

Enhanced Integer Permutation based Genetic Algorithm for Optimization of Tube-Fin Heat Exchanger Circuitry with Splits and Merges

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ABSTRACT

Tube-fin heat exchangers (HXs) are widely used in air-conditioning and heat pump applications. The performance of these heat exchangers is strongly influenced by the refrigerant circuitry. Studies have proved that by optimizing the refrigerant circuitry, the performance of HXs can be significantly improved. In our previous research, an Integer Permutation based Genetic Algorithm (IPGA) was developed to obtain the optimal circuitry designs. Our previous research showed that IPGA demonstrates superior capability to obtain better refrigerant circuitries with lower computational cost than the other methods in literature. And the optimal circuitry designs obtained from IPGA are manufacturable with the available tooling. However, the IPGA developed previously cannot generate designs with splitting and merging of circuits. To remedy this limitation, a new chromosome which can represent circuitry with splitting and merging of circuits is developed. In addition to the six genetic operators implemented previously, two new genetic operators are developed to generate splits and merges. As a result, the enhanced IPGA can explore the solution space more thoroughly than the previous IPGA. A case study using an evaporator from an A-type indoor unit shows that, given the similar capacity improvements obtained from the enhanced IPGA compared with the previous IPGA, the refrigerant pressure drop reduction obtained from the enhanced IPGA is 26.5% compared against 1.0% pressure drop reduction from the previous IPGA. The benchmark of the enhanced IPGA with other methods in literature demonstrates that the enhanced IPGA can generate circuitry designs with performance superior to those obtained from other methods.

1. INTRODUCTION

Tube-fin heat exchangers (TFHXs) are prominent components in air conditioning and heat pump systems. This type of heat exchanger consists of a bundle of tubes with fin sheets. The performance of TFHXs is greatly affected by a large number of structural parameters (tube diameter, tube length, fin type, fin thickness, etc.) and therefore, conducting optimization on such parameters can significantly improve its performance (Huang *et al.*, 2015). In addition to those parameters, the configuration of tube connections, i.e. refrigerant circuitry, determines the refrigerant flow path, and also significantly impacts the heat exchanger performance (Chwalowski *et al.*, 1989; Wang *et al.*, 1999a; Ding *et al.*, 2011; Ye and Lee, 2012; Joppolo *et al.*, 2015). For an existing TFHX design with a given geometry, performing optimization on refrigerant circuitry is more convenient and cost effective than varying other structural parameters. For example, changing the circuitry is a matter of changing U-bend length and locations, whereas changing tube-spacing requires new fin dies, which is costly and has much longer lead time.

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Researchers have tried to use different tube-fin HX circuitry optimization approaches to obtain the best circuitry under a specific operating condition. Domanski *et al.* (1989) developed an optimization tool called ISHED (Intelligent System of Heat Exchanger Design). This optimization scheme switches between Evolutionary learning and Symbolic learning. They performed comprehensive studies to optimize circuitry of TFHXs as evaporators or condensers (Domanski and Yashar, 2007a; Domanski and Yashar, 2007b; Yashar *et al.*, 2012). Their maximum reported HX capacity improvement from optimization runs and experimental validations are 13% from CO₂ gas cooler (Cho and Domanski, 2016) and 2.2% from a R410A evaporator (Yashar *et al.*, 2015), respectively. Wu *et al.* (2008a & 2008b) developed another optimization approach, which switches between knowledge-based GA and Simulated Annealing. Moreover, the group created effective operators to improve the convergence. They performed circuitry optimization on 3 evaporators and 1 condenser, and the maximum predicted capacity improvement in their study is 7.4%. Ploskas *et al.* (2018) represented circuitry using an adjacency matrix and constrained the hairpins to be on one side of the HX. By comparing five different derivative-free optimization algorithms, they concluded that TOMLAB/glcDirect and TOMLAB/glcSolve can efficiently find optimal or near-optimal circuitries. Li *et al.* (2019) presented an integer permutation-based Genetic Algorithm (IPGA) for optimizing circuitry with manufacturability and operating constraints. Six genetic operators are designed such that all chromosomes generated by IPGA can be mapped to a valid circuitry. A constraint-dominated sorting technique is used in the fitness assignment stage to handle manufacturability constraints. Overall, a 2.4–14.6% increase in heat exchange capacity is observed by applying IPGA to an A-shaped indoor unit.

However, the IPGA developed in Li *et al.* (2019) cannot generate designs with splitting and merging of circuits. To remedy this limitation, this paper proposes a new chromosome design and new genetic operators to enhance the capability of IPGA. The remainder of the paper is organized as follows. Section 2 details the new algorithm. Section 3 presents a case study to show the improvements of the enhancement IPGA against previous IPGA. Section 4 compares the enhanced IPGA with previous IPGA and other methods in literature. Section 5 draws the conclusions.

2. ENHANCED INTEGER PERMUTATION BASED GENETIC ALGORITHM

2.1 Chromosome representation

To conduct optimization using Genetic Algorithm, the first step is to find a mathematical representation of the circuitry. A good individual representation (chromosome) can not only reduce the searching space, but can also simulate the nature of the problem (Deb, 2012). In our previous research, IPGA represents tubes as a sequence of integers (i.e. integer permutation). For an integer permutation, each integer (i.e., tube number) appears exactly once, thus, any chromosome generated by the Genetic Algorithm can be mapped to a valid circuitry and the size of the search space is dramatically reduced by the elimination of redundant designs. Li *et al.* (2019) has shown that IPGA demonstrates superior capability to obtain better refrigerant circuitries with lower computational cost than the other methods in literature.

However, the previous chromosome cannot represent designs with splitting and merging of circuits. In order to remedy this limitation, a new chromosome representation called ‘Split Branch Chromosome’ (Figure 1 (a)) is developed and implemented in the enhanced IPGA. The new chromosome representation uses the concept of jagged arrays, with each element of this array representing a branch of tubes. Each branch contains 3 parts. The 1st part of the branch represents the tube sequence. The 2nd part of the branch represents the upstream tube. The 3rd part of the branch represents the downstream tube. Dummy tube number ‘0’ is used as the place holder in the 2nd part, if the 1st tube of the branch is an inlet tube of a circuit. And dummy tube number ‘-1’ is used as the place holder in the 3rd part, if the last tube of the branch is an outlet tube of a circuit.

Figure 1 (b) shows the chromosome representation of the 24-tube heat exchanger in Figure 1 (a). This sample heat exchanger has one splitting tube (tube # 3) and one merging tube (tube #20). With the ‘Split Branch Chromosome’, number of circuits is flexible, i.e., the optimal number of circuits is also an output from optimization, and it can represent circuitry designs with splitting and merging circuits.

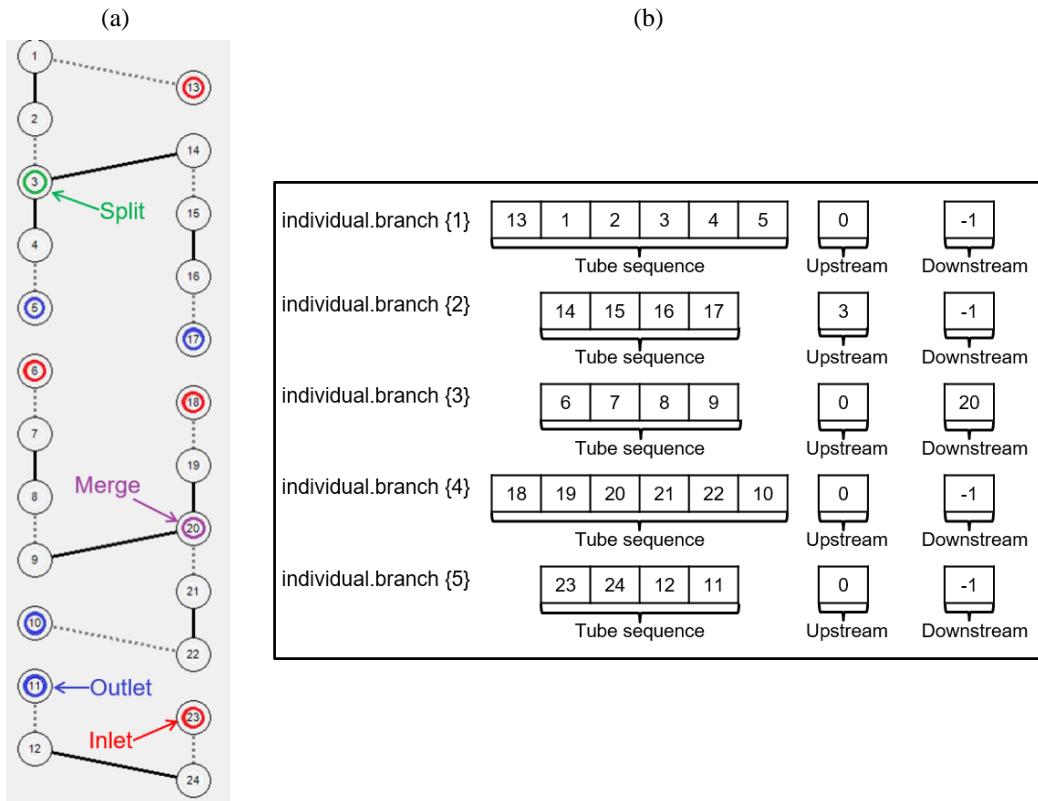


Figure 1 Chromosome Representation: (a) 24-tube heat exchanger sample; (b) Split branch chromosome.

2.2 GA Operators

In Genetic Algorithm, the creation of new individuals relies on the genetic operators. By transforming the selected individual to a new individual with potentially better fitness, the genetic operators direct the search and drive the optimization process.

To adapt the new chromosome representation and extend IPGA's capability to generate designs with splitting and merging circuits, the enhanced IPGA requires new genetic operators. Therefore, two novel genetic operators (Figure 2) are developed. In order to generate circuit splitting, the split operator in Figure 2 (a) randomly determine one tube as the splitting tube and make a sub-section of tubes to form a new branch, thus creates a split. Oppositely, the merge operator in Figure 2 (b) randomly determines one tube as the merging tube and makes a sub-section of tubes to form a new branch, thus creates a merging of circuits.

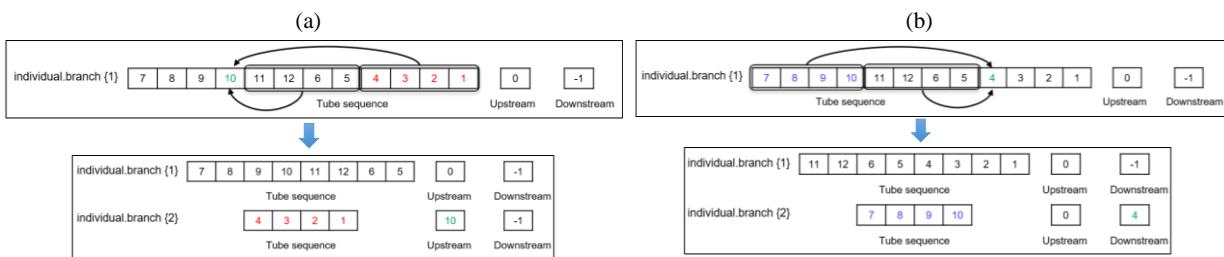


Figure 2: (a) Split operator; (b) Merge operator.

In order to adapt to the change of the chromosome representation, the six GA operators from previous version of IPGA are modified to form six new GA operators (Figure 3). The 'gene sequence inversion' operator in Figure 3 (a) chooses a random subsection of a branch and inverts the order of the tubes. The 'gene insertion' operator in Figure 3 (b) puts a randomly selected tube into a random position. The 'in-branch transposition' operator in Figure 3 (c) reverses two

randomly chosen tubes inside a single branch, while the ‘cross-branch transposition’ operator in Figure 3 (d) reverses two randomly selected tubes between different branches. The ‘branch detachment’ operator in Figure 3 (e) splits a randomly selected branch into two branches. The ‘branch union’ operator in Figure 3 (f) chooses two branches and unites them into one branch. With the ‘branch union’ and ‘branch detachment’ operators, the number of circuits can be varied and optimized in a generic fashion.

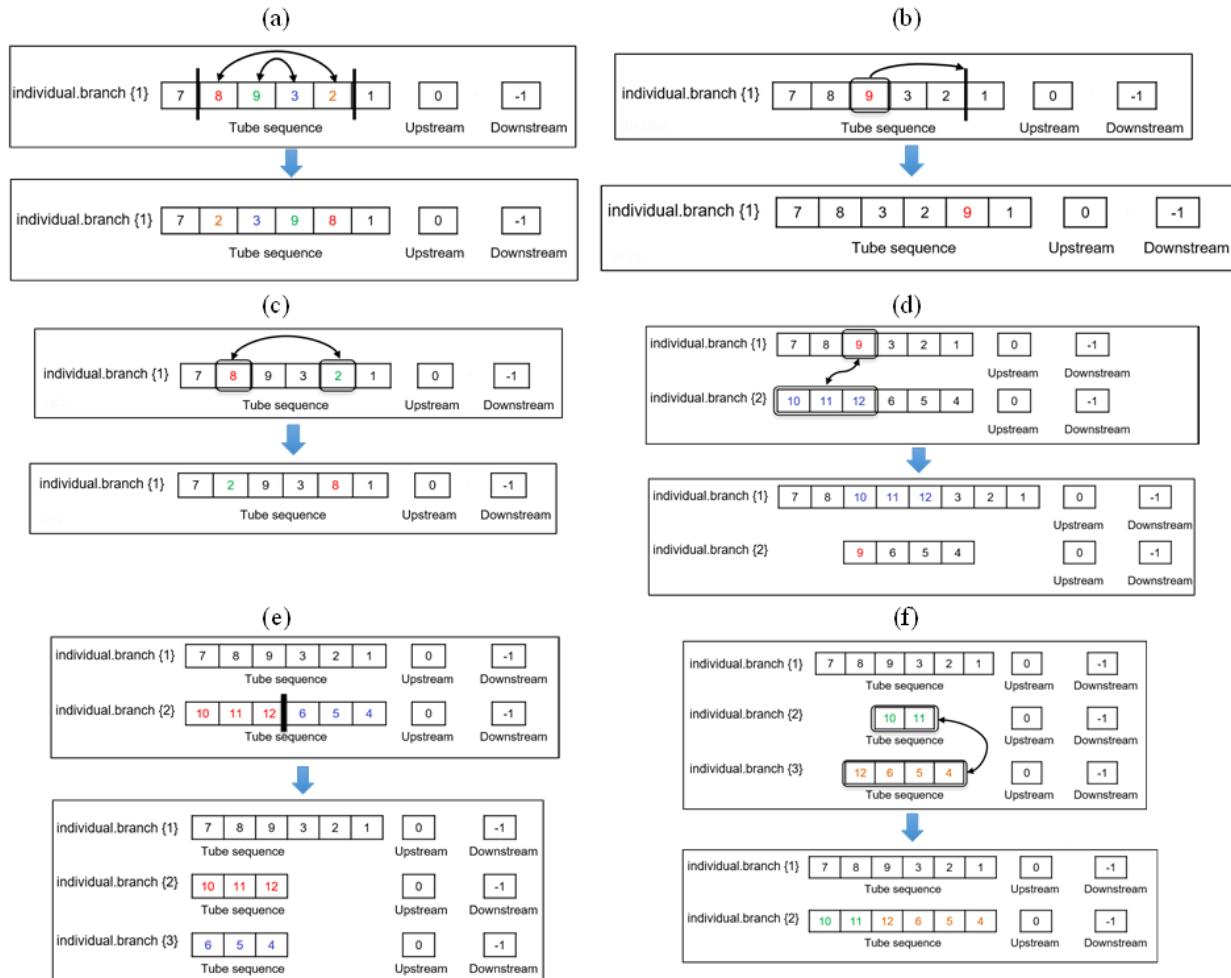


Figure 3: GA Operators: (a) Gene sequence inversion; (b) Gene insertion; (c) In-branch transposition; (d) Cross-branch transposition; (e) Branch detachment; (f) Branch union.

2.3 New Manufacturability Constraints

For tube-fin circuitry optimization problem, it is crucial to have options to add manufacturability constraints and guarantee that the optimal designs can be manufactured in a cost-effective manner. Four manufacturability (mfg.) constraints are incorporated in the method. The first three mfg. constraints are available in the previous version of IPGA. The first three mfg. constraints are introduced briefly as below. More details can be referred to Li *et al.* (2019).

1. Inlets and outlets on the same side of HX. Figure 4 (a) shows a preferable design. Figure 4 (b) shows an unpreferable design.
2. Limit or avoid long U-bends: Figure 4 (c) shows some long U-bends which will be avoided by IPGA. The maximum allowed length of ‘long U-bend’ is set by the user of IPGA.
3. Limit or avoid U-bend crossovers: Figure 4 (d) shows different types of U-bends crossovers that IPGA is capable to avoid.

The 4th constraint is newly implemented in the enhanced version of IPGA.

4. Limit the number of tubes splitting or merging from a single tube: Figure 4 (e) illustrates a heat exchanger design where the refrigerant from four tubes (tube #5, #6, #13 and #14) merges into tube #9. While the refrigerant at tube #12 splits into two tubes (tube #8 and #16). The proposed algorithm provides the option for the user to set maximum number of tubes splitting or merging from a single tube.

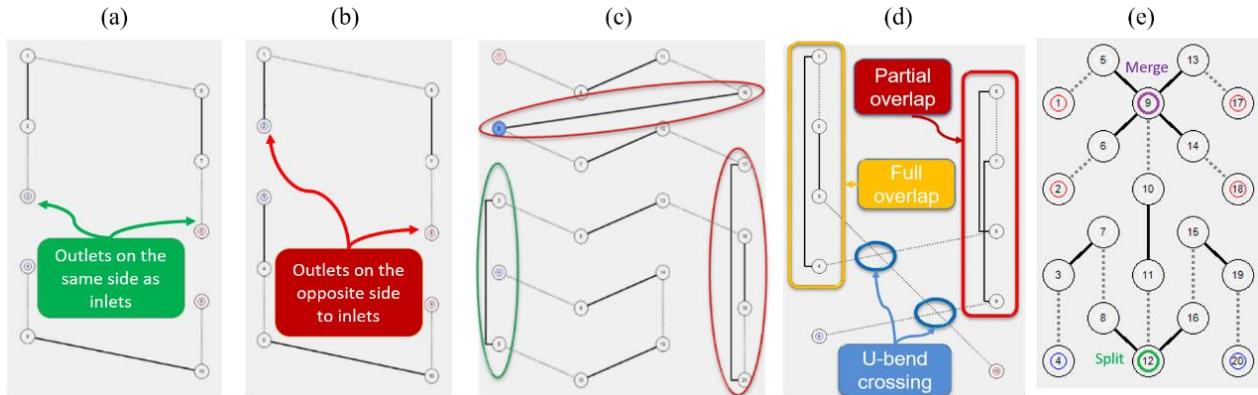


Figure 4: Manufacturability Constraints: (a) Inlet & Outlet on the same side; (b) Inlet & outlet on the opposite side; (c) Long U-bends; (d) U-bend crossings; (e) Tubes splitting or merging from a single tube.

3. CASE STUDY

3.1 Baseline Heat Exchangers

An evaporator from an A-type indoor unit (Figure 5 (a)) is used as the baseline for circuitry optimization, because it is experimentally validated in our laboratory. Table 1 shows the structural parameters for the baseline indoor coil and its operating condition. The air velocity flow distribution is uniform. Table 2 lists the empirical correlations used for local heat transfer and pressure drop calculations during the performance evaluation of the HX.

Table 1: Structural Parameters and Operating Conditions of Baseline Evaporator

Structural Parameters	Value	Operating Conditions	Value
Tube Outer Diameter	9.5 mm	Refrigerant	R410A
Fins per inch	15 FPI	Refrigerant Inlet Pressure	1154.5 kPa
Fin Type	Wavy Louver	Refrigerant Inlet Quality	0.22
Tube Length	0.503 m	Refrigerant Mass Flow Rate	0.0312 kg/s
Vertical Spacing	20.0 mm	Air Volume Flow Rate (Uniformly Distributed)	600 ft ³ /min
Horizontal Spacing	25.0 mm	Air Pressure	101.325 kPa
Number of Tube Banks	4	Air Temperature	26.42 °C
Number of Tubes Per Bank	22	Air Relative Humidity	50.97 %

Table 2: Correlations Adopted in HX Simulation

Operating Mode	Heat Transfer Correlations	Pressure Drop Correlations
Refrigerant - Liquid Phase	Dittus and Boelter, 1985	Blasius, 1907
Refrigerant - Two Phase	Jung <i>et al.</i> , 1989	Jung and Radermacher, 1989
Refrigerant - Vapor Phase	Dittus and Boelter, 1985	Blasius, 1907
Air	Wang <i>et al.</i> , 1998	Wang <i>et al.</i> , 1998

At the fitness evaluation stage, a finite volume heat exchanger model (Jiang *et al.*, 2006) is used to simulate the performance of the heat exchanger with different circuitries. The model can account for the refrigerant flow maldistribution among different circuits by iterating on the pressure residual at the outlet of each circuit. This HX

model was validated with measured data (Alabdulkarem *et al.*, 2015) for the same baseline HX, and the deviation between simulations and experiments are below 5% as shown Figure 5 (b).

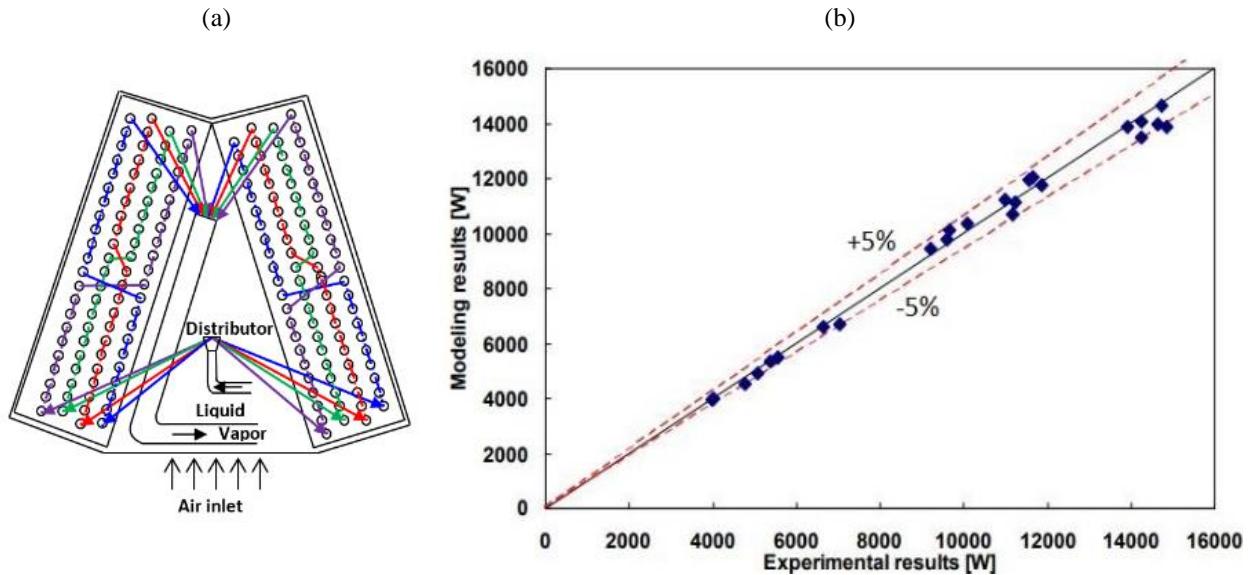


Figure 5: (a) Baseline Circuitry; (b) Experiment Tests vs Simulations

3.2 Problem Formulation

Equation (1) shows the optimization problem formulation. The objective is to maximize the HX capacity. The problem formulation implements 7 constraints. The first 4 operating constraints limit capacity, refrigerant pressure drop, outlet refrigerant saturation ΔT and sensible heat ratio in acceptable ranges as compared with the baseline. The remaining 3 constraints are manufacturability constraints. Specifically, the 5th constraint ensures the inlet and the outlet tubes on the same side of HX. The 6th constraint avoids long U-bends spanning over 2 tube rows. The last constraint avoids partially overlapped U-bends crossing.

$$\text{Objective : Maximize}(Q)$$

Subject to :

$$Q \geq Q_{\text{baseline}}$$

$$\Delta P_{\text{refrigerant}} \leq \Delta P_{\text{refrigerant,baseline}}$$

$$|\Delta T_{\text{sat}} - \Delta T_{\text{sat,baseline}}| \leq 1K \quad (1)$$

$$|SHR - SHR_{\text{baseline}}| \leq 3\%$$

Inlets and outlets on the same side of HX

No long U-bend spanning over 2 tube rows

No overlapped U-bend crossing

3.3 Results

To compare the performance of the previous IPGA with the enhanced IPGA. Two optimization runs are conducted using these two versions of IPGA, respectively. For both of the optimization runs, the GA population size is set to 200, the number of generations is set to 500 and the population replacement ratio is 20% (i.e. the elitism reservation ratio is 80%).

Table 3 details the performance of the baseline and the two optimal designs. Figure 6 shows the circuitry of baseline and two optimal designs. In the circuitry plots, different color represents different circuit. A solid line represents a U-bend on the near end of the heat exchanger, while a dotted line represents a U-bend on the far end. The red tubes are the inlets, while the blue ones are outlets. Merging tubes and splitting tubes are represented by yellow and black solid circles, respectively.

As a result, the optimal design from enhanced IPGA offers slightly more capacity improvement (2.5%) than the previous IPGA (2.4%). More significantly, the optimal design from the enhanced IPGA reduces the refrigerant pressure drop by 26.5%, which is more prominent than the 1.0% pressure drop reduction obtained from the previous IPGA. This attributes to the capability of enhanced IPGA to generate splits and merges. As shown in Figure 6, the baseline (Figure 6 (a)) and the optimal design from previous IPGA (Figure 6 (b)) have 4 inlets and 4 outlets without split or merge. However, the optimal from the enhanced IPA has 6 inlets and 5 outlets. There is one merge tube (the solid yellow tube) in its bottom circuit (Figure 6 (c)). This case study demonstrates that the enhanced IPGA explores the solution space more thoroughly with the capability of generating splits and merges.

Table 3: Evaporator Optimization Results using Previous IPGA and Enhanced IPGA

Case	Baseline	Previous IPGA (Avoiding splits or merges)	Enhanced IPGA (Allow splits and merges)
Capacity [W]	5294	5421 (2.4%↑)	5424 (2.5%↑)
Ref. DP [kPa]	11.8	11.7 (1.0%↓)	8.6 (26.5%↓)
SHR	79.6%	80.6%	80.1%
U-bends L1	81	62	43
U-bends L2	1	22	40
U-bends \geq L3	2	0	0
Collinear U-bends	0	0	0

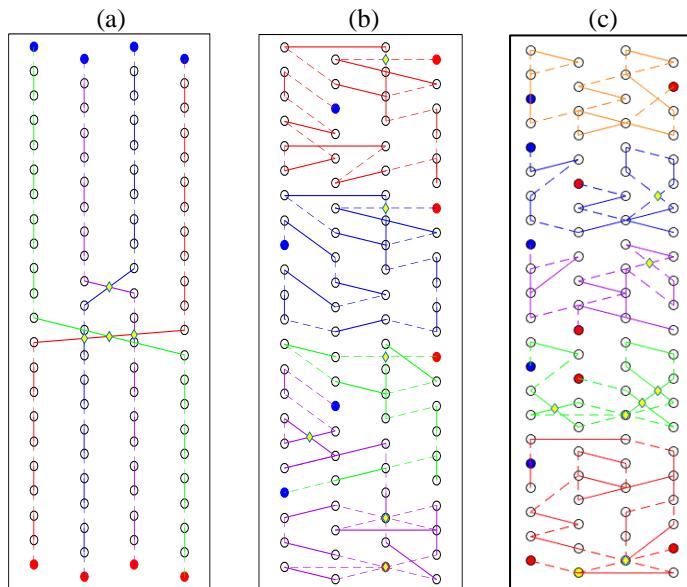


Figure 6: (a) Baseline; (b) Optimal Design from Previous IPGA; (c) Optimal Design from Enhanced IPGA.

4. COMPARISON WITH METHODS FROM LITERATURE

In this section, the enhanced IPGA is benchmarked against Domanski *et al.*(2004) and Wu *et al.*(2008b)'s methods as well as the previous IPGA (Li *et al.* (2019)) by optimizing the same heat exchanger. In previous researches, Domanski *et al.*(2004), Wu *et al.*(2008b) and Li *et al.* (2019) applied their methods to optimize the same 36-tube R22 baseline evaporator as shown in Figure 7 (a) under uniform airflow distribution.

To be consistent with the GA settings used by previous researchers, the population size for this optimization run is 15 and number of generations is 200. The boundary condition to simulate this evaporator is specified evaporator outlet condition. According to Domanski *et al.*(2004), the evaporator outlet pressure is fixed at a saturation pressure corresponding to 7.2°C saturation temperature, while the outlet superheat is 5°C. The evaporator inlet enthalpy is determined based on the outlet enthalpy of a condenser, to be specific, the enthalpy at 40°C condensation temperature with 5°C subcooling. The HX structural parameters, operating conditions and correlations are provided in Domanski *et al.*(2004).

Figure 7 (b) shows the optimal solution as reported in Domanski *et al.*(2004). It achieves 4.2% capacity increase, accompanied by 27.7% refrigerant pressure drop increase. There is one partial overlap U-bend crossover highlighted in red rectangular and the author suggested to manually modify the circuitry to improve its manufacturability. Figure 7 (c) shows optimal solution from Wu *et al.*(2008b). The capacity increase is 4.1%, the manufacturability is better than the former counterpart since there is no long U-bend or partial overlapped collinear U-bends. However, the refrigerant pressure drop also increases by 26.0%. Figure 7 (d) and Figure 7 (e) shows the optimal solutions from the previous IPGA (i.e. Li *et al.* (2019)) and the enhanced IPGA, respectively.

The optimal from previous IPGA yields 5.9% capacity increase and 8.0% refrigerant reduction. The optimal from the enhanced IPGA yields 6.2% capacity increase and 0.4% refrigerant reduction. Considering the optimization objective is to maximize the heat exchanger capacity, the optimal design from the enhanced IPGA outperforms all the other optimal designs. The optimal design from the enhanced IPGA has 1 inlet tube, 2 outlet tubes and one split tube shown as the solid black tube in Figure 7 (e). While, all the other designs have two inlet tubes and two outlet tubes without split or merge. This benchmark emphasizes the superiority of the enhanced IPGA and the necessity to investigate circuitry designs with splits and merges.

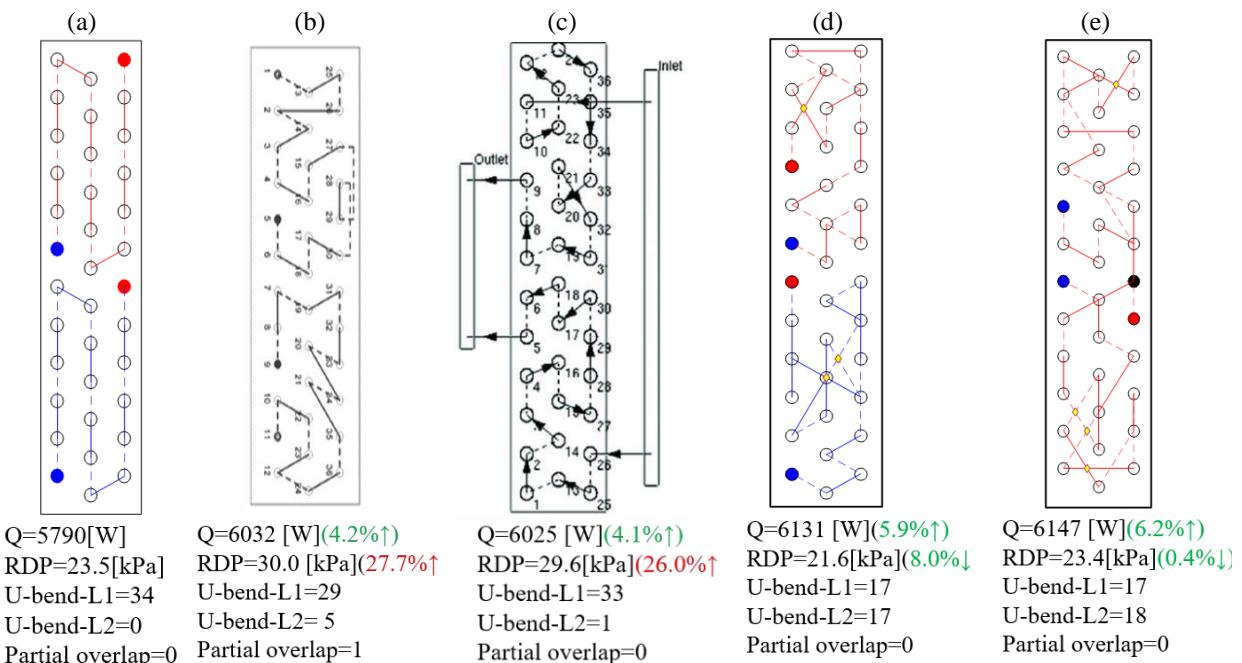


Figure 7: Optimal Solutions: (a) Baseline; (b) Domanski *et al.* (2004); (c) Wu *et al.*(2008b); (d) Previous IPGA; (e) Enhanced IPGA.

5. CONCLUSIONS

This research presents an enhanced Integer Permutation based Genetic Algorithm (IPGA) to optimize the refrigerant flow path of air-to-refrigerant heat exchangers. The algorithm has novel features such as to (i) generate circuitry designs with splitting and merging of circuits, (ii) use effective chromosome representations and GA operators to guarantee the chromosome (genotype) can be mapped to valid heat exchanger designs (phenotype), and (iii) incorporate real-world manufacturability constraints to ensure the optimal designs are manufacturable with the available tooling. Case studies have demonstrated that the optimal designs obtained from this algorithm exhibiting higher capacity, lower pressure drop and better manufacturability. Comparison with other algorithms in literature shows that the enhanced IPGA exhibits higher quality optimal solutions than the previous version of IPGA and outperforms the other methods.

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