Unknown Unknowns: How to talk about geometric uncertainty in inverse-Abel computations of radiographic densities

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The Challenge

Can we:

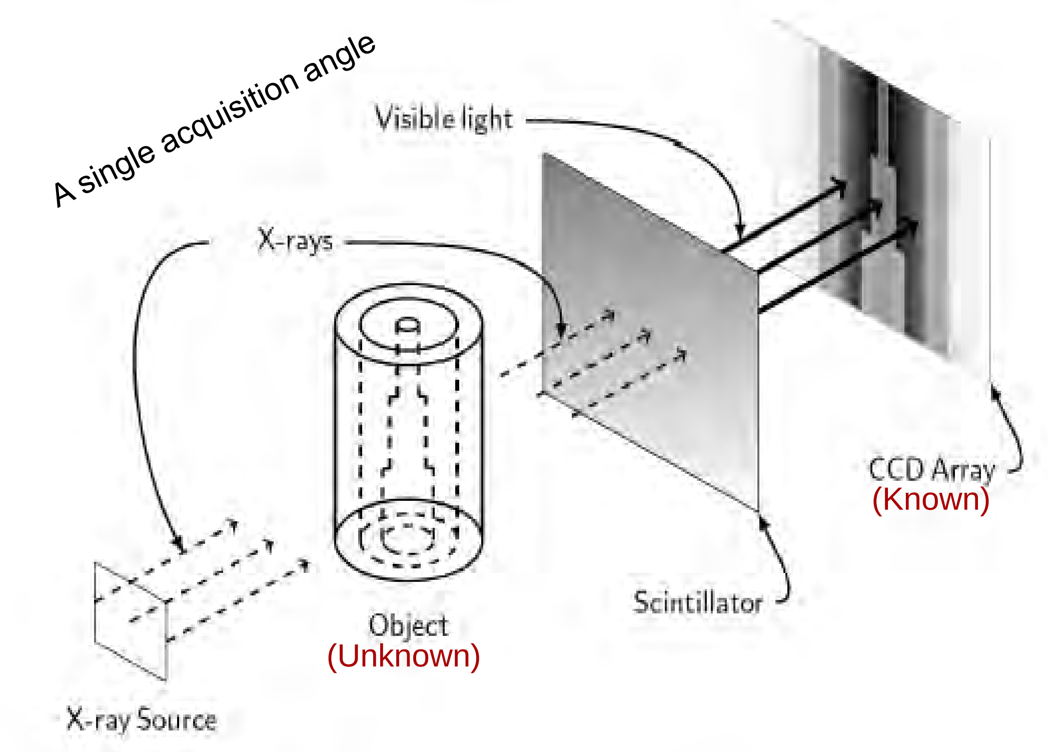
1. Accurately quantify perturbations to rotational symmetry from a single forward projection?

2. Quantify the uncertainty in single-view reconstructions, as a function of that eccentricity?

3. Can we develop a data-driven technique to better-reconstruct mission-relevant targets, without *assuming* rotational symmetry of the target?

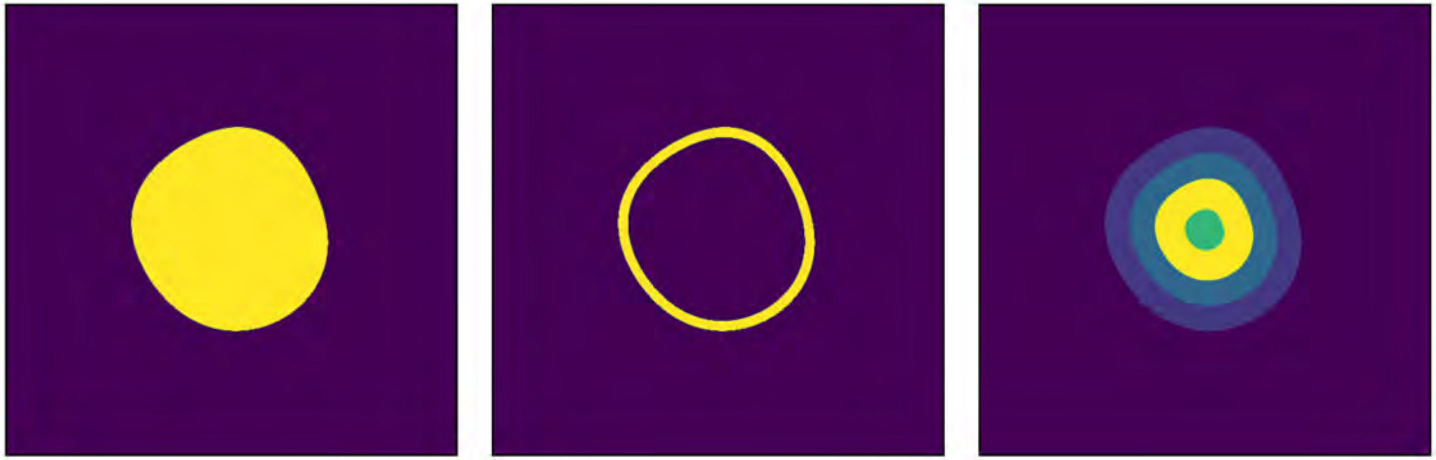
4. Can we determine the minimum requirements sufficient (from an imaging system) to accurately reconstruct eccentric mission-relevant targets using the latest tomographic reconstruction techniques?

5. Investigate interesting problems we happen upon, along the way?



Scoring Target Eccentricity, In Target Space

- “Can we accurately quantify rotational symmetry / asymmetry from a single forward projection?”
  - First, we need a generalized **scale, translation,** and **rotation-invariant** eccentricity score in the *target space.*
  - Further, we limit the scope of study to axial deviations that are continuous (as a function of angle, centered at the object’s center-mass.)
    - Eliminates: ”Is the deviation noise?”, “Is it an errant object?”



Scoring Target Eccentricity, In Target Space

- "Can we accurately quantify rotational symmetry / asymmetry from a single forward projection?"

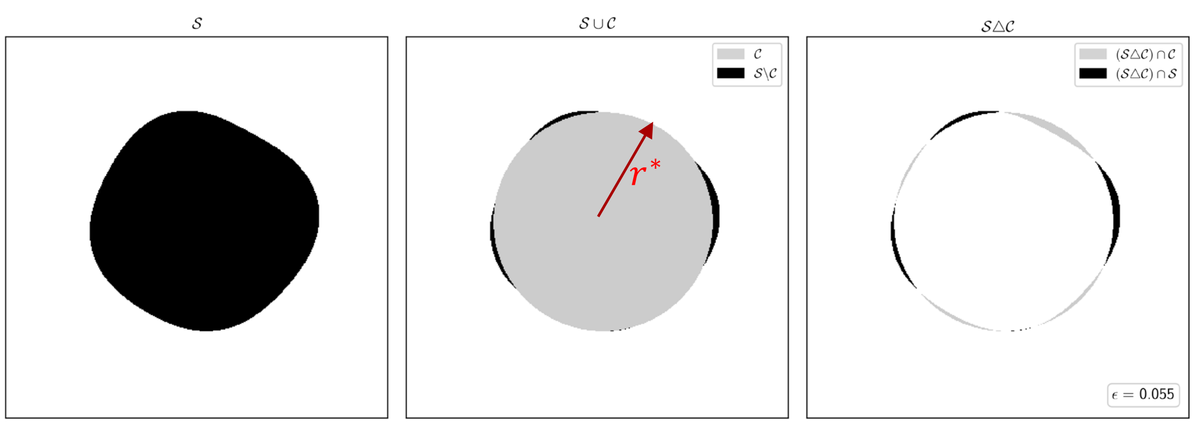
The *Symmetric Difference* between two sets

$$A \,\Delta\, B = A \cup B \;-\; A \cap B$$

Find $r^* > 0$ such that:

$$S\Delta C(r^*) \cap C(r^*) = S\Delta C(r^*) \cap S$$

$$\epsilon(S) := \frac{m(S\Delta C(r^*))}{m(C(r^*))}$$



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SDRD SITE-DIRECTED RESEARCH & DEVELOPMENT

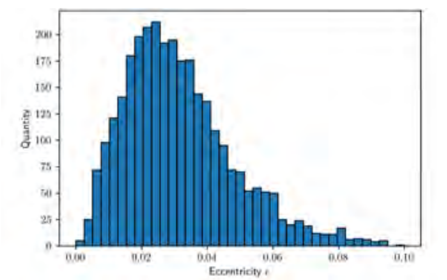
Technical Approach

- "Can we quantify the uncertainty in Abel reconstructions, as a function of that eccentricity?
  - Christian Bombara (UNR / NNSS Intern) completed an uncertainty quantification study on a regularized inverse-Abel method (N=1), varying **only** the degree of eccentricity present in a variety of target objects.

Regularized Abel Reconstruction Problem (solved via primal-dual method):

$$u^* = min_{\{u \in BV(\Omega)\}} ||u||_{TV(\Omega)} + \frac{\lambda}{2}||\mathcal{A}u - d||^2_{L^2(\Omega)}$$

Abel Operator $\mathcal{A}$: Onion-layer method.



Shape *Profile* Dataset:

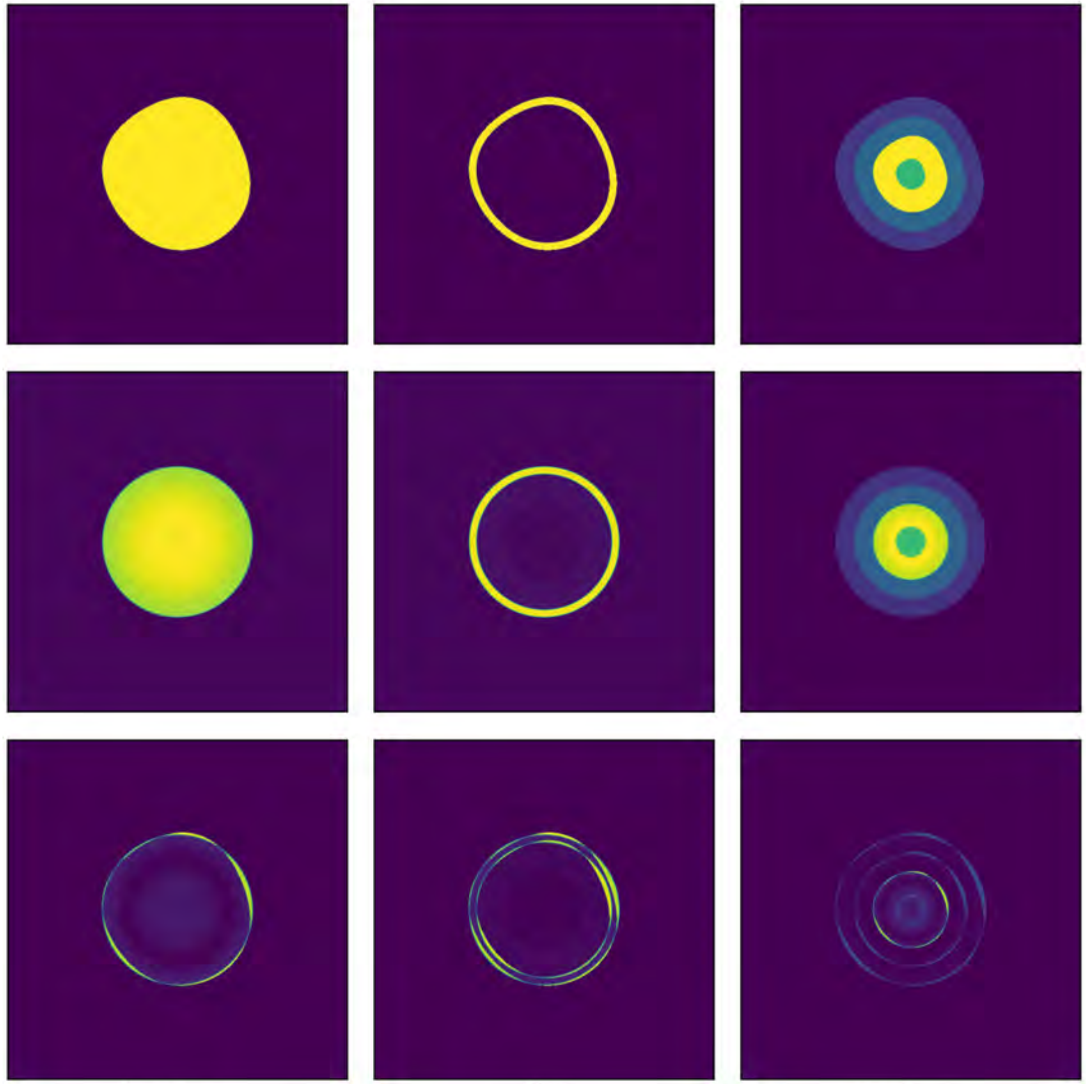
- 15k Geometry profiles considered
- Resolution: 1MP (1000x1000)
- Eccentricities are distributed (roughly) Poisson, varying $0 < \epsilon < 0.1$.

Technical Approach

Truth ($\epsilon \approx 0.06$)

Abel Reconstruction

Absolute Spatial Error



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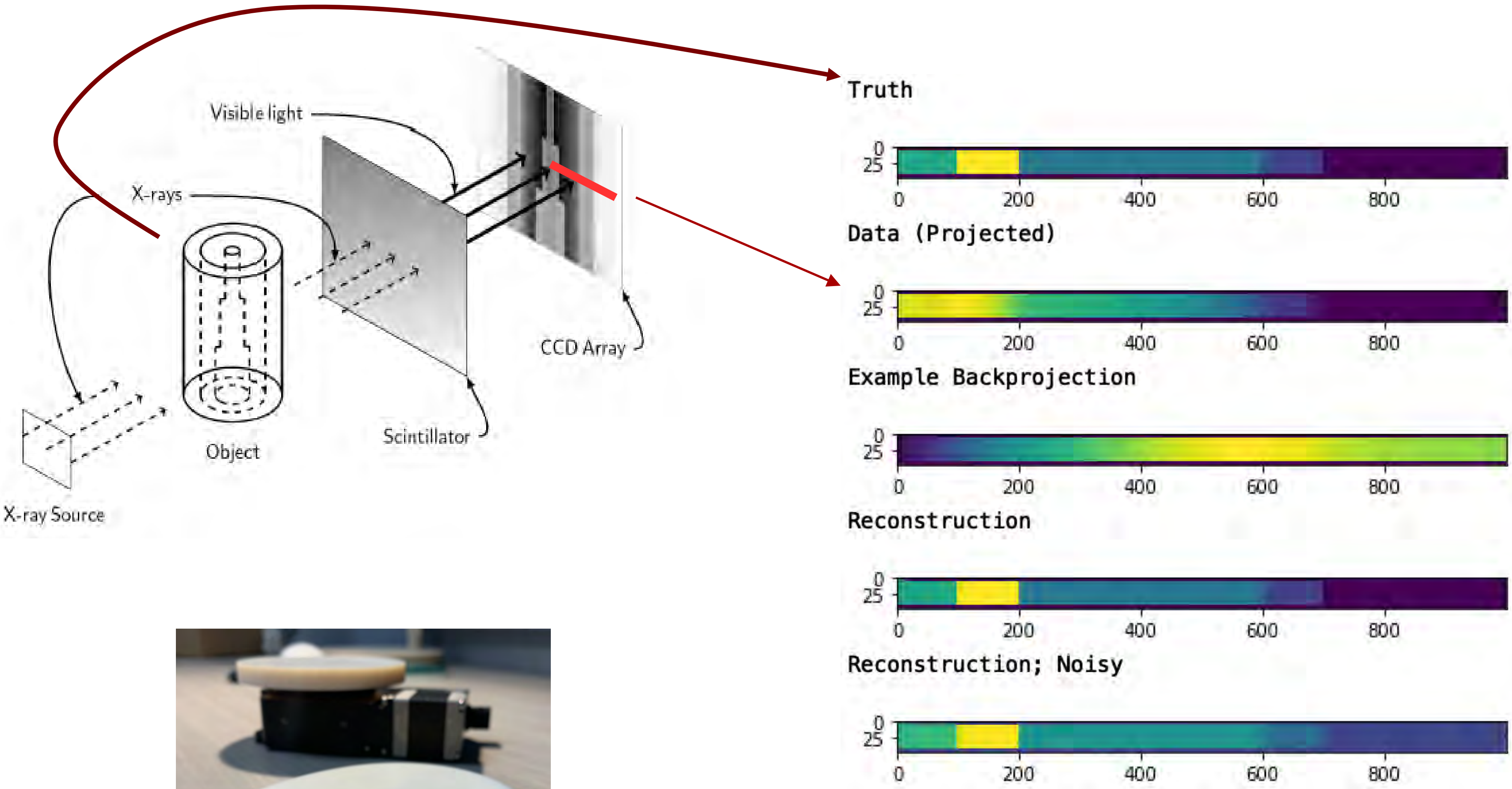
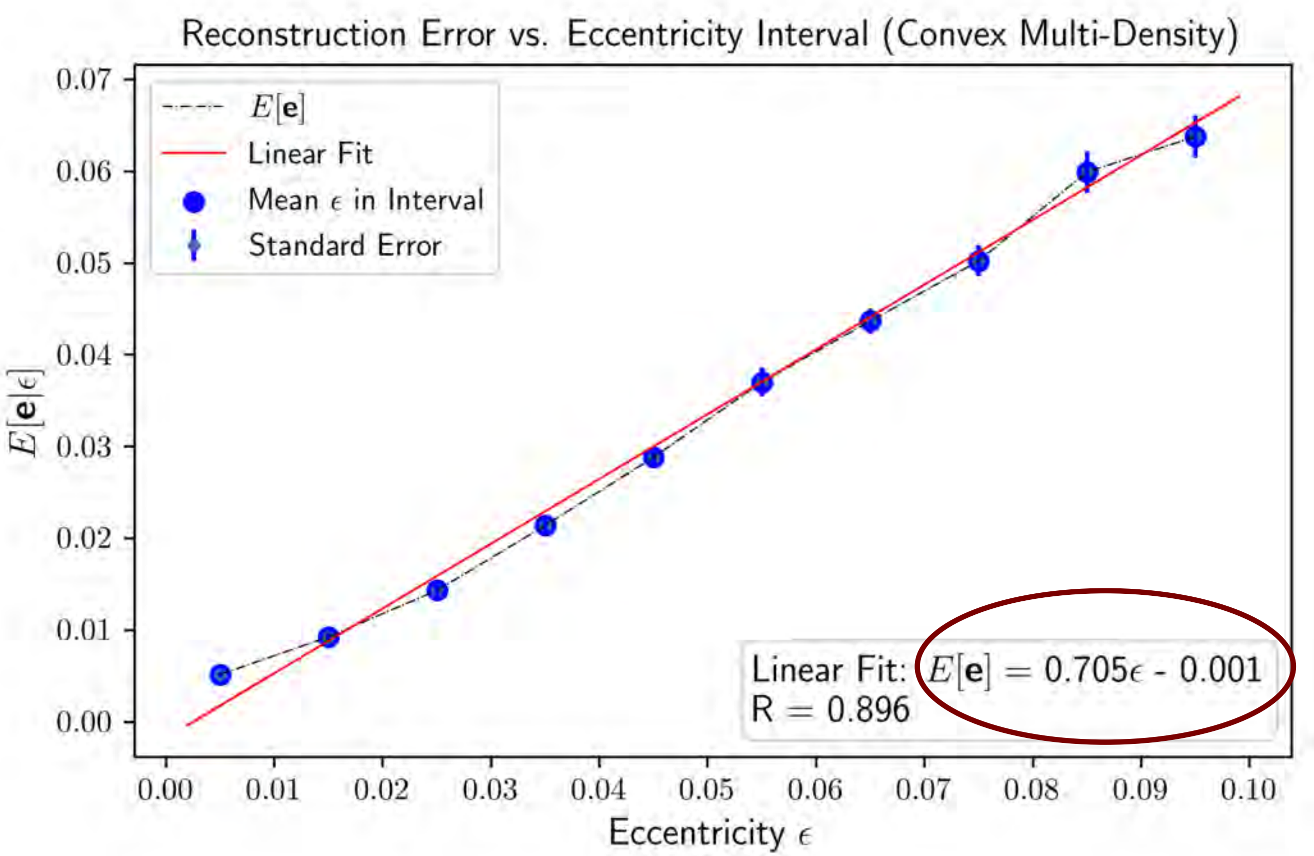
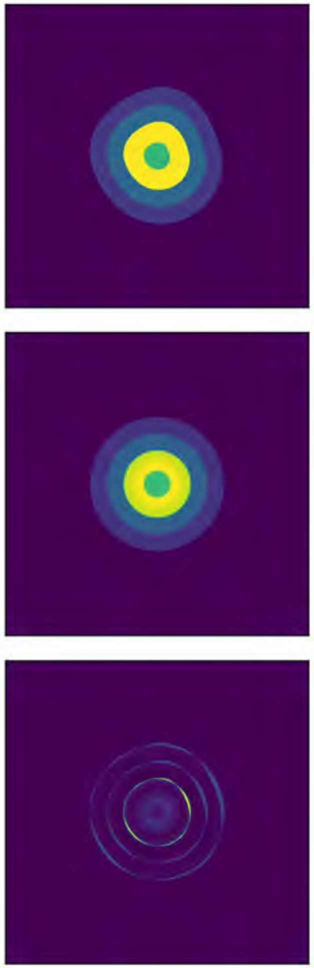
Technical Approach



(Some) Results

$$Normalized\ L^2(\Omega)\ Error: \frac{||u - v||}{||v||}$$



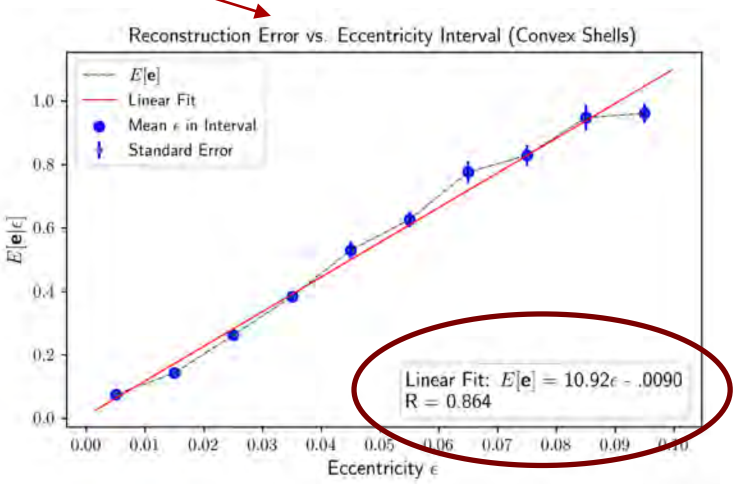


Results

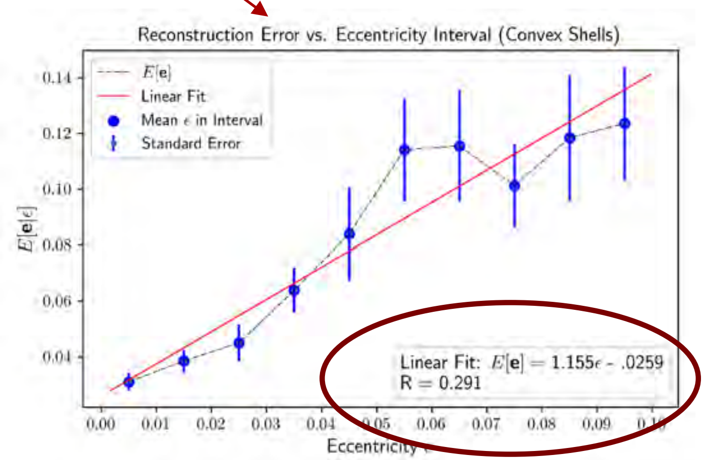
$$Normalized\ L^2(\Omega)\ Error: \frac{||u - v||}{||v||}$$

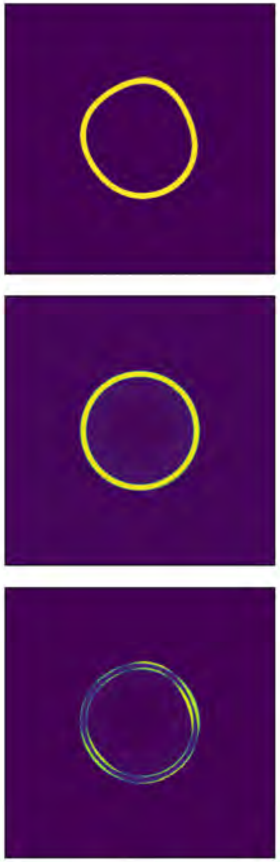
$$Contextualized\ L^2\ (\Omega)\ Error : \frac{|\|u\| - \|v\||}{\|v\|}$$

Normalized error expectation
has a slope of approximately 11!



Contextualized error expectation
slope is closer to 1.16.

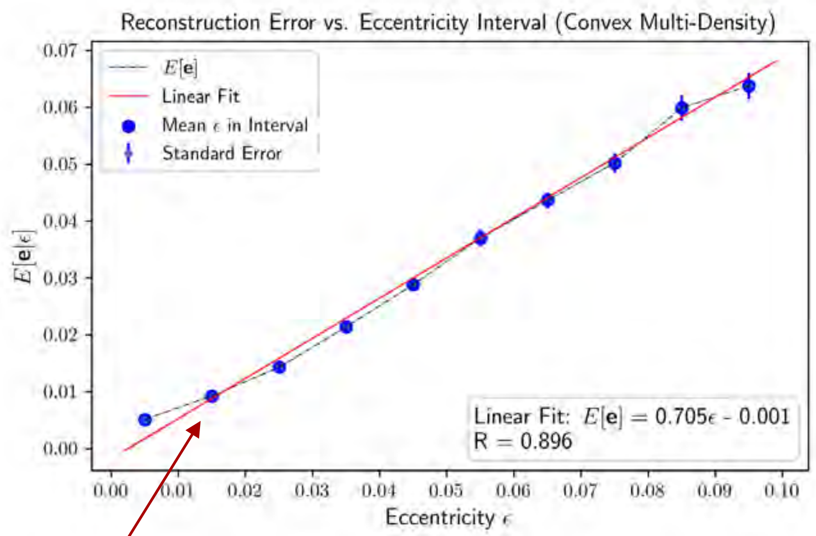


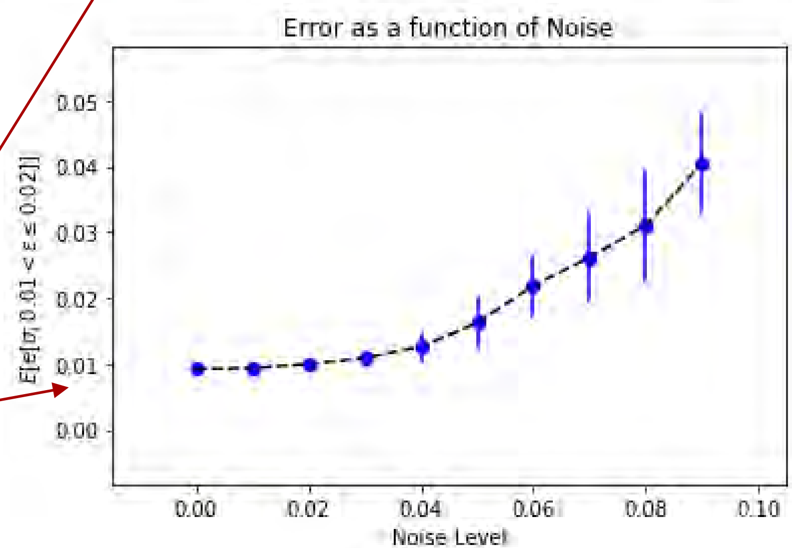


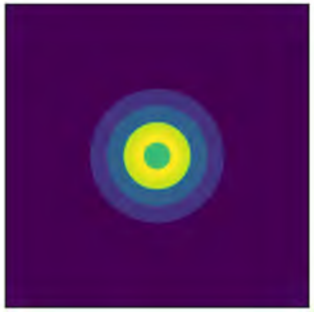
Actively Acquiring Results

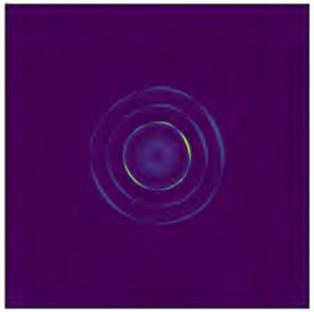
We expect the noise and geometric uncertainties to be independent.

Monte Carlos are still running.









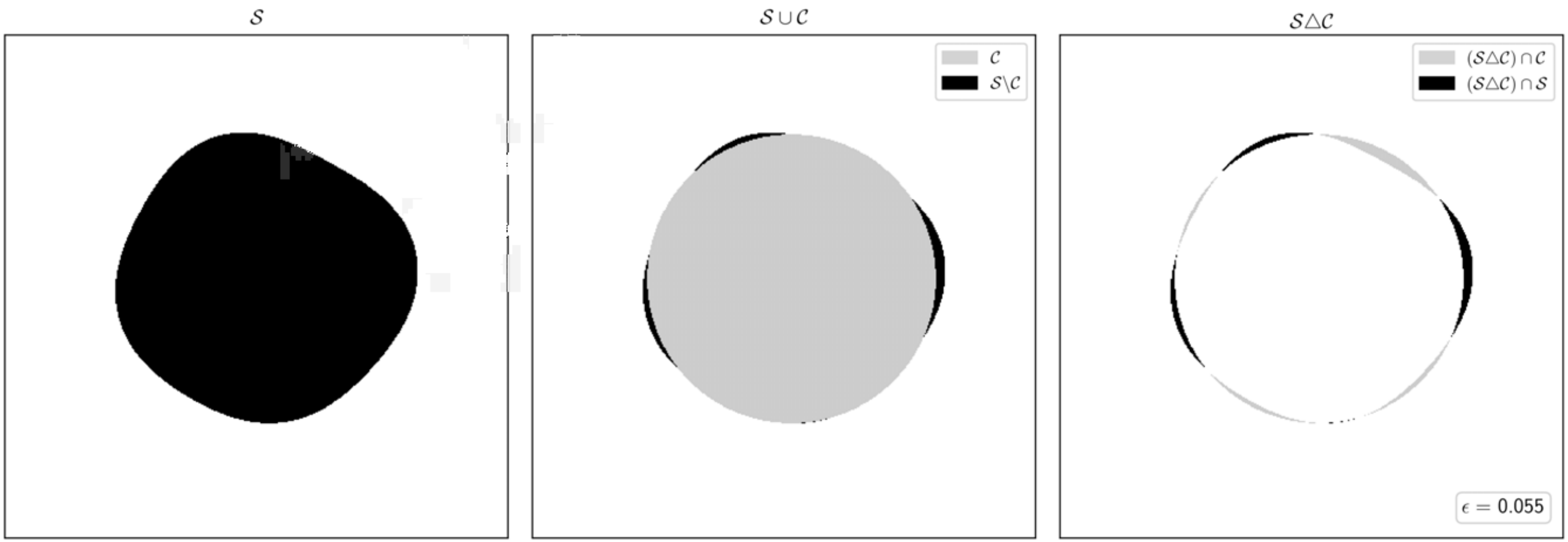
Problems ahead

Can this eccentricity score be determined post-hoc, from one projection?

Accurately? Possibly.

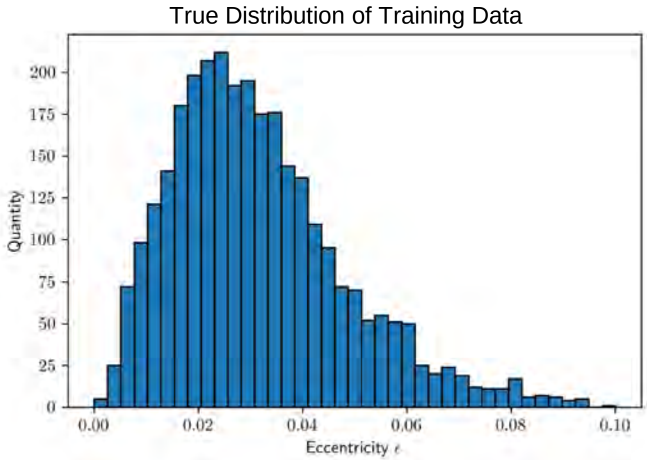
How about from two or more?

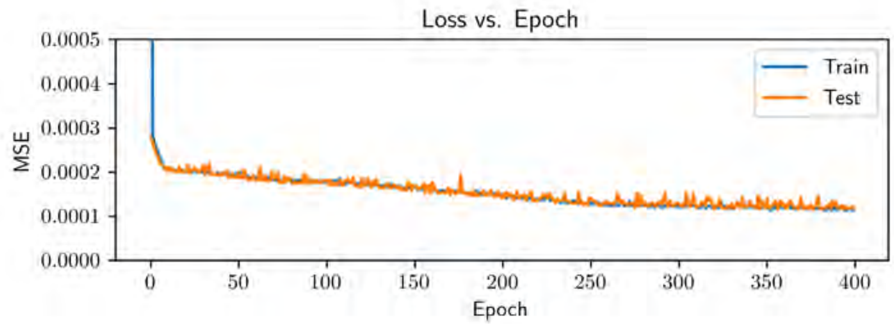
Accurately? Yes.

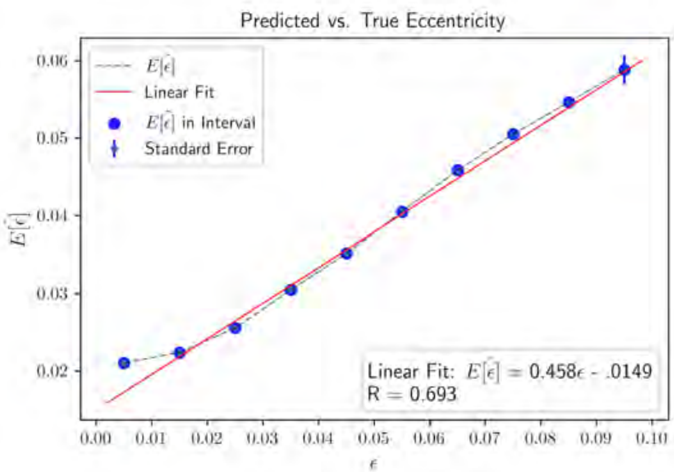


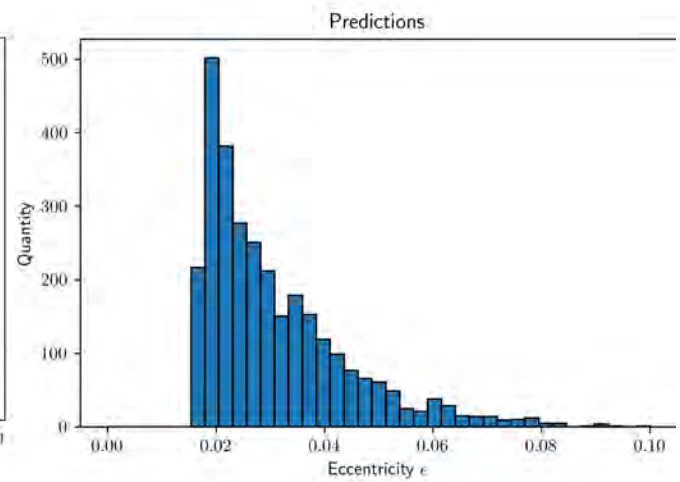
Results

- We trained convolutional neural nets (CNNs) to estimate that eccentricity score from 1, 2, or 3 projection views. (N=1 presented below)
- By N=3 (-60, 0, and 60 degrees) our results are nearly perfect.









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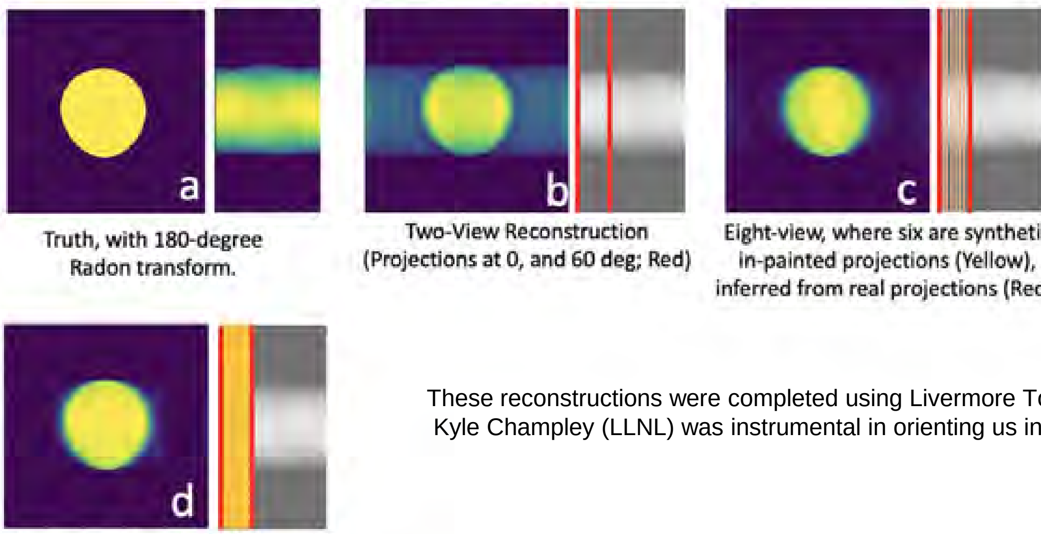
Technical Approach

- Can we develop a data-driven technique to better-reconstruct volumetric density of nearly-radially symmetric targets, without *assuming* rotational symmetry of the target? In short, yes*.



a: Truth, with 180-degree Radon transform.

b: Two-View Reconstruction (Projections at 0, and 60 deg; Red)

c: Eight-view, where six are synthetic in-painted projections (Yellow), inferred from real projections (Red).

d: Seventeen view, where fifteen are synthetic in-painted projections (Yellow), inferred from real projections (Red).

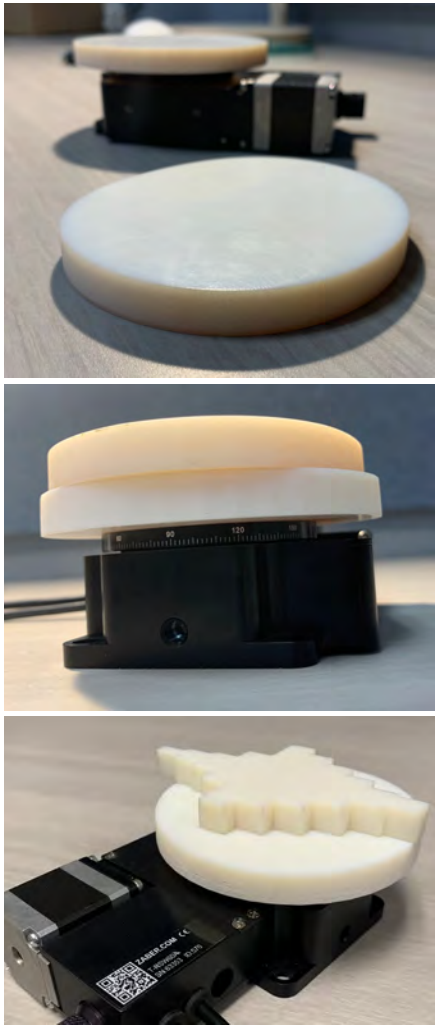
These reconstructions were completed using Livermore Tomography Tools (LTT). Kyle Champley (LLNL) was instrumental in orienting us in the software's usage.

* If you have at least two views, or alternative spatial data.

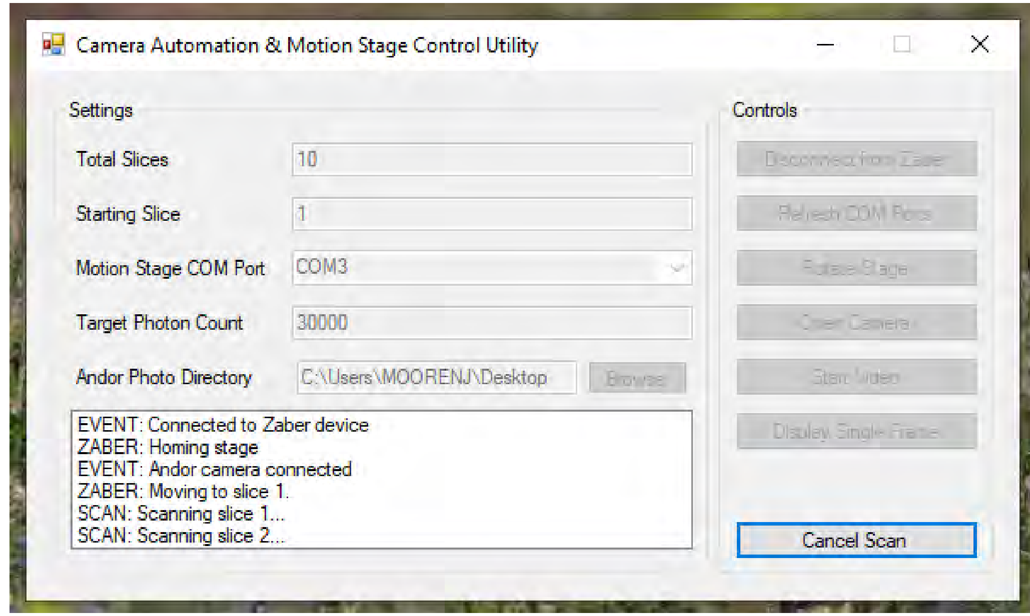
Discussion

- Major lesson learned:
  - Even when restricting the target object to **extremely simple classes of shapes**, we have not been able to use a CNN-based method to reliably divine a second view from one, alone.
    - An alternative approach is being developed, wherein we consider a single radiographic view, and a data-informed “guess” of the target object’s target geometry.
  - With two+ views, we see a substantial improvement to qualitative reconstruction with CNN-inpainting. Particularly, when the target objects are convex.
    - We are currently structuring a geometric uncertainty quantification study of N-view methods (with and without the aid of CNN-inpainting).
    - Our SEALab data will be vital for this study.

Discussion



Huge thanks to Martin Palagi (SEO Mission Support) and Christian Bombara

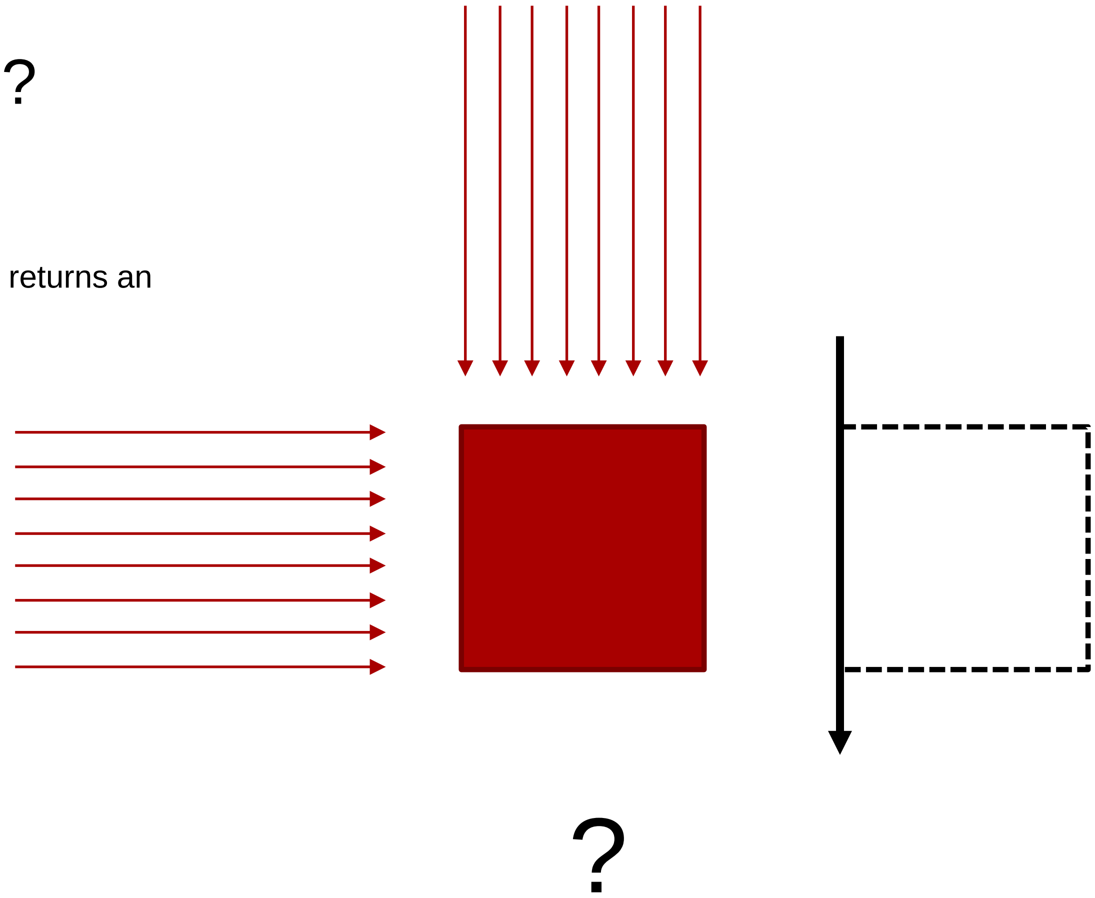


Huge thanks to Nolan Moore (SEO Mission Support)

I am prepared to travel to collect these images, ASAP.

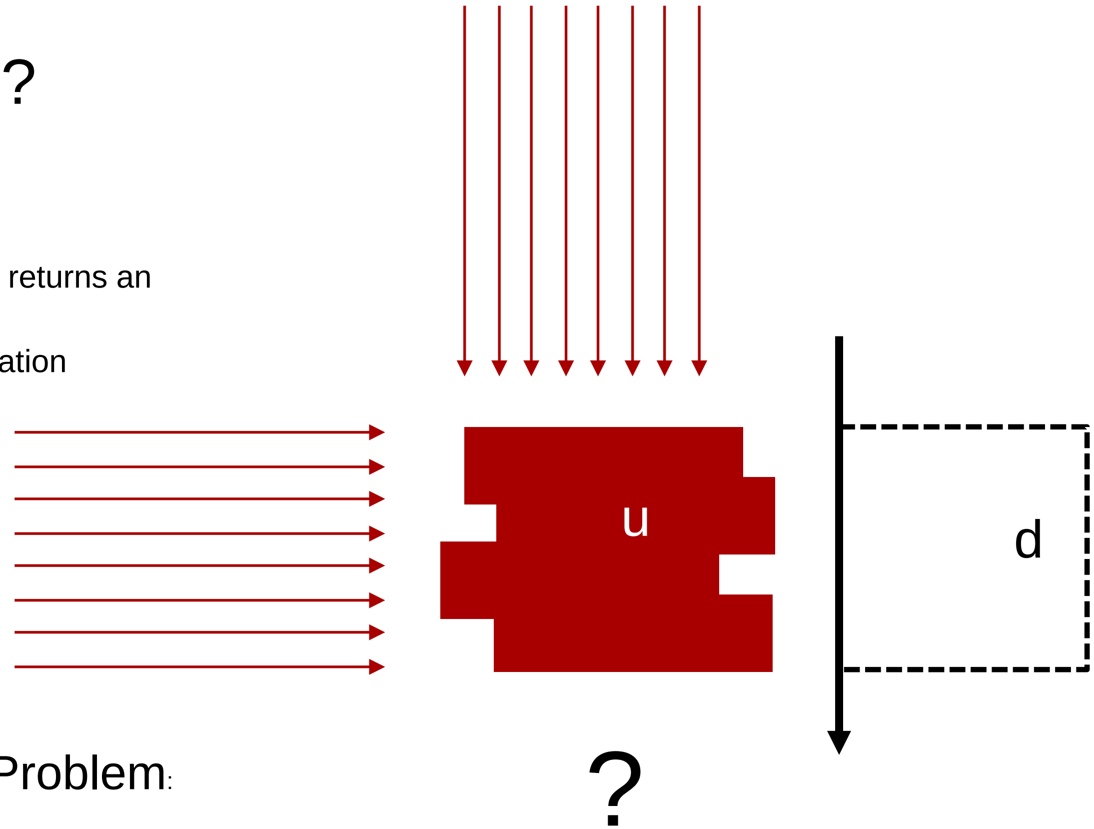
Discussion

- Where is this going?
  Two places.
  - Single-view Reconstruction
    - Neural-net based software returns an averaged result.



Discussion

- Where is this going?
  Two places.
  - Single-view Reconstruction
    - Neural-net based software returns an averaged result.
    - Solve a secondary optimization problem!



Constrained Optimization Problem:

$$u^* = argmin\, \epsilon(u)$$
$$subject\ to$$
$$Au = d$$

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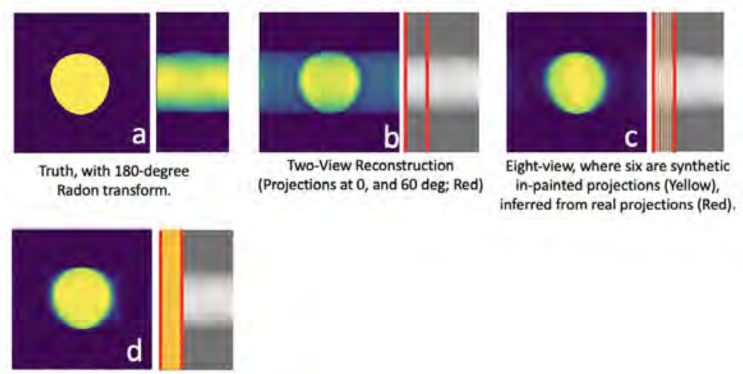
The dimensionality is high, but I think I see a way forward.

SDRD SITE-DIRECTED RESEARCH & DEVELOPMENT

Discussion

- Where is this going?
  Two places.
  - Single-view Reconstruction
    - Neural-net based software returns an averaged result.
    - Solve a secondary optimization problem!
  - Multi-View Reconstruction
    - Neural-net guided sinogram inpainters are showing promise.
    - Threre are many pre-processing, post-processing, and parameter choices to explore, still.



Truth, with 180-degree Radon transform.

Two-View Reconstruction (Projections at 0, and 60 deg; Red)

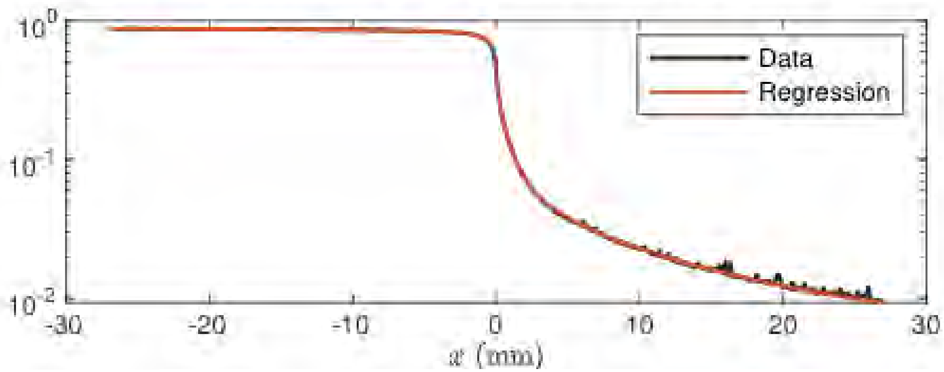
Eight-view, where six are synthetic in-painted projections (Yellow), inferred from real projections (Red).

Seventeen view, where fifteen are synthetic in-painted projections (Yellow), inferred from real projections (Red).

Technical Approach (Blind Deconvolution)

Radiographic Image of Rolled Edge Target

Real and Synthetic Edge Spread Function (Semilog)



$$f_1(a,x) = \frac{1}{2}\left(1 + \operatorname{erf}(ax)\right),$$

$$f_2(a,x) = \left(\frac{1}{2} + \frac{1}{\pi}\arctan\left(x/a\right)\right),$$

$$f_3(a,x) = \frac{1}{2}\left(1 + \frac{x}{\sqrt{x^2 - a^2}}\right),$$

$$F(\mathbf{a},\mathbf{b},r) = b_1 f_1(a_1,r) + b_2 f_2(a_2,r) + b_3 f_3(a_3,r) + b_4 f_3(a_4,r).$$

Technical Approach (Blind Deconvolution)

While investigating the following numerical optimization method to solve inverse-Abel problems:

$$\min_{u} \|\mathcal{A}u - d\|_{L^2(\Omega)}^2 + \lambda \|Lu\|_{L^p(\Omega)}^p$$

We discovered literature applying the L^p model to blind deconvolution (deblurring) problems :

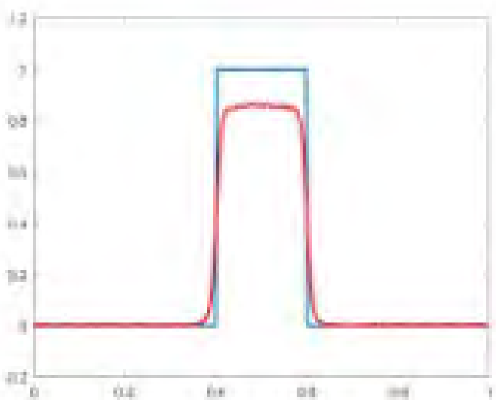
$$\min_{u,a,b} \|K(a,b)u - d\|_{L^2(\Omega)}^2 + \lambda \|Lu\|_{L^p(\Omega)}^p$$

This method returns both the deblurred image, and the approximate spot K(a,b). We have found that this class of methods shows great promise in our settings.

Numerical example: Our true synthetic blur kernel consists of four functions (totaling to eight parameters.)

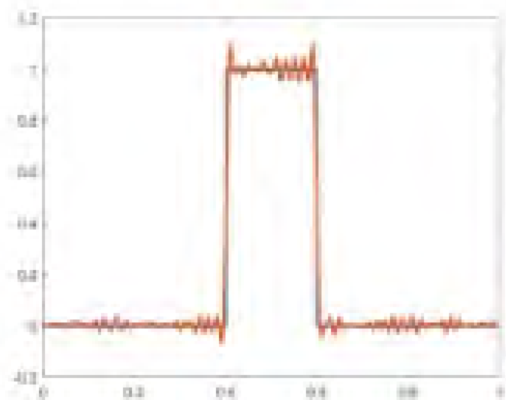
For p=1/2, we see *POINTWISE* convergence to our true spot K(a,b)!

Blue: True solution. Red: Blurred Solution.



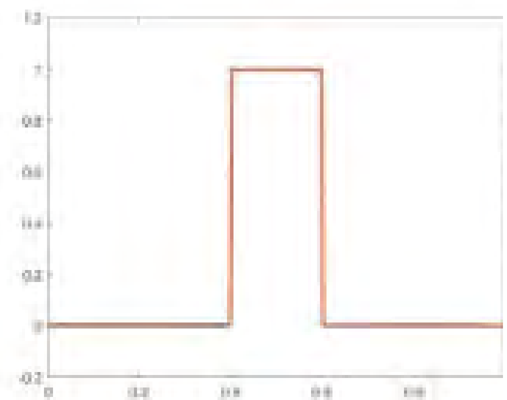
a)

Blue: True solution. Orange: p=2



b)

Blue: True solution. Orange: p=1/2



c)

Impact

- We have three manuscripts in preparation for submission by the end of the fiscal.
  - Daniel Champion's Image Chasing (2D Reconstruction from views.)
  - Sean Breckling & Christian Bombara's Geometric Uncertainty Study
  - Malena Espanol & Sean Breckling's Multi-Parameter Blind Deconvolution Method

- Given the relaxing of COVID restrictions, and the maturity of the work enumerated, this work will be presented at conferences in the following year.

- The usage of Livermore's Tomography Tools have been a great boon for the project, as has Kyle Champley's (LLNL) assistance. LTT will become a staple of NNSS radiographic analysis for the foreseeable future.