

Exceptional service in the national interest



Sandia theory perspective: detail, completeness, and consistency

S. Hansen, T. Nagayama, T. Gomez, R. More*,
T. Hentschel**, C. Jennings, A. Baczewski, and a great community ***

Sandia National Laboratories (DOE/OFES early career & LDRD)

**private company*

***Cornell University*

**** LLNL, LANL, CEA...*



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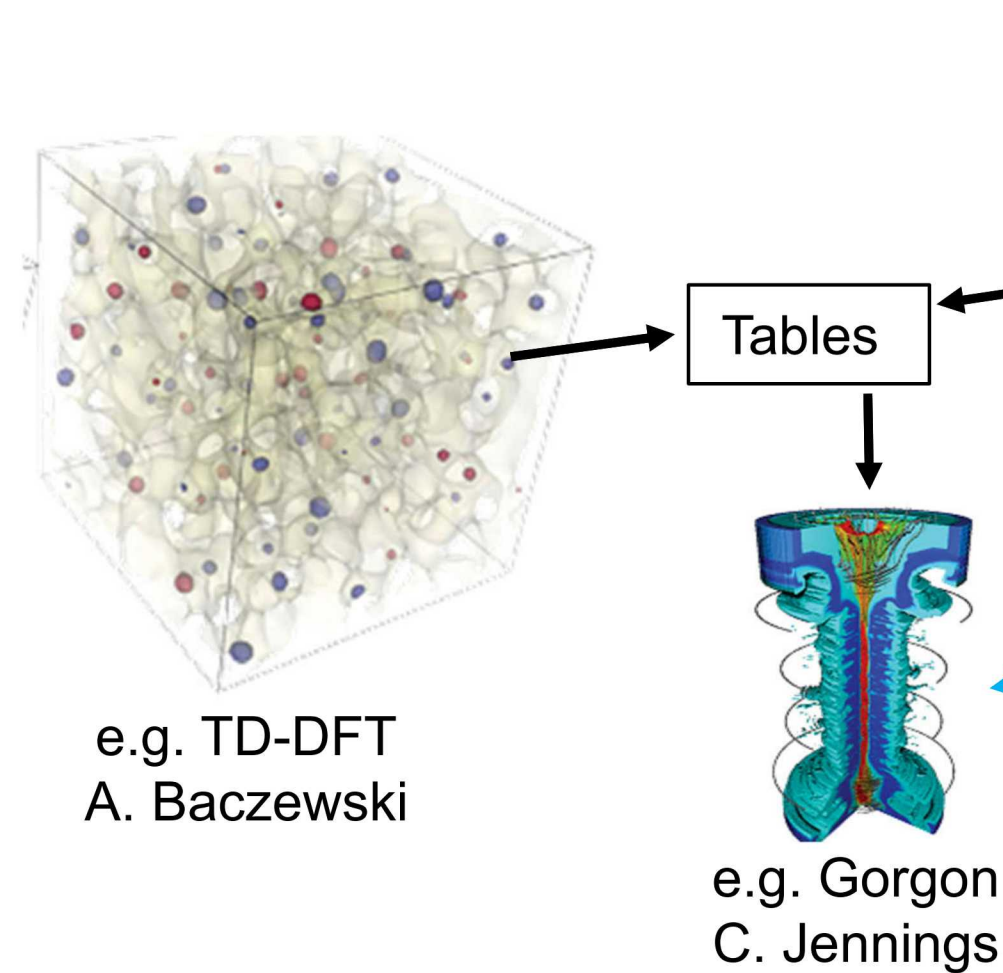
High-precision measurements drive innovation and improvements in atomic models

- **HEDP uses models for experimental design and diagnostics**
 - We can adjust model detail to match applications:
 - Low-resolution models are ok for rad-hydro design
 - High-resolution models are needed for spectroscopic diagnostics
 - For all applications, we care about accuracy, which requires completeness and consistency, and about tractability, which often requires sacrifices in detail
- **High-precision measurements from carefully prepared experiments set the standard:**
 - XFEL fluorescence (e.g. Vinko et al)
 - Continuum lowering & dense plasma effects
 - Opacity (Z, NIF, Omega) (e.g. Bailey et al)
 - One- and two-photon cross sections, line shapes
 - ICF: fluorescence and absorption (e.g. Hansen et al, Jiang et al)
 - Line shifts and dense plasma effects
- **Where would we like to see models go?**
 - Internally consistent, spectroscopically detailed, rigorously complete... and tractable

A brief tour of the atomic model zoo

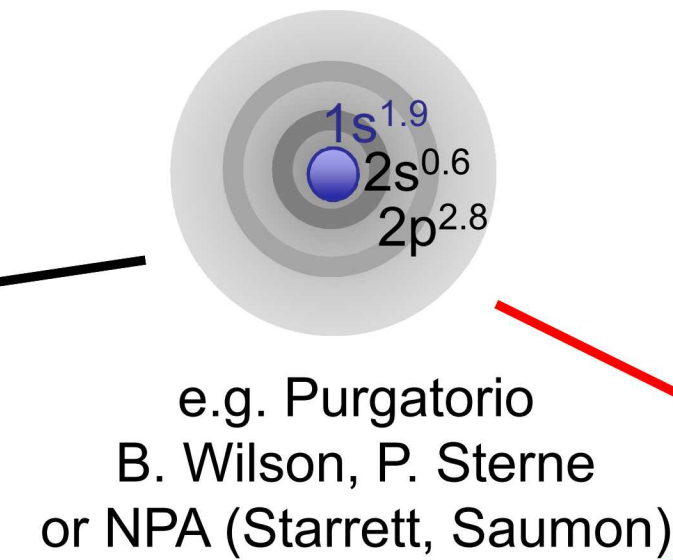
Density functional theory molecular dynamics (DFT-MD)

Accurate for EOS & transport
Quite expensive & undetailed
Natively consistent & complete
Limited to LTE



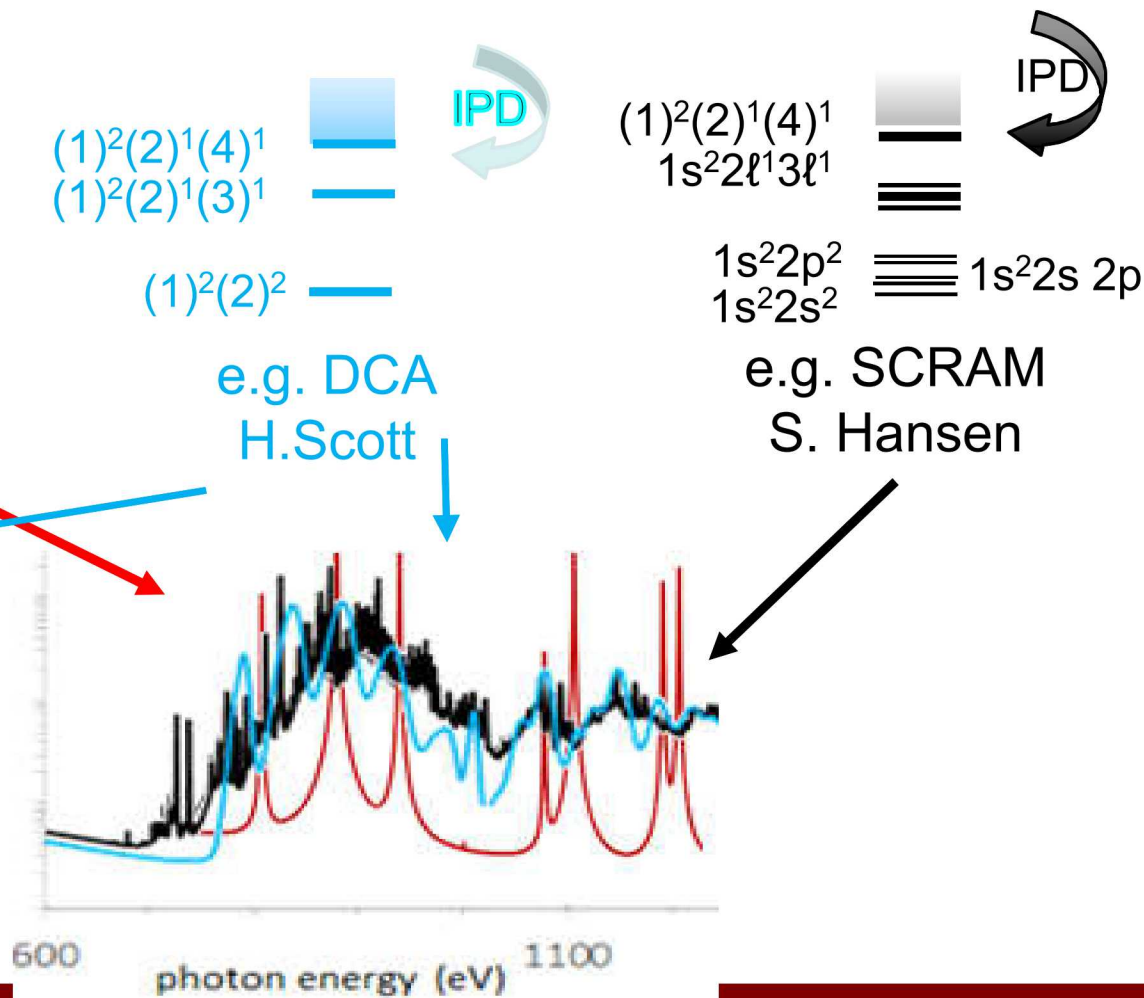
Density functional theory Average atom (DFT-AA)

Accurate-ish for EOS & transport
Quite cheap & undetailed
Natively consistent & complete
Limited to LTE



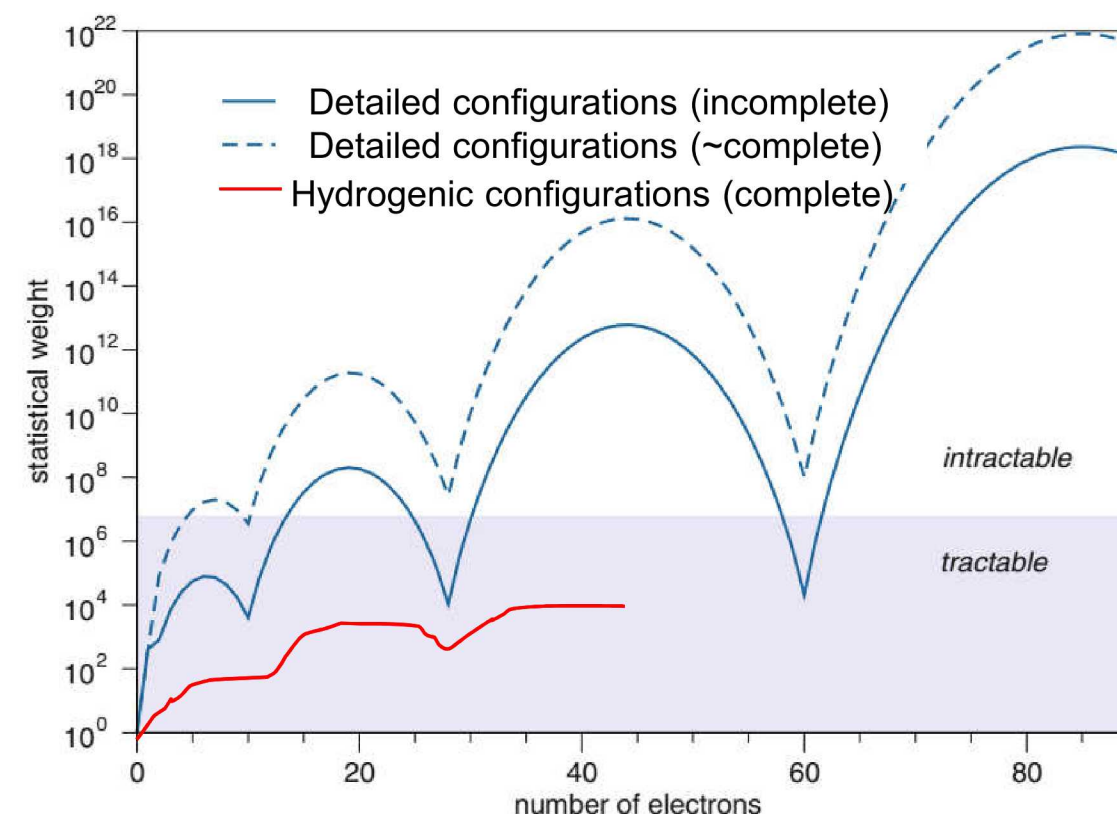
Configuration-based collisional-radiative models

Accurate for EOS & spectra
Variable cost & detail
Can be inconsistent, incomplete
Can handle non-LTE

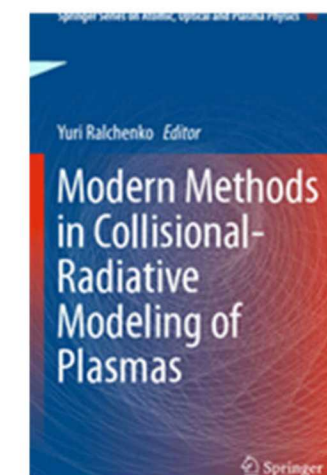


Detail and completeness compete with tractability; consistency is often an afterthought

- Combinatorics of completeness are unforgiving:
 - Ion number and electronic states challenge DFT-MD at high Z, T
 - Electronic configurations & rates challenge collisional-radiative models at high Z
 - Sacrificing completeness degrades accuracy
- Sacrifices can be made in detail/resolution:
 - DFT-AA models access all Z, T easily
 - Hydrogenic collisional-radiative models are fast enough to run inline in simulations
 - Rough rate approximations can be ok
- Consistency also matters:
 - Density effects/structure/rates
 - Rates/fields/lineshapes
 - EOS/transport/opacity
 - Non-LTE & non-local effects



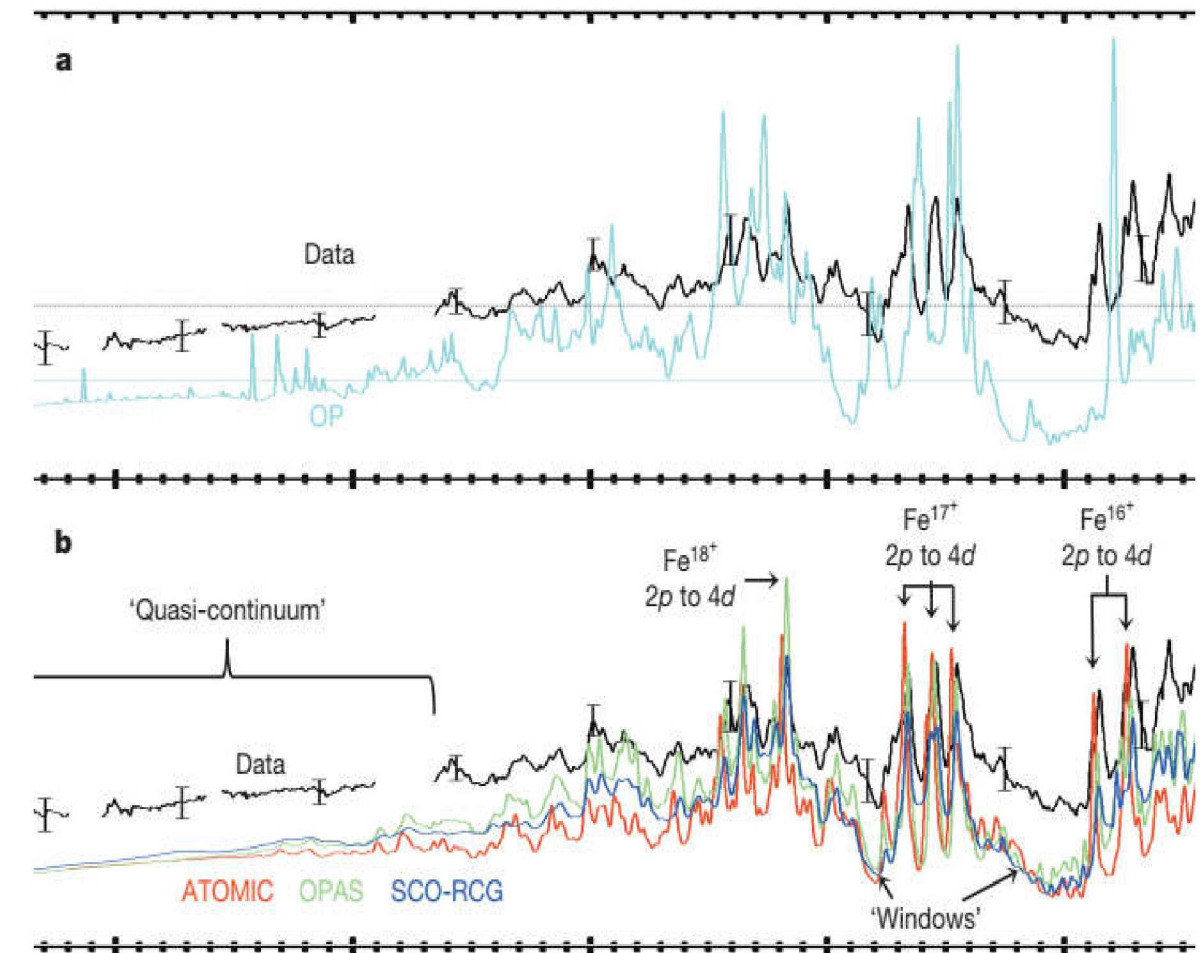
2016
Ed. Yu. Ralchenko



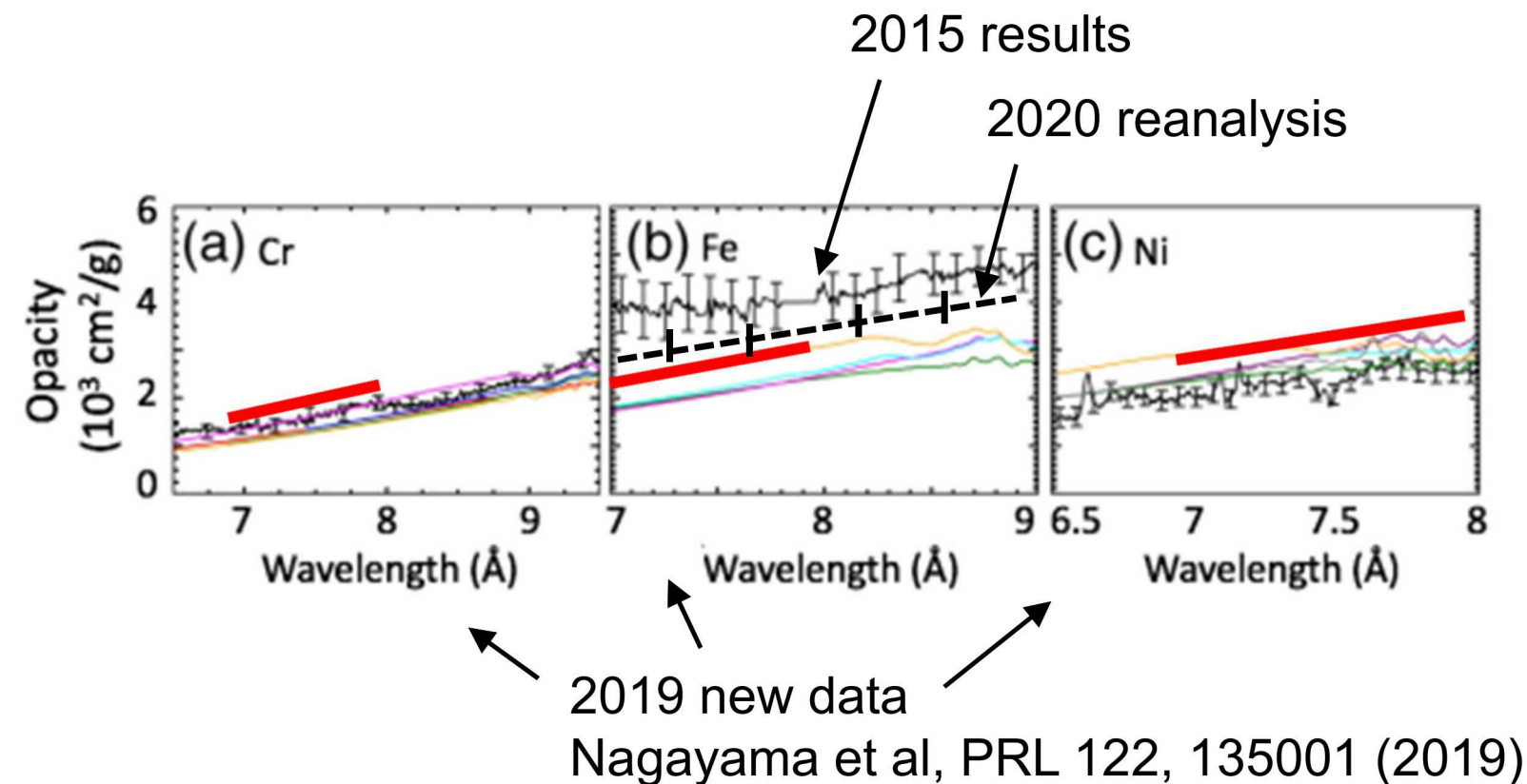
High-precision measurements help us know what's "good enough," but tend to require high resolution

Bailey et al's 2015 iron opacity measurements stimulated extensive investigations

- Good agreement with detailed models at lower T , n_e
→ OP not sufficiently detailed
- Disagreement in widths, windows & continuum at higher T , n_e
→ Better cross sections? (Nahar & Pradhan)
→ More complete states? (Iglesias, Hansen)
→ More complete rates? (More, Pain, Kruse & Iglesias)
→ Better line shapes? (Gomez, Nagayama)
→ Refined analysis? (Nagayama)
→ More experiments? (Bailey, Loisel, Perry, Heeter...)

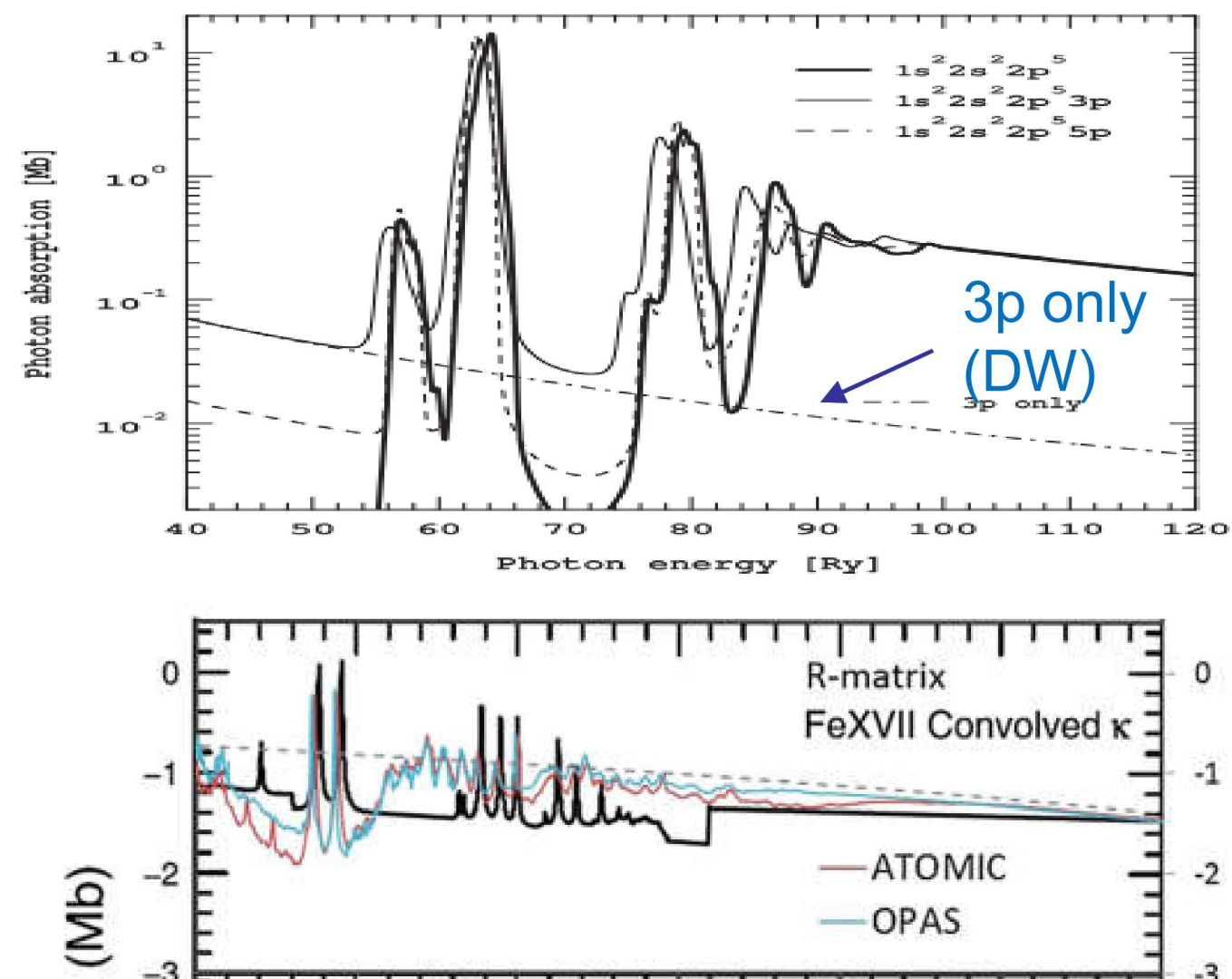
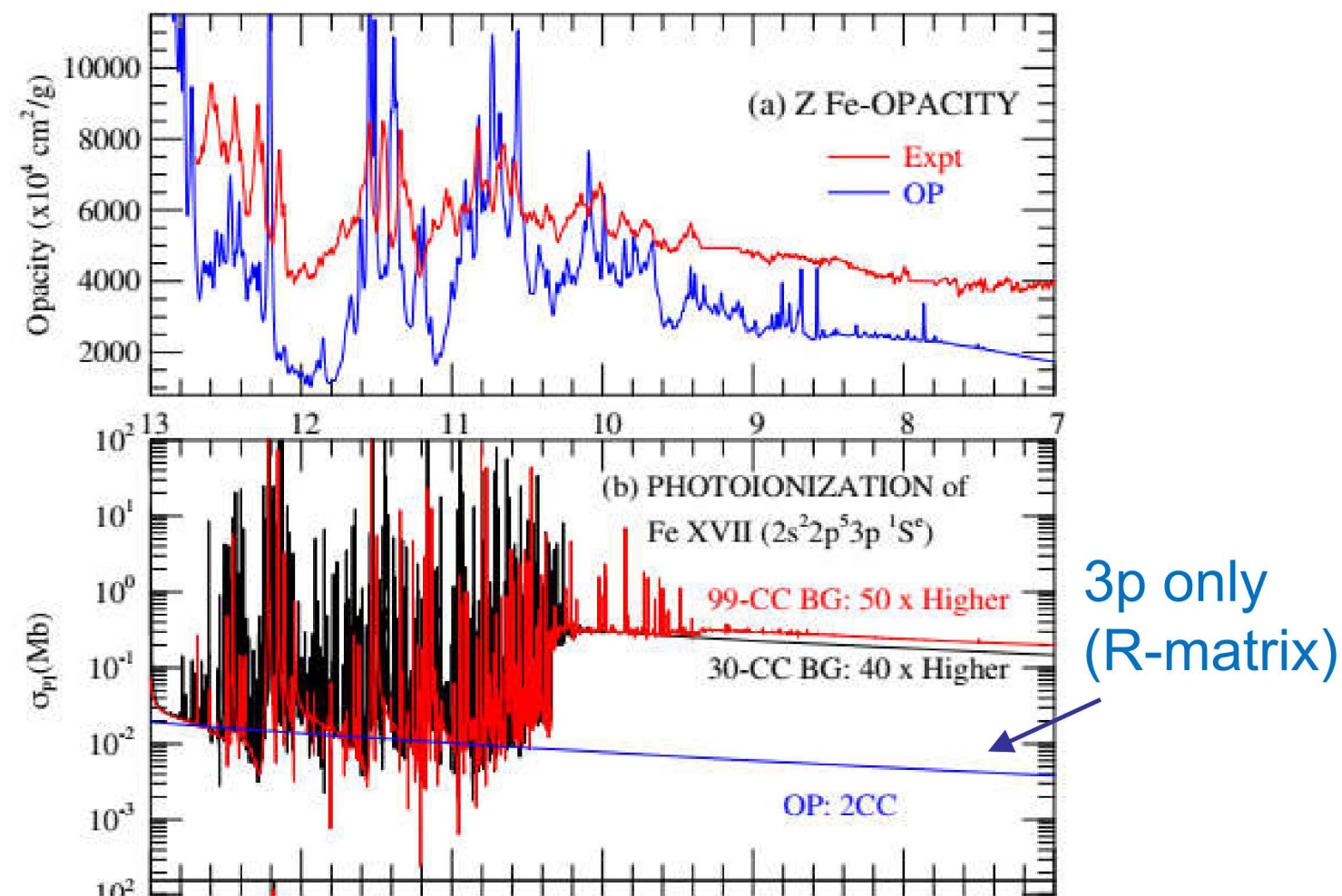


High iron continuum was especially worrisome



- Iglesias HEDP 15, 4 (2015): Sum rule violation?
 - Measurement range was insufficient for rigorous test
 - Absent other processes or redistribution, higher-than-cold σ^{PI} indicates $N_{\text{L-shell}} > 8$
- No similar discrepancy observed for Cr, Ni, or lower- T, n_e Fe – why?
- Reanalysis lowered Fe measurement, but discrepancy remains:
 - States: detail/accuracy? Completeness?
 - Rates: detail/accuracy? Completeness?

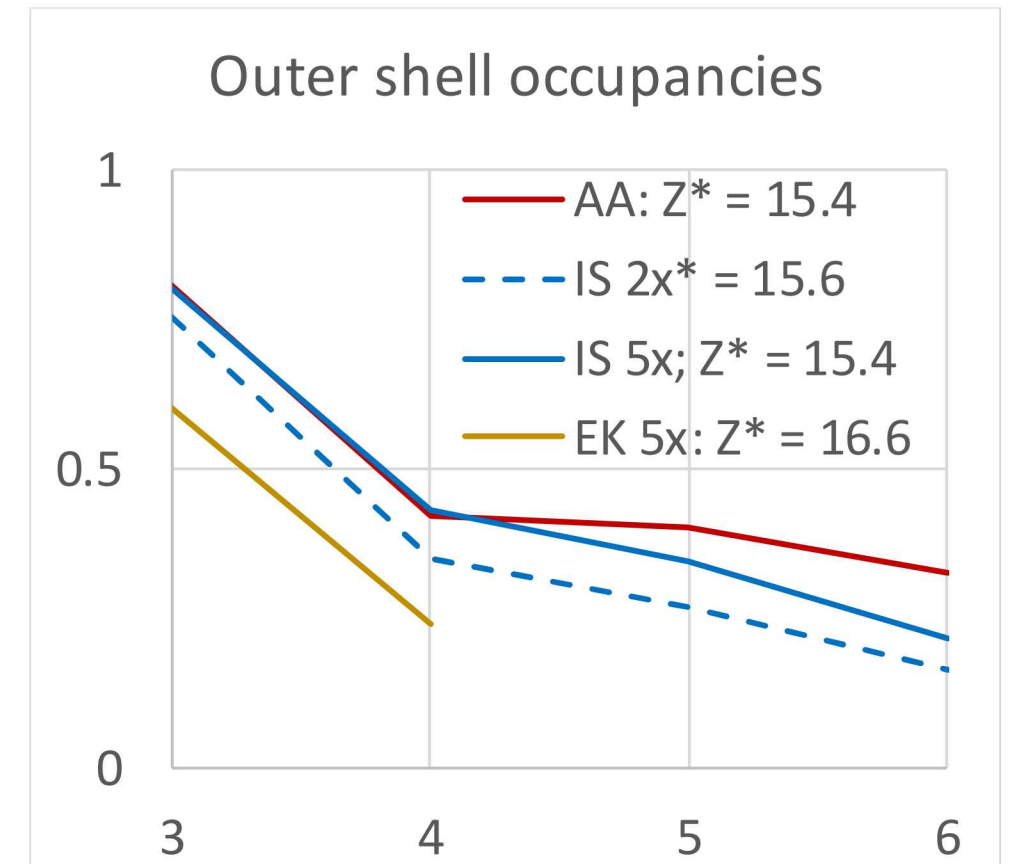
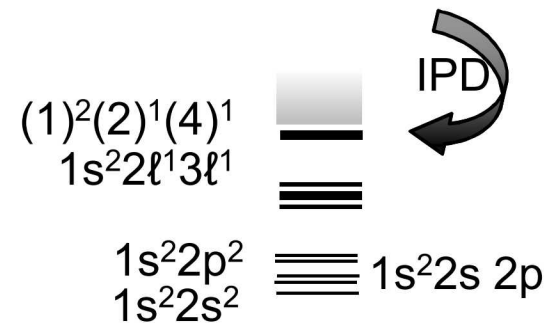
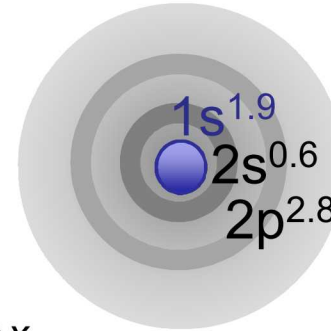
Rates: Do close-coupling/R-matrix calculations help?



- Nahar and Pradhan, PRL 116 (2016) proposed that close-coupling/R-matrix cross sections might be more accurate than distorted wave (DW), thereby enhancing σ^{PI}
- Iglesias & Hansen (ApJ 835, 2017) showed that incompleteness in previous CC models accounted for the reported enhancement
- Blancard et al (PRL 117, 2016) showed that DW is adequate (& much easier to make complete!)

States: Are models including enough excitations?

- Average-atom models
 - Native and rigorous completeness
 - Native treatment of continuum lowering
 - Based on non-integer-occupied orbitals $(nl)^x$
- Configuration-based models $(nl)^N$
 - Externally imposed continuum lowering
 - SCRAM uses ion-sphere for consistency with average atom
 - Completeness can be rigorously assessed
 - SCRAM uses systematic excitation of all electrons from all shells until $dN < \varepsilon$ (cf. James Colgan)
 - Completeness is also needed at low density for dielectronic recombination (cf. Howard Scott)



Rates: are models missing an important process?

A two-photon interlude

- Two-photon absorption had been discounted as negligible for K-shell (More & Rose, 1991)
- Increases in strength with n^8 *and with state complexity* so maybe not negligible here
- Has its own internal sum rule
- Multi-photon absorption/ionization/decay is well known for optical processes (NLO)
- X-ray two-photon ionization has been measured in XFEL experiments

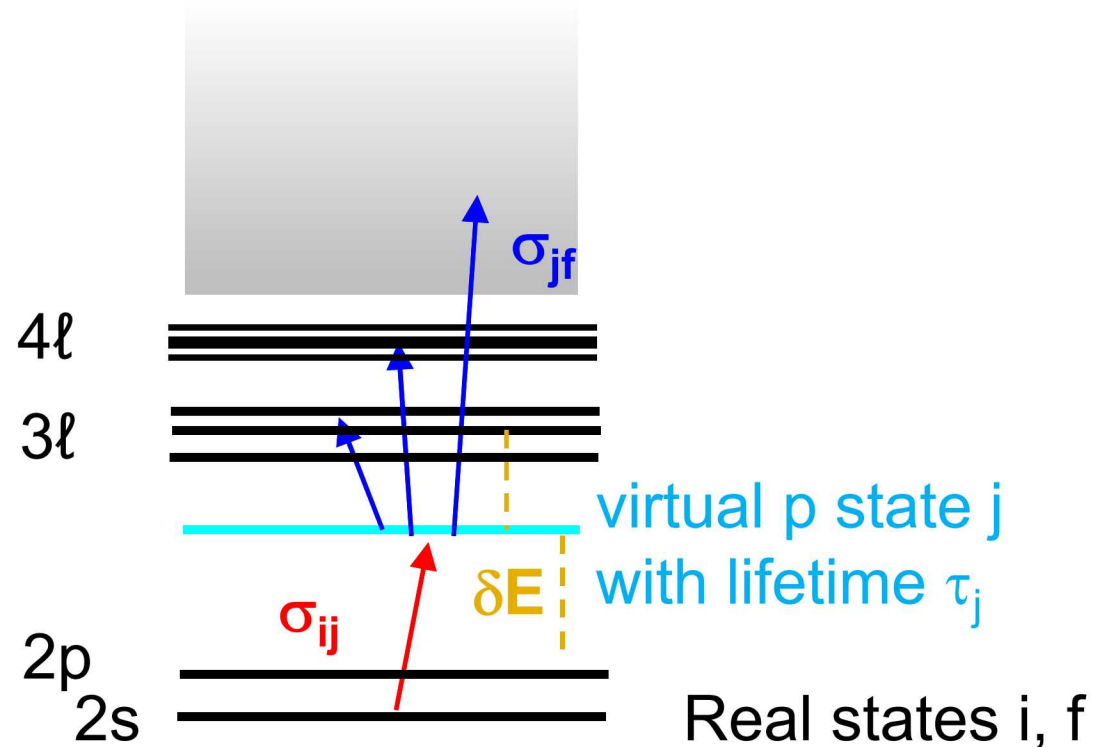
The 2015 iron results stimulated significant work:

- More et al HEDP 24, 44 (2017): initial calculations
- Pain HEDP 26, 23 (2018): hydrogenic, one-color
- Kruse & Iglesias HEDP 31, 28 (2019): total two-photon from 2s
- Colgan & Pindzola (private communication): TDSE cross sections
- More et al, HEDP 34, 100717 (2020): f-f matrix elements
- Baggot, Rose, & Mangles (2020): $\gamma + \gamma$ & $e + \gamma$; line shapes

Accuracy, detail, and completeness all matter, and including all together is challenging

- More, Kruse & Iglesias, and Pain all compute accurate QM two-photon cross sections
- I tried something reckless: the terrible-for-opacity-but-natively-complete average atom plus some stuff from nonlinear optics (Lambropoulos, Adv. At. Mol. Phys. 12, 87 (1976))

$$\sigma^{\text{TPA}} \sim \sigma_{ij}(\mathbf{h}\mathbf{v}_1) \tau_j \sigma_{jf}(\mathbf{h}\mathbf{v}_2) [\text{cm}^4 \text{ s}],$$



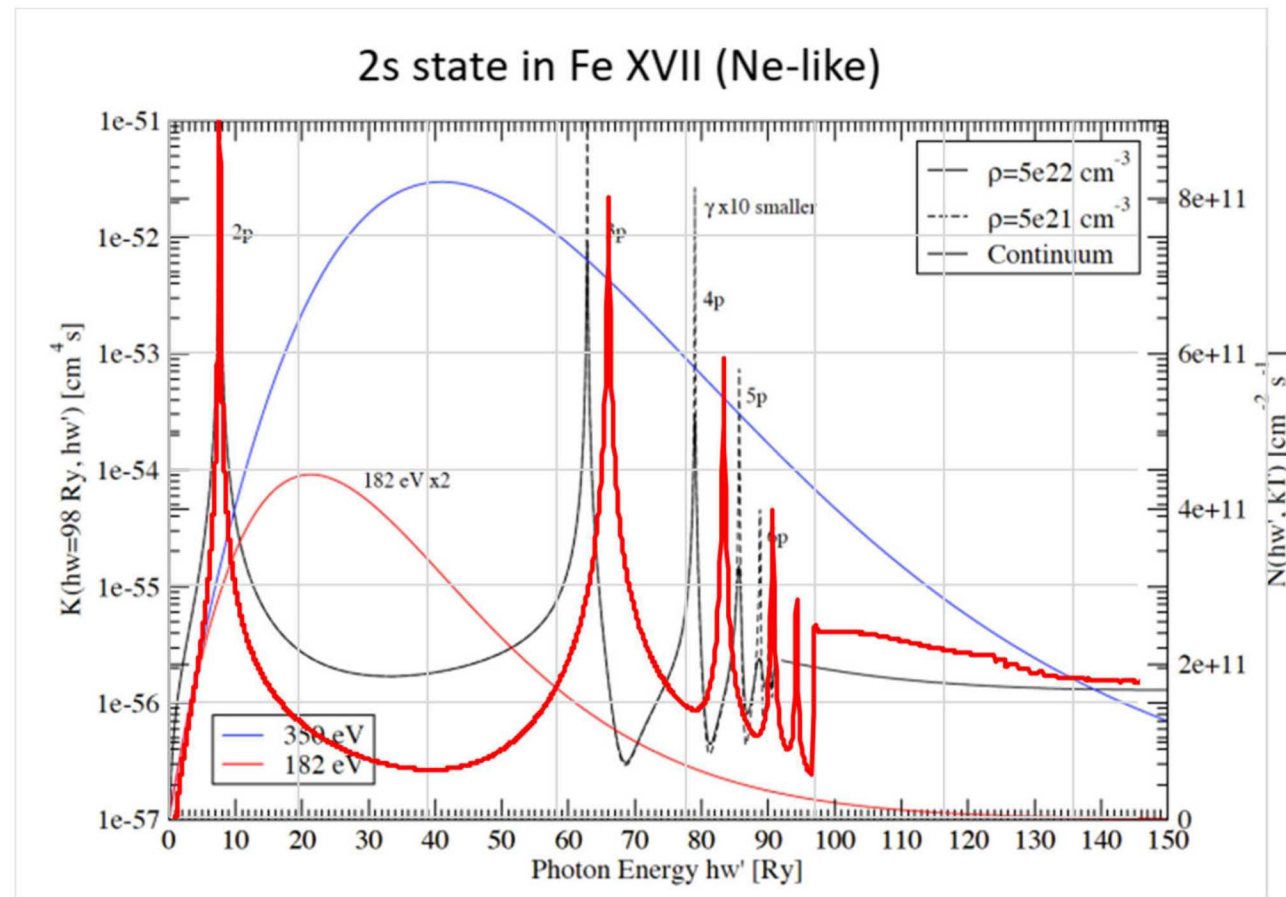
- σ are one-photon cross sections ($\sim 10^{-20} \text{ cm}^2$)
- τ are lifetimes of virtual states ($\hbar/\delta E + \tau_c \sim 10^{-14} \text{ s}$)
- $\rightarrow \sigma^{\text{TPA}} \sim 10^{-54} \text{ cm}^4 \text{ s}$
- Must sum over all virtual and final states
- Backlighter photons can be either $\mathbf{h}\mathbf{v}_1$ or $\mathbf{h}\mathbf{v}_2$, so we must integrate *twice* over local rad field FdE ($\sim \text{eV}/\text{eV-s-cm}^2$)

This approach tracks a *complete* set of states and $i - j - f$ channels, but with much less detail than multiconfiguration one-photon models

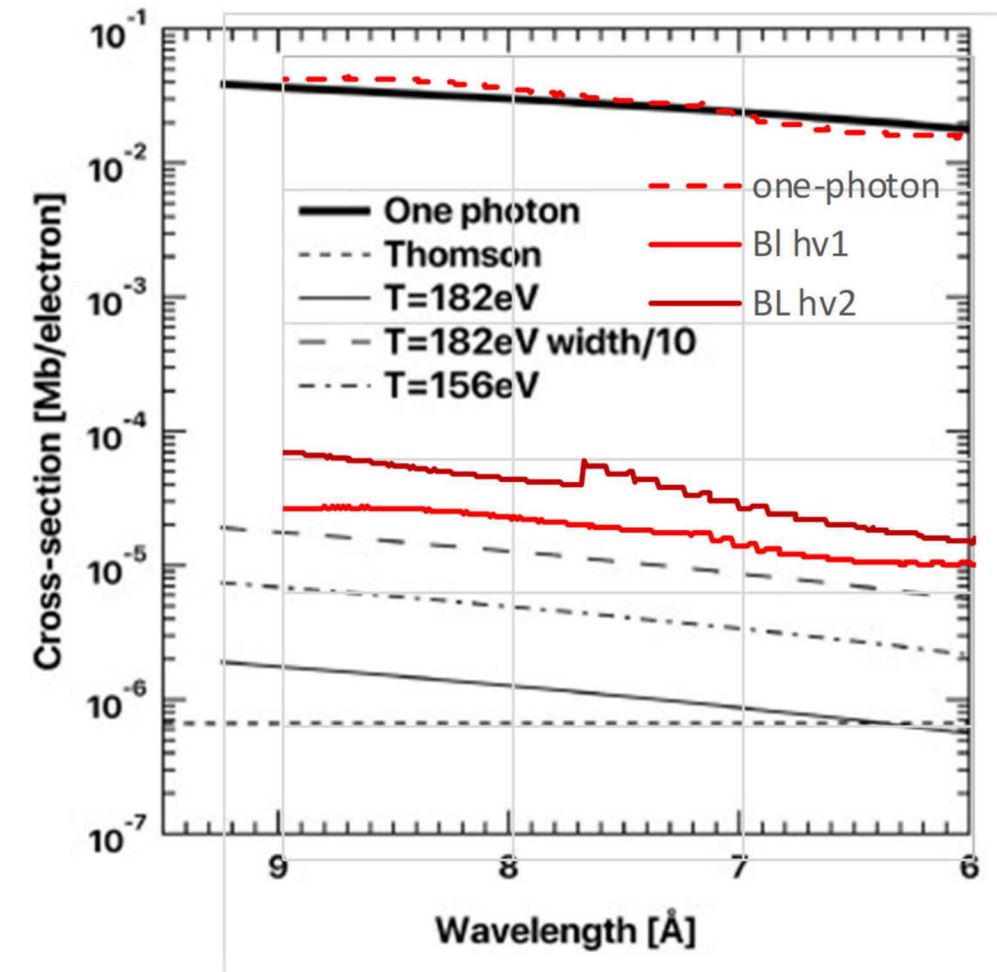
Sparse states that maximize δE will minimize two-photon contributions

Test approximation for $h\nu_1 = 1.3$ keV backlighter photon from $2s$ against accurate $2s^2 2p^6 - 2s 2p^6 nd$ calculations

This is the case considered by Kruse & Iglesias

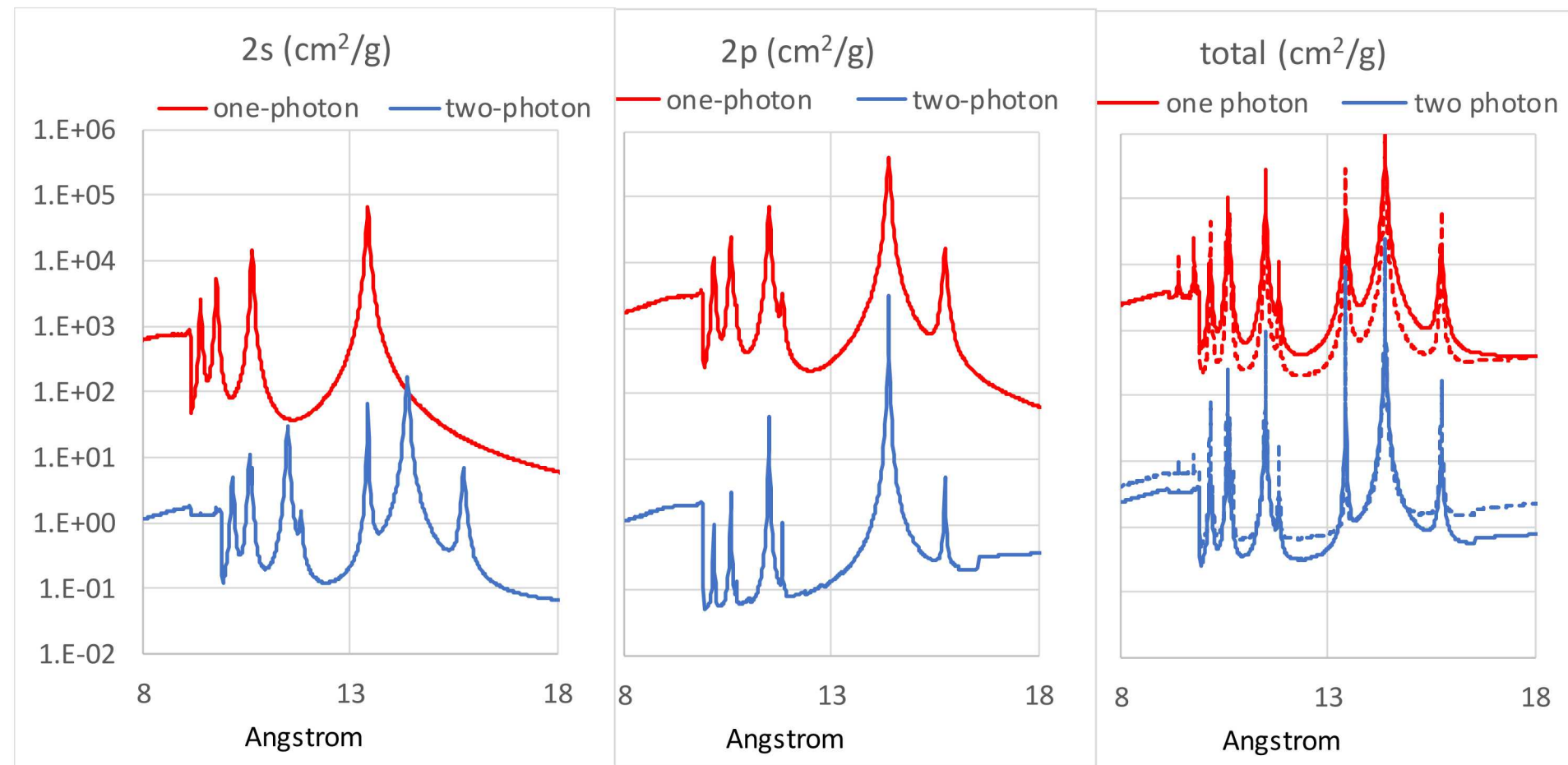


The approximate AA cross section is similar in character to Kruse & Iglesias' accurate calculation for detailed Ne-like Fe

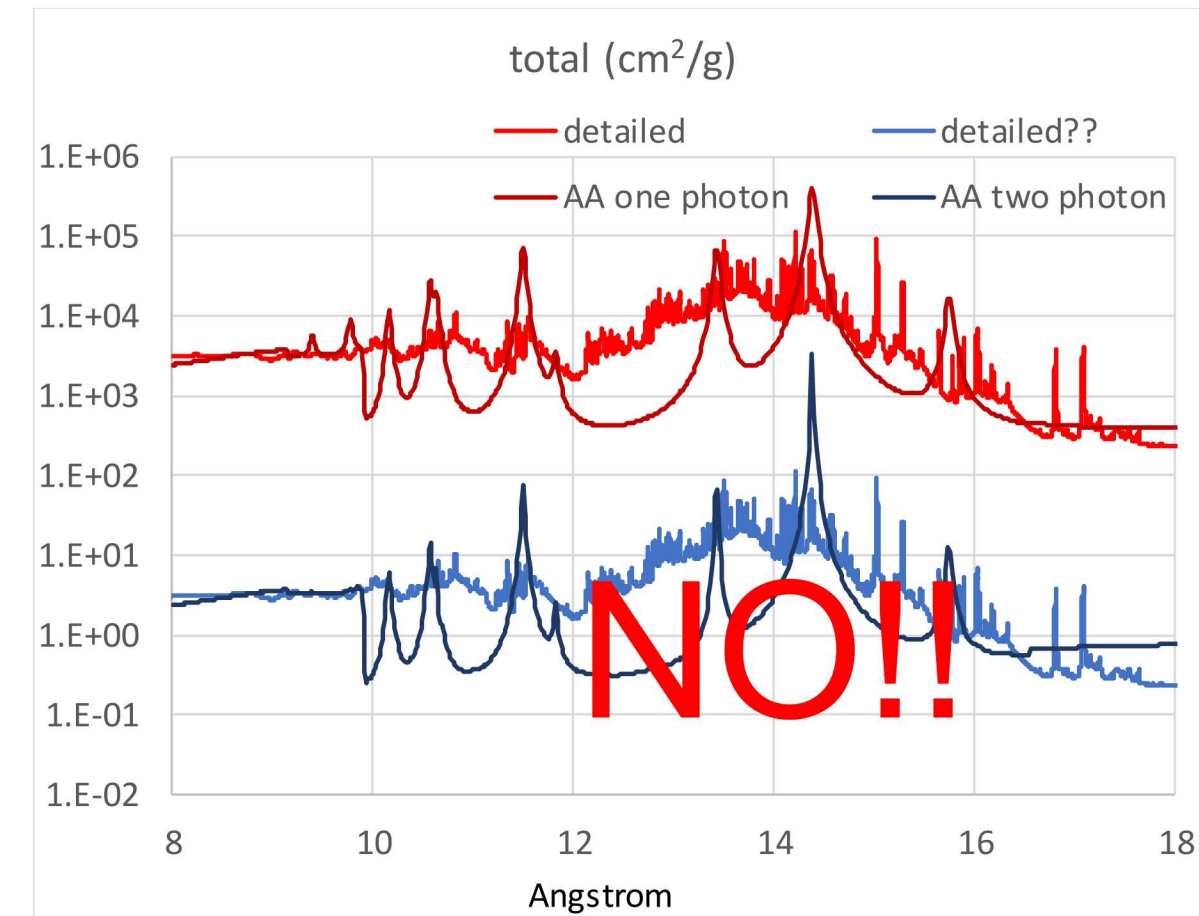


First integral is similar to Kruse & Iglesias
 Second integral (backlighter photon $h\nu_2$) is $\sim 2\times$ first
 Add together

Approximate two-photon absorption from undetailed average atom is small, but detailed structure *matters*

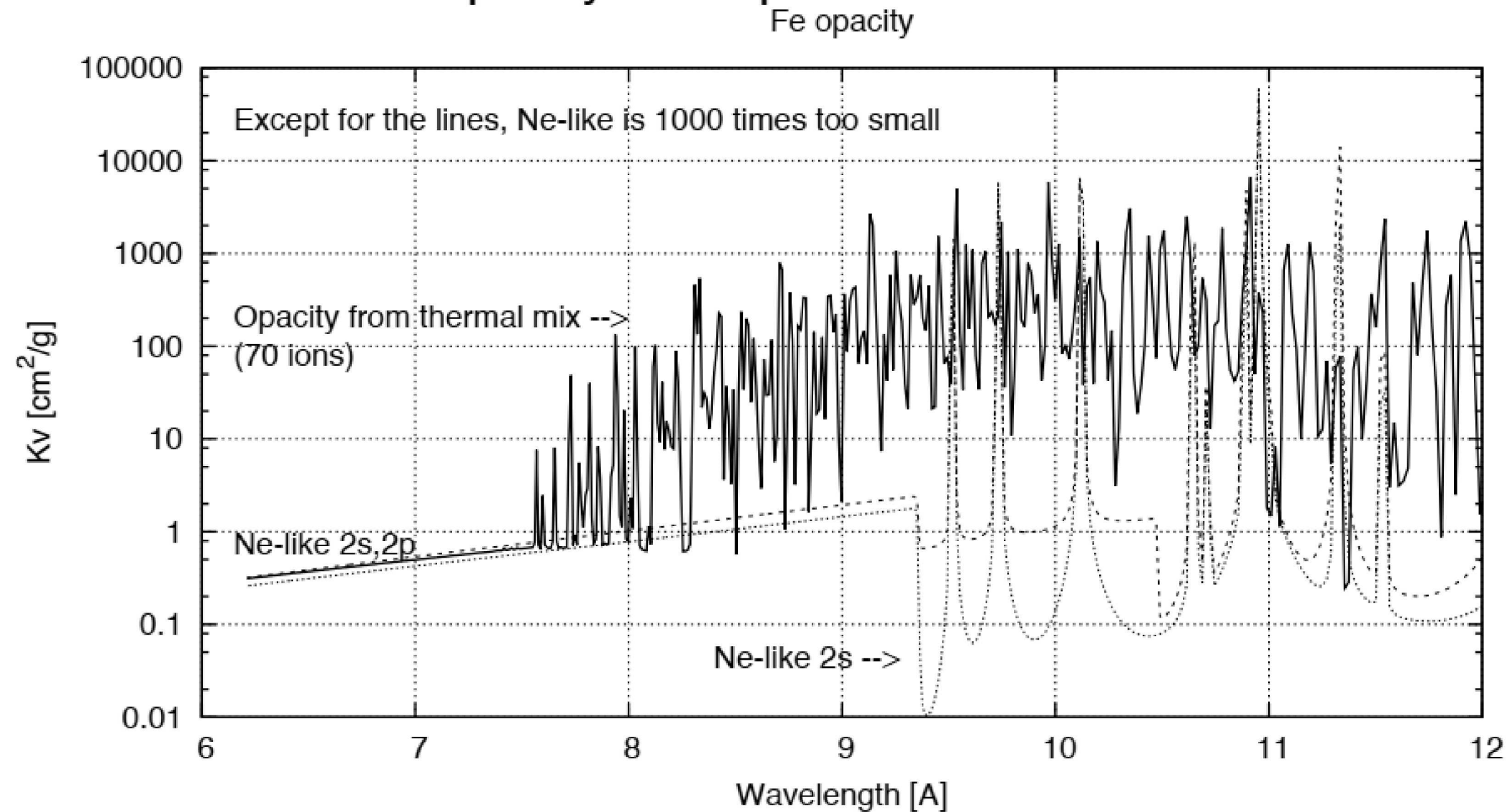


- 2p, 2s, & total two-photon cross sections from the average atom are all 1000x smaller than one-photon
- Like Kruse & Iglesias, AA results are dependent on line widths



While σ_{ij} and σ_{jf} both obey sum rules and preserve strength under splitting into detailed structure, the two-photon $\sigma_{ij} (h/\delta E) \sigma_{jf}$ cross section can get **much larger** with more detailed structure

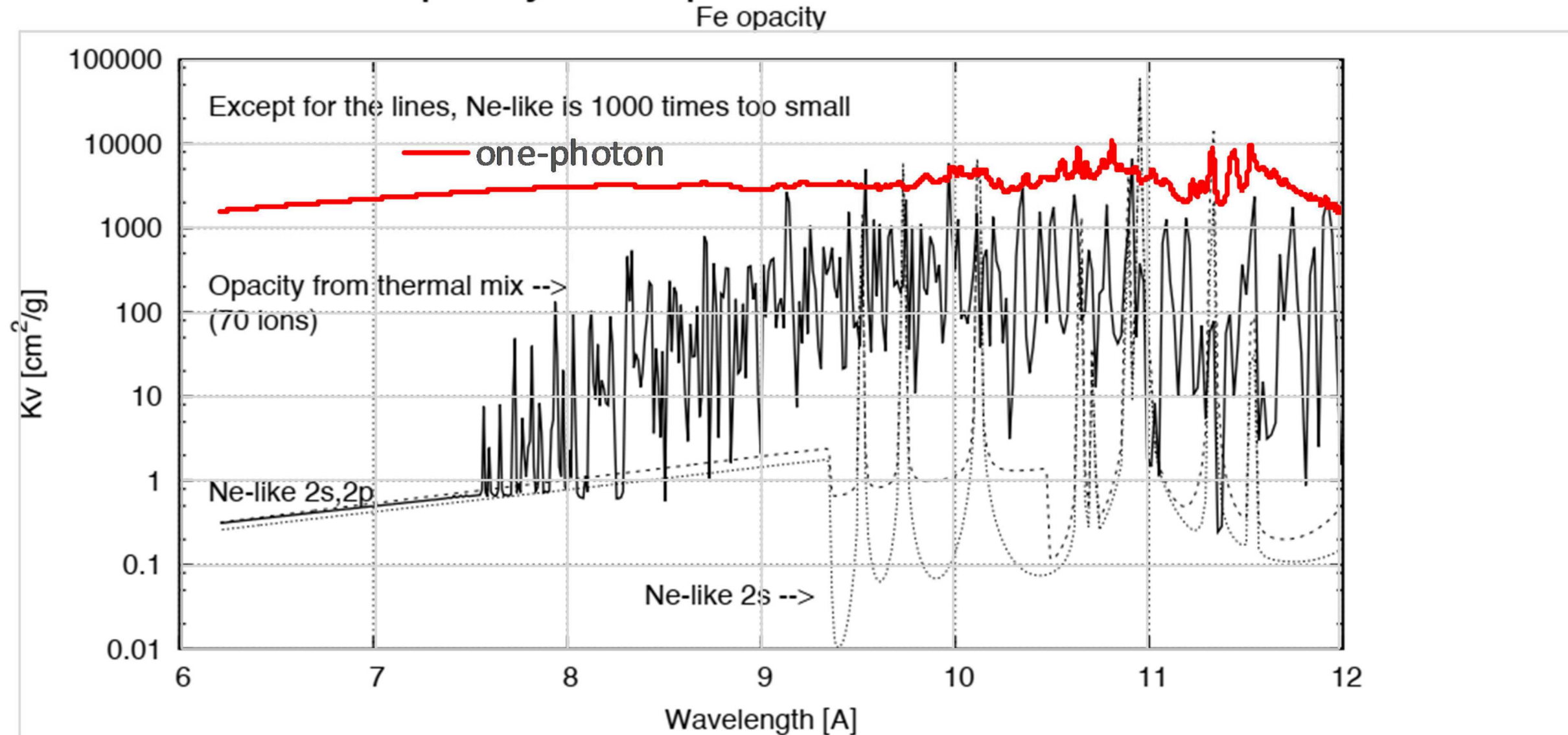
Fe opacity - two-photon and Raman effect



Calculations from
Richard More

Ne-like ground-state opacity is hopelessly small (on this range).
Certain excited states of Ne-like ion have larger cross-sections.

Fe opacity - two-photon and Raman effect

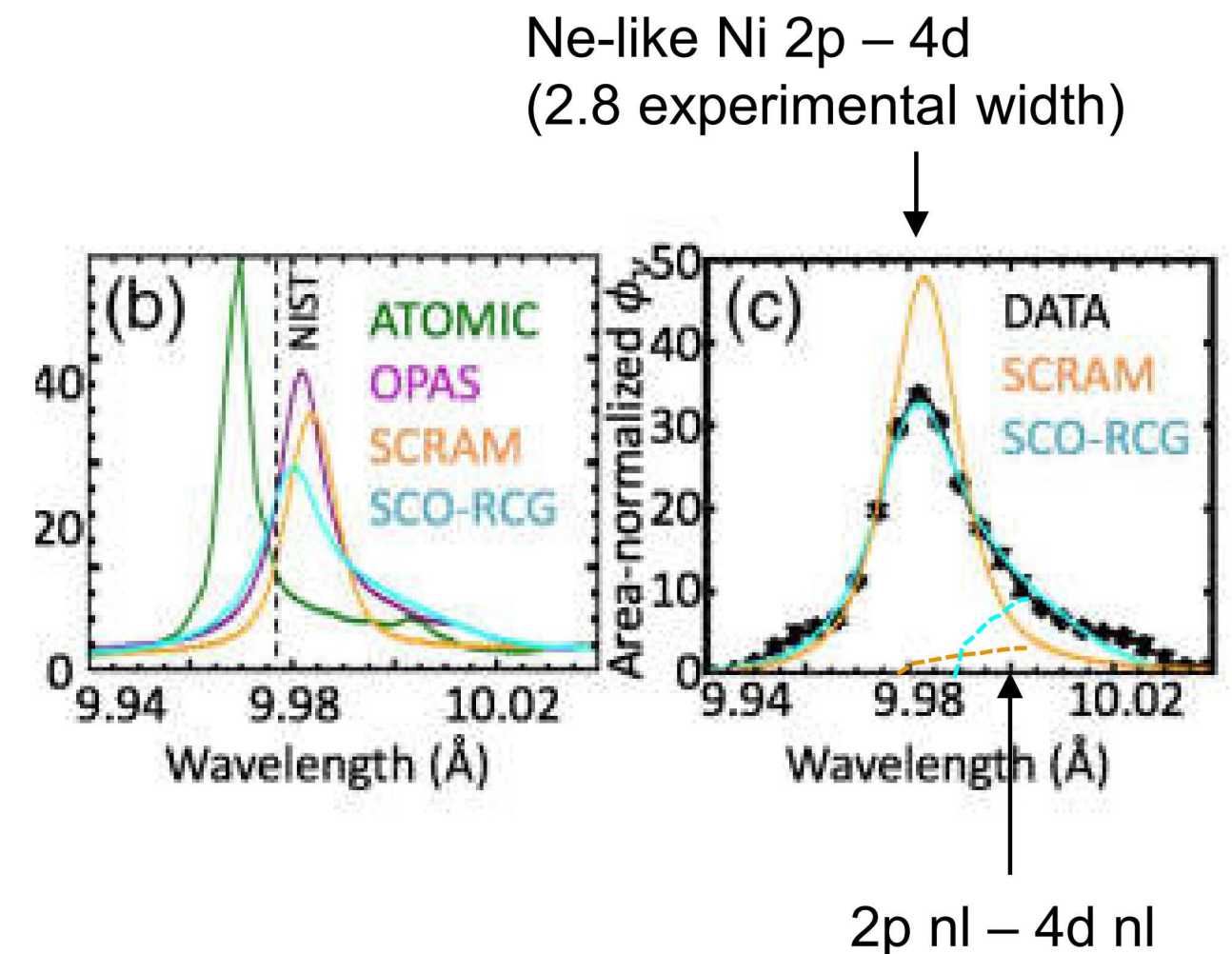


Ne-like ground-state opacity is hopelessly small (on this range).
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Calculations from
Richard More
& **SCRAM**

Line shapes matter for two-photon absorption, and completeness, detail, and consistency matter for line shapes

- Completeness & detail
 - Instrumental & thermal broadening (0.4, 0.1 eV)
 - Stark broadening from external fields (1-3 eV)
 - Natural broadening (0.7 eV)
 - Collisional broadening from collisional ex/dex (0.7 - 1 eV; must include $\Delta n=0$)
 - Additional collisional broadening from ion/rec? (~ 0.05 eV for $\Delta Z = 1$)
Gomez et al PRL 214, 055043 (2020)
 - **Complete set of spectator satellites**
Satellites seem best captured by SCO-RCG:
SCRAM's hybrid-structure satellites are weak/broad and too close to resonant 2p-4d line, while ATOMIC's nlj satellites appear too far from line
- **Are collisions, fields, and structure consistent?**



Nagayama et al, PRL 122, 135001 (2019)
J-C Pain and F Gilleron: SCO-RCG

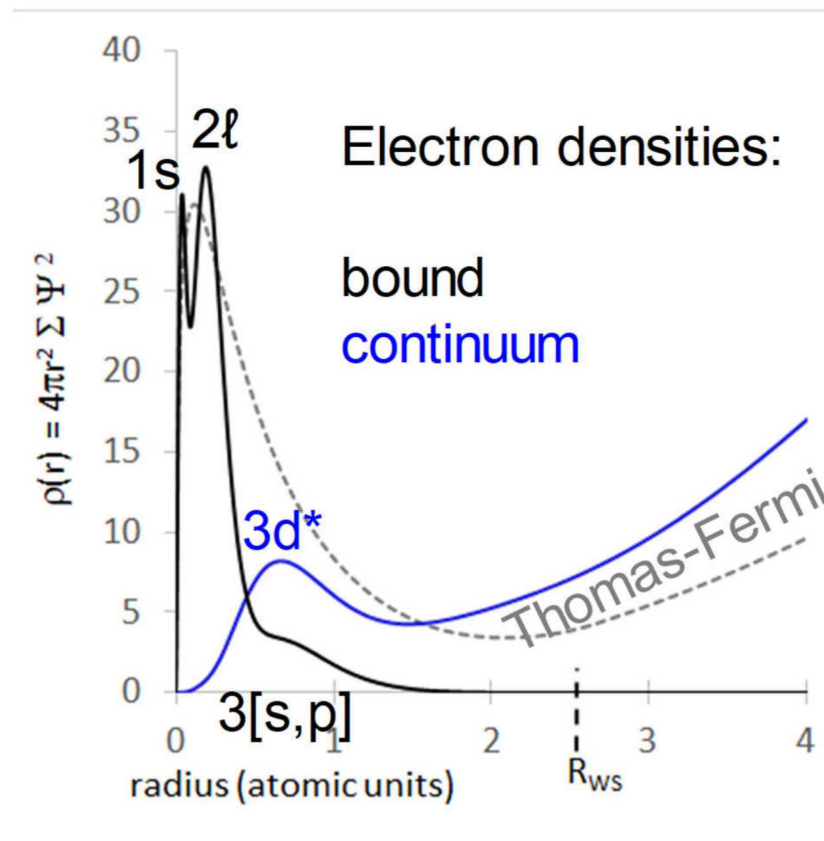
Consistency: what makes a consistent model, and why should we care?

- Consistency has multiple aspects
 - Density effects on electronic structures/wavefunctions & rates
→ average atom/DFT/self-consistent-field models
 - Electronic screening effects on ion structure
→ Neutral pseudo atom (e.g. Starrett & Saumon)
 - Lines shapes consistent with electronic & ionic structure
→ rates from electronic structure, Stark fields from ionic structure
 - Simulations often draw from separate models for EOS, transport, opacity
→ can we build a single model that generates everything?

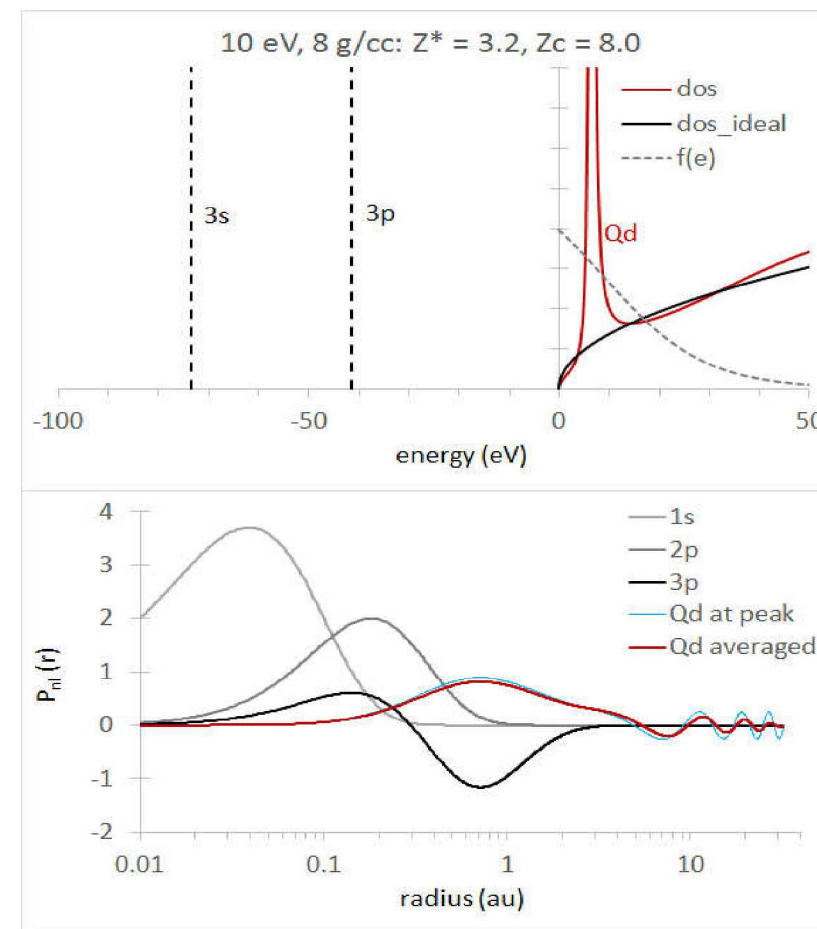
Rigorously enforcing consistency within models can help us constrain models even in regimes without high-precision measurements

Our approach: enforce consistency wherever possible

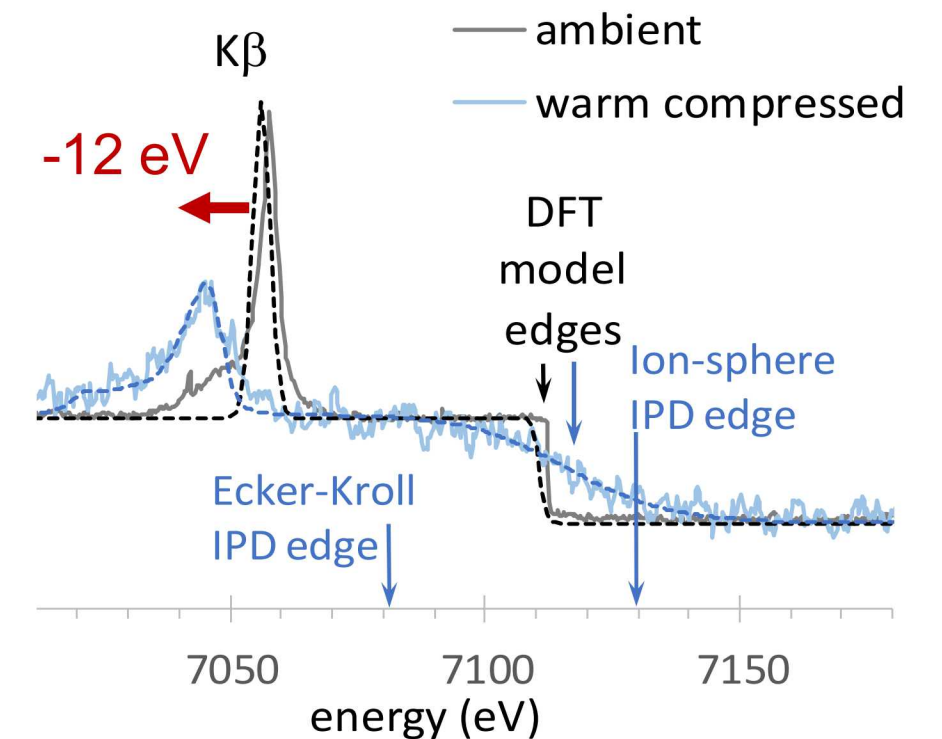
- In wavefunctions, atomic structure/EOS, and rates



Self-consistent field model lets electronic wavefunctions respond to changes in density and temperature



Continuum lowering is natively incorporated in structure & EOS (nothing ad-hoc)



Rates and transition energies respond to changes in the wavefunctions

Hansen et al HEDP 2017
Jiang et al PRE 2020

Our approach: enforce consistency wherever possible

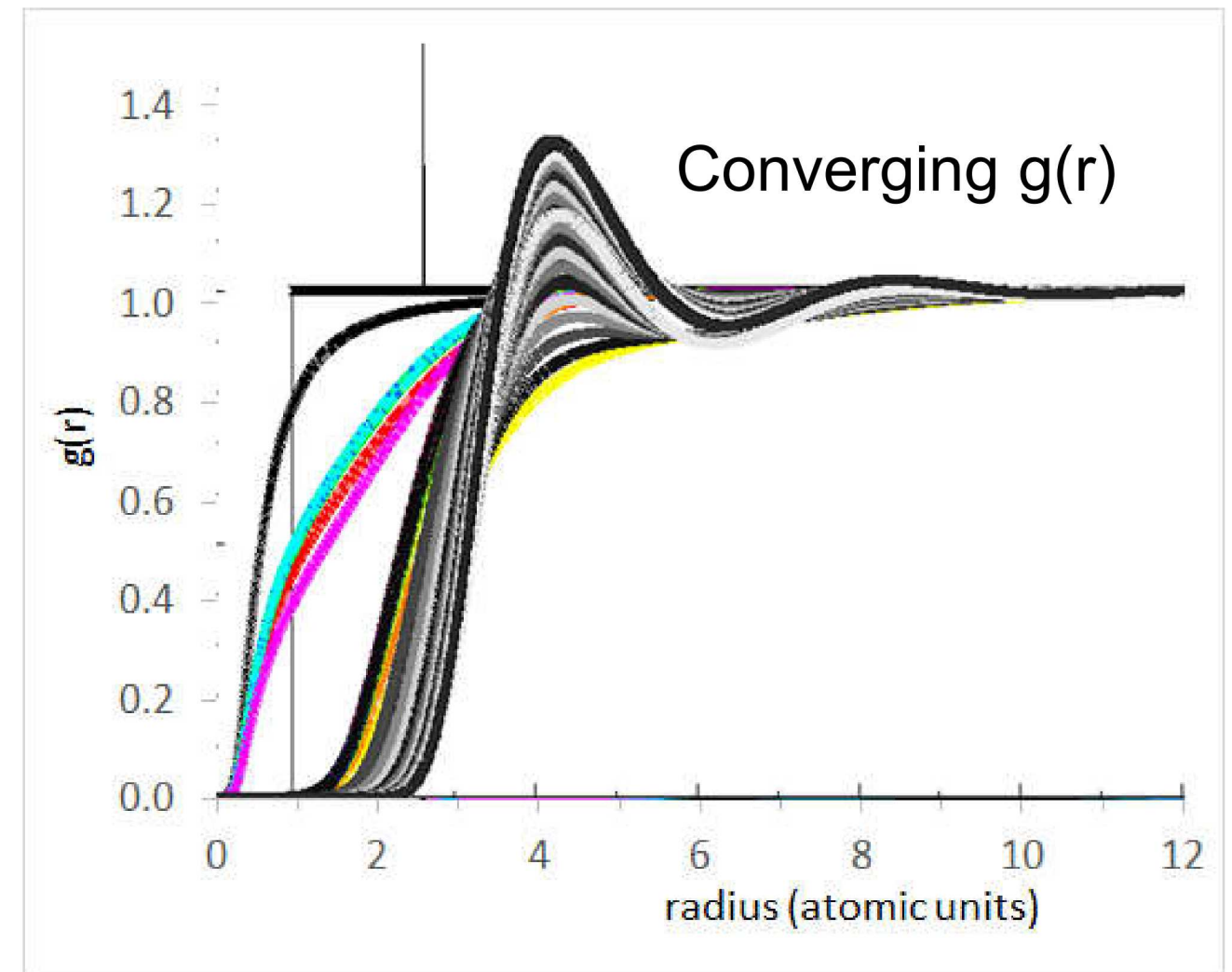
- Between electronic structure and ionic structure

The screening electron density determines the ion-ion interaction potential:

$$\beta V(k) = \frac{4\pi\beta}{k^2} Z^2 - n_e^{\text{scr}}(k) C_{\text{le}}(k)$$

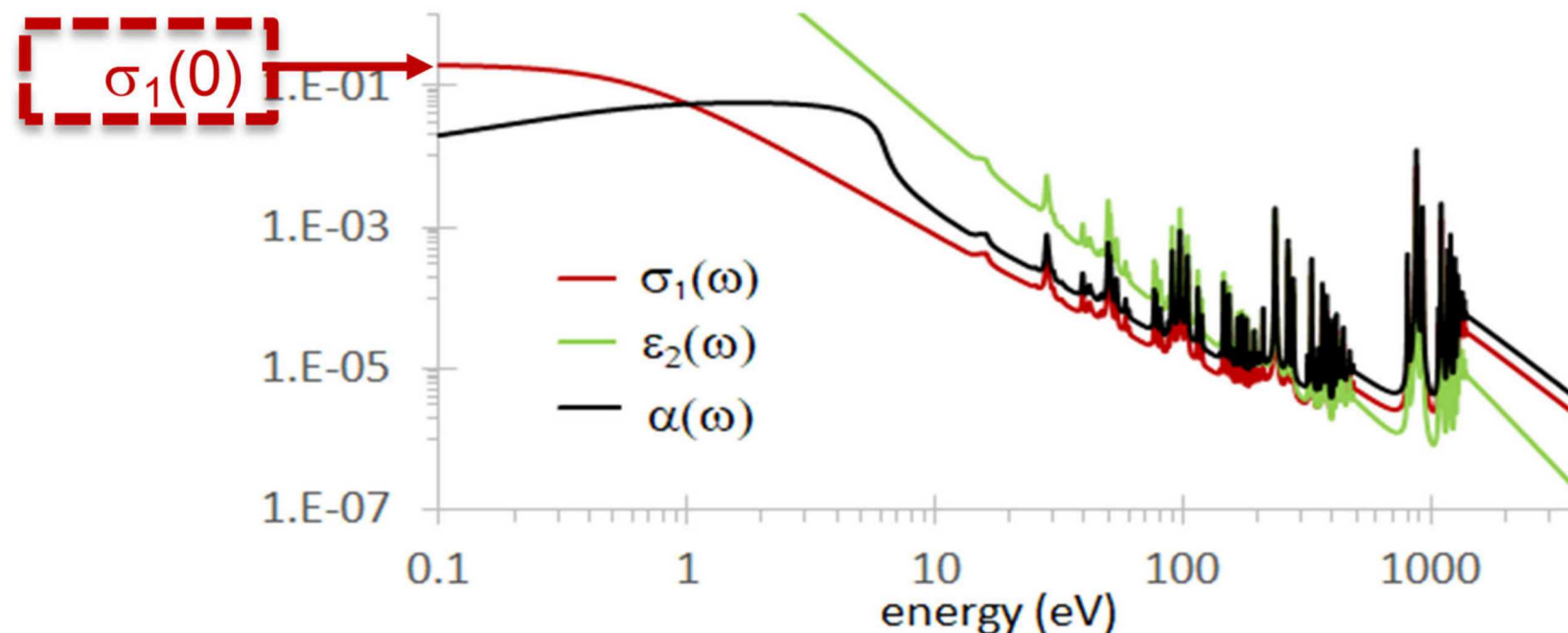
This potential constrains the ion distribution through the quantum Ornstein-Zernicke equations – or through classical MD simulations

Solid-density iron, 10 eV



Our approach: enforce consistency wherever possible

- Between electronic & ionic structure and transport quantities



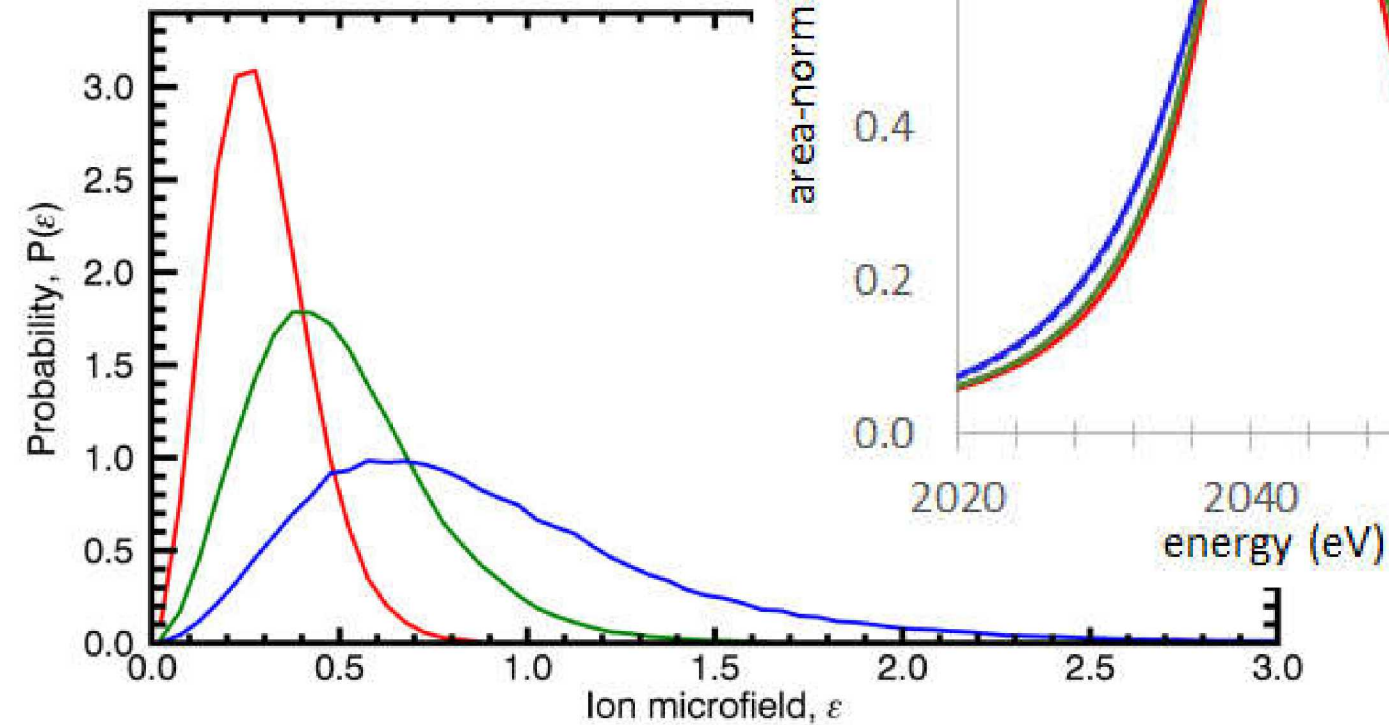
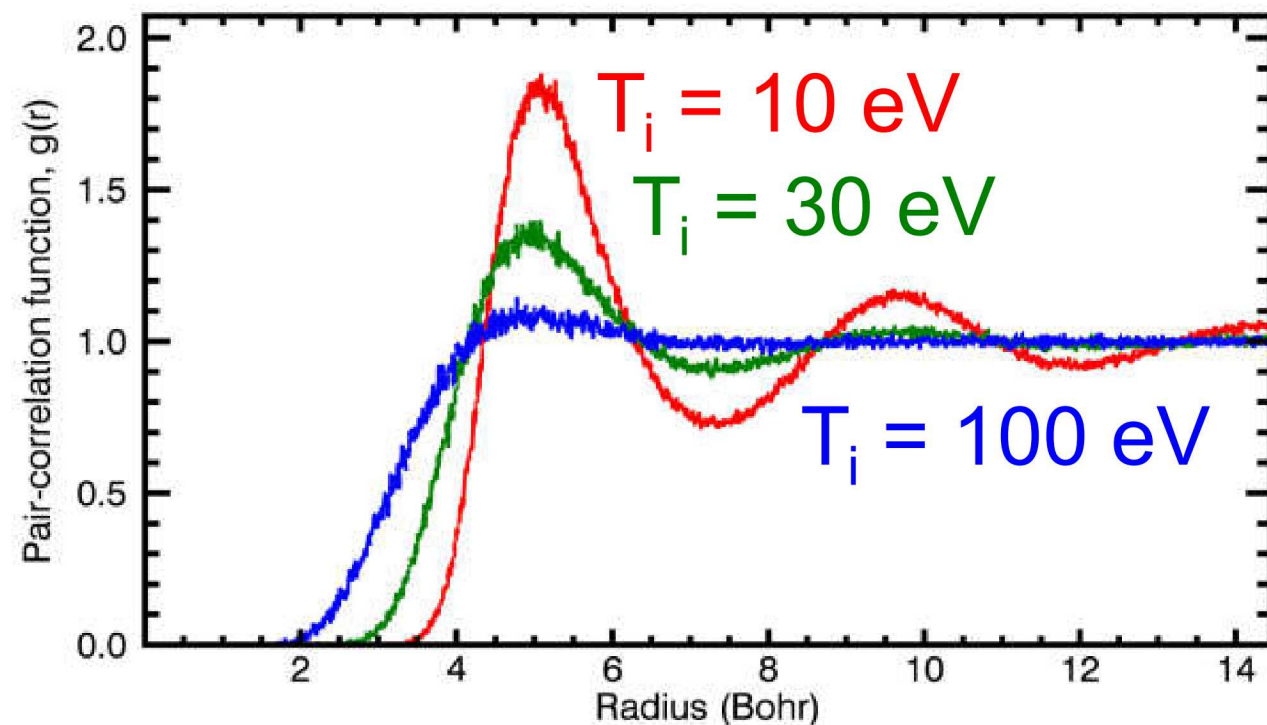
Response functions based on matrix elements of average-atom wavefunctions and ion $g(r)$ constrain:

- Electrical and thermal conductivities (cf. M. Desjarlais)
- X-ray Thomson scattering signatures (A. Baczewski)
- Stopping powers (T. Hentschel)
- Collision frequencies

Our approach: enforce consistency wherever possible

- Between electronic & ionic structure and line shapes

Solid-density Al, $T_e = 250$ eV

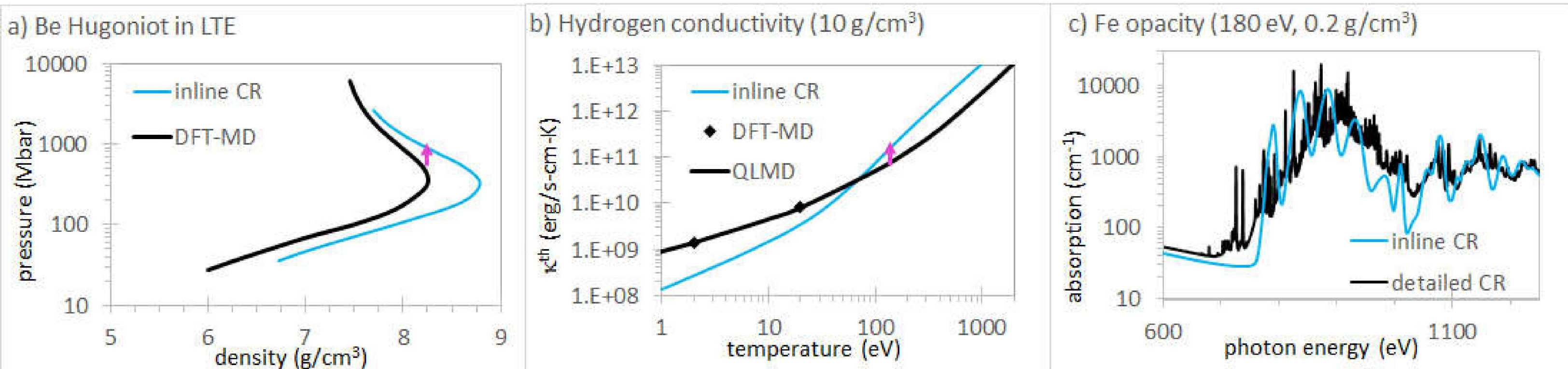


T. Gomez

T. Nagayama, (M. Murillo, L. Stanek)
cf. Starrett's PAMD

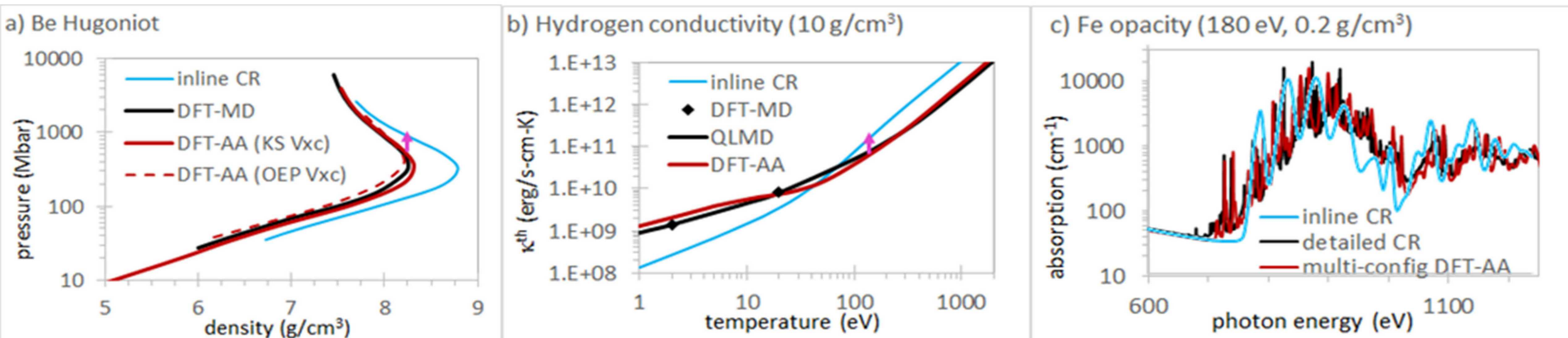
Why we care: the effects of inconsistency on simulations is not well understood

- Simulations draw different quantities (EOS/transport/opacity) from different tables and models
- Parameterized quantities (or inline non-LTE) can increase consistency, at the expense of discontinuities and/or reduced resolution/detail



Our goal: a spectroscopically detailed, rigorously complete, and internally consistent model

- Use AA wavefunctions to build multiconfiguration models
- Extensible to non-LTE + additional processes/quantities as needed (e.g. two-photon)
- Validated against DFT-MD, TD-DFT, and detailed CR models, where available
- Verified against high-precision measurements, where available
- Tractable enough for tabulation (cf. H. Scott)

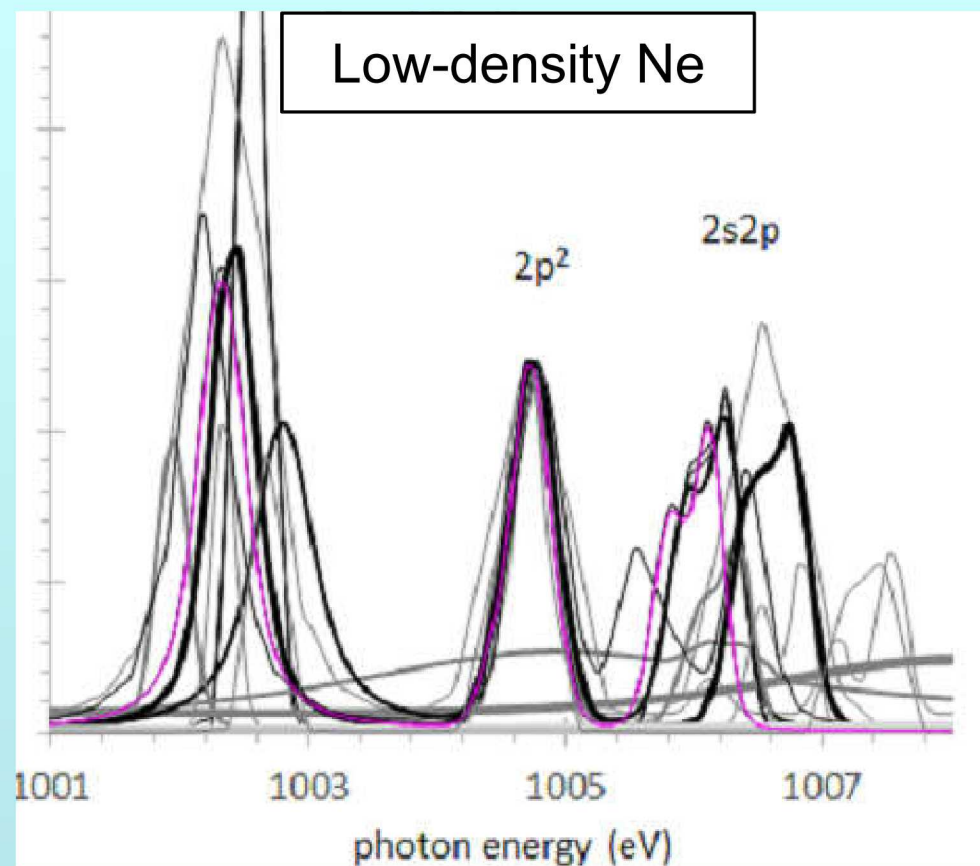


Thank you!

Questions?

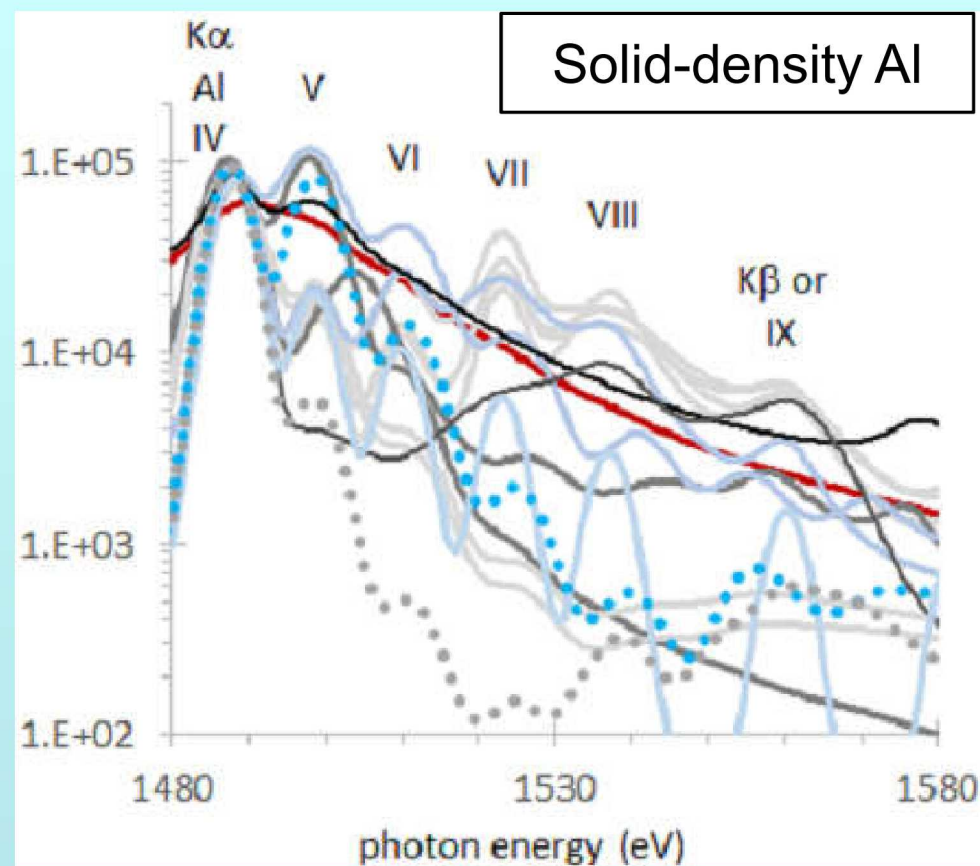
Consistency: it is hard to use one model for everything

Detailed models enable precision diagnostics for plasmas at modest densities



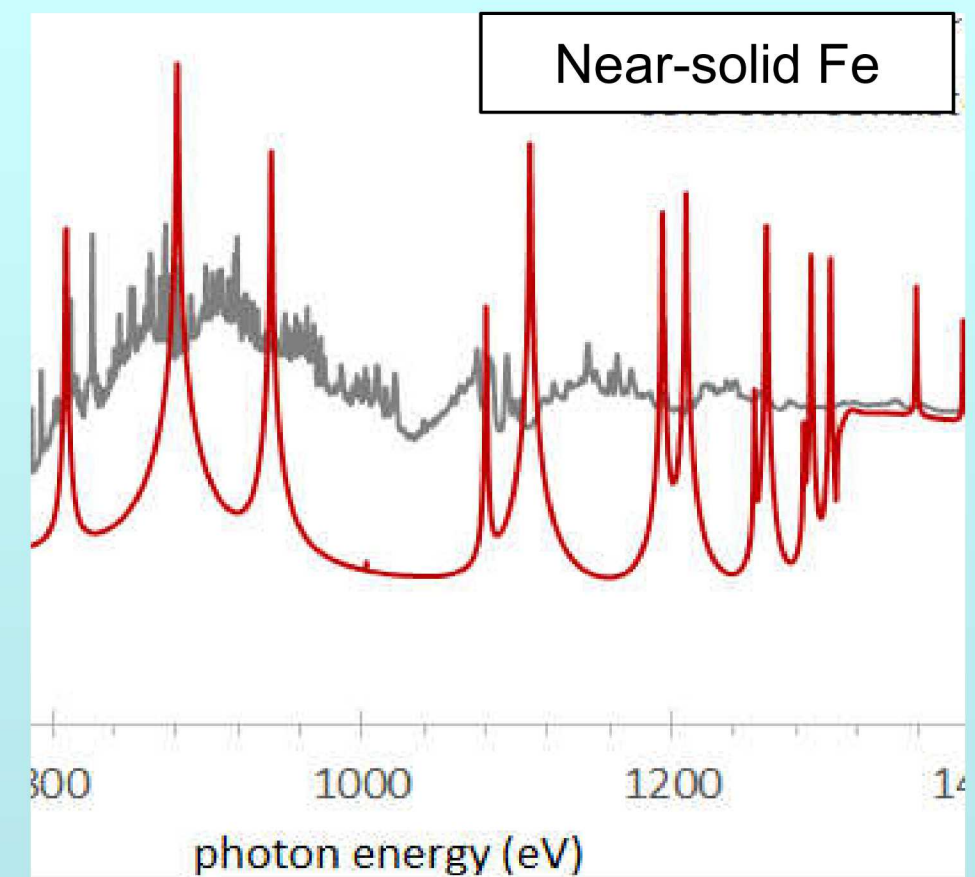
S. Hansen *et al*,
Review of the 10th NLTE code comparison workshop, HEDP (2019)

At high densities, *ad-hoc* corrections lead to incoherence



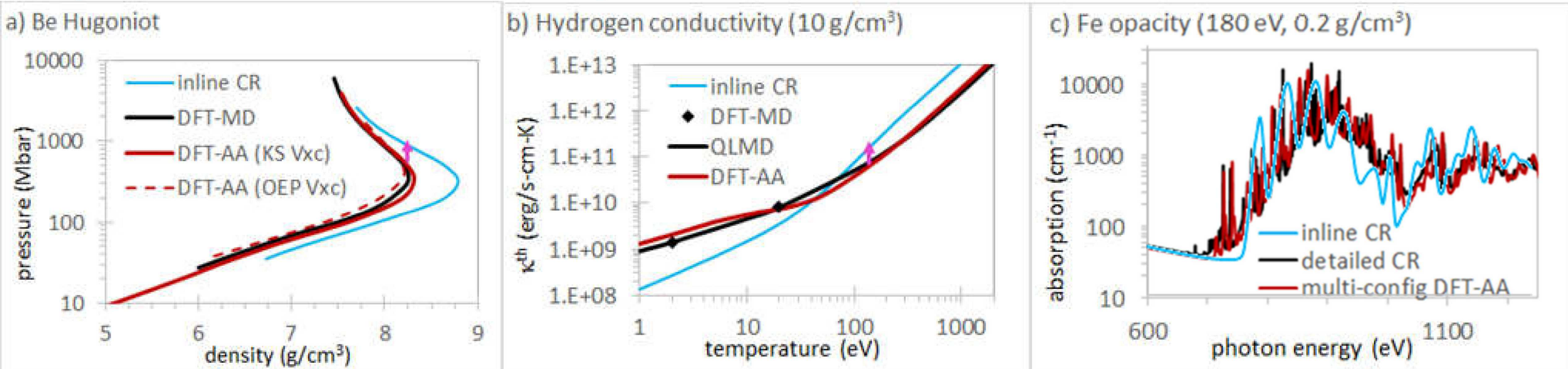
cf. O. Ciricosta *et al*,
PRL **109**, 065002 (2012)

Models that natively include density effects tend to lack detail



cf. P. Sterne *et al*, HEDP **19**, 1 (2007)

Extending existing DFT-AA model to non-LTE will offer unprecedented consistency in material data



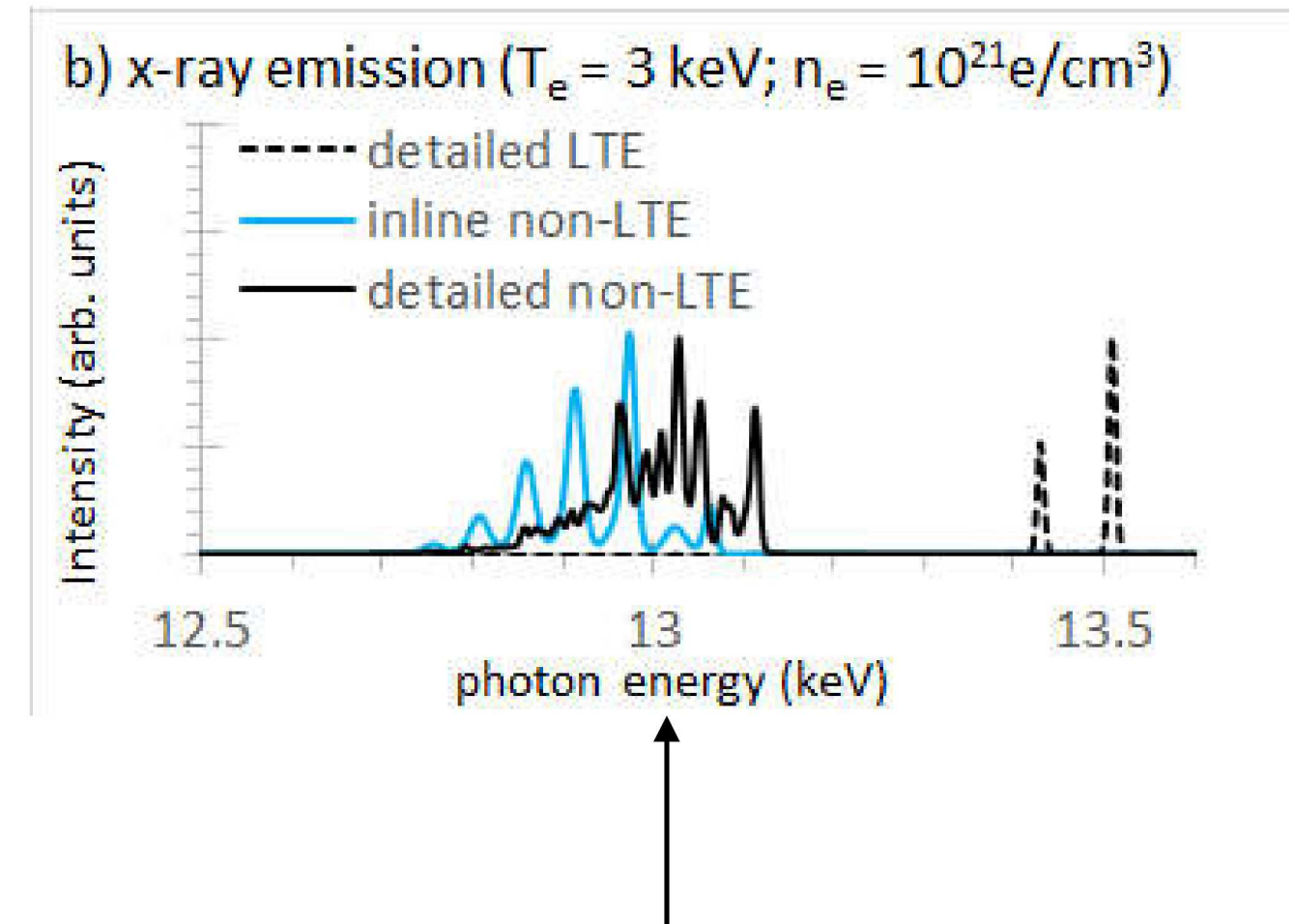
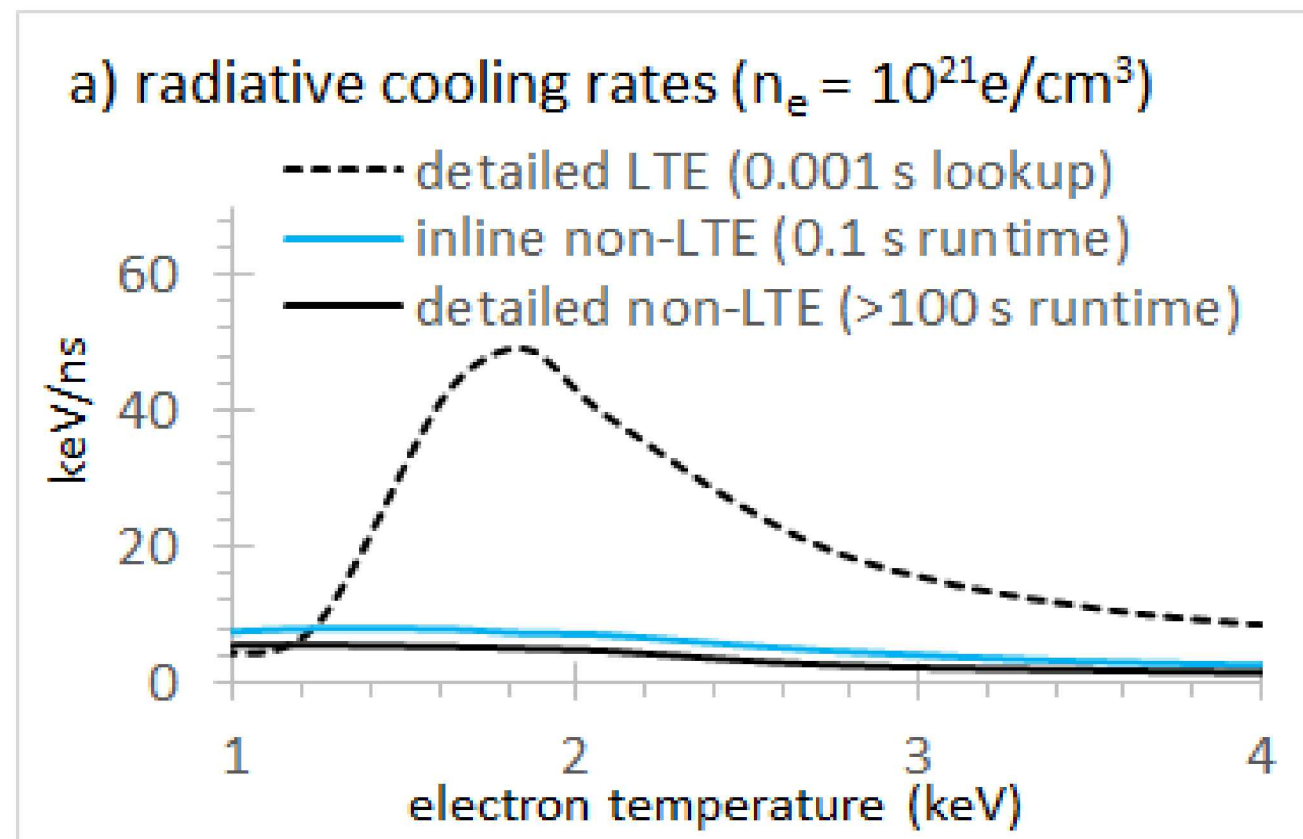
- Rigorous internal consistency/ DFT foundations eliminate “seams” & ill-quantified uncertainties (1)
- Detail could revolutionize design cycles, enabling routine generation quality synthetic diagnostics (3)
- UQ could be undertaken on *whole tables* rather than piecemeal quantities (outyears)

To be done:

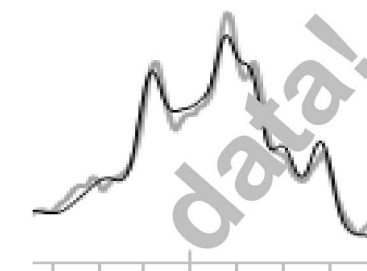
Add rates to multi-configuration model → CR + new self-consistent electron & ion
 Add configuration-interaction + implement line shapes in select levels for diagnostics
 Find and enforce mutual constraints among properties (e.g. line shapes, $\partial E/\partial x$, σ)
 Compare to existing tables / DFT-MD/ TD-DFT/ detailed CR models to benchmark

High-T and high-Z plasmas (RES, ICF mix/diagnostic) are particularly sensitive to non-LTE effects


Exemplar: Kr-doped MagLIF
LTE ($T_r = T_e$) v. non-LTE ($T_r = 0$)



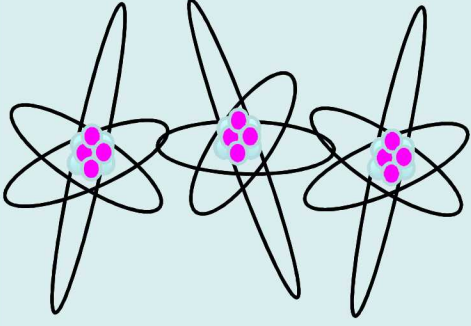
Compared to the DFT models
used for LTE tables, inline CR
models are *slow* and *bad*



... and inform stopping powers (essential for self-heating)



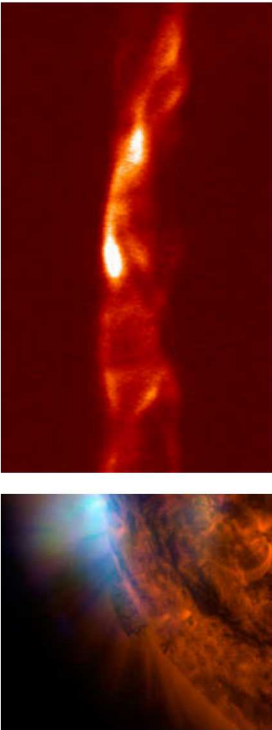
$$L(\rho, V) = \frac{i}{\pi \omega_0^2} \int_0^\infty \frac{dk}{k} \int_{-kV}^{kV} \omega d\omega \left(\frac{1}{\epsilon(k, \omega)} - 1 \right)$$
$$\left(\frac{dE}{dx} \right) = - \frac{4\pi}{m} \left(\frac{Ze^2}{V} \right)^2 \int_0^\infty \rho(r) L(\rho, V) 4\pi r^2 dr$$



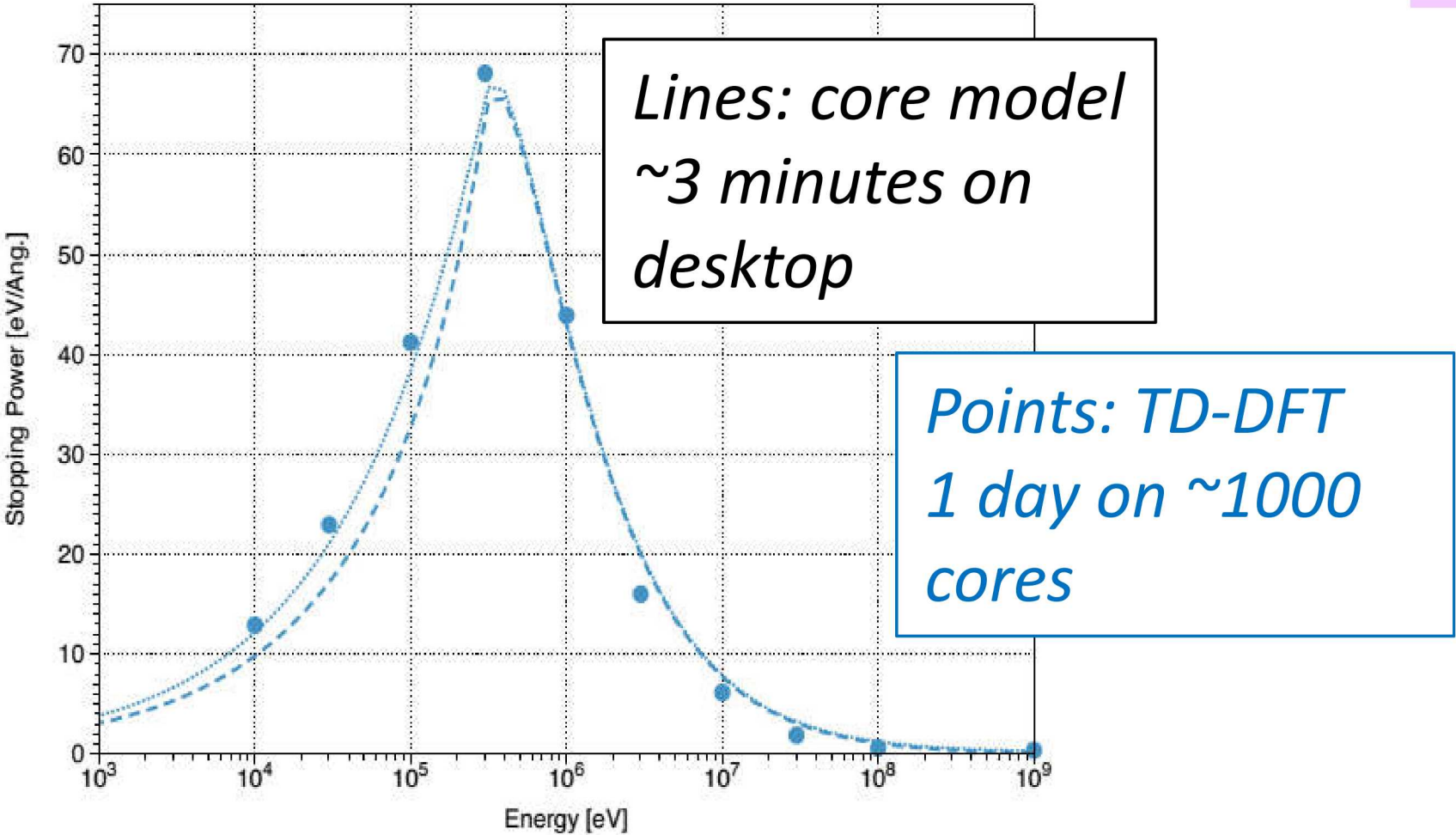
Core model:
quantum
average atom +
ion correlations

$$\sigma(\omega), \epsilon(\omega), \kappa^{\text{th}}, \partial E / \partial x$$

Astrophysics & ICF




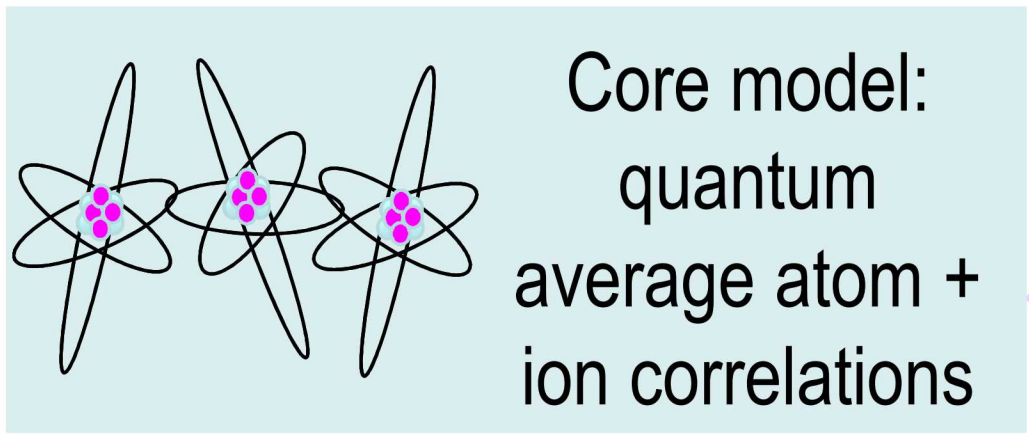
Protons
stopping in
10 g/cc
deuterium



TD-DFT stopping:
A. Cangi &
A. Baczewski

... and X-ray scattering (used to diagnose laboratory plasmas)

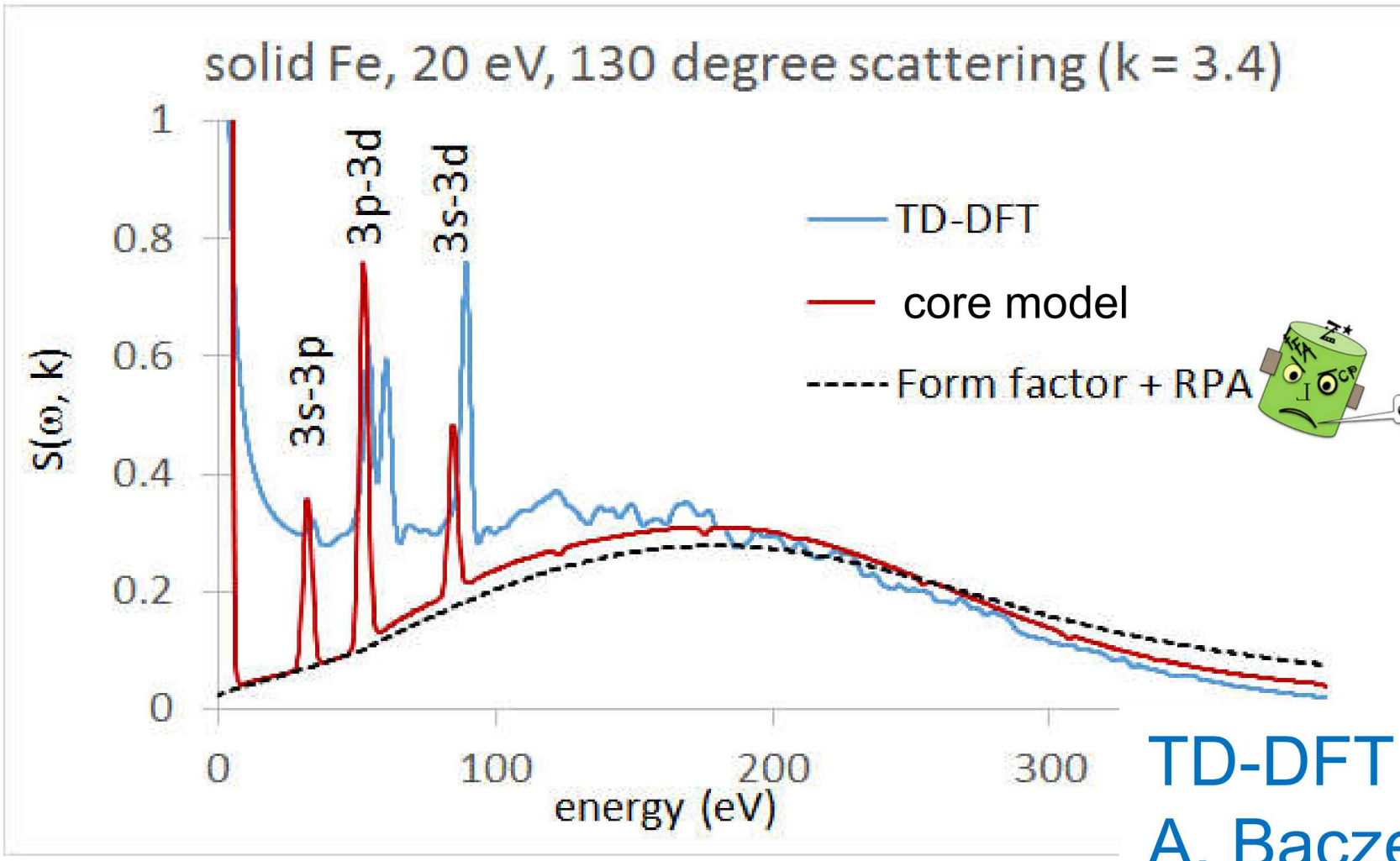
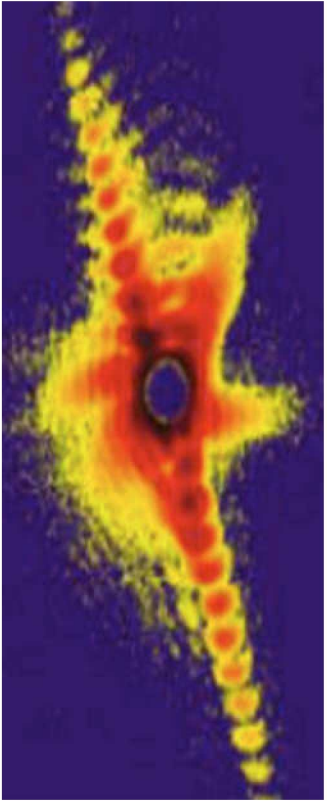
$$S(k, \omega) \propto \text{Im} \left[\frac{1}{\epsilon(k, \omega)} \right]$$




$$\sigma(\omega), \epsilon(\omega), \kappa^{\text{th}}, \partial E / \partial x$$

X-ray
scattering

XFEL



TD-DFT scattering:
A. Baczewski

Chihara decomposition:

$$S(k, \omega) = \underbrace{|f_I(k) + q(k)|^2 S_{ii}(k, \omega)}_{\text{elastic}} + \underbrace{\bar{Z} S_{ee}(k, \omega)}_{\text{free-free}} + \underbrace{S_{bf}(k, \omega)}_{\text{bound-free}} + \underbrace{S_{bb}(k, \omega)}_{\text{bound-bound}}$$