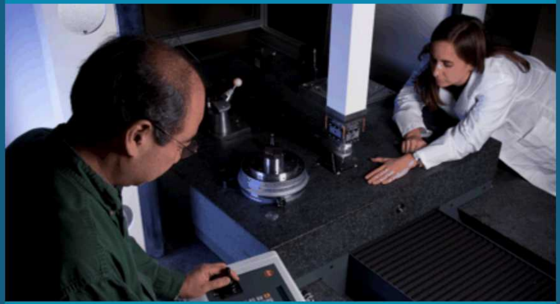
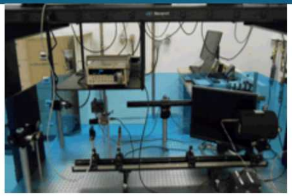
SAND2020-8025C

Evaluating risk in an abnormal world





PRESENTED BY

Collin J. Delker, Primary Standards Lab

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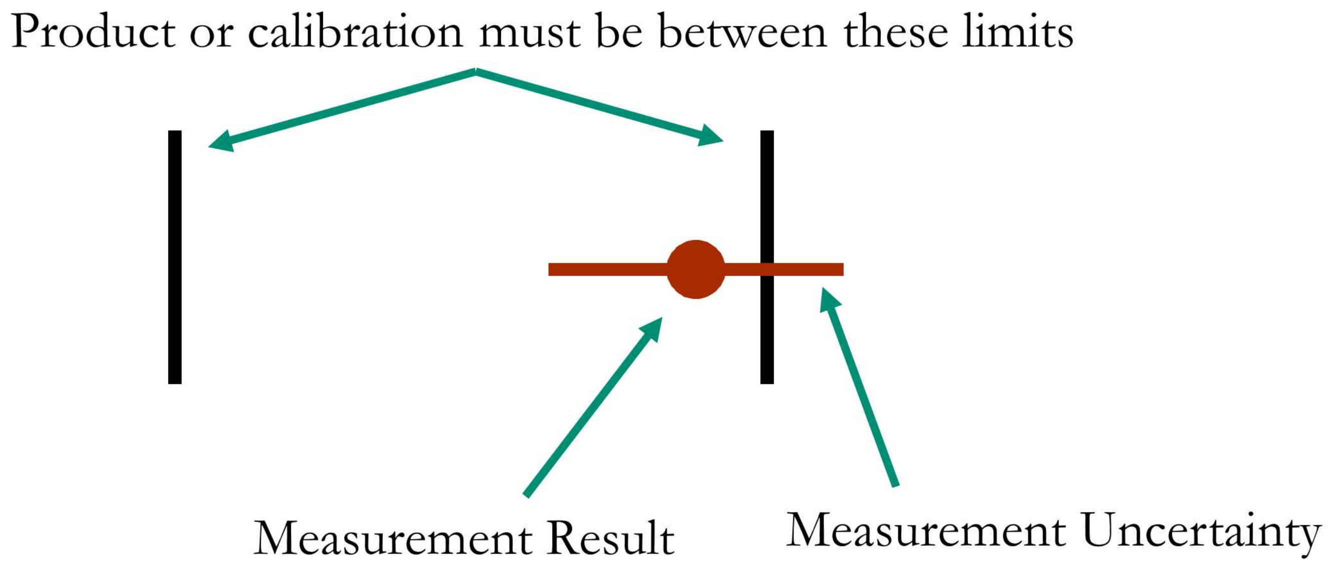
Outline

- Measurements in Decision Making
- Specific Risk and Global Risk
- Test Uncertainty Ratio
- Non-normal distributions in measurement and risk
- Examples
- Skew and Kurtosis
- Guardbanding with non-normal distributions
- Conclusions

Every measurement decision has uncertainty of being incorrect

Measurement results are used to make decisions – pass/fail, accept/reject, go/no-go

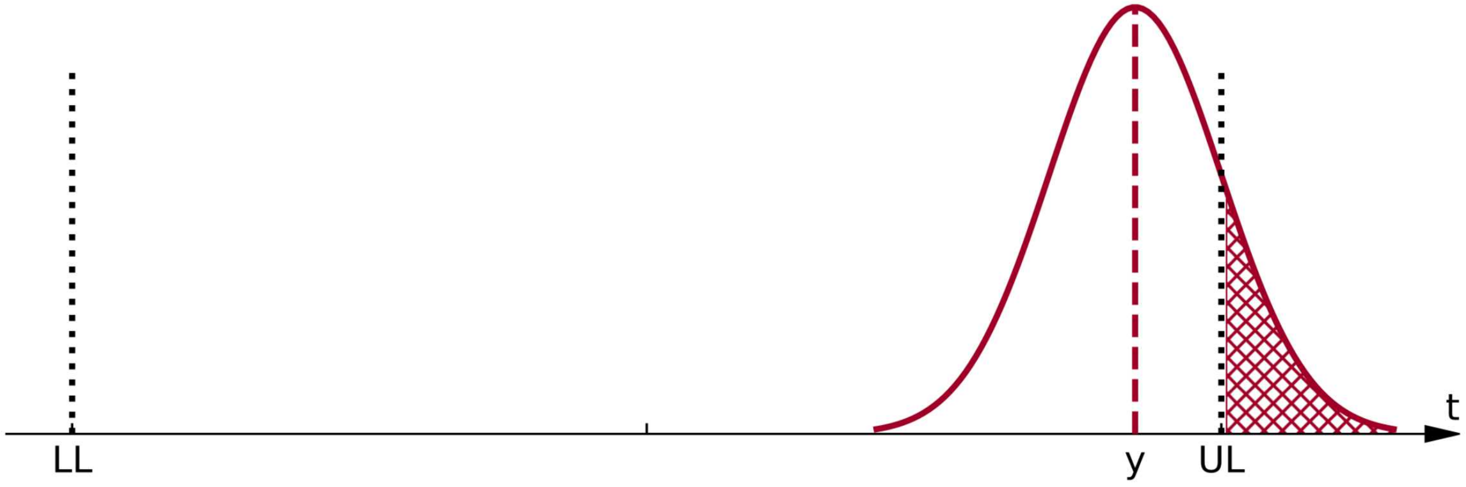
All measurements have uncertainty.



Specific Risk – Probability that decision based on a *specific* measurement is incorrect

- Requires knowledge of measurement uncertainty probability distribution: p_{test}.

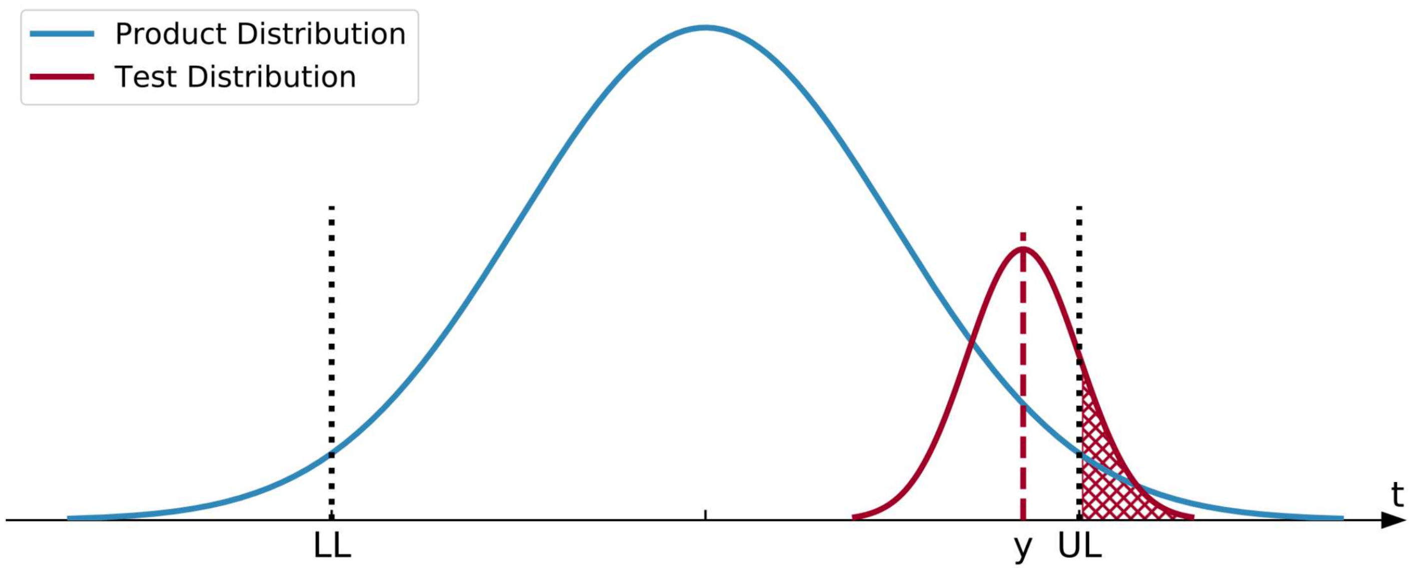
$$p_{FA}|_y = \int_{-\infty}^{LL} p_{test}(t-y)\,dt + \int_{UL}^{\infty} p_{test}(t-y)\,dt$$



Global Risk – Probability that a decision based on measurement of *any* product is incorrect

- Risk of a *future* measurement result being incorrect
- Combines specific risk with probability of encountering a product at the measured value

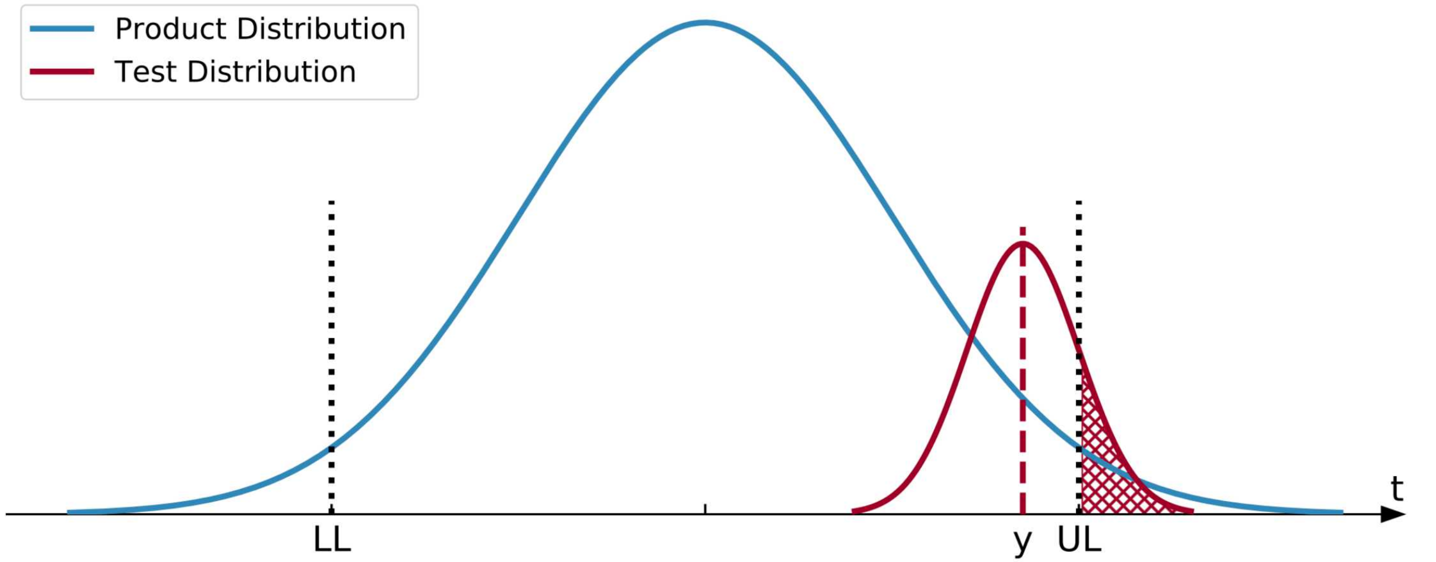
$$PFA = \int_{LL}^{UL} \int_{-\infty}^{LL} p_{test}(t-y) p_{uut}(t)\, dt\, dy + \int_{LL}^{UL} \int_{UL}^{\infty} p_{test}(t-y) p_{uut}(t)\, dt\, dy$$



Global Risk – Probability that a decision based on measurement of *any* product is incorrect

- Requires some prior knowledge about the product, p_{uut}.
- This data can be hard to get

$$PFA = \int_{LL}^{UL} \int_{-\infty}^{LL} p_{test}(t-y) p_{uut}(t)\, dt\, dy + \int_{LL}^{UL} \int_{UL}^{\infty} p_{test}(t-y) p_{uut}(t)\, dt\, dy$$

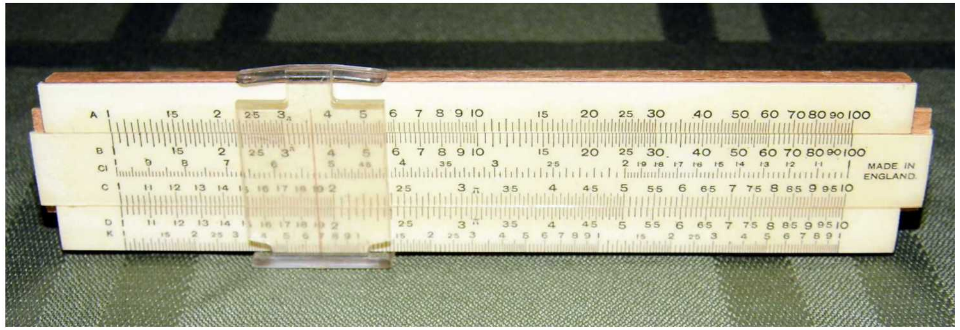


Test Uncertainty Ratio – The easy risk evaluation…

But that math is scary! Let's introduce a risk metric we can calculate easily:

$$TUR = \frac{\pm\, Product\ Tolerance}{\pm\, Measurement\ Uncertainty\ (k = 2)}$$

Now we can evaluate risk with our slide rules.



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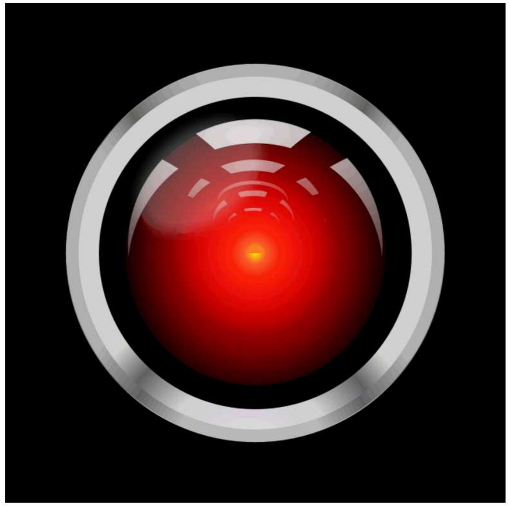
We have computers than can crunch that scary integral in microseconds!



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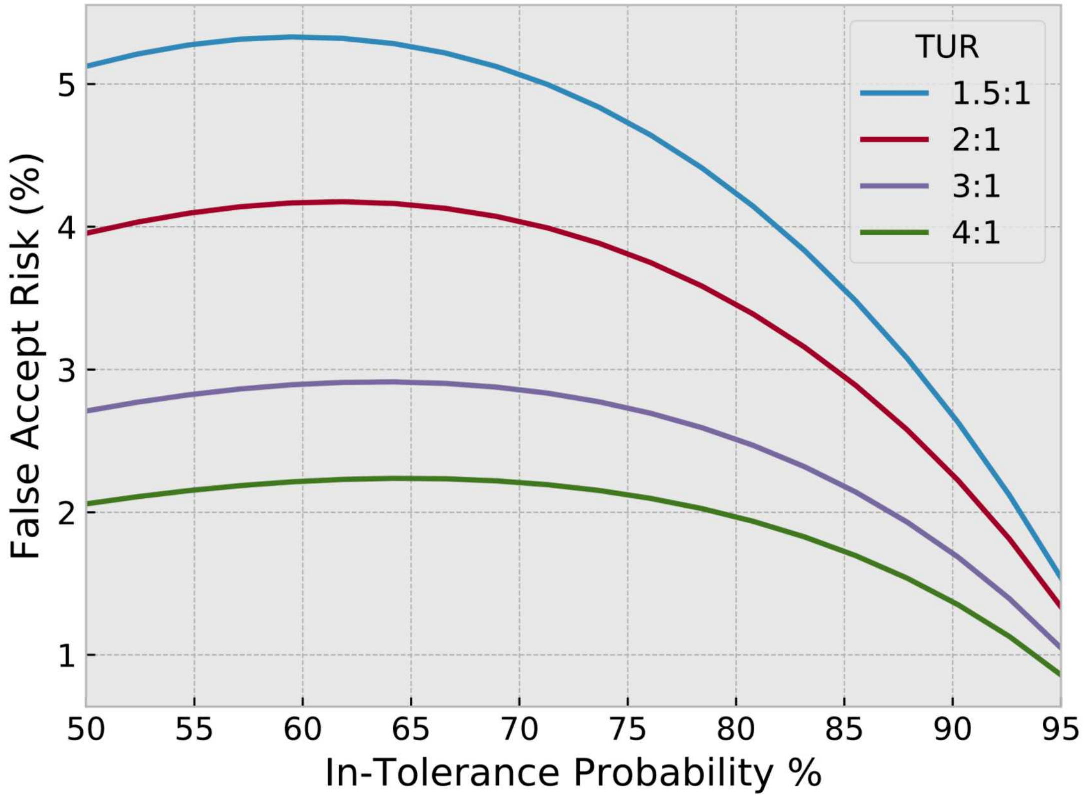
Test Uncertainty Ratio – makes questionable assumptions

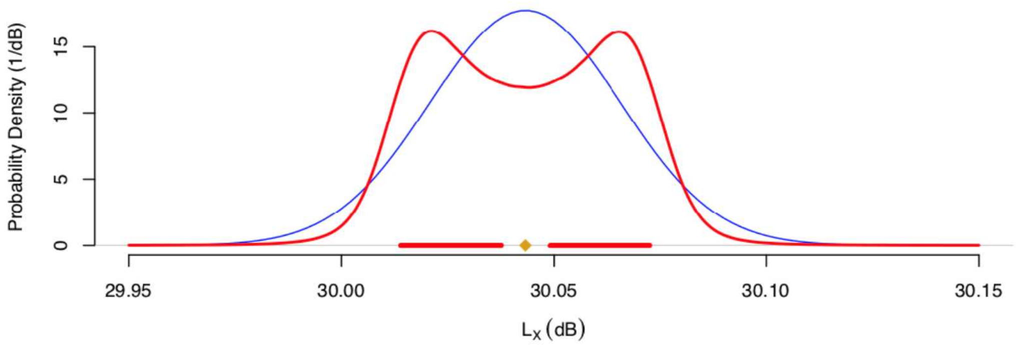
- Assumes 2% is a reasonable PFA for everybody
- 4:1 TUR does NOT guarantee < 2% PFA
- Assumes sufficient data to characterize both distributions
- Not helpful for one-sided limits
- **Assumes distributions are normal and unbiased**



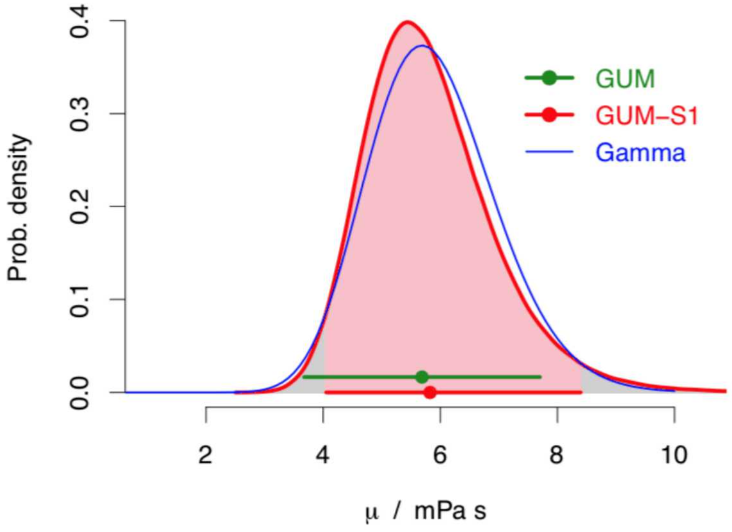
- In-tolerance probability → width of product distribution
- TUR → width of measurement uncertainty

Distributions aren't always normal

- Nonlinearity in measurement model
- Non-normal input distributions
- Physical limitations
- Many examples in JCGM, NIST-1900, etc.



NIST.TN.1900



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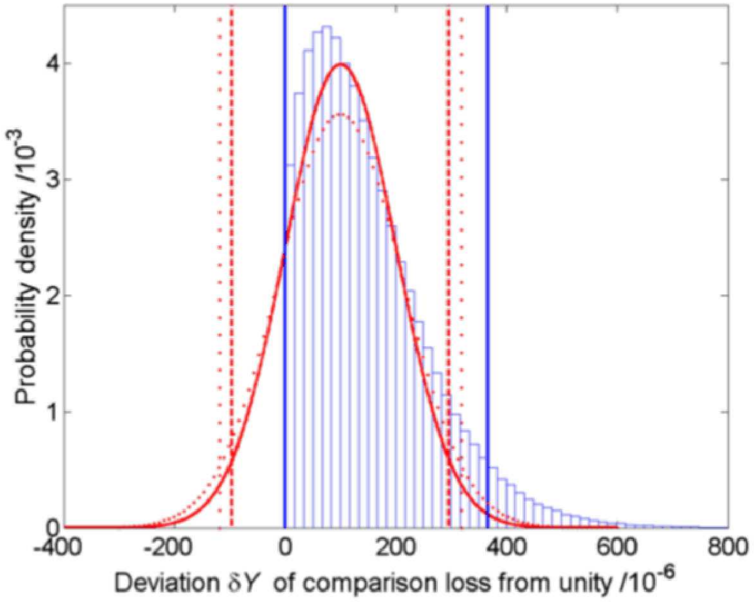


JCGM 101-2008 (GUM-S1)

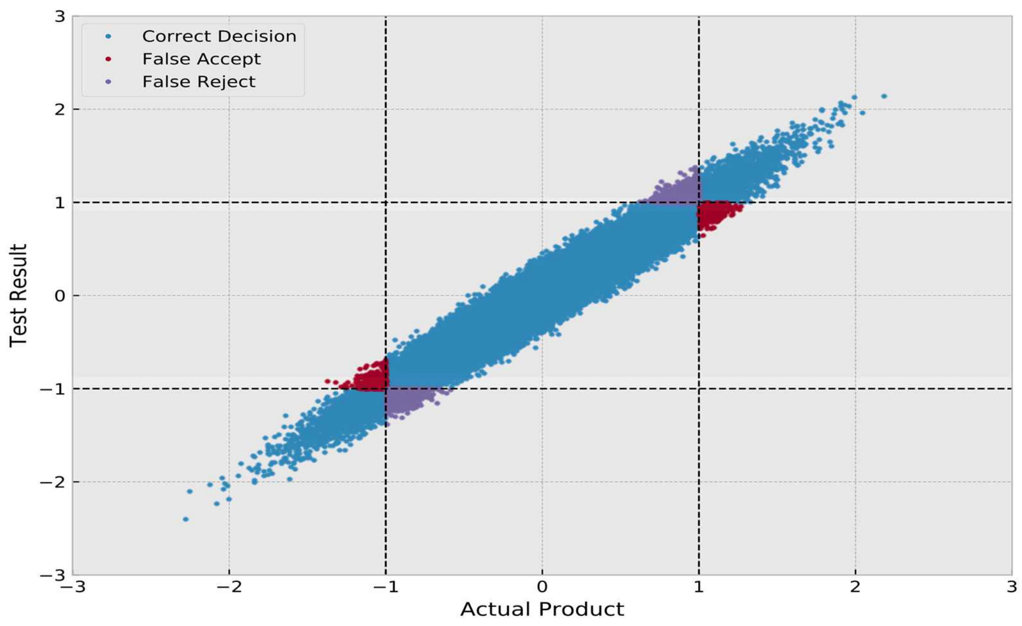
Risk under ANY distribution can be evaluated using numeric integration or Monte Carlo method

- Numeric Integration:

$$PFA = \int_{LL}^{UL} \int_{-\infty}^{LL} p_{test}(t-y) p_{uut}(t)\, dt\, dy + \int_{LL}^{UL} \int_{UL}^{\infty} p_{test}(t-y) p_{uut}(t)\, dt\, dy$$

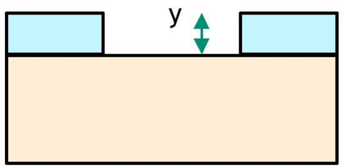
  - For example, in Python:
    - `scipy.integrate.dblquad(func, a, b, gfun, hfun)`

- Monte Carlo Evaluation:
  - Count the dots!



- Sandia Uncertainty Calculator, Risk Tool does both methods, no coding required.
  - https://sandiapsl.github.io
  - Open-source, GPL

Example – Specific risk is dependent on non-normality

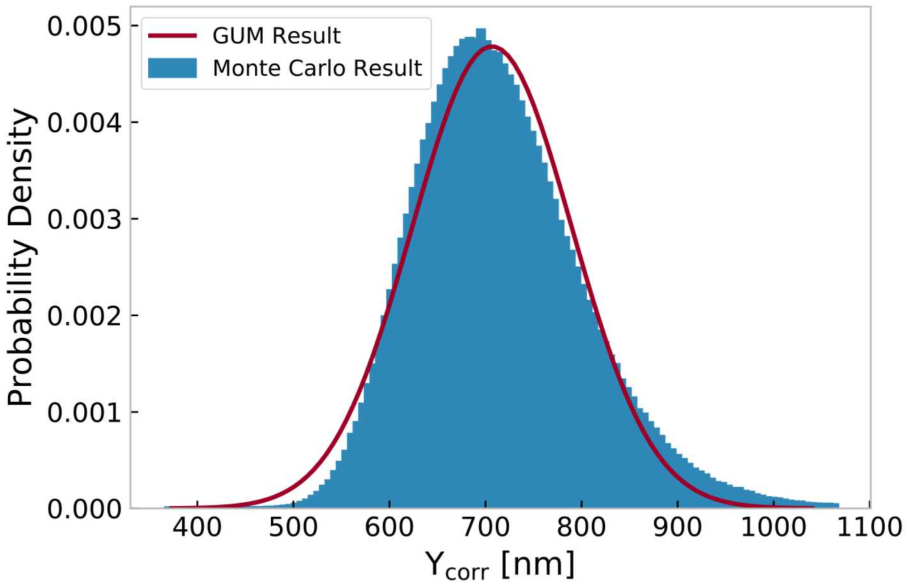


$$Y_{corr} = \frac{X_{cal}}{X_{meas}} Y_{meas}$$

$$Y_{meas} = 698\,nm \;\pm 20\,nm$$
$$X_{meas} = 180\,nm \;\pm 20\,nm$$
$$X_{cal} = 182\,nm \;\pm 5\,nm$$

Normal, k = 1

Measurement Uncertainty, GUM vs MC



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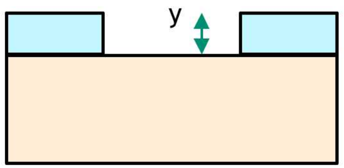

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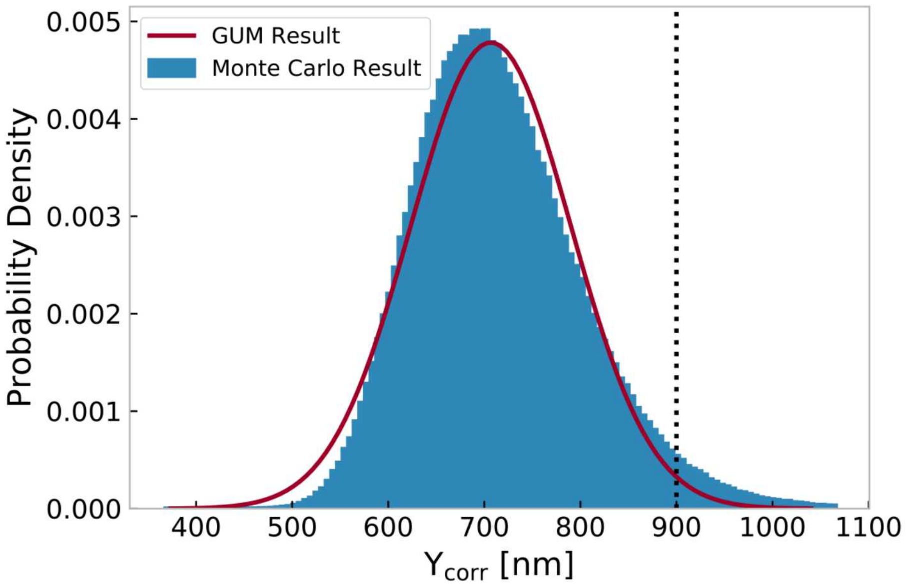

Specific Risk Calculated Using:

Limit	Normal PDF (GUM)	Histogram PDF (MC)
< 900 nm	0.90%	2.97%

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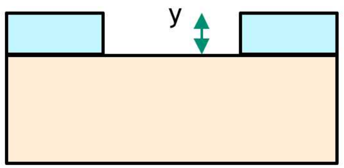

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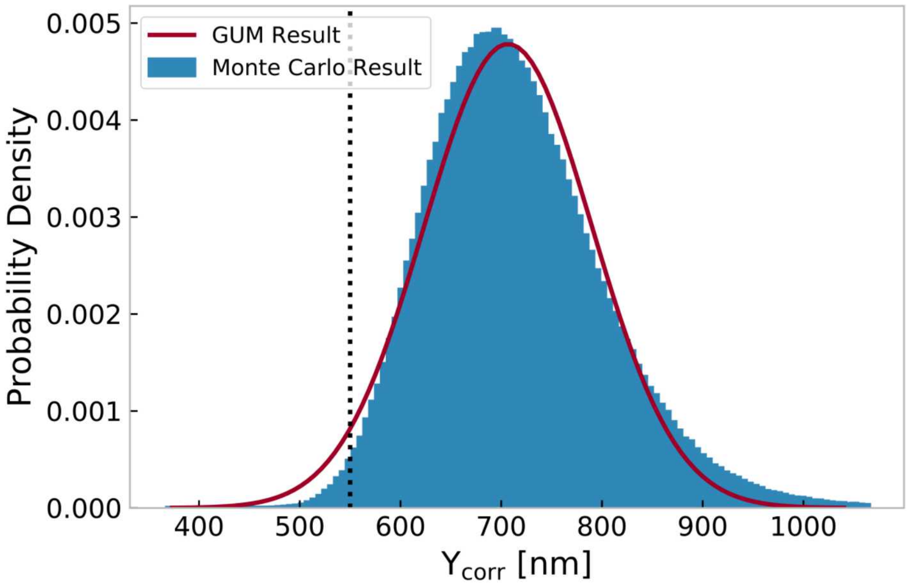


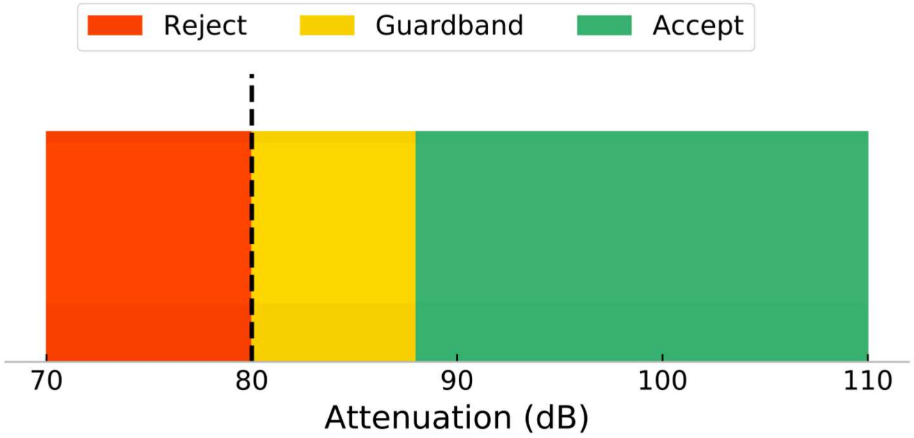
Specific Risk Calculated Using:

Limit	Normal PDF (GUM)	Histogram PDF (MC)
< 900 nm	0.90%	2.97%
> 550 nm	2.75%	0.80%

Example – global risk on a one-sided measurement

Attenuation measurement for product acceptance

- Requirement: > 80 dB
- Measurement Uncertainty: 4 dB (k = 1)
- Typical one-sided guardband: SL + U^{95} = 80 + 8 = > 88 dB



Example – global risk on a one-sided measurement

But there is good historical data on these products!

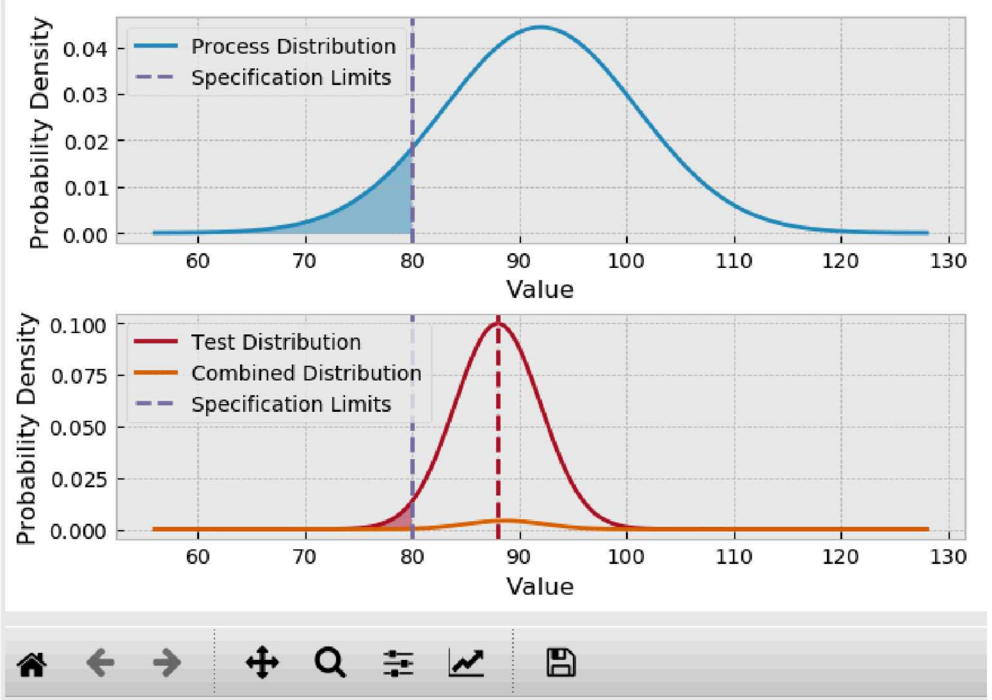
- Mean = 92 dB
- Standard Deviation = 9.0 dB

With 8 dB guardband:

- PFA = 0.05 %
- PFR = 25.0 %

With no guardband:

- PFA = 2.0 %
- PFR = 4.0 %



Process Risk	Test Measurement Risk	Global Risk
Process Risk: 9.1%	TUR: inf	Total PFA: 2.0%
Upper limit risk: 0.0%	Measured value: 88	Total PFR: 4.0%
Lower limit risk: 9.1%	Result: ACCEPT	
Process capability index (Cpk): 0.44	Specific FA Risk: 2.3%	

Sandia Uncertainty Calculator

Example – global risk on a one-sided measurement

A lognormal distribution is a better fit to the historical data

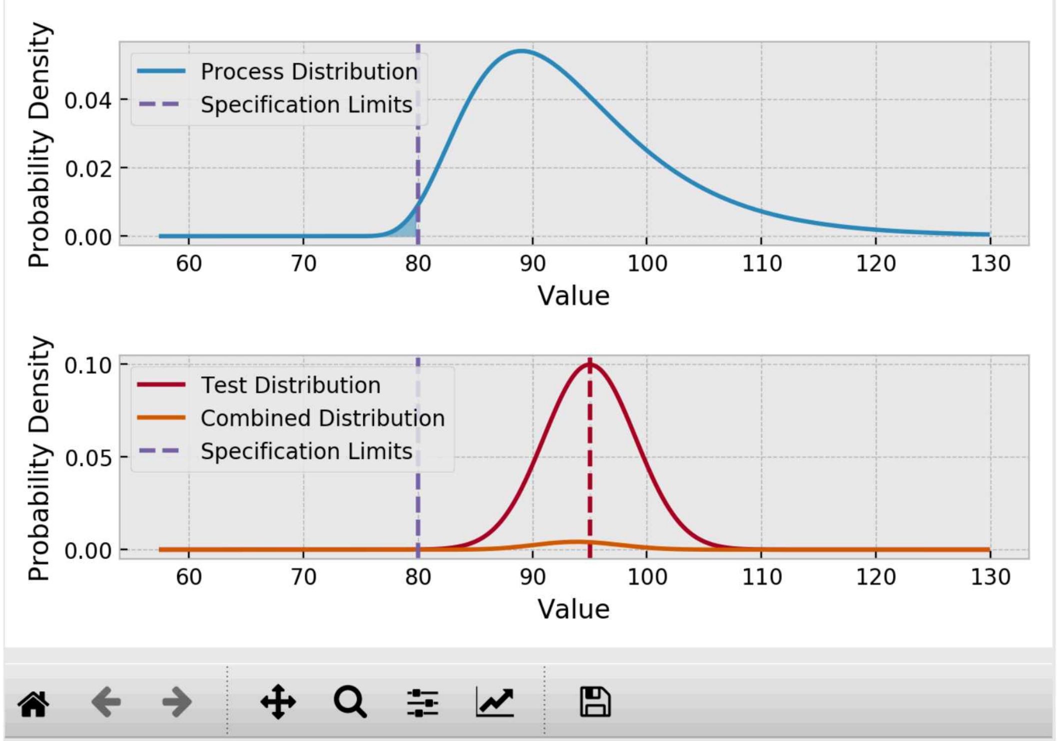
With 8 dB guardband:

- PFA = 0.015 %
- PFR = 29.0 %

With no guardband:

- PFA = 0.44 %
- PFR = 4.0 %

Using the non-normal distribution justifies removing guardband – dropping PFR from 29% to 4%.



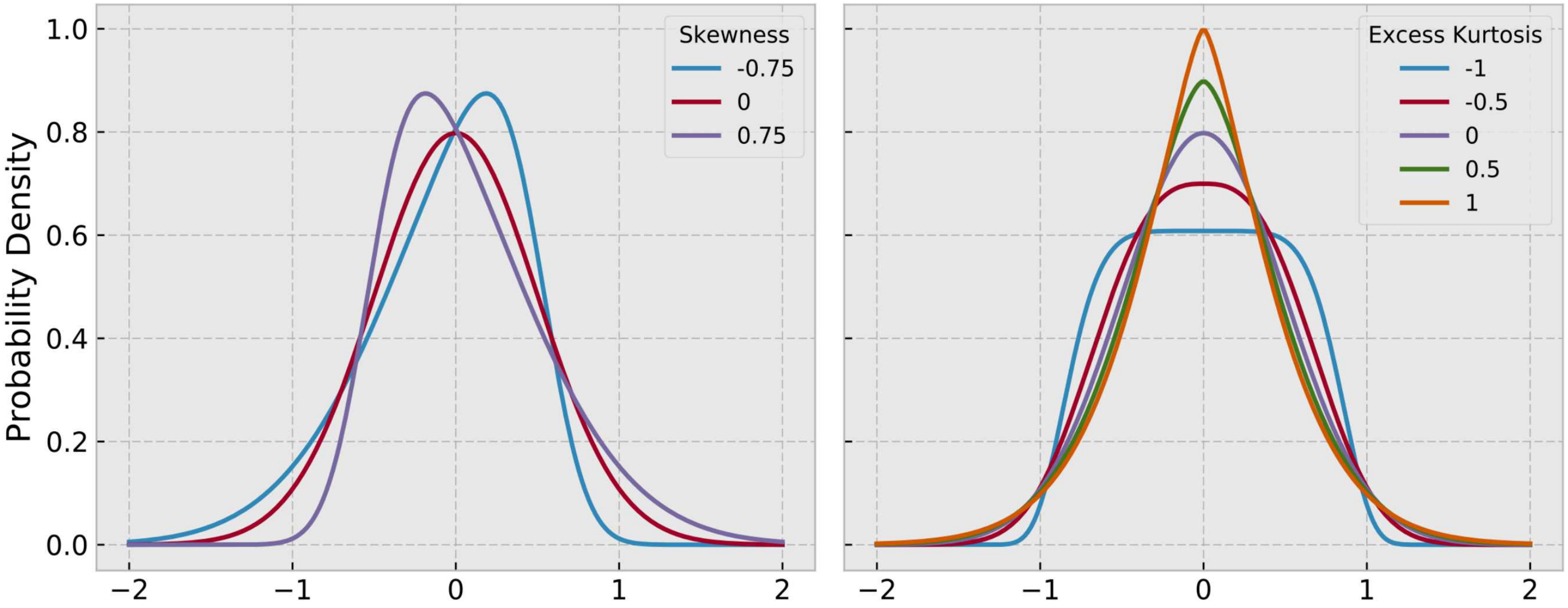
Process Risk	Test Measurement Risk	Global Risk
Process Risk: 1.1%	TUR: inf	Total PFA: 0.44%
Upper limit risk: 0.0%	Measured value: 95	Total PFR: 4.0%
Lower limit risk: 1.1%	Result: ACCEPT	
Process capability index (Cpk): 0.76	Specific FA Risk: 0.0088%	

Sandia Uncertainty Calculator

Skew and Kurtosis – two statistics for quantifying non-normality

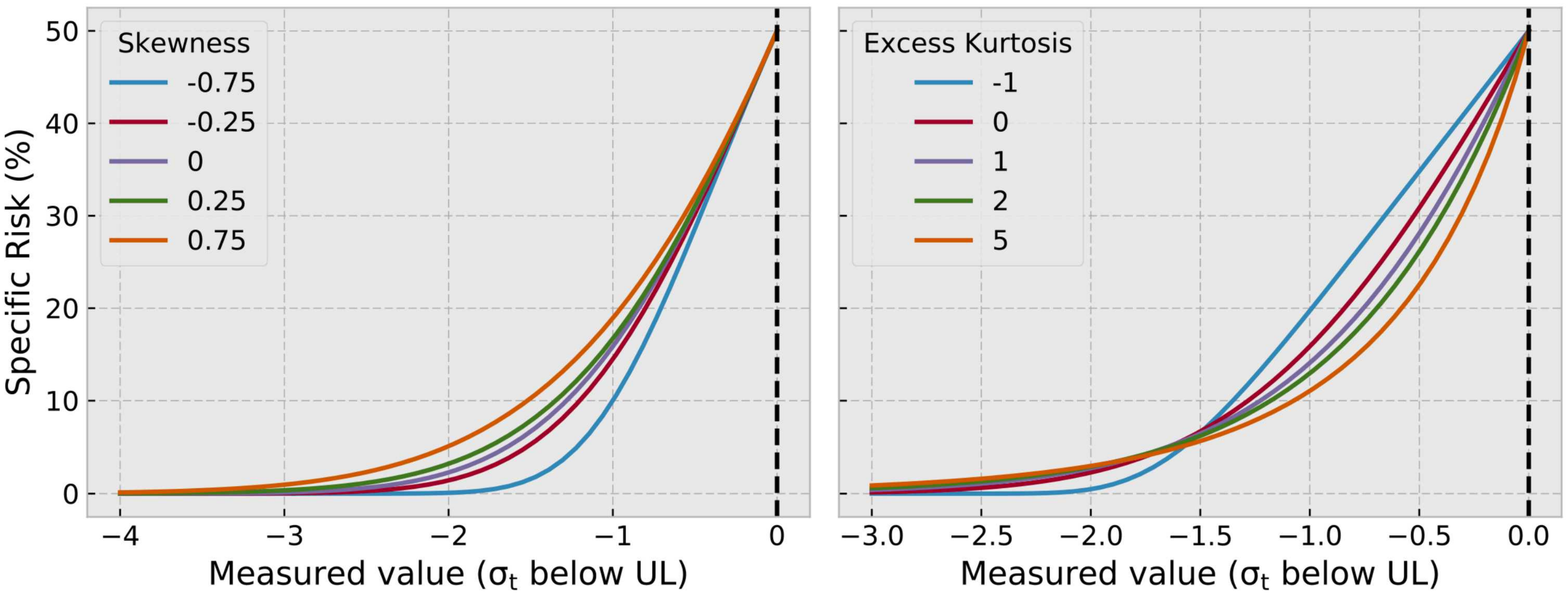
Skewness: measures symmetry of a distribution

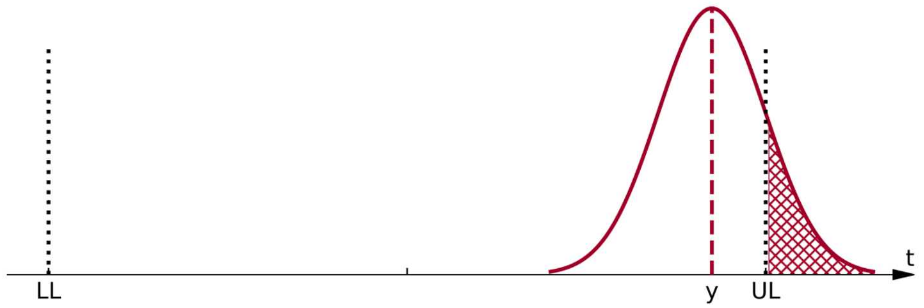
Kurtosis: compares strength of peak vs. tails of distribution



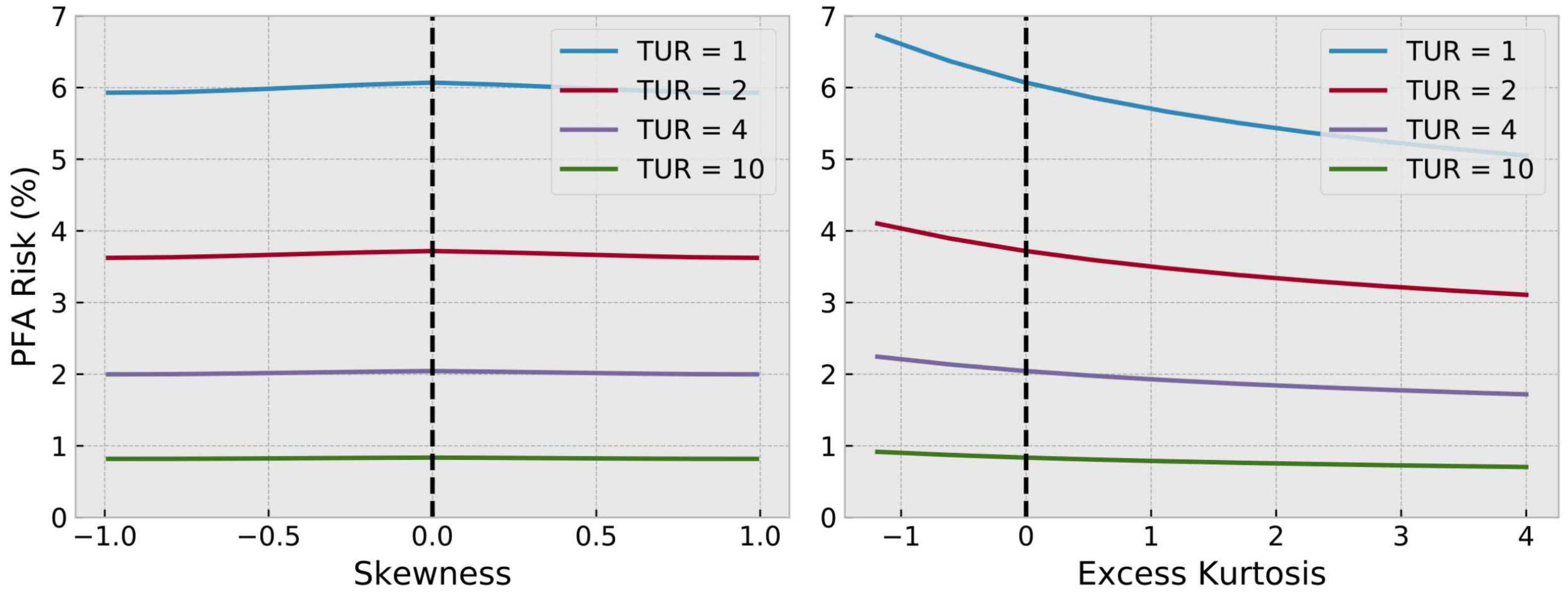
All curves have same median and standard deviation

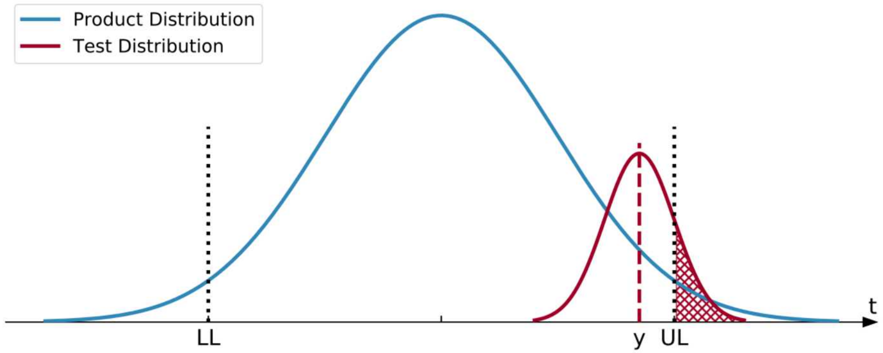
Skew and kurtosis affect specific risk





Kurtosis affects global risk, but skew does not

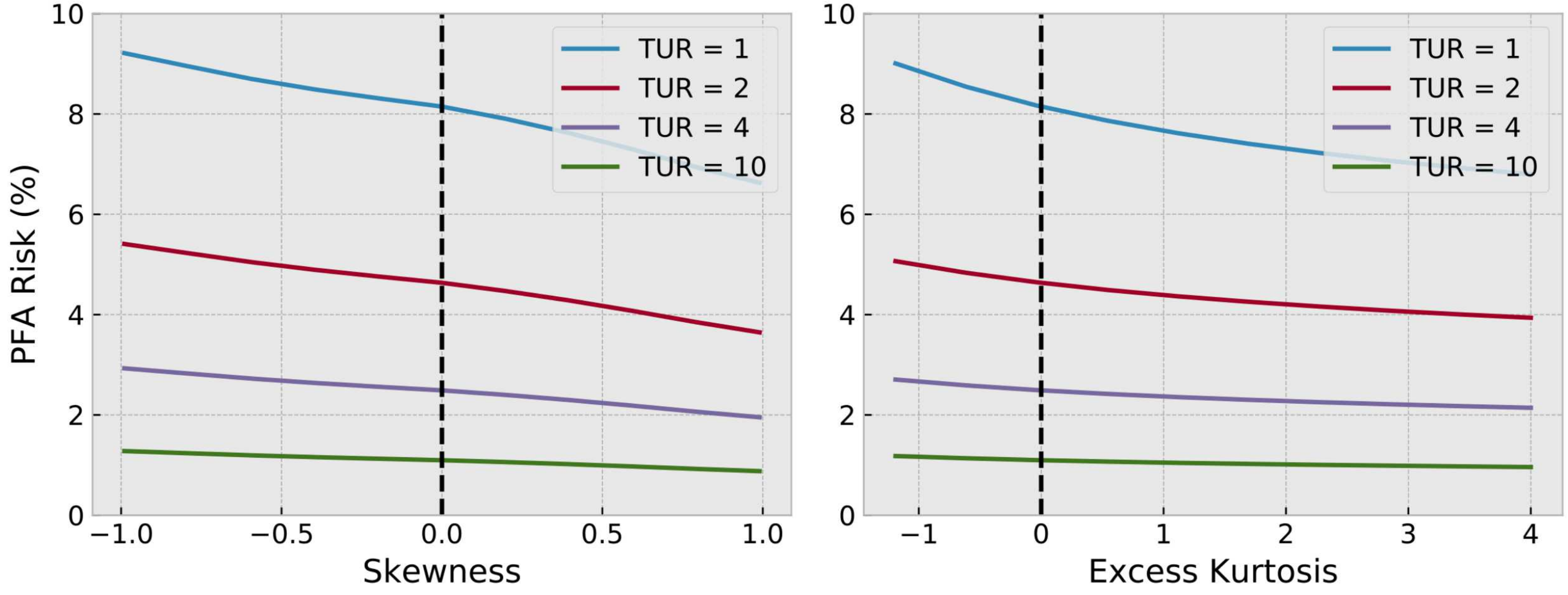


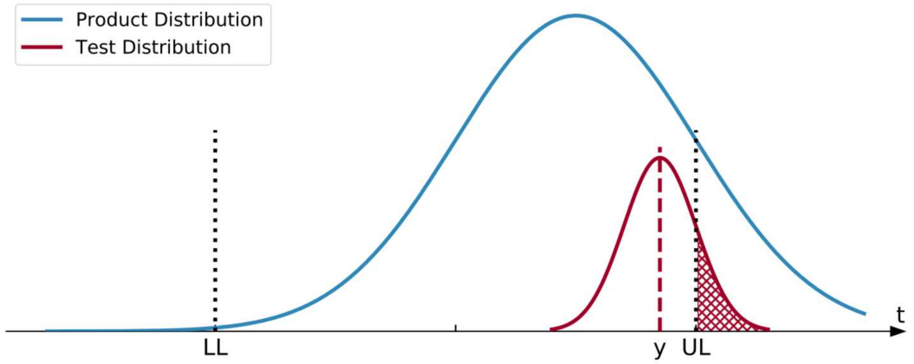


Product Distribution: Normal, 75% itp

Yet skew DOES affect global risk if the product distribution is biased

Bias: Product distribution not centered between limits



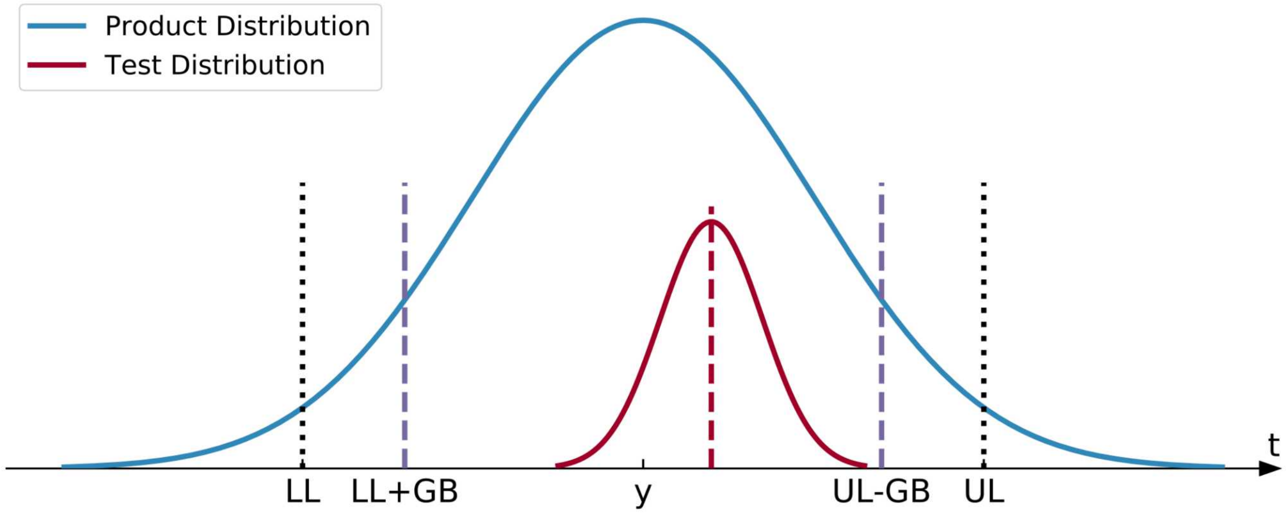


Product Distribution: Normal, 75% itp, 80% bias

Guardband factors can be solved for numerically to target a desired PFA

- Specify an acceptable PFA, numerically solve for GB
- Use numerical minimization techniques

$$PFA_{GB} = \int_{LL+GB}^{UL-GB} \int_{-\infty}^{LL} p_{test}(t-y) p_{uut}(t)\, dt\, dy + \int_{LL+GB}^{UL-GB} \int_{UL}^{\infty} p_{test}(t-y) p_{uut}(t)\, dt\, dy$$



Conclusions

- Risk calculations require good prior knowledge of product distribution and measurement uncertainty
- Know the limitations and assumptions of the 4:1 TUR rule. Use it when the assumptions are met (or if a full risk calculation cannot be done due to insufficient data about the product).
- Non-normal behavior, including bias, can affect global and specific risk.
- It is not difficult to account for non-normal effects with modern computing.

Acknowledgements

Sandia Primary Standards Lab

- Otis Solomon
- Ricky Sandoval
- Raegan Johnson
- Roger Burton

Sandia Statistical Sciences Department

- Steve Crowder

Sandia Uncertainty Calculator

- https://sandiapsl.github.io