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Nonlinear Model Predictive Control for Hypersonic Vehicles

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Contents

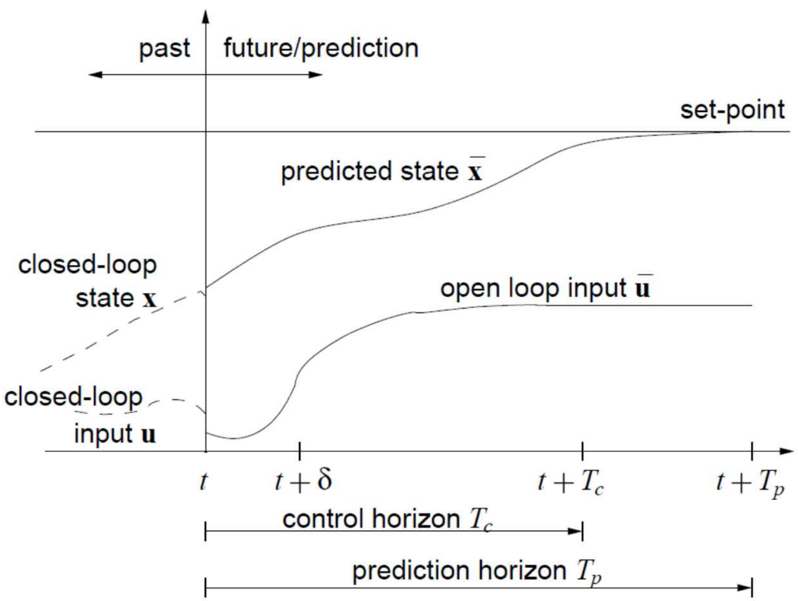
Project Objective

- Develop a guidance and control algorithm for hypersonic vehicle flight based on nonlinear optimization achieving:
  - Nonlinear system dynamics with uncertainties
  - State and control constraints: No-fly zone, waypoints, target, and limited actuation
  - Real-time implementation, stability, and robustness for hypersonic flight

Image Credit: Sandia National Laboratories

Nonlinear Model Predictive Control

- Nonlinear Model Predictive Control (NMPC)
  - Constrained optimal control problem over a receding horizon
  - Nonlinear system dynamics and constraints integration into NMPC framework [Findeisen and Allgöwer 2002]



$$\min_{u(\cdot)} \; J(x(\cdot),u(\cdot)),$$

where $J(x(\cdot),u(\cdot)) = \Phi(x(t+N)) + \sum_{k=t}^{t+N-1} L(x(k),u(k)),$

subject to

$$x(k+1) = f(x(k),u(k)), \quad f: \mathrm{R}^{n+m} \to \mathrm{R}^n,$$
$$x(t) = x_t, \quad x_t \in \mathrm{R}^n,$$
$$C(x(k),u(k)) \le 0, \quad C: \mathrm{R}^{n+m} \to \mathrm{R}^l,\ k = t,\cdots,t+N-1,$$
$$\bar{C}(x(k))) \le 0, \quad \bar{C}: \mathrm{R}^n \to \mathrm{R}^q,\ k = t,\cdots,t+N.$$

  - **Main challenge:** computational complexity for real-time implementation

Method for Real-Time Computation

- Rapid optimization approaches and solvers
  - Advantageous for
    - Nonlinear systems
    - Systems with changing parameters

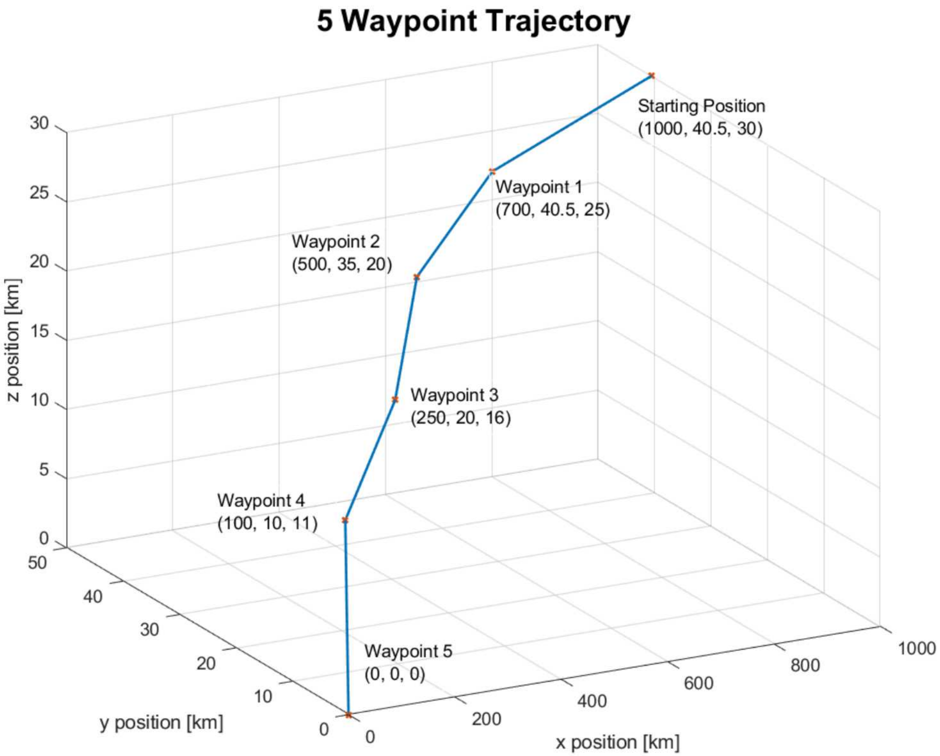
- Interior Point Optimizer (IPOPT) [Wächter & Bigeler 2005]
  - Interior Point (IP) Method
    - Effective method for solving nonlinear constrained optimization problems
    - Directly integrates equality and inequality constraints into objective function optimization
  - Widely available in solvers in commercial and open source spaces
    - MATLAB/Simulink
    - COIN-OR IPOPT

Mission Structure

- Hypersonic Glide Vehicle (HGV) governed by 3 degrees-of-freedom (DoF) equations of motion
- No propulsion power

Mission Structure

1. Hypersonic release (no engine thrust)
2. Hypersonic glide through No-Fly-Zone corridor (Corridor is 300km in length and 1km in width)
3. Waypoint 1 at the end of No-Fly-Zone corridor (altitude decreases)
4. Waypoint following sequence from 1 to 5 decreasing in altitude



3DoF HGV Model

- Equations of motion [Jorris 2007, Hood et al. 2019]

$$\dot{x} = V\cos(\gamma)\cos(\theta)$$

$$\dot{y} = V\cos(\gamma)\sin(\theta)$$

$$\dot{h} = V\sin(\gamma)$$

$$\dot{V} = -\frac{\rho V^2 S C_L^*(1 + c_l^2)}{4E^* m} - g\sin(\gamma)$$

$$\dot{\gamma} = \frac{\rho c_l C_L^* S V\cos(\sigma)}{2m} - \frac{S\sin(\sigma)}{mV} - \frac{g\cos(\gamma)}{V}$$

$$\dot{\theta} = \frac{\rho c_l C_L^* S V\sin(\sigma)}{2m\cos(\gamma)} + \frac{S\cos(\sigma)}{mV\cos(\gamma)}$$

$$\boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} x \\ y \\ h \\ V \\ \gamma \\ \theta \end{bmatrix}, \boldsymbol{u} = \begin{bmatrix} c_l \\ \sigma \end{bmatrix} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

		Symbol	Description	Value
State Variables	x_1	x	Position in x direction	
	x_2	y	Position in y direction	
	x_3	h	Altitude	
	x_4	V	Velocity	
	x_5	γ	Flight path angle	
	x_6	θ	Heading angle	
Control Input	u_1	c_l	Coefficient of lift ratio C_L/C_L^*	$0 \le c_l \le 2$
	u_2	σ	Bank angle	$-\frac{\pi}{3} \le \sigma \le \frac{\pi}{3}$
Constant Parameters		S	Wing reference area	$0.458\ m^2$
		m	Mass	$907\ kg$
		E^*	Max L/D Ratio	3.45
		C_L^*	Coefficient of lift for Max L/D Ratio	0.45

*Note: Air density is recalculated at every sampling time
$\rho = \rho_{sl} e^{-\beta h}$
where $\rho_{sl} = density\ at\ sea\ level\ \left(1.225\ \frac{kg}{m^3}\right)$,
$\beta = air\ density\ decay\ factor\ (-0.14 km^{-1}),\ h = altitude\ (km)$

Optimization Problem

- NMPC problem formulation

Minimize $$J(x(\cdot),u(\cdot)) = (x(t+N)-x_f)^T P(x(t+N)-x_f) + \sum_{k=t}^{t+N-1} (x(k)-x_f)^T Q(x(k)-x_f) + u(k)^T R u(k),$$

subject to
$$x(k+1) = x(k) + T_s f(x(k),u(k)),$$
$$x(t) = x_i,$$
$$0 \le u_1(k) \le 2,$$
$$-\frac{\pi}{3} \le u_2(k) \le \frac{\pi}{3},$$
$$40 \le x_2(k) \le 41, \quad \text{if } 700 \le x_1(k) \le 1000$$
$$x(t+N) = x_f \quad x_f \text{ is updated from waypoints}$$

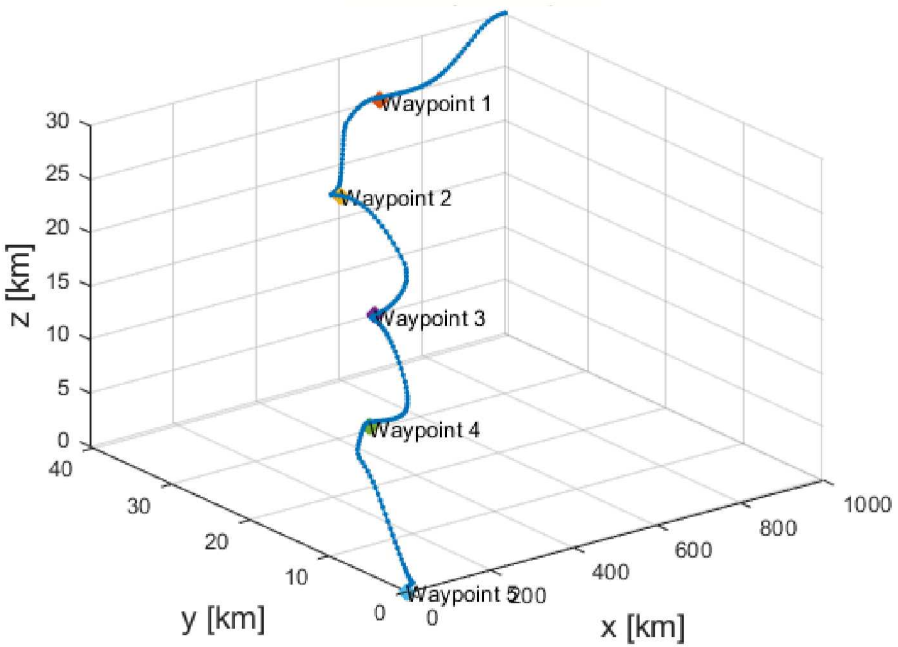
- Simulation setup

Symbol	Parameters	Value	Symbol	Parameters	Value
V_i	Initial velocity	$4.08\ km/s$ (Mach 12)	P	Terminal cost matrix	$2Q$
Q	Weighting matrix	$diag([0.0002, 0.033, 0.033, 0, 0, 0])$	N	Prediction horizon	20
R	Weighting matrix	$diag([1, 1])$	T_s	Sampling time	2 sec

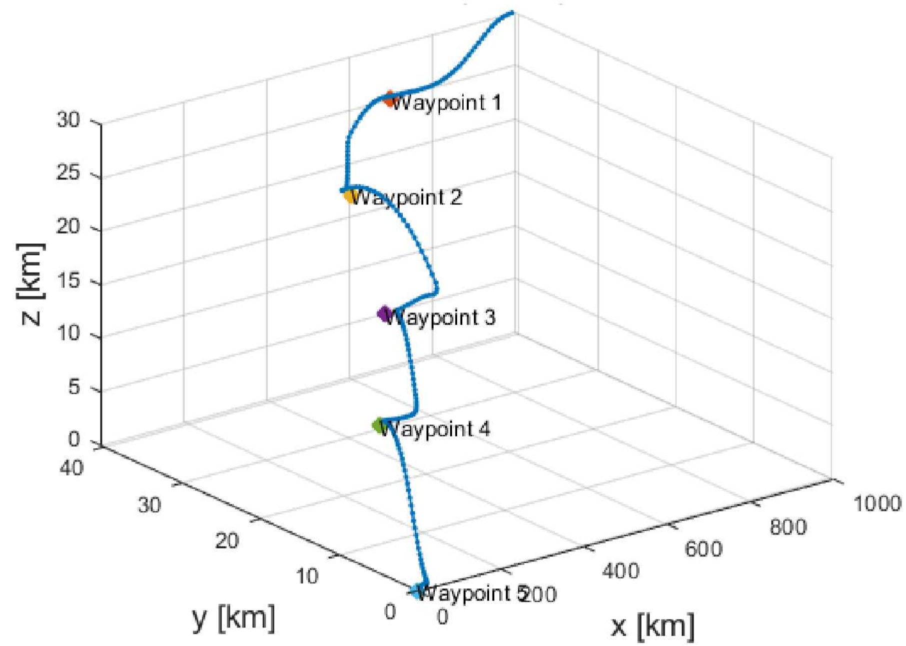
Simulation Results: 5 Waypoint Trajectory (1)

- Compare results by MATLAB fimincon function with results by IPOPT Simulink
- Trajectory comparison

fmincon



IPOPT Simulink

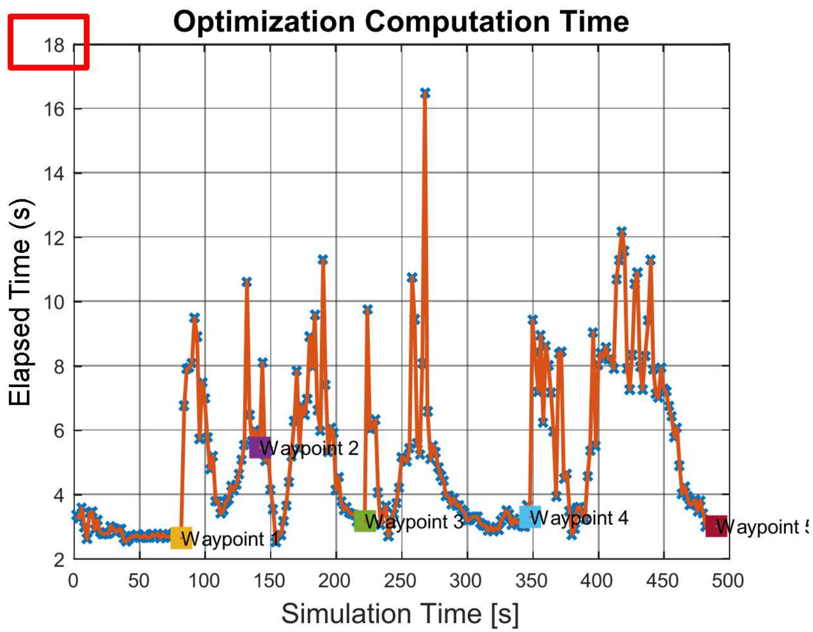


HGV takes discrete turns in y and z before completing x state goal

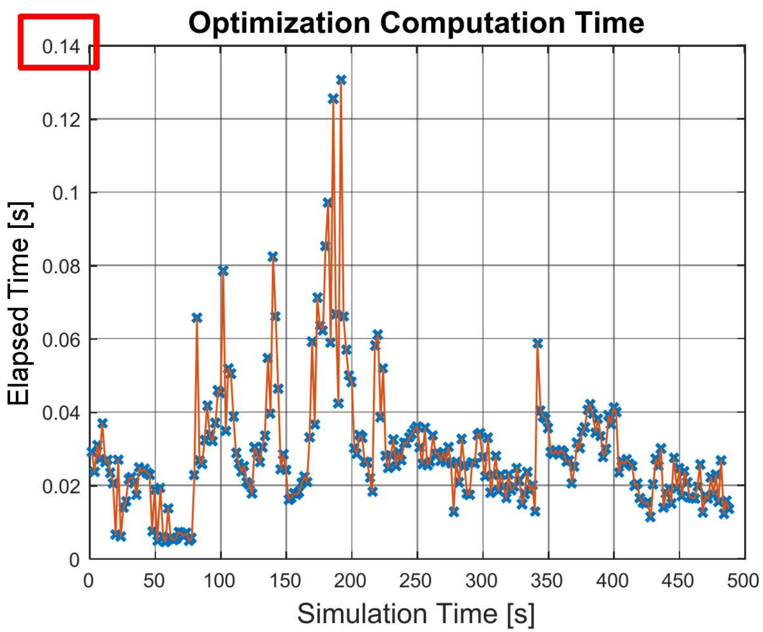
Simulation Results: 5 Waypoint Trajectory (2)

- Computation time
  - Simulations on a computer with Intel(R) Core i7 CPU @ 1.60 GHz

fmincon



IPOPT Simulink

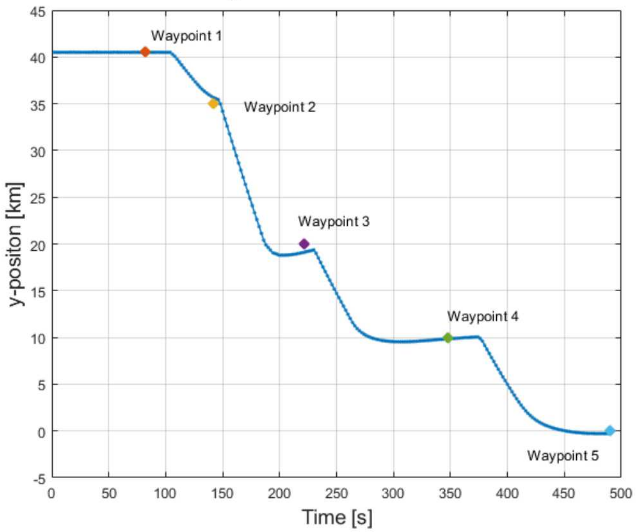


Computation Time [sec]	fmincon	IPOPT Simulink
Average	5.08	**0.029**
Maximum (worst case)	16.49	**0.131**

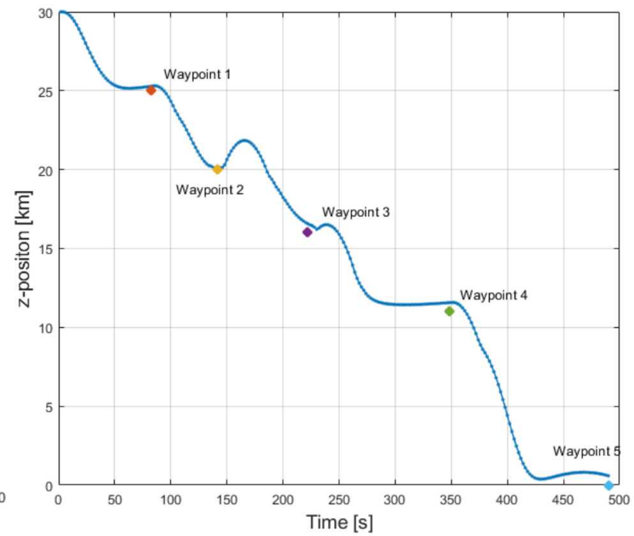
Sampling time: 2 sec

Simulation Results: 5 Waypoint Trajectory (3)

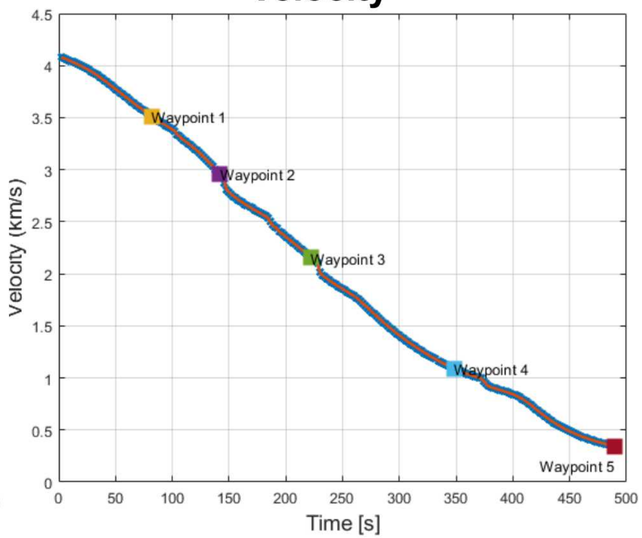
y Position



Altitude

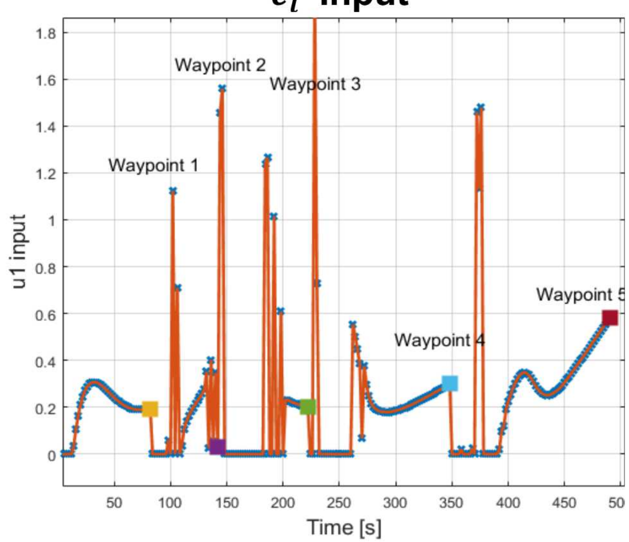


Velocity

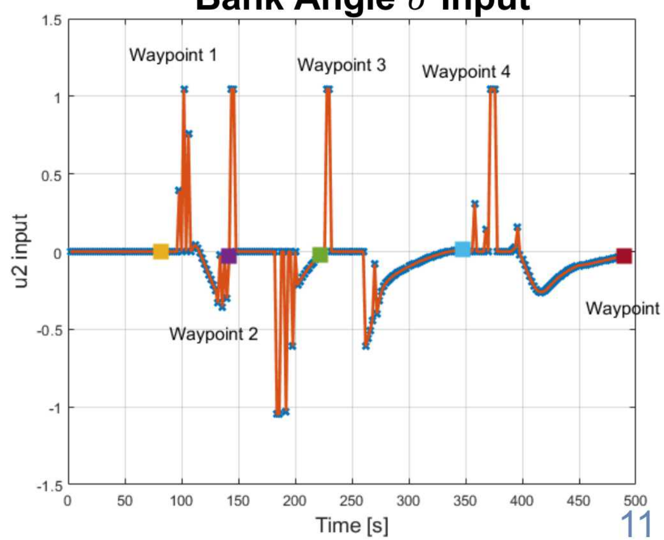


- Velocity is well maintained
- Control inputs in large bursts as the model maneuvers before and after waypoint checks

c_l Input



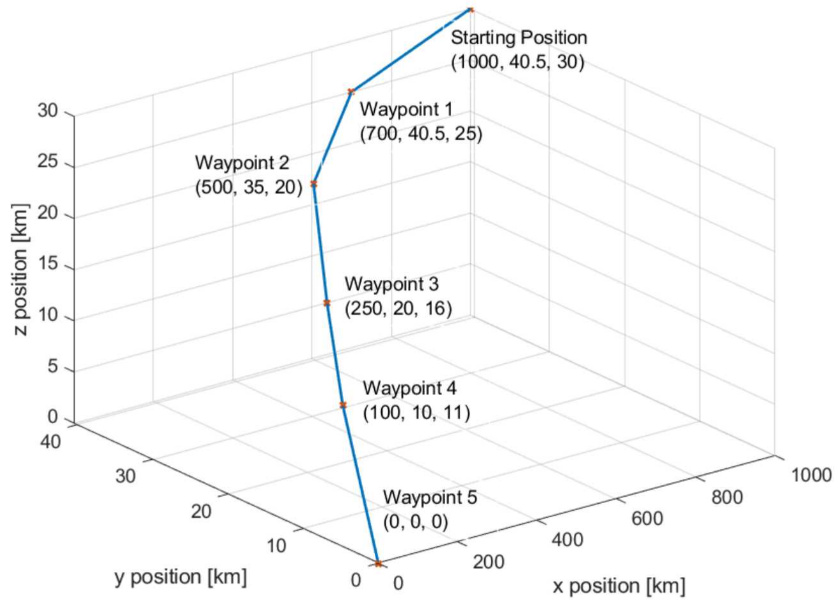
Bank Angle σ Input



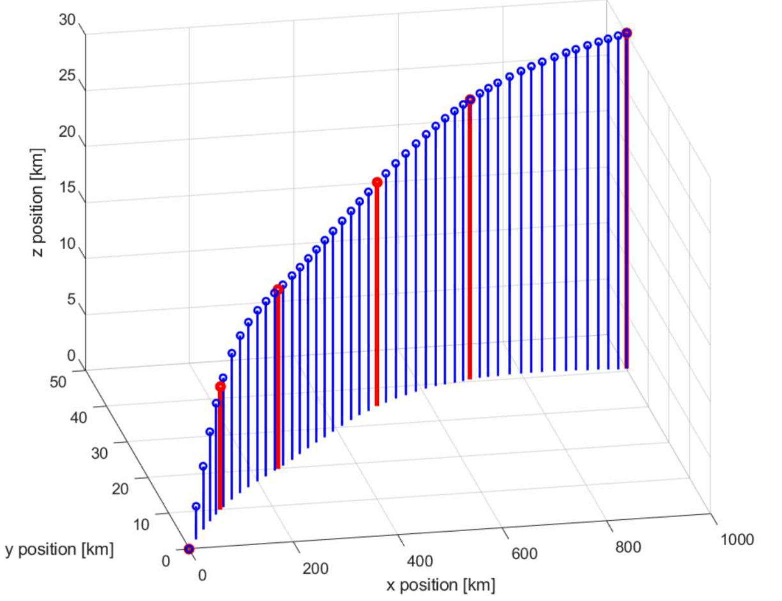
Spline Trajectory - Waypoint Reformulation

- Waypoint refined to generate a smother trajectory
  - 5 waypoints were used to generate a smooth spline of several dozen waypoints to enforce strict waypoint following
- Ad-hoc method to generate sub-waypoints

Reformulated 5 Waypoints



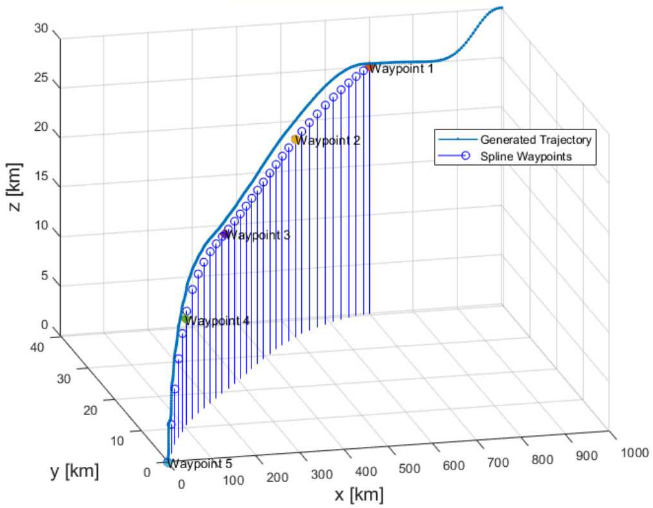
50 Sub-Waypoint Point Spline



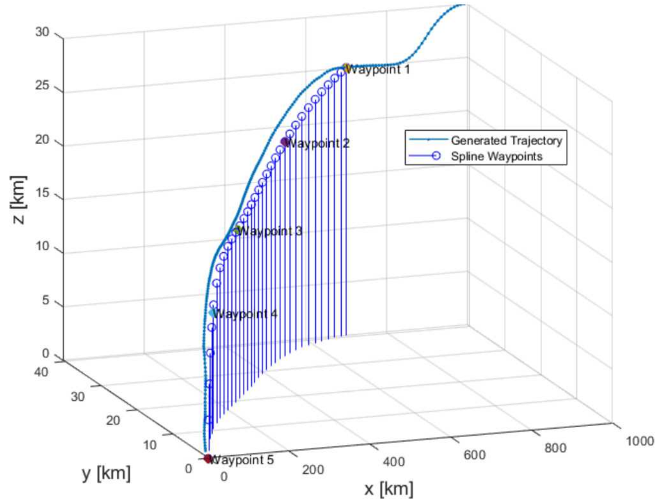
Simulation Results: Spline Trajectory (1)

- Trajectory comparison

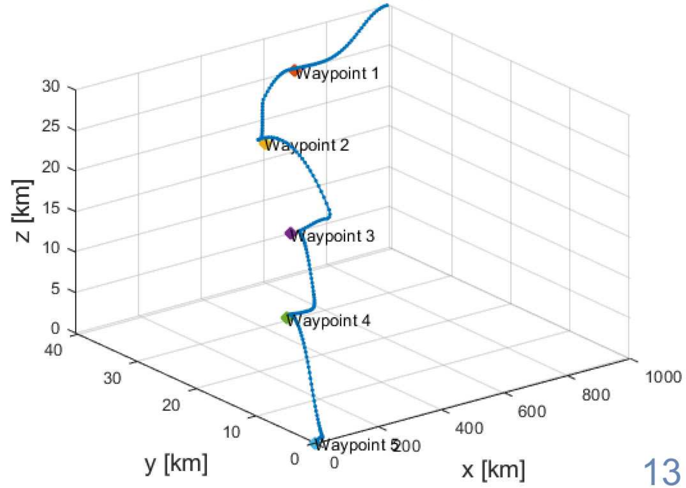
fmincon



IPOPT Simulink



5 WP Trajectory

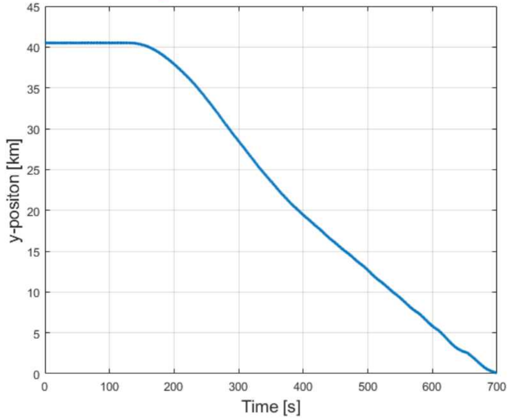


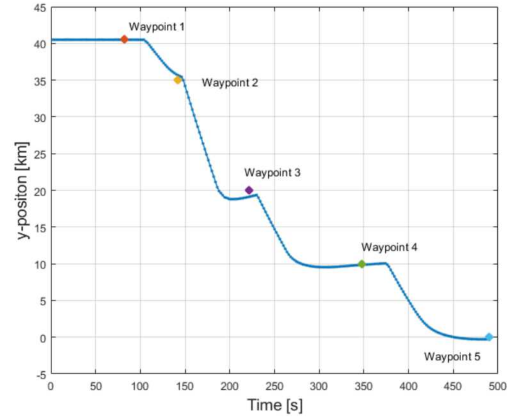
  - Trajectory smoothness greatly improved
  - NMPC framework for hypersonic flight works better with many short-term state objectives rather than few long-term objectives
  - Spline model has high accuracy for final waypoint (within 50m)

Simulation Results: Spline Trajectory (2)

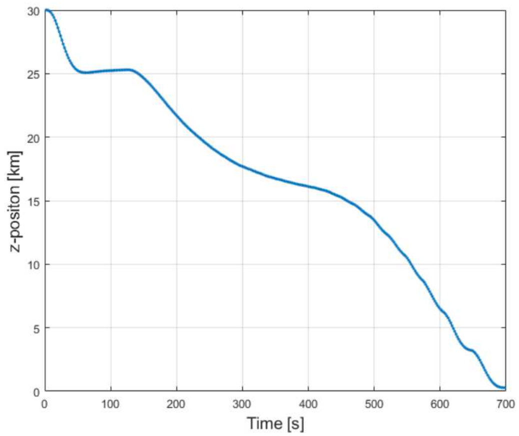
Spline model | **5 WP model**

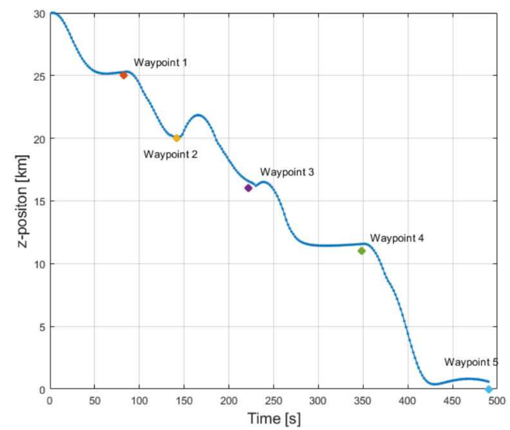
y **position:**





Altitude:



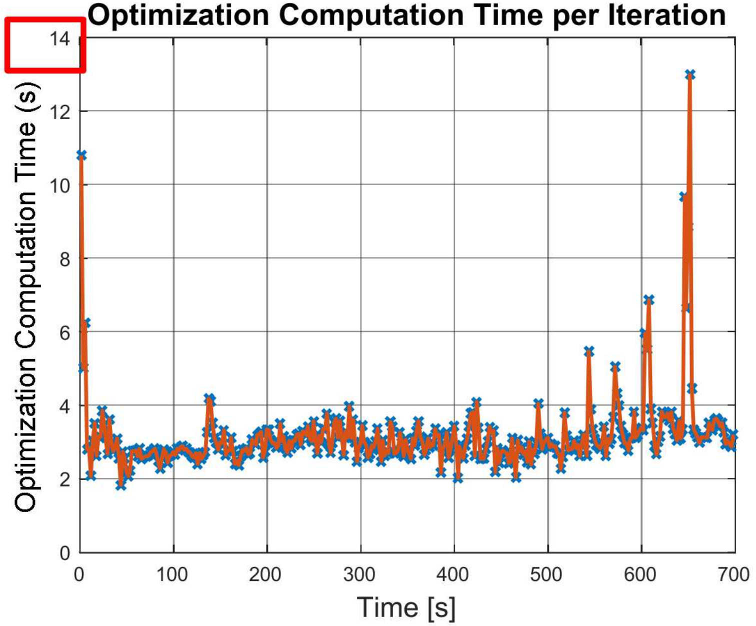


– Spline model is much improved
– Less slope variation (i.e. smoother trajectory)
– Improvements in efficiency & reduction in structural loading

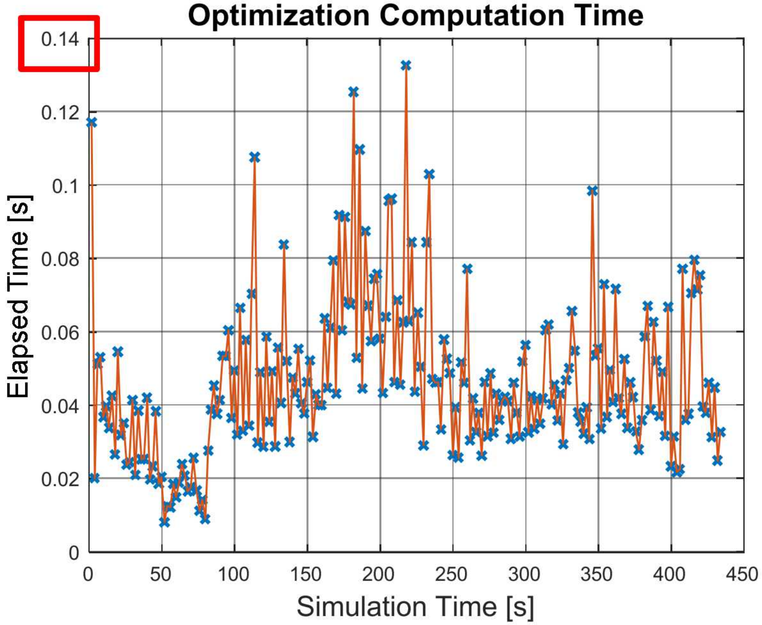
Simulation Results: Spline Trajectory (3)

- Computation time
  - Simulations on a computer with Intel(R) Core i7 CPU @ 1.60 GHz

fmincon



IPOPT Simulink

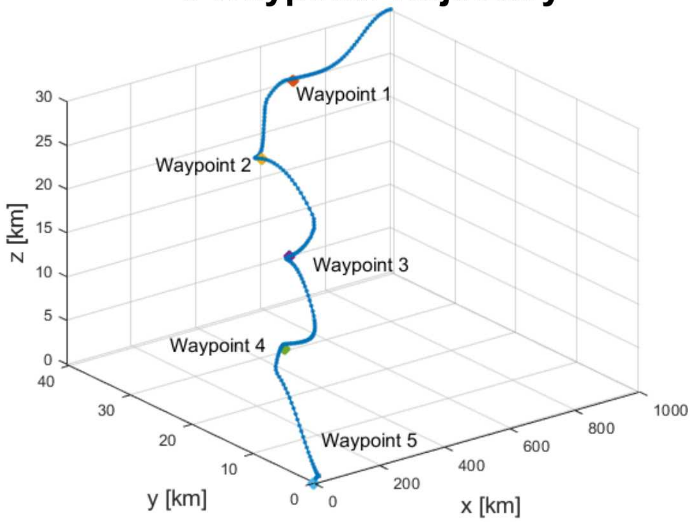


Computation Time [sec]	fmincon	IPOPT Simulink
Average	3.15	**0.046**
Maximum (worst case)	12.99	**0.133**

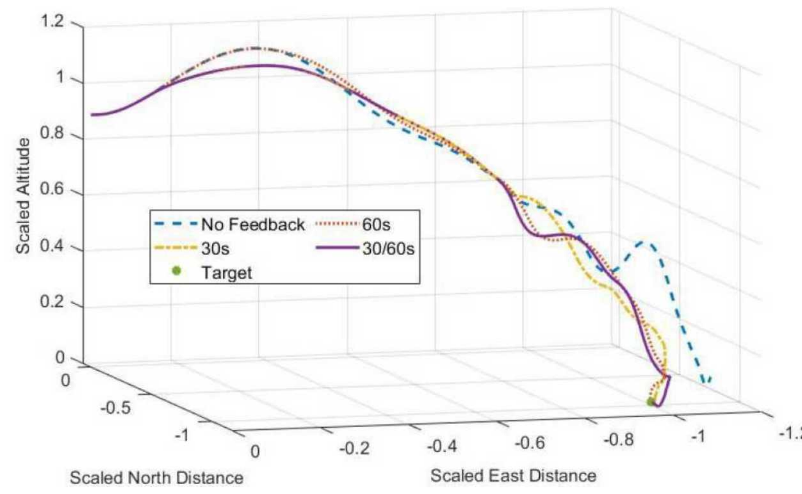
Sampling time: 2 sec

Literature Comparison for 3DoF Model

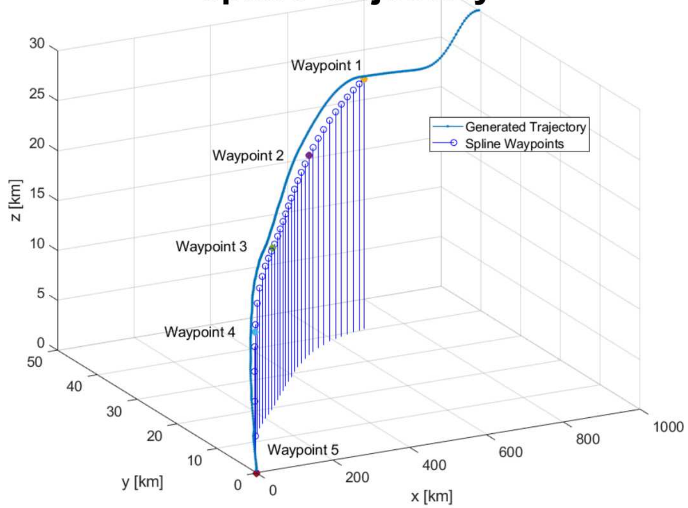
5 Waypoint Trajectory



Full Optimization Trajectory [Hood et al. 2019]



Spline Trajectory



- 5 Waypoint results show similar trajectory variations to 3DoF model with pseudospectral optimization [Hood et al. 2019]
- Large improvements made by spline model

Summary & Future Work

- Summary
  - NMPC control approaches have developed for 3DoF model
  - Spline trajectory following strategy has been implemented
  - Real-time implementability is verified using Simulink-based simulations

- On-going work
  - 6DoF hypersonic glider model formulation
    - Control-oriented model & High-fidelity model
  - Stability proof for NMPC closed-loop optimization
    - Terminal penalty cost function & Terminal constraint region

- Future work
  - Development of a rigorous waypoint following strategy
  - Implementation of NMPC for 6DoF models
  - Development of robust NMPC approaches handling uncertainties

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Thank you

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Q & A

Backup Slides

3DoF Motion and State Space Model

Hood Reformulation

$$\dot{x} = V\cos(\gamma)\cos(\theta)$$
$$\dot{y} = V\cos(\gamma)\sin(\theta)$$
$$\dot{h} = V\sin(\gamma)$$
$$\dot{V} = -\frac{D}{m} - g\sin(\gamma)$$
$$\dot{\gamma} = \frac{L\cos(\sigma) - S\sin(\sigma)}{mV} - \frac{g\cos(\gamma)}{V}$$
$$\dot{\theta} = \frac{L\sin(\sigma) + S\cos(\sigma)}{mV\cos(\gamma)}$$

Modified Formulation

$$\dot{x} = V\cos(\gamma)\cos(\theta)$$
$$\dot{y} = V\cos(\gamma)\sin(\theta)$$
$$\dot{h} = V\sin(\gamma)$$
$$\dot{V} = -\frac{\rho V^2 S C_L^*(1 + c_l^2)}{4E^* m} - g\sin(\gamma)$$
$$\dot{\gamma} = \frac{\rho c_l C_L^* S V\cos(\sigma)}{2m} - \frac{S\sin(\sigma)}{mV} - \frac{g\cos(\gamma)}{V}$$
$$\dot{\theta} = \frac{\rho c_l C_L^* S V\sin(\sigma)}{2m\cos(\gamma)} + \frac{S\cos(\sigma)}{mV\cos(\gamma)}$$

where
$$L = \frac{1}{2}\rho V^2 c_l C_L^* S$$
$$D = \frac{1}{2}\rho V^2 \frac{C_L^*(1 + c_l^2)}{2E^*} S$$

*Note: Multiple formulations of 3D model were evaluated. Flight path angle small angle & Three waypoints

*Note: Air density is recalculated at every sampling time

$(\rho = \rho_{sl} e^{-\beta h})$

$\rho_{sl} = density\ at\ sea\ level\ \left(1.225\ \frac{kg}{m^3}\right)$

$\beta = air\ density\ decay\ factor\ (-0.14 km^{-1})$

$h = altitude\ (km)$

$$\boldsymbol{x} = \begin{bmatrix} x \\ y \\ h \\ V \\ \gamma \\ \theta \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix}$$

$$\boldsymbol{u} = \begin{bmatrix} c_l \\ \sigma \end{bmatrix} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$\dot{\boldsymbol{x}} = \begin{bmatrix} x_4\cos(x_5)\cos(x_6) \\ x_4\cos(x_5)\sin(x_6) \\ x_4\sin(x_5) \\ -\frac{\rho C_L^* S x_4^2(1 + u_1^2)}{4mE^*} - g\sin(x_5) \\ \frac{\rho C_L^* S x_4 u_1\cos(u_2)}{2m} - \frac{S\sin(u_2)}{m x_4} - \frac{g\cos(x_5)}{x_4} \\ \frac{\rho C_L^* S x_4 u_1\sin(u_2)}{2m\cos(x_5)} + \frac{S\cos(u_2)}{m x_4\cos(x_5)} \end{bmatrix}$$

Symbol	Parameter	Value
S	Wing reference area	0.458 m^2
m	Mass	907 kg
E^*	Max L/D Ratio	3.45
C_L^*	Coeff. of Lift for Max L/D Ratio	0.45

[Jorris 2007] [Hood 2019]

Aerodynamic Heating

- Formulation for nose heating was added to the model to guide future considerations for thermal or structural constraints

$$\dot{q} = \frac{k}{\sqrt{r_{nose}}}\left(\frac{\rho}{\rho_{sl}}\right)\left(\frac{V}{\sqrt{g_0 r_0}}\right)^3$$

$$where\ k = 17000\ \frac{BTU \cdot ft^{\frac{-3}{2}}}{s}$$

- Note: $\dot{q}$ is calculated in BTU/s and converted to Watts afterwards

[Jorris 2007]

Symbol	Parameter
k	Heating constant
r_{nose}	Radius of aircraft nose
ρ	Local air density
ρ_{sl}	Sea level air density
V	Velocity
g_0	Earth Gravity
r_0	Radius from earth center

Simulation Parameters – 5 Waypoints

MATLAB fmincon Implementation

Optimization Parameters

n_s = 6 - Number of State Variables
n_u = 2 - Number of Control Variables
n_a = 8 - State and Control Variables
n_c = 7 - Number of Constraints
T_s = 2 - Sampling Time [sec]
N = 20 - Prediction Horizon

Weighting matrices
Q = diag[0.005, 0.03, 0.03, 0, 0, 0]
R = [1,1]
P = $2 \cdot Q$

For waypoint 4 to 5 $P = 3 \cdot Q$

Initial Conditions

x_1 (x-pos) initial = 1000 km
x_2 (y-pos) initial = 40.5 km
x_3 (z-pos) initial = 30 km
x_4 (V) initial =4.08 km/s (Mach 12)
x_5 (γ) initial = 0 rad (0 deg)
x_6 (θ) initial = π rad (180 deg)

Waypoints (Target)

WP1: (700, 40.5, 23) [km]
WP2: (500, 35, 18) [km]
WP3: (300, 20, 10) [km]
WP4: (100, 10, 6) [km]
WP5: (0, 0, 0) [km]

Simulation Parameters – Spline Trajectory

MATLAB fmincon Implementation

Optimization Parameters

n_s = 6 - Number of State Variables
n_u = 2 - Number of Control Variables
n_a = 8 - State and Control Variables
n_c = 7 - Number of Constraints
T_s = 2 - Sampling Time [sec]
N = 20 - Prediction Horizon

Weighting matrices
Q = diag[0.0002, 0.25, 0.25, 0, 0, 0]
R = [1,1]
$P = 2 \cdot Q$

Initial Conditions

x_1 (x-pos) initial = 1000 km
x_2 (y-pos) initial = 40.5 km
x_3 (z-pos) initial = 30 km
x_4 (V) initial =2.72 km/s (Mach 8)
x_5 (γ) initial = 0 rad (0 deg)
x_6 (θ) initial = π rad (180 deg)

Waypoints (Target)

33 Sub-waypoints from end of No-Fly-Zone (700, 40.5, 25) to old Waypoint 5 (0,0,0)

Spline Trajectory Results

- Velocity is well maintained despite lower initial velocity and constant cl control input
- Aerodynamic heating reaches a maximum of approx. 60 kW
- Control input in constant fluctuations with no maximum constrained inputs