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Title: Reliability Prediction using FMEA, FTA, and Related Techniques

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Reliability Prediction using FMEA, FTA, and Related Techniques

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Reliability prediction steps

1. Define the boundary of the system under analysis—what counts as part of the system, what does not?
2. Sketch a preliminary reliability block diagram (RDB) to define "components" of the system (which could be events or functions)
3. Perform a failure modes and effects analysis (FMEA)
   – Iterate with RDB to insure all components are covered
   – Estimate component failure probabilities (point estimates or probability distributions)
4. Perform a fault tree analysis (FTA)
5. Quantify results from FTA and RBD
6. Design/initiate component reliability, aging and compatibility tests
   – And/or utilize data from previous testing
   – Update reliability estimates if necessary
7. Iterate if necessary (typically will be necessary)

Defining our terms

- ***Failure Mode***: One of the ways in which a component or subsystem can fail.
  - One of its weaknesses, deficiencies, or defects
- ***Failure effect***: For a given failure mode, what are the consequences to the system? How critical are they? Is repair or workaround possible?
- ***Failure cause***: Is it random? Caused by something wearing our? Caused by external stress (heat, mechanical shock, radiation, etc.)?

Exercise

What other information is useful regarding failure modes?

Pick a fairly simple component (could be anything you have knowledge of, from a weapon component to an automobile tire)

– List all the failure modes, with their effects

– What can cause each failure?

– Order your list by by criticality/severity

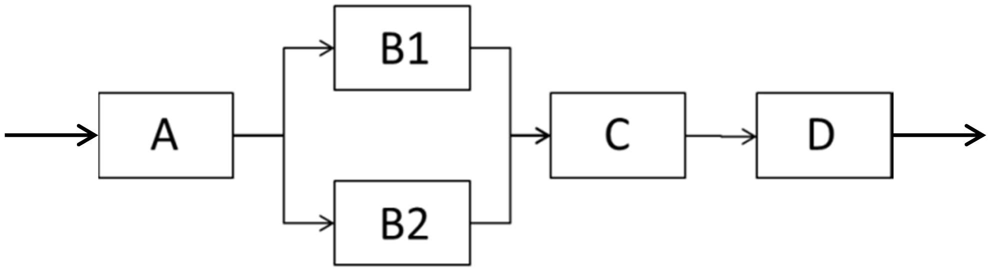
Analyzing failure modes

- We have many tools at our disposal—statistical, and just commonsense
- We may care about the frequency of the failure, the severity, or the chance of detection

To be discussed:

- Reliability block diagrams (RDB)
- Failure mode and effect analysis (FMEA)
  - Or Failure mode, effect and criticality analysis (FMECA)
- Fault tree analysis (FTA)
  - And success tree analysis
- Monte Carlo simulation and Bayesian analysis for quantifying uncertainty about system reliability
- All of these require eliciting information from subject matter experts, and we discuss how this is done

Reliability block diagram (RDB)



$$R_B = 1 - (1 - R_{B1})(1 - R_{B2})$$

$$R_{System} = R_A \times R_B \times R_C \times R_D$$

- Structural decomposition of the system
  - May be performed at varying levels of granularity
  - Can be done hierarchically– decompose single input/single output block into subblocks
  - May include interfaces (e.g., cables) as components
- Alternative to, or in addition to, fault tree
- Component reliabilities can be point estimates or distributions
  - Often captured using FMEA (next slide)
- RDB analysis may miss interactions and “common cause” failures

Failure modes and effects analysis (FMEA)

Component	Failure Mode	Cause	Effect/Severity	Probability

- Thorough FMEA helps insure consideration of all failure modes
- May include failures caused by defects introduced in production or assembly
- May include common-cause failure modes (e.g., common power bus)
- Failure mode probabilities based on component tests, industry databases, historical experience, elicitation of expert knowledge, etc.
- Could also capture sources of data to reduce uncertainty, mitigations for failure modes, etc.
- Elicitation of failure modes and probabilities from subject matter experts is labor-intensive, but critical

Elicitation of expert judgment

- Elicitation: A structured process for gathering quantitative information and uncertainty estimates on a given topic from informed experts, in a form useful for analysis or decision-making
- Used to supplement "hard" quantitative data with subjective information from subject-matter experts
  - Or when no quantitative data exist

"I know nothing about the subject, but I'm happy to give you my expert opinion."

- Examples of information elicited:
  - Probability of an event, odds on an event
  - Rank ordering of probabilities for different events
  - Uncertainty (error bar, probability distribution)
  - Ratio of odds or probabilities for two events
  - Relative or absolute cost or benefit of an event

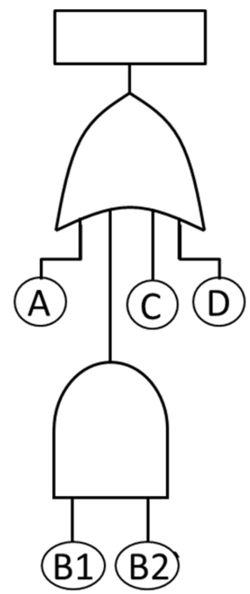
The elicitation process

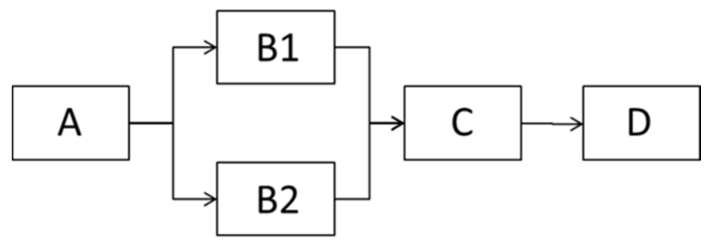
1. What information is needed? What specific questions need to be answered? In what form (point estimate, distribution, range, . . .)?
2. What expertise is needed? Which experts can answer these questions? Can we elicit quantitative information from them?
3. Do the experts have biases? Can we adjust for biases?
4. What process should be used (questionnaire, individual interviews, Delphi, interactive meeting, . . .)?
5. Do we need a practice run? What are the logistics of the elicitation?
6. Can we aggregate data from multiple experts? What if they disagree?
7. How do we quantify uncertainty in the expert judgments?

See Meyer and Booker (1991) for process details.

Fault tree/success tree analysis

- Functional (event-based) decomposition of the system
  - Events may be failures (fault tree) or successes (success tree)
  - May be performed at varying levels of granularity
  - Can be done hierarchically– decompose events into component events
- If the "events" are component failures, then the tree is isomorphic to an equivalent RDB
- Use fault tree or success tree, whichever makes best sense (see next slide)
- The "tree" can be a directed acyclic graph to capture common cause failures





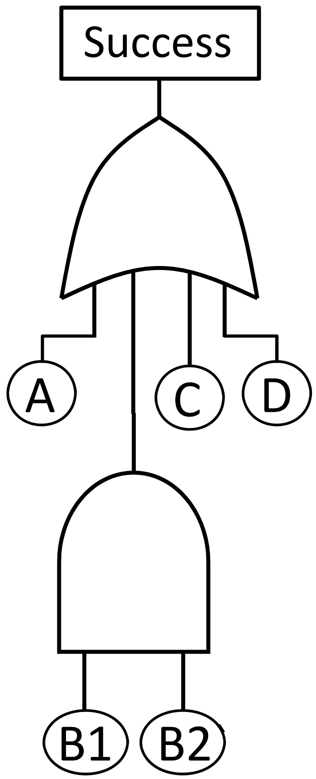
System success if
A∧(B1∨B2)∧C∧D

$$R_B = 1 - (1 - R_{B1})(1 - R_{B2})$$

$$R_{System} = R_A \times R_B \times R_C \times R_D$$

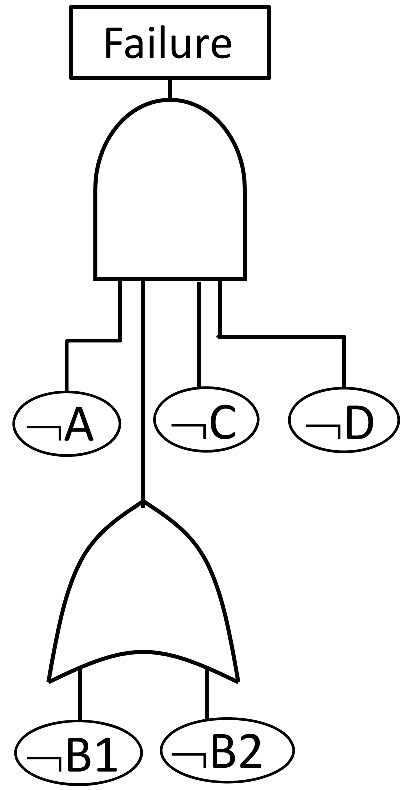
Fault tree and equivalent success tree

Success if $A \wedge (B1 \vee B2) \wedge C \wedge D$



$$R_B = 1 - (1 - R_{B1})(1 - R_{B2})$$
$$R_{System} = R_A \times R_B \times R_C \times R_D$$

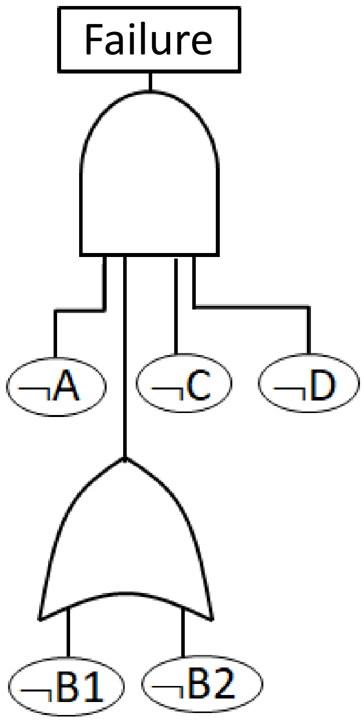
Failure if $\neg A \vee (\neg B1 \wedge \neg B2) \vee \neg C \vee \neg D$

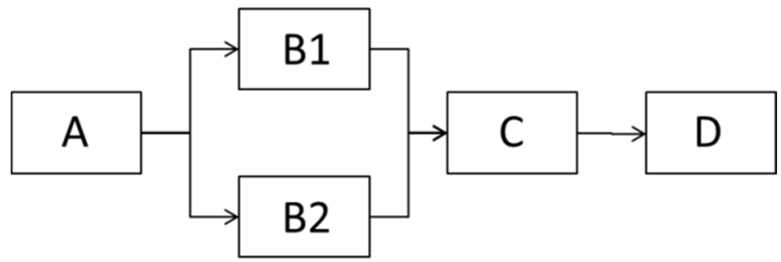


$$P_B = P_{B1} \times P_{B2} = 1 - R_B$$
$$P_{System} = 1 - R_{system} =$$
$$1 - (1 - P_A)(1 - P_B)(1 - P_C)(1 - P_D)$$

Fault tree and equivalent RBD

- Assume here that events in the fault tree are failures of components in the reliability block diagram
  - As on the previous slide, we could instead use a success tree





System failure if

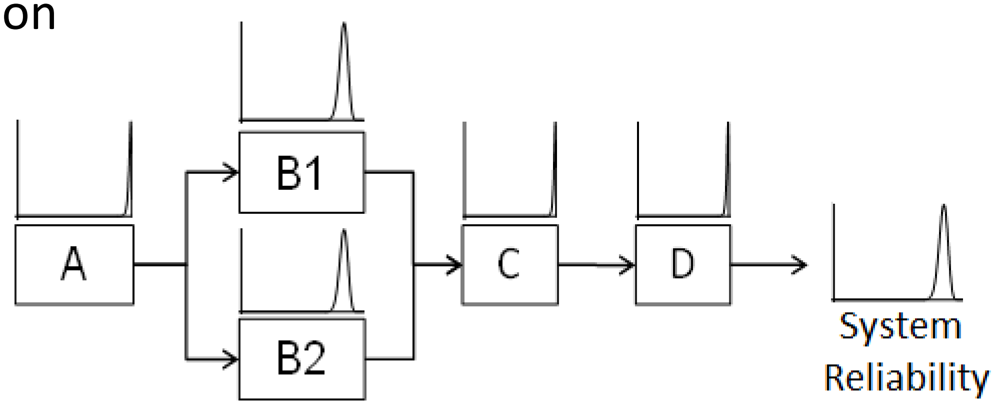
¬A∨(¬B1∧¬B2)∨¬C∨¬D

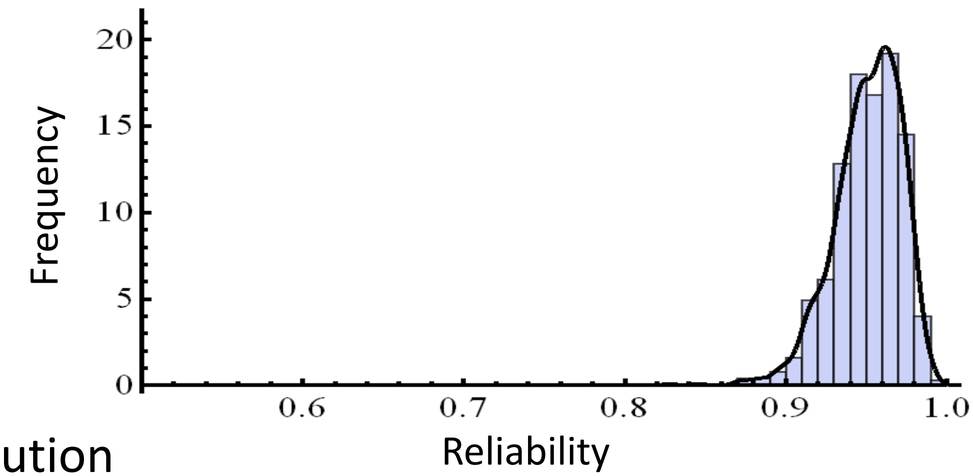
$$P_B = P_{B1} \times P_{B2} = 1 - R_B$$

$$P_{System} = 1 - R_{system} = 1 - (1 - P_A)(1 - P_B)(1 - P_C)(1 - P_D)$$

Monte Carlo estimation (with uncertainty) of R_{System} for the RDB

1. Assign probability distribution of reliability for each block
   – Simplest assumption is that all component reliabilities are independent
2. Draw random sample from each component, calculate R_{System}
   – If component reliabilities are dependent, sample from joint distribution
3. Repeat (2) n times (e.g., n = 10,000), estimate distribution of R_{System} from empirical quantiles

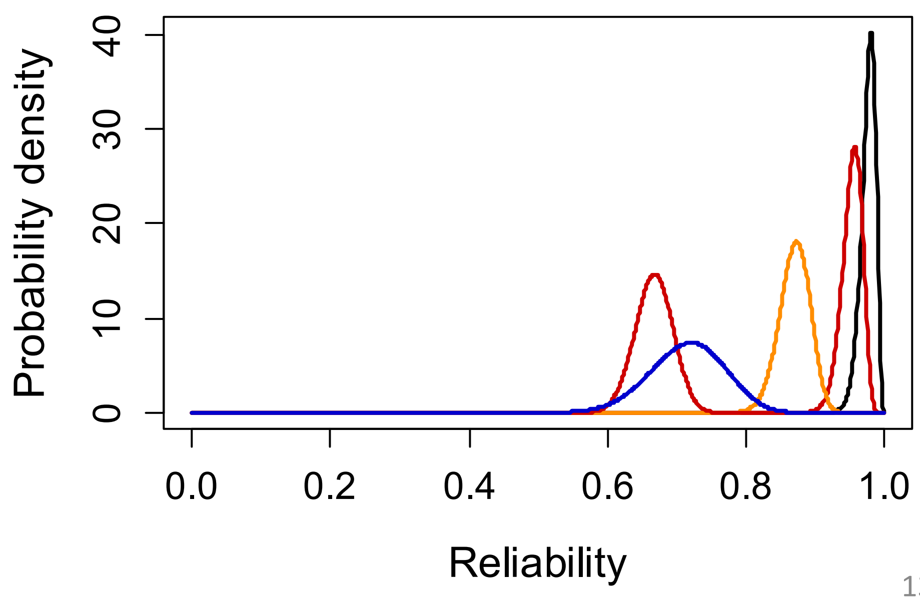




Equivalent analysis can be done with fault or success tree

Elicitation and use of probabilities

- If only point reliability or failure probability estimates are used, deriving a system reliability estimate by propagation through a fault/success tree or reliability block diagram is straightforward
- To estimate *uncertainty* in a complex reliability model (RDB or FT) we need to estimate a probability distribution over reliability or failure probability at each node
  - Must be supported on [0, 1]
  - Characterized in a way that facilitates setting distribution parameters based on expert judgment
  - Facilitates combining expert judgment with test results using Bayesian methods
- Alternative: elicit upper/lower bounds, use interval analysis



Combining prior knowledge and test data

- In the absence of sufficient test data, distribution parameters may be estimated *a priori* based on expert judgment or physical models
- These estimates can be used to develop Bayesian prior distributions, which are updated with available data:

$$\pi(p \mid \mathcal{D}) = \frac{L(p \mid \mathcal{D})\pi(p)}{\int L(p \mid \mathcal{D})\pi(p)dp} \qquad \text{(Bayes' theorem)}$$

  Assume p (failure probability) is the parameter of interest; $\pi(p)$ is the prior distribution, $L(p \mid \mathcal{D})$ is the likelihood function of the data, and the denominator normalizes the expression to a proper probability density function (pdf).

- Note the parameter is treated as a random variable; think of this as epistemic uncertainty.
- $\pi(p \mid \mathcal{D})$ is the posterior (pdf) for p, used to calculate the posterior predictive density for future reliability.

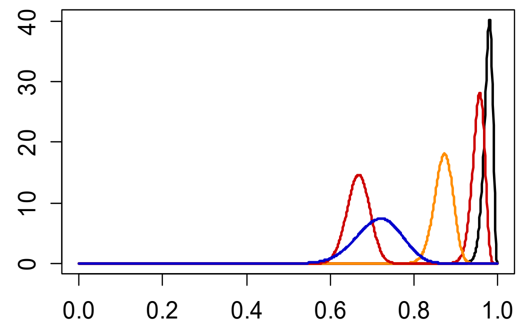
"Probability of reliability" – Binomial/beta distributions

- Given a constant probability p of failure on one test, the probability of k failures in n tests is (binomial distribution)

$$f(k \mid n, p) = \binom{n}{k} p^k (1-p)^{n-k} = \frac{n!}{k!(n-k)!} p^k (1-p)^{n-k}$$

- Commonly used prior probability distribution for p is the beta:

$$f(p \mid \alpha, \beta) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} p^{\alpha-1} (1-p)^{\beta-1}$$



- Conjugate prior for binomial distribution ("conjugate" meaning the posterior has the same form as the prior)
  - Assume prior belief is that α failures would be observed in $\alpha + \beta$ tests
  - In current data, k failures are observed in n tests
  - pdf of posterior distribution is Beta($\alpha + k$, $\beta + n - k$)

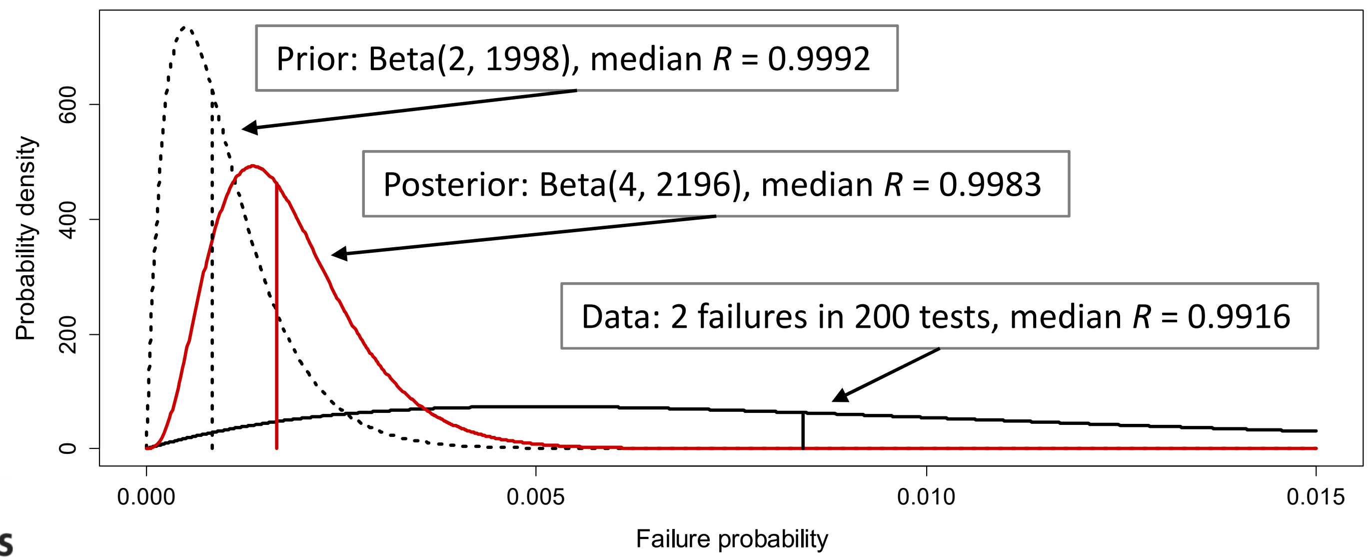
$$f(p \mid \alpha, \beta) = \frac{\Gamma(\alpha+k+\beta+n-k)}{\Gamma(\alpha+k)\Gamma(\beta+n-k)} p^{\alpha+k-1} (1-p)^{\beta+n-k-1}$$

Bayesian analysis of binomial failure data

- Elicit beta prior based on expert judgment or historical experience
  - Assume prior belief is that α failures would be observed in $\alpha + \beta$ tests
  - In current data, k failures are observed in n tests
  - pdf of posterior distribution is Beta($\alpha + k$, $\beta + n - k$)

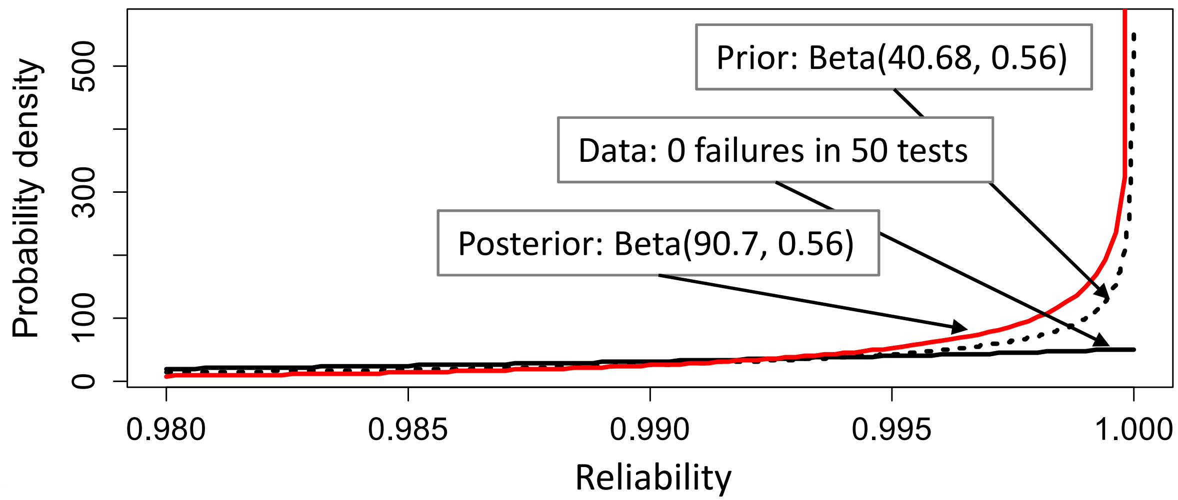
$$f(p \mid \alpha, \beta) = \frac{\Gamma(\alpha + k + \beta + n - k)}{\Gamma(\alpha + k)\Gamma(\beta + n - k)} p^{\alpha+k-1}(1-p)^{\beta+n-k-1}$$

- Example (in this case, estimating posterior distribution of p; could also estimate distribution of $R = 1 - p$)



Bayesian analysis of binomial failure data

- Alternative elicitation of a Beta prior distribution: prior belief in percentiles of the reliability distribution uniquely determines Beta parameters*
  - "With 90% confidence, I think the reliability should be between 0.95 and 0.9999" (note here we are counting successes, not failures)
  - I.e., assuming symmetric confidence interval, 5th percentile is 0.95, 95th percentile is 0.9999
  - So prior is Beta(40.68, 0.56)
  - If the observed test data is 0 failures in 50 tests, the posterior is Beta(90.68, 0.56) – median *R* = 0.9969 (add 50 to the number of successes)



*Solving for the Beta parameters requires numerical optimization

Summary

- We presented a reliability analysis framework
- Point estimates of reliability using reliability block diagrams, fault trees, success trees
- Estimates with uncertainty using expert elicitation, Monte Carlo simulation, Bayesian analysis
- Expert elicitation of failure modes and probabilities is labor-intensive, but critical
- Bayesian analysis updates information from expert elicitation with data from reliability and aging tests (aging/compatibility data are needed to estimate lower-bound reliabilities at end of life)
- Estimation by more than one method helps insure consistency and accuracy

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