SAND2020-8164PE

Amnesia Radius Versions of Conditional Point Sampling for Radiation Transport in 1D Stochastic Media

PRESENTED BY

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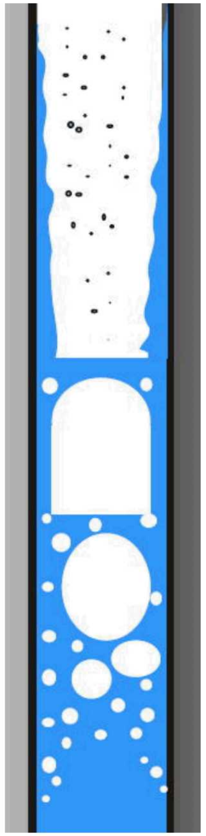
Outline

- **Introduction and Problem Statement**
- Amnesia Radius in Conditional Point Sampling
- Leakage Results
- Numerical Studies
- Conclusion and Future Work

Stochastic Media

- Spatially heterogenous mixing
  - Random material type
  - Rayleigh-Taylor instabilities
  - BWR Coolant
  - Porous materials



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Problem Statement

- Stochastic Transport Equation

$$\mu \frac{\partial \psi(x,\mu,\omega)}{\partial x} + \Sigma_t(x,\omega)\psi(x,\mu,\omega) = \frac{\Sigma_s(x,\omega)}{2}\int_{-1}^{1} d\mu' \psi(x,\mu',\omega) \tag{1a}$$

$$0 \le x \le L; -1 \le \mu \le 1 \tag{1b}$$

$$\psi(0,\mu) = 2, \mu \ge 0; \psi(L,\mu) = 0, \mu < 0 \tag{1c}$$

- x, μ, ω – spatial, angular, and stochastic dependence
- $\Sigma_t(x, \omega)$ – total cross section
- $\Sigma_s(x, \mu, \omega)$ – angular cross section
- L – domain length
- Isotropic boundary source on “left” boundary, otherwise vacuum BCs

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Markovian-Mixed Media Statistics

- Correlation length based on chord lengths of each material $(\Lambda_\alpha, \Lambda_\beta)$:

$$\Lambda_c = \frac{\Lambda_\alpha \Lambda_\beta}{\Lambda_\alpha + \Lambda_\beta} \tag{2}$$

- Average number of pseudo-interfaces per distance r:

$$I = \frac{r}{\Lambda_c} \tag{3}$$

- Pseudo-interfaces are Poisson-distributed, so their frequency is:

$$f(k, I) = e^{-I} \frac{I^k}{k!} \tag{4}$$

- Probability of material at a random point in a realization:

$$P_\alpha = \frac{\Lambda_\alpha}{\Lambda_\alpha + \Lambda_\beta} \tag{5}$$

P. SWITZER, "A random set process in the plane with a Markovian property," *Annals of Mathematical Statistics*, **36**, 1859–1863 (1965).

S. D. PAUTZ, B. C. FRANKE, A. K. PRINJA, and A. J. OLSON, "Solution of stochastic media transport problems using a numerical quadrature-based method," in "M&C 2013," American Nuclear Society, Sun Valley, ID (May 2013).

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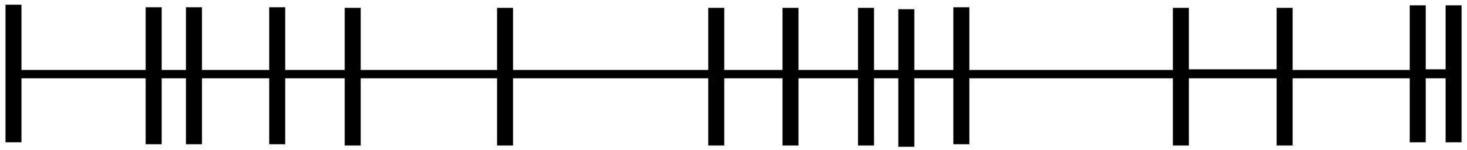
Constructing Binary Markovian-Mixed Media

Material α

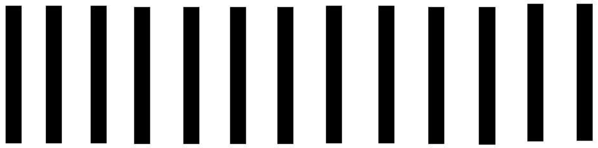
Material β

$$P_\alpha = \frac{\Lambda_\alpha}{\Lambda_\alpha + \Lambda_\beta}; P_\beta = 1 - P_\alpha = \frac{\Lambda_\beta}{\Lambda_\alpha + \Lambda_\beta}; \lambda_{\alpha,i} = \Lambda_\alpha \log\left(\frac{1}{\xi}\right)$$

$$\Lambda_C = \frac{\Lambda_\alpha \Lambda_\beta}{\Lambda_\alpha + \Lambda_\beta}; \quad I = \frac{r}{\Lambda_C}; \quad f(k, I) = e^{-I}\frac{(I)^k}{k!}$$



…aces in domain.



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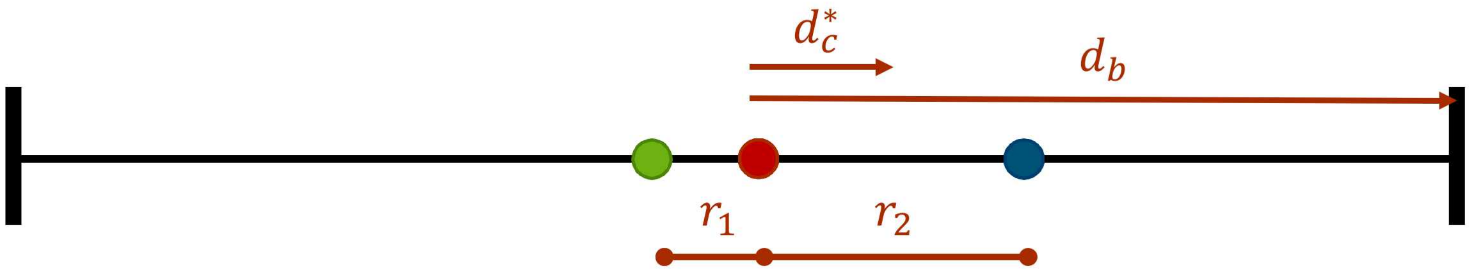
Conditional Point Sampling: Algorithm

Material α

Material β



$\mu = \cos\theta$

$d_b = domain$

$d_c^* = -\frac{1}{\Sigma_t^*}\log(\xi)$

$P_\alpha = \frac{\Lambda_\alpha}{\Lambda_\alpha + \Lambda_\beta}$

$P_{col} = \frac{\Sigma_t}{\Sigma_t^*}$

1. Begin Woodcock Monte Carlo Algorithm.
   - Initialize x and μ.
2. Sample distance to ***potential*** collision, d_c^*, and distance to boundary, d_b.
3. Stream particle based on, $d_{min} = min(d_c^*, d_b)$.
   - If external boundary is crossed, terminate particle.
   - If particle streams to potential collision site, sample material at that point using P_α or $conditional\ probability\ function$.
4. Sample against P_{col} to determine if collision is accepted.
   - If collision is rejected, continue streaming particle by returning to step 2.
   - If collision is accepted, evaluate collision.

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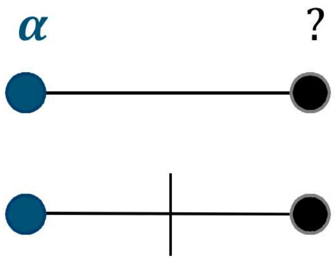
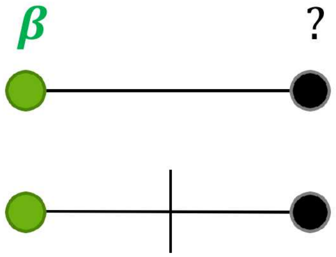
Conditional Point Sampling: Deriving 2-Point Conditional Probability Function (CoPS2)



$$\pi(\alpha,\alpha|\alpha,\kappa) = (1)f(k=0,r=r_1) + (P_\alpha)f(k>0,r=r_1) \quad (6a)$$

$$\pi(\alpha,\alpha|\alpha,\kappa) = 1 - P_\beta\left(1 - e^{-\frac{r_1}{\Lambda_c}}\right) \quad (6b)$$

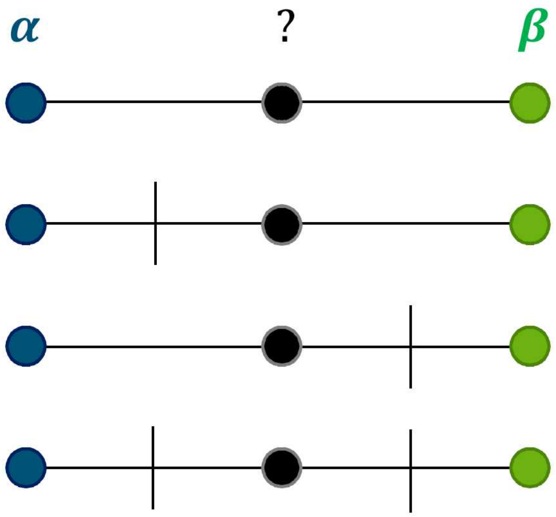


$$\pi(\beta,\alpha|\alpha,\kappa) = (0)f(k=0,r=r_1) + (P_\alpha)f(k>0,r=r_1) \quad (7a)$$

$$\pi(\beta,\alpha|\alpha,\kappa) = P_\alpha\left(1 - e^{-\frac{r_1}{\Lambda_c}}\right) \quad (7b)$$

E. H. VU and A. J. OLSON, "Conditional Point Sampling: A novel Monte Carlo method for radiation transport in stochastic media," *Trans. Am. Nucl. Soc.*, **120** (2019).

Conditional Point Sampling: Deriving 3-Point Conditional Probability Function (CoPS3)



$$\pi(\alpha|\boldsymbol{m},\boldsymbol{r})$$
$$\boldsymbol{m} = \{m_1, m_2\}$$
$$\boldsymbol{r} = \{r_1, r_2\}$$

$$\pi(\alpha|\{\alpha,\beta\}) = \Big[(0) f(k>0, r=r_1) f(k=0, r=r_2) + (1) f(k=0, r=r_1) f(k>0, r=r_2) + (P_\alpha) f(k>0, r=r_1)\, f(k>0, r=r_2)\Big] \Big/ \big(1 - f(k=0, r=r_1+r_2)\big) \quad (8)$$

$$\pi(\alpha|\{\alpha,\alpha\}) = 1 - P_\beta\left(1 - e^{-\frac{r_1}{\Lambda_c}}\right)\left(1 - e^{-\frac{r_2}{\Lambda_c}}\right) \quad (9)$$

$$\pi(\alpha|\{\beta,\beta\}) = P_\alpha\left(1 - e^{-\frac{r_1}{\Lambda_c}}\right)\left(1 - e^{-\frac{r_2}{\Lambda_c}}\right) \quad (10)$$

$$\pi(\alpha|\{\alpha,\beta\}) = \frac{\left(1 - e^{-\frac{r_2}{\Lambda_c}}\right)\left[1 - P_\beta\left(1 - e^{-\frac{r_1}{\Lambda_c}}\right)\right]}{1 - e^{-\frac{r_1+r_2}{\Lambda_c}}} \quad (11)$$

E. H. VU and A. J. OLSON, "Conditional Point Sampling: A novel Monte Carlo method for radiation transport in stochastic media," *Trans. Am. Nucl. Soc.*, **120** (2019).

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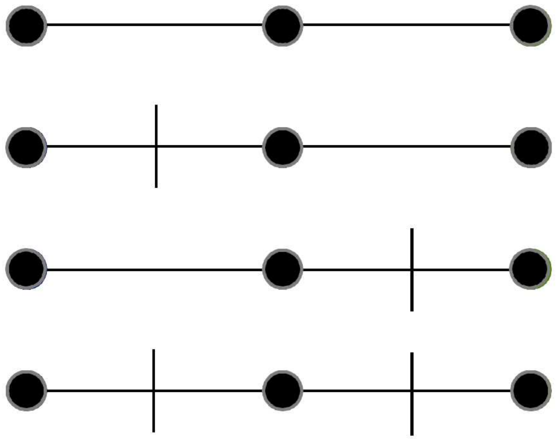
Conditional Point Sampling: Deriving Errorless 3-Point Conditional Probability Function (CoPS3PO)

Scenarios of Pseudo – Interfaces



Use Bayes′ Theorem to derive conditional probability function

$$\pi(\alpha,\alpha,\alpha|\alpha,?,\alpha) = \frac{\pi(\alpha,?,\alpha|\alpha,\alpha,\alpha)\pi(\alpha,\alpha,\alpha)}{\pi(\alpha,?,\alpha)} \quad (12)$$

$$\pi(\beta,\alpha,\beta|\beta,?,\beta) = \frac{\pi(\beta,?,\beta|\beta,\alpha,\beta)\pi(\beta,\alpha,\beta)}{\pi(\beta,?,\beta)} \quad (13)$$

$$\pi(\alpha,\alpha,\beta|\alpha,?,\beta) = \frac{\pi(\alpha,?,\beta|\alpha,\alpha,\beta)\pi(\alpha,\alpha,\beta)}{\pi(\alpha,?,\beta)} \quad (14)$$

Possible Material Combinations

$\boldsymbol{\alpha},\boldsymbol{\alpha},\boldsymbol{\alpha}$	$\boldsymbol{\alpha},\boldsymbol{\beta},\boldsymbol{\alpha}$
$\boldsymbol{\beta},\boldsymbol{\alpha},\boldsymbol{\beta}$	$\boldsymbol{\beta},\boldsymbol{\beta},\boldsymbol{\beta}$
$\boldsymbol{\beta},\boldsymbol{\alpha},\boldsymbol{\alpha}$	$\boldsymbol{\beta},\boldsymbol{\beta},\boldsymbol{\alpha}$
$\boldsymbol{\alpha},\boldsymbol{\alpha},\boldsymbol{\beta}$	$\boldsymbol{\alpha},\boldsymbol{\beta},\boldsymbol{\beta}$

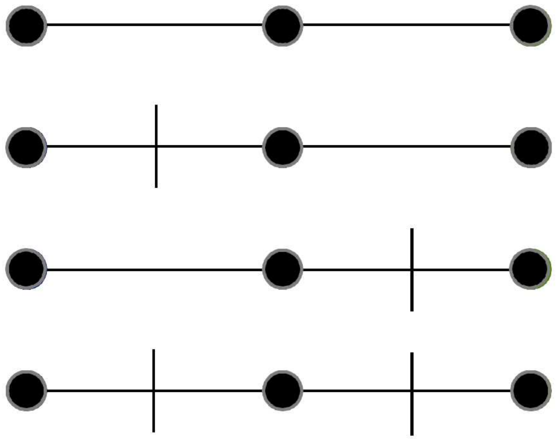
Bayes′ Theorem Terms

$$\pi(m_1,?,m_2|m_1,m_M,m_2) = 1$$

$$\pi(m_1,?,m_2) = \pi(m_1,m_1,m_2) + \pi(m_1,m_2,m_2)$$

Conditional Point Sampling: Deriving Errorless 3-Point Conditional Probability Function (CoPS3PO)

Scenarios of Pseudo – Interfaces



$$\pi(\alpha,\alpha,\alpha|\alpha,?,\alpha) = 1 - \frac{P_\beta\left(1-e^{-\frac{r_1}{\Lambda_c}}\right)\left(1-e^{-\frac{r_2}{\Lambda_c}}\right)}{1-\frac{P_\beta}{P_\beta-1}e^{-\frac{r_1+r_2}{\Lambda_c}}} \quad (15)$$

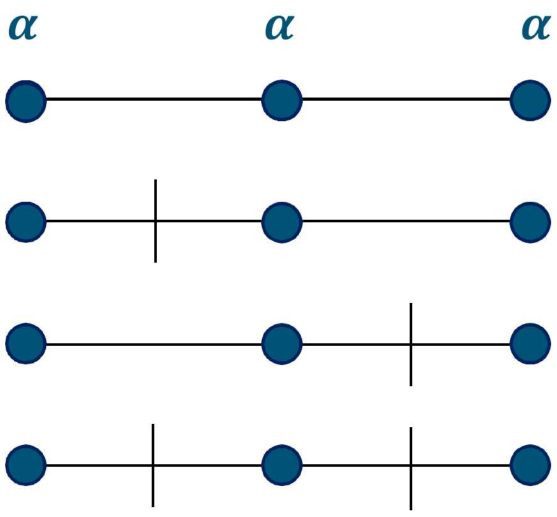
$$\pi(\beta,\alpha,\beta|\beta,?,\beta) = \frac{P_\alpha\left(1-e^{-\frac{r_1}{\Lambda_c}}\right)\left(1-e^{-\frac{r_2}{\Lambda_c}}\right)}{1-\frac{P_\alpha}{P_\alpha-1}e^{-\frac{r_1+r_2}{\Lambda_c}}} \quad (16)$$

Possible Material Combinations

$\boldsymbol{\alpha,\alpha,\alpha}$	$\boldsymbol{\alpha,\beta,\alpha}$
$\boldsymbol{\beta,\alpha,\beta}$	$\boldsymbol{\beta,\beta,\beta}$
$\boldsymbol{\beta,\alpha,\alpha}$	$\boldsymbol{\beta,\beta,\alpha}$
$\boldsymbol{\alpha,\alpha,\beta}$	$\boldsymbol{\alpha,\beta,\beta}$

$$\pi(\alpha,\alpha,\beta|\alpha,?,\beta) = \frac{\left(1-e^{-\frac{r_2}{\Lambda_c}}\right)\left[1-P_\beta\left(1-e^{-\frac{r_1}{\Lambda_c}}\right)\right]}{1-e^{-\frac{r_1+r_2}{\Lambda_c}}} \quad (17)$$

Conditional Point Sampling: Deriving Errorless 3-Point Conditional Probability Function (CoPS3PO)



$$\pi(\alpha,\alpha,\alpha) =$$

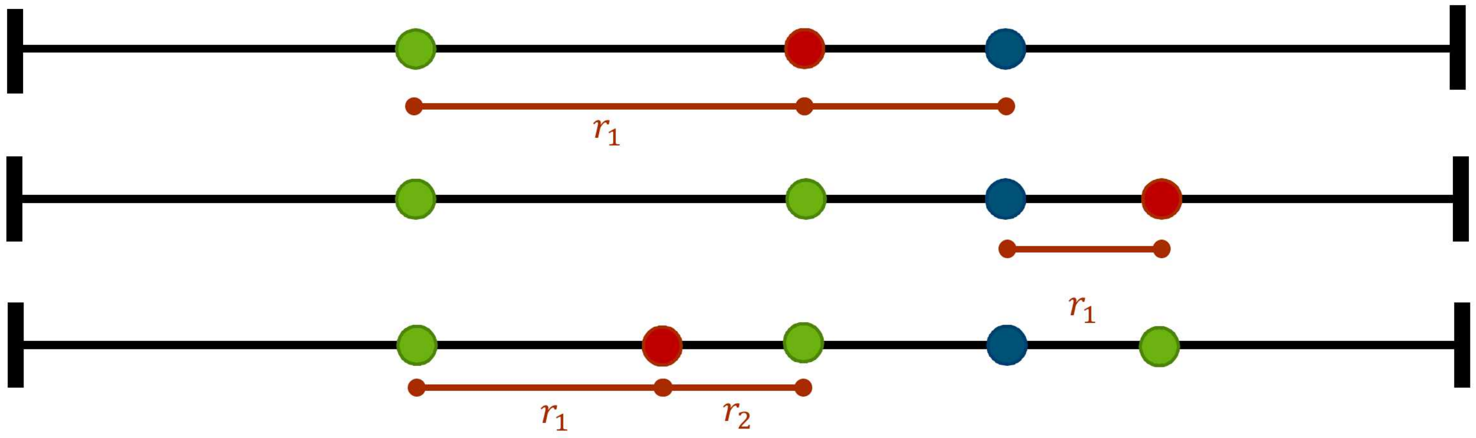
$$P_\alpha\, f(k=0, r=r_1+r_2)$$
$$+\quad [P_\alpha f(k>0, r=r_1)][P_\alpha f(k=0, r=r_2)]$$
$$+\quad [P_\alpha f(k=0, r=r_1)][P_\alpha f(k>0, r=r_2)]$$
$$+P_\alpha\, [P_\alpha f(k>0, r=r_1)]\, [P_\alpha f(k>0, r=r_2)] \tag{20}$$

$$\pi(\alpha,\alpha,\alpha) = P_\alpha e^{-\frac{r_1+r_2}{\Lambda_c}} + P_\alpha^2\left(e^{-\frac{r_2}{\Lambda_c}}\right)\left(1-e^{-\frac{r_1}{\Lambda_c}}\right) + P_\alpha^2\left(e^{-\frac{r_1}{\Lambda_c}}\right)\left(1-e^{-\frac{r_2}{\Lambda_c}}\right) + P_\alpha^3\left(1-e^{-\frac{r_1}{\Lambda_c}}\right)\left(1-e^{-\frac{r_2}{\Lambda_c}}\right) \tag{21}$$

Conditional Point Sampling: Amnesia Radius Versions

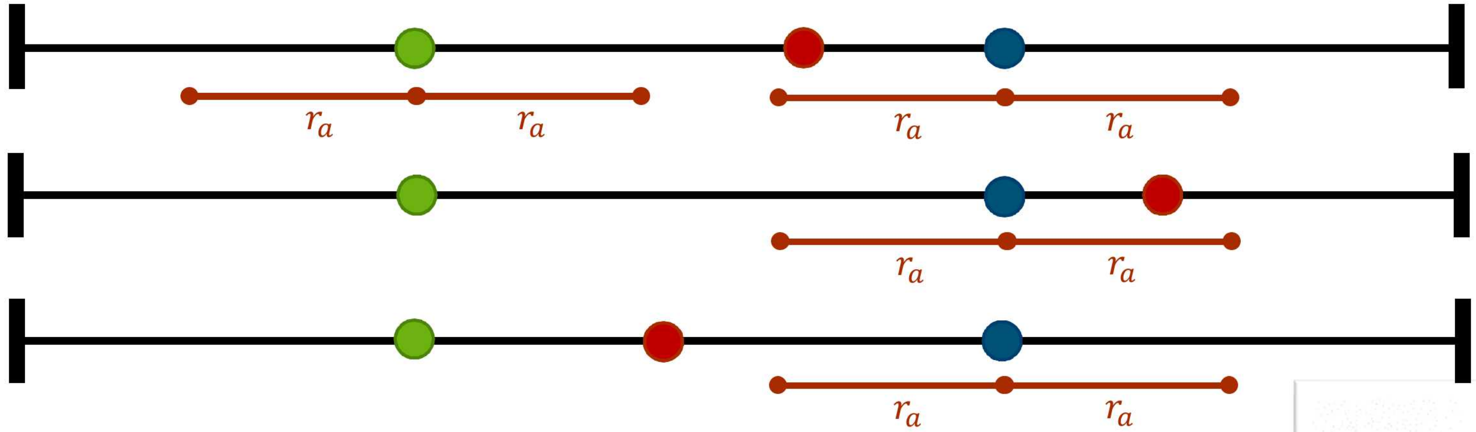
Conventional

r_1, r_2



Amnesia Radius

r_a



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Benchmark Suite Problem Parameters

- Adams Benchmark Suite Problem Parameters

Case Number	$\Sigma_{t,0}$	$\Sigma_{t,1}$	Λ_0	Λ_1
1	10/99	100/11	0.99	0.11
2	10/99	100/11	9.9	1.1
3	2/101	200/101	5.05	5.05

Case Letter	c_0	c_1	Slab Thickness	
a	0.0	1.0	$L =$	10.0
b	1.0	0.0		
c	0.9	0.9		

- $\Sigma_{t,j}$ – total cross section
- Λ_j – average chord length
- c_j – scattering cross section
- L – slab length
- $j \in \{0,1\}$ – material type

M. L. ADAMS, E. W. LARSEN, and G. C. POMRANING, "Benchmark results for particle transport in a binary Markov statistical medium," *J. Quant. Spectrosc. and Rad. Transfer*, **42**, *4*, 253–266 (1989).

P. S. BRANTLEY, "A benchmark comparison of Monte Carlo particle transport algorithms for binary stochastic mixtures," *J. Quant. Spectrosc. and Rad. Transfer*, **112**, 599–618 (2011).

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Calculating Error Metrics

- Relative Error

$$E_{R_i} = \frac{x_i - x_{approx_i}}{x_i}, \qquad i \in \{1a, 1b, 1c, 2a, 2b, 2c, 3a, 3b, 3c\} \qquad (14)$$

- Root Mean Squared Error

$$RMS\ E_R = \sqrt{\frac{1}{N} \sum_i {E_R}_i^2} \qquad (15)$$

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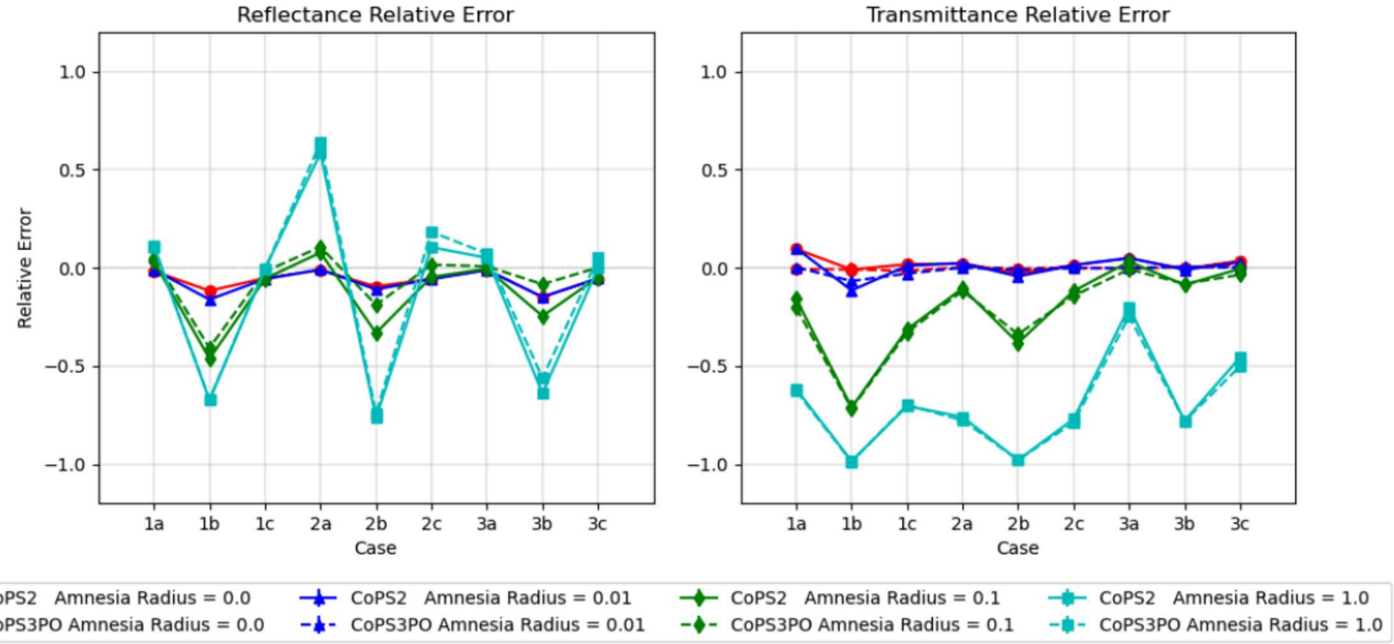
Results & Analysis: Accuracy Comparison of Mean Leakage Results using CoPS2 and CoPS3PO and Cohort Size of 1



	Reflectance				Transmittance			
Amnesia Radius	0.0	0.01	0.1	1.0	0.0	0.01	0.1	1.0
CoPS2	0.083	0.094	0.215	0.451	0.052	0.058	0.294	0.731
CoPS3PO	0.008	0.014	0.161	0.446	0.014	0.013	0.296	0.741

M. L. ADAMS, E. W. LARSEN, and G. C. POMRANING, "Benchmark results for particle transport in a binary Markov statistical medium," *J. Quant. Spectrosc. and Rad. Transfer*, **42**, *4*, 253–266 (1989).

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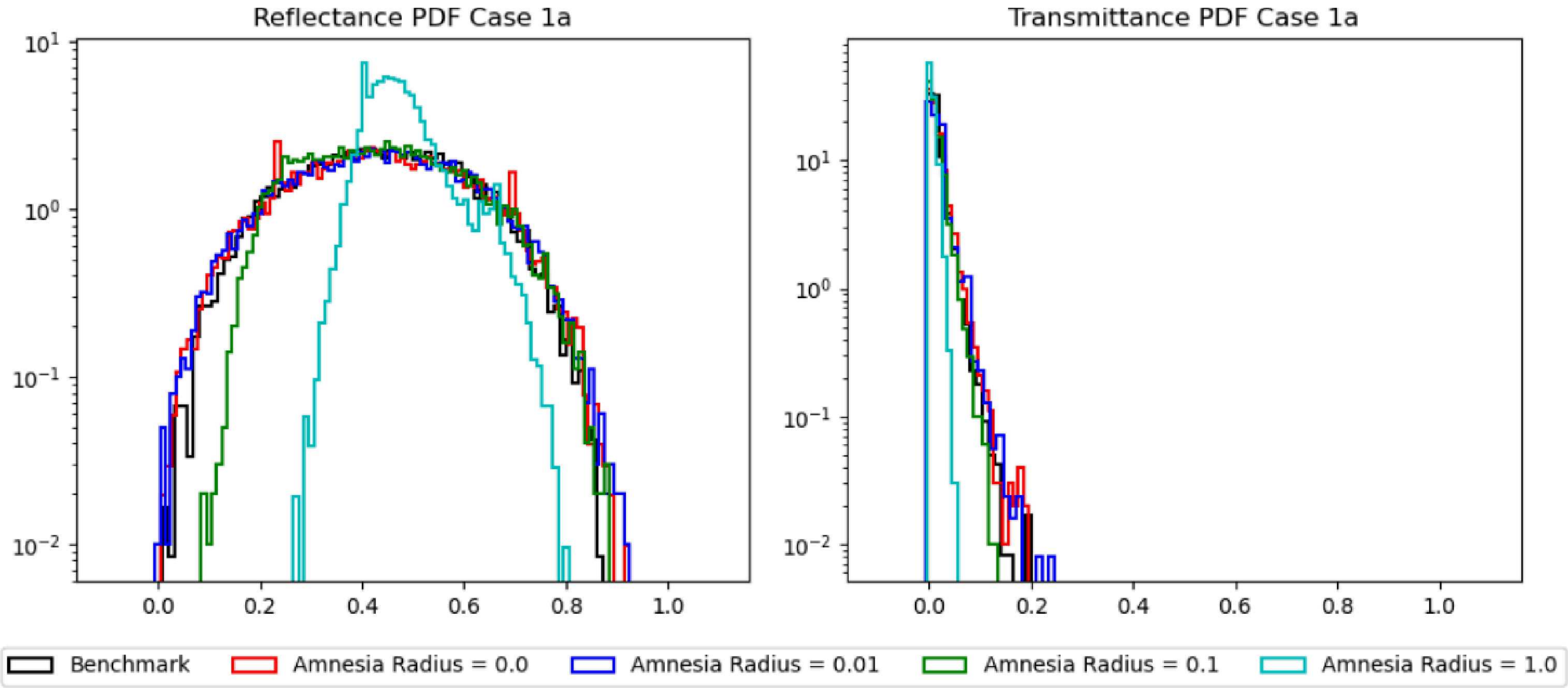
Results & Analysis: Probability Density Function of Case 1a using CoPS3PO and Cohort Size of 100



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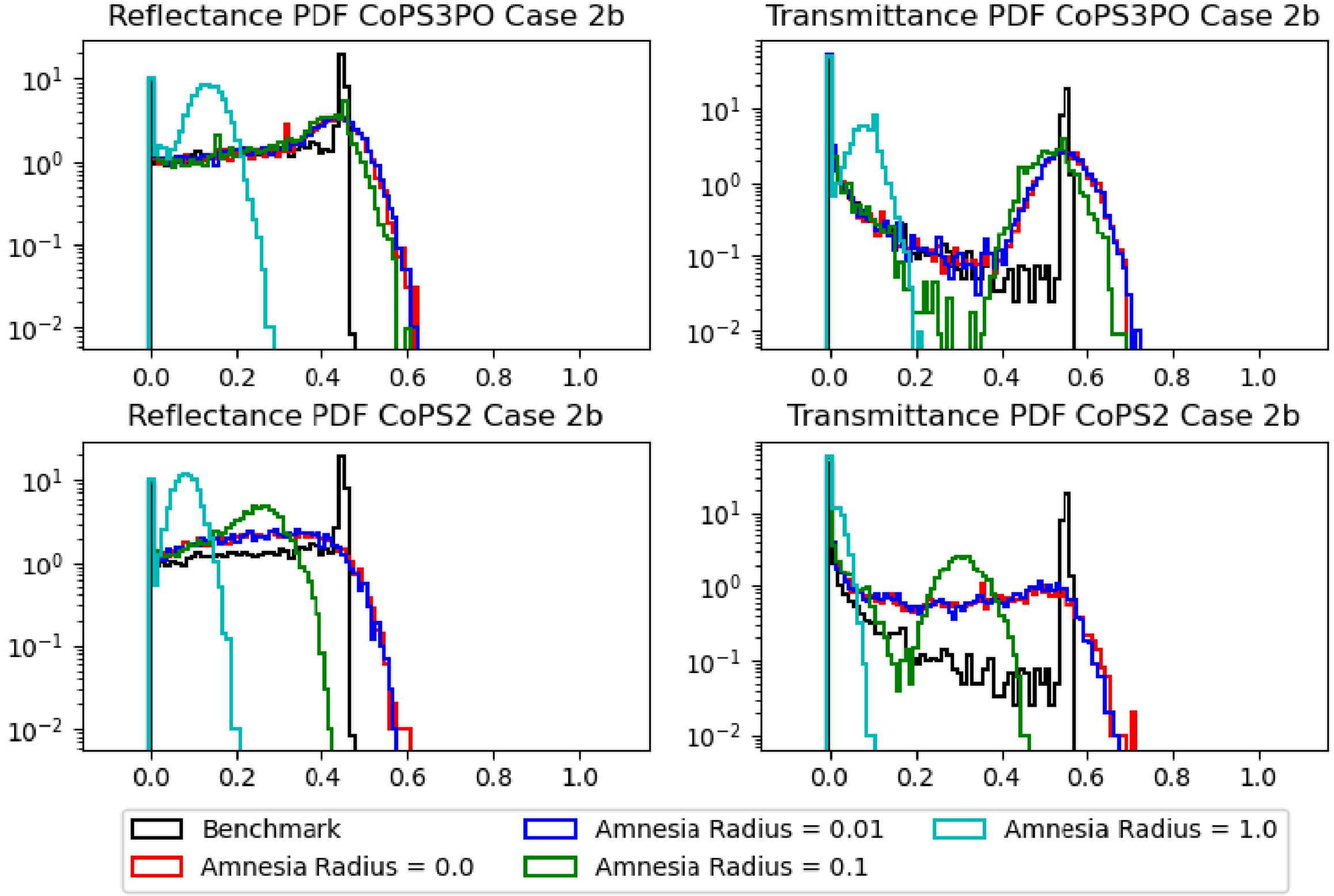
Results & Analysis: Probability Density Function of Case 2b using CoPS3PO and CoPS2 and Cohort Size of 100



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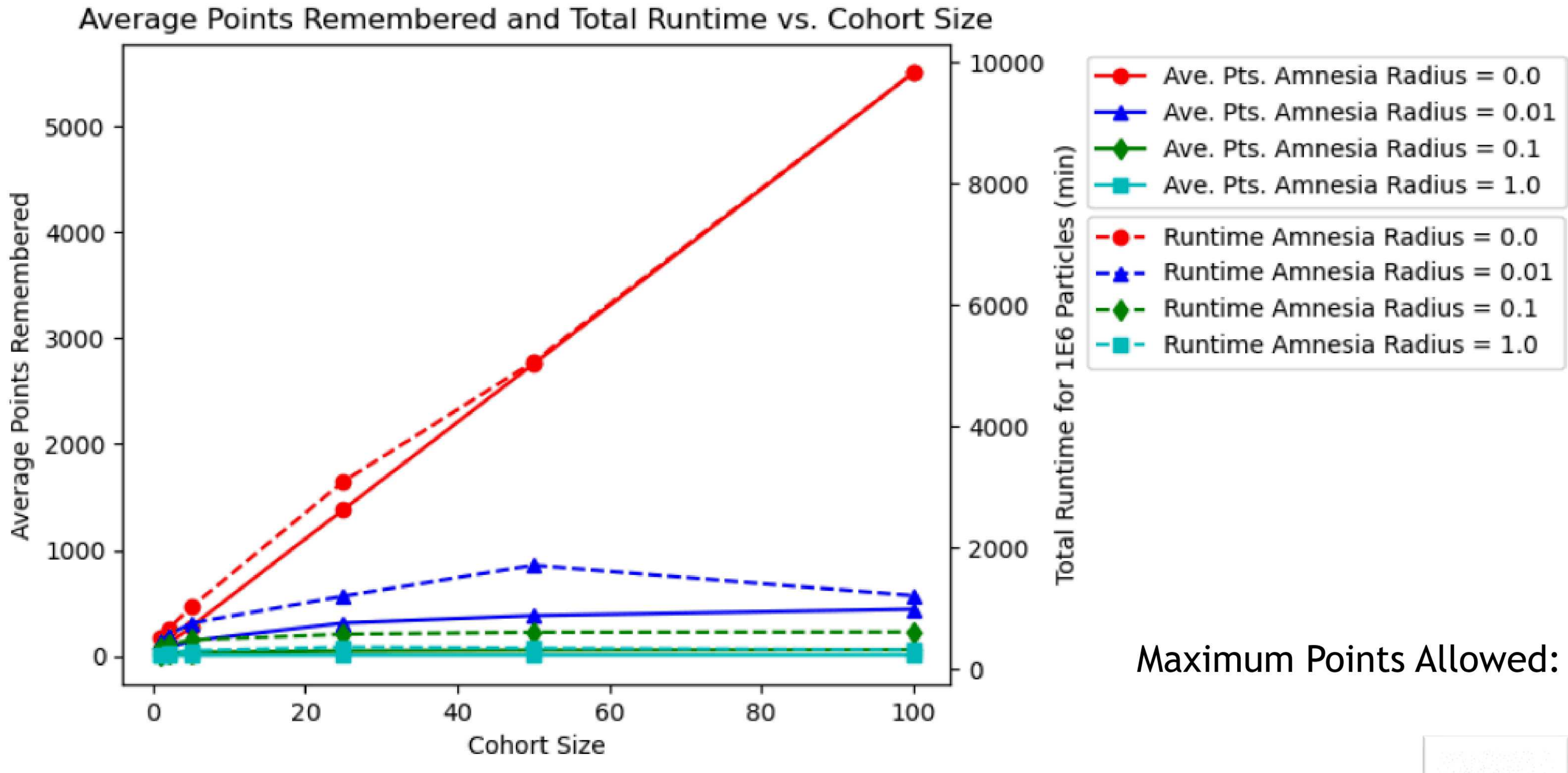
Results & Analysis: Memory and Runtime Numerical Study using CoPS3PO



Maximum Points Allowed: $int\left(\frac{L}{r_a}\right)$

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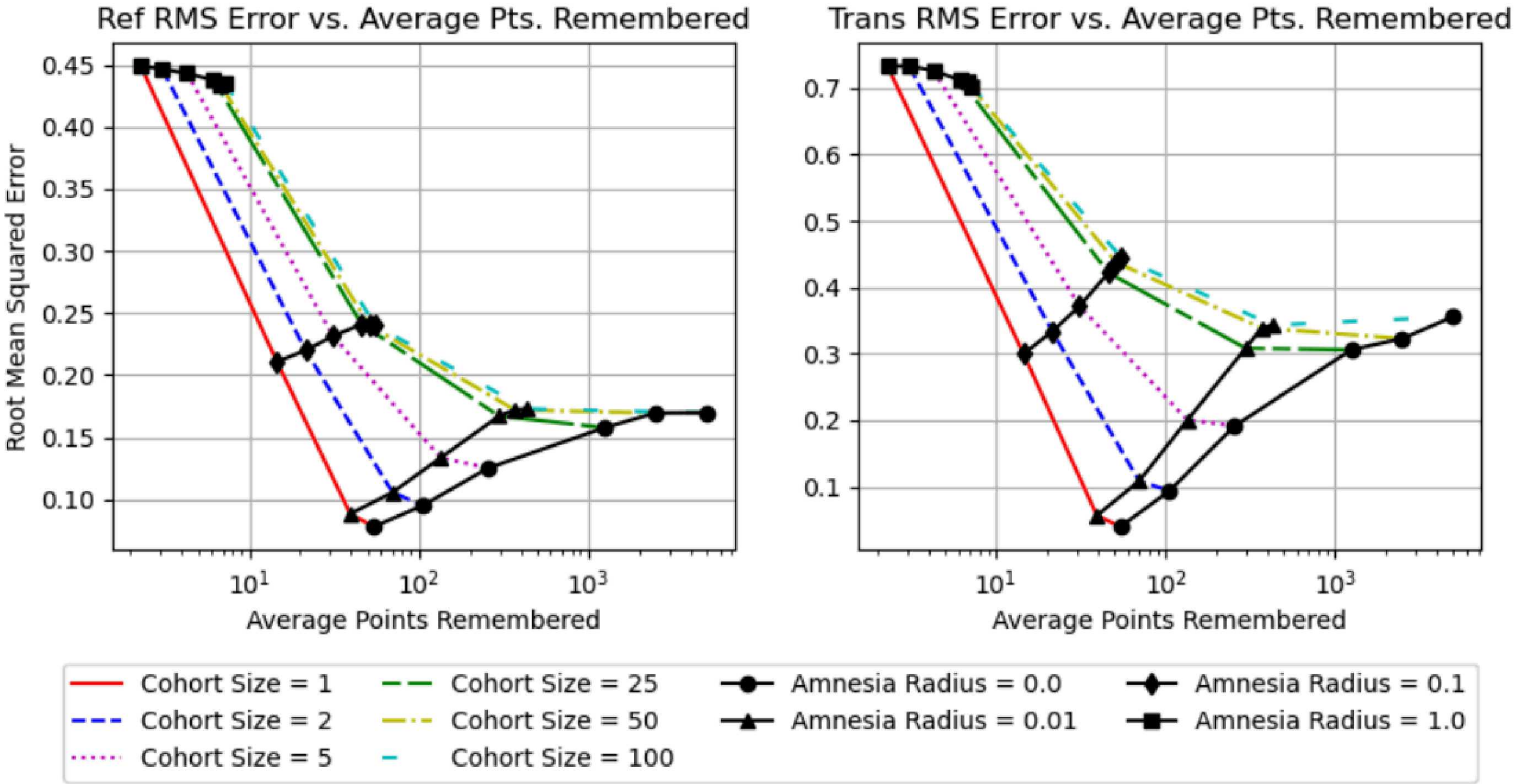
Results & Analysis: Accuracy and Memory Numerical Study using CoPS2



Maximum Points Allowed: $int\left(\frac{L}{r_a}\right)$

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Conclusion

- Demonstrated CoPS's ability to produce accurate mean leakage results and probability density functions using cohorts and amnesia radii.
- CoPS3PO consistently produces less error than CoPS2 using a cohort size of 1. Accuracy is comparable at amnesia radius of at least 0.1.
- More error is introduced in PDFs when using an imperfect conditional probability function (CoPS2) than when using a modest amnesia radius of 0.1 in CoPS3PO. No noticeable deviation from benchmark PDFs when using an amnesia radius of 0.01.
- Limit on total runtime and required computer memory is imposed when using an amnesia radius $r_a \neq 0$.
- Yield significant savings in runtime and memory when using an amnesia radius of 0.01 while retaining a high degree of accuracy.

Future Work

- Investigate hybrid memory-reduction techniques
- Extend memory-reduction techniques to multi-dimensional CoPS
- Non-Markovian-mixed media
- CoPS benchmarks for multi-material media

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Acknowledgements

- Sandia National Laboratories
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- Internship Manager
  - Dr. Joe Castro

- Internship Mentor
  - Dr. Aaron Olson

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Questions?

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