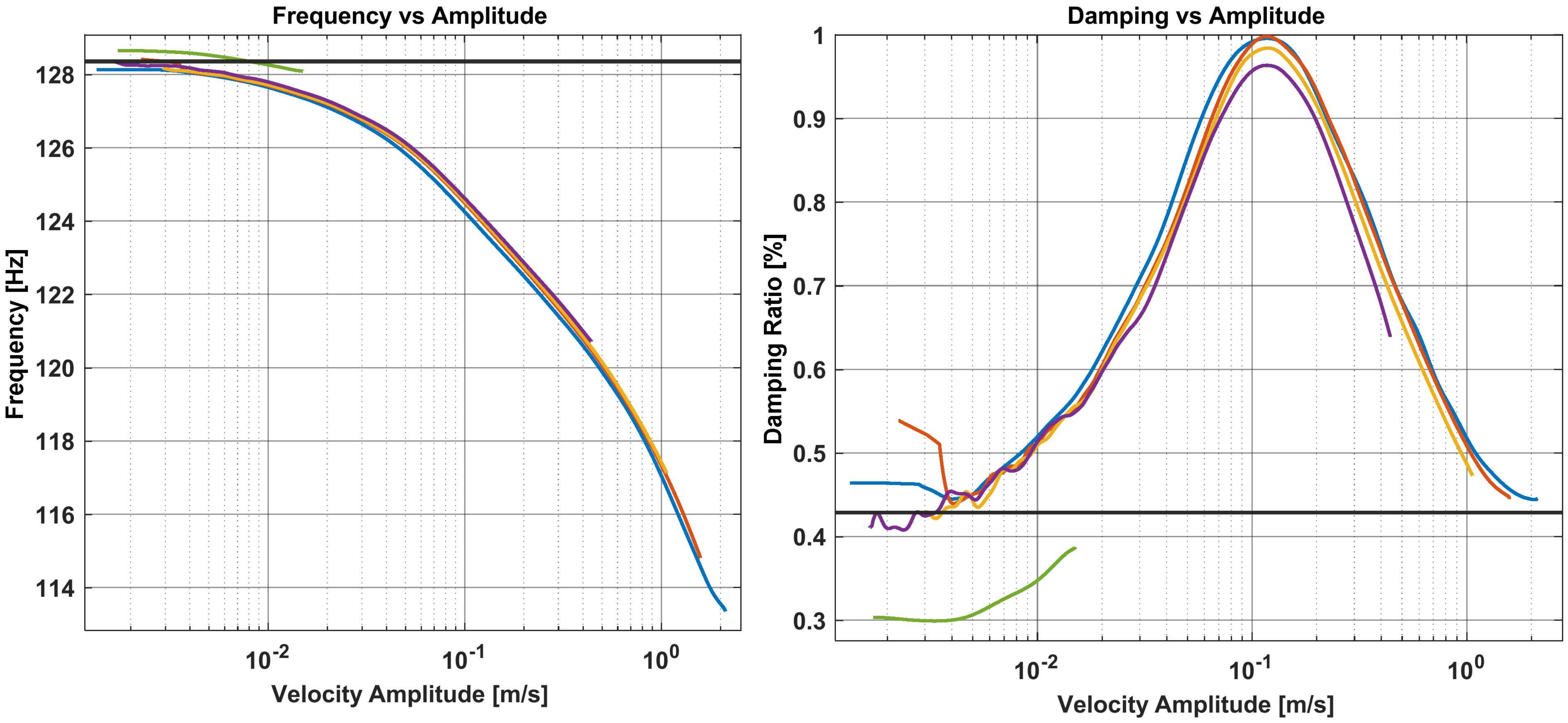


Sort by Response Amplitude





Sort by Response Amplitude



Equations for Directly Determining the System Parameters c and k

FORCEVIB Derivation

- $\ddot{Y}(t) + 2h_0(A)\dot{Y}(t) + \omega_0^2(A)Y(t) = X(t)/m$

- $Y(t) = A(t)e^{i\psi(t)}$
- $\dot{Y}(t) = \left(\dot{A}(t) + i\omega(t)A(t)\right)e^{i\psi(t)}$
- $\ddot{Y}(t) = \left(\ddot{A}(t) - \omega(t)^2A(t) + 2i\omega(t)\dot{A}(t) + i\dot{\omega}(t)A(t)\right)e^{i\psi(t)}$

Determined with Hilbert Transform

- $\frac{Y}{A} = e^{i\psi}$
- $\dot{Y} = \left(\frac{\dot{A}}{A} + i\omega\right)Y$
- $\ddot{Y} = \left(\frac{\ddot{A}}{A} - \omega^2 + 2i\omega\frac{\dot{A}}{A} + i\dot{\omega}\right)Y$

FORCEVIB Derivation

- $\ddot{Y} + 2h_0\dot{Y} + \omega_0^2 Y = X/m$
- $\left(\frac{\ddot{A}}{A} - \omega^2 + i2\omega\frac{\dot{A}}{A} + i\dot{\omega}\right)Y + 2h_0\left(\frac{\dot{A}}{A} + i\omega\right)Y + \omega_0^2 Y = X/m$
- $\frac{\ddot{A}}{A} - \omega^2 + i2\omega\frac{\dot{A}}{A} + i\dot{\omega} + 2h_0\left(\frac{\dot{A}}{A} + i\omega\right) + \omega_0^2 = \frac{1}{m}\frac{X}{Y}$ ⟶ $\frac{X}{Y} = \alpha(t) + i\beta(t)$
- Split into Real and Imaginary Parts and Solve for Stiffness and Damping
- $\left(\frac{\ddot{A}}{A} - \omega^2 + 2h_0\frac{\dot{A}}{A} + \omega_0^2\right) + i\left(2\omega\frac{\dot{A}}{A} + \dot{\omega} + 2h_0\omega\right) = \frac{1}{m}\left(\alpha(t) + i\beta(t)\right)$
  - $\omega_0^2(t) = \omega^2 + \frac{\alpha(t)}{m} - \frac{\beta(t)\dot{A}}{A\omega m} - \frac{\ddot{A}}{A} + \frac{2\dot{A}^2}{A^2} + \frac{\dot{A}\dot{\omega}}{A\omega}$
  - $h_0(t) = \frac{\beta(t)}{2\omega m} - \frac{\dot{A}}{A} - \frac{\dot{\omega}}{2\omega}$

Differential Eqs. for Amplitude and Phase

- $m\ddot{x} + c\dot{x} + kx = f$
- $m(\ddot{\theta} + \dot{\theta}^2)e^{\theta} + c\dot{\theta}e^{\theta} + ke^{\theta} = f$
- $\ddot{\theta} = -\dot{\theta}^2 - \frac{c}{m}\dot{\theta} - \frac{k}{m} + \frac{f}{mx}$
- $\ddot{b} + \mathrm{i}\ddot{a} = -(\dot{b}^2 - \dot{a}^2 + 2\mathrm{i}\dot{a}\dot{b}) - \frac{c}{m}(\dot{b} + \mathrm{i}\dot{a}) - \frac{k}{m} + \frac{f}{mx}$
- Split into Real and Imaginary Parts
  - $\ddot{b} = \dot{a}^2 - \dot{b}^2 - \boxed{\frac{c}{m}}\dot{b} - \boxed{\frac{k}{m}} + \frac{1}{m}\frac{f_r x_r + f_i x_i}{x_r^2 + x_i^2}$ ← Amplitude
  - $\ddot{a} = -2\dot{a}\dot{b} - \boxed{\frac{c}{m}}\dot{a} + \frac{1}{m}\frac{f_i x_r - f_r x_i}{x_r^2 + x_i^2}$ ← Phase

- $x = e^{\theta(t)}$
- $\dot{x} = \dot{\theta}e^{\theta(t)}$
- $\ddot{x} = (\ddot{\theta} + \dot{\theta}^2)e^{\theta(t)}$
  - $\theta(t) = b(t) + \mathrm{i}a(t)$
  - $\dot{\theta}(t) = \dot{b}(t) + \mathrm{i}\dot{a}(t)$
  - $\ddot{\theta}(t) = \ddot{b}(t) + \mathrm{i}\ddot{a}(t)$

- $x = e^{\theta(t)} = x_r + ix_i$
- $f = e^{\sigma(t)} = f_r + if_i$
- $Re\left(\frac{f}{x}\right) = \frac{f_r x_r + f_i x_i}{x_r^2 + x_i^2}$
- $Im\left(\frac{f}{x}\right) = \frac{f_i x_r - f_r x_i}{x_r^2 + x_i^2}$

Equations for c and k

- $\ddot{a} = -2\dot{a}\dot{b} - \frac{c}{m}\dot{a} + \frac{1}{m}\frac{f_i x_r - f_r x_i}{x_r^2 + x_i^2}$
- $\frac{c}{m} = -2\dot{b} - \frac{\ddot{a}}{\dot{a}} + \frac{1}{m\dot{a}}\frac{f_i x_r - f_r x_i}{x_r^2 + x_i^2}$ ← Damping

- $\ddot{b} = \dot{a}^2 - \dot{b}^2 - \frac{c}{m}\dot{b} - \frac{k}{m} + \frac{1}{m}\frac{f_r x_r + f_i x_i}{x_r^2 + x_i^2}$
- $\frac{k}{m} = \dot{a}^2 - \dot{b}^2 - \ddot{b} - \left(-2\dot{b} - \frac{\ddot{a}}{\dot{a}} + \frac{1}{m\dot{a}}\frac{f_i x_r - f_r x_i}{x_r^2 + x_i^2}\right)\dot{b} + \frac{1}{m}\frac{f_r x_r + f_i x_i}{x_r^2 + x_i^2}$
- $\frac{k}{m} = \dot{a}^2 + \dot{b}^2 - \ddot{b} + \ddot{a}\frac{\dot{b}}{\dot{a}} - \frac{\dot{b}}{m\dot{a}}\frac{f_i x_r - f_r x_i}{x_r^2 + x_i^2} + \frac{1}{m}\frac{f_r x_r + f_i x_i}{x_r^2 + x_i^2}$ ← Stiffness

FORCEVIB = Mine

- $\frac{c}{2m} = \frac{1}{2m\dot{a}} \frac{f_i x_r - f_r x_i}{x_r^2 + x_i^2} - \dot{b} - \frac{\ddot{a}}{2\dot{a}}$
- $h_0 = \frac{\beta(t)}{2\omega m} - \frac{\dot{A}}{A} - \frac{\dot{\omega}}{2\omega}$

$$\frac{f_r x_r + f_i x_i}{x_r^2 + x_i^2} = \alpha(t) \quad ; \quad \frac{f_i x_r - f_r x_i}{x_r^2 + x_i^2} = \beta(t)$$

$$\dot{a} = \omega \quad ; \quad \ddot{a} = \dot{\omega}$$

$$b = \ln(A) \quad ; \quad \dot{b} = \frac{\dot{A}}{A} \quad ; \quad \ddot{b} = \frac{\ddot{A}}{A} - \frac{\dot{A}^2}{A^2}$$

- $\frac{k}{m} = \dot{a}^2 + \frac{1}{m} \frac{f_r x_r + f_i x_i}{x_r^2 + x_i^2} - \frac{\dot{b}}{\dot{a} m} \frac{f_i x_r - f_r x_i}{x_r^2 + x_i^2} - \ddot{b} + \dot{b}^2 + \ddot{a} \frac{\dot{b}}{\dot{a}}$
- $\omega_0^2(t) = \omega^2 + \frac{\alpha(t)}{m} - \frac{\beta(t)\dot{A}}{A\omega m} - \frac{\ddot{A}}{A} + \frac{2\dot{A}^2}{A^2} + \frac{\dot{A}\dot{\omega}}{A\omega}$