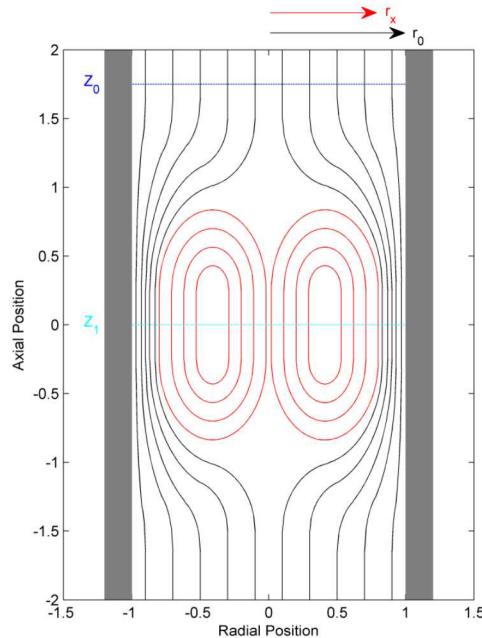


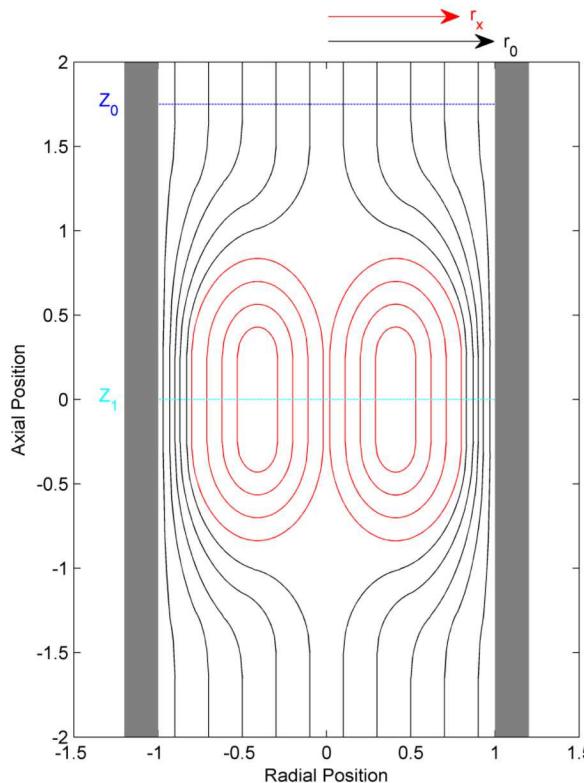
Liner driven FRC implosions



Z Fundamental Science Workshop
Sandia Labs, August 3, 2020
Stephen A. Slutz and Mathew R. Gomez
Sandia National Laboratories

Sandia National Laboratories is a multimission laboratory managed and operated by National Technology & Engineering Solutions of Sandia, LLC, a wholly owned subsidiary of Honeywell International Inc., for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-NA0003525.

Field Reversed Configuration (FRC) have been studied for magnetic confinement fusion



FRC formation: create a plasma with an axial B field and then apply the opposite polarity field.

The plasma is heating by reconnection during the formation.

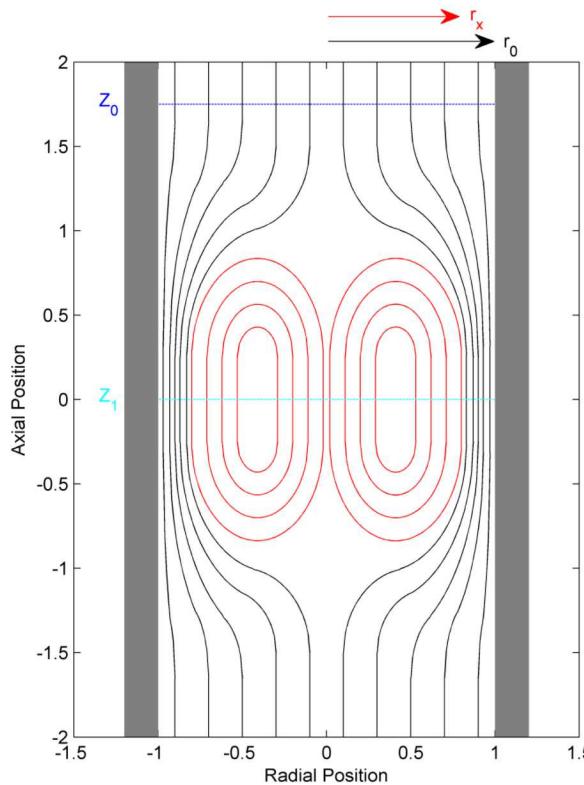
The peak plasma pressure \sim peak magnetic pressure

Plasma temperature is typically < 1 keV
(too low for fusion)

Compression of the FRC would raise the temperature and density

We present an analytic model of the fusion gain of liner driven FRC implosions

An FRC could be made inside an AutoMag liner



The initial field could be provided by the field coils used to magnetize MagLIF experiments

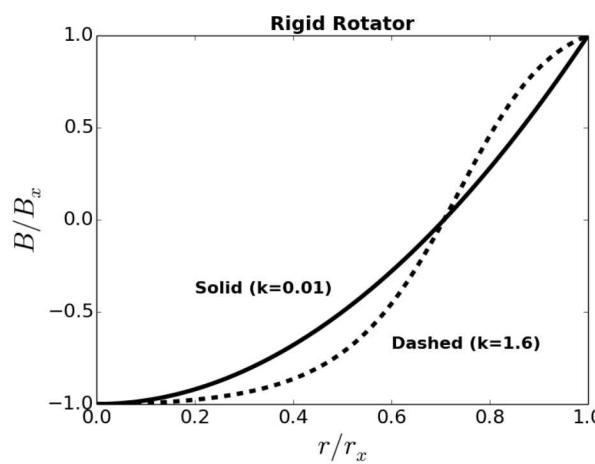
An AutoMag liner has helically conducting paths so that a low current foot pulse can generate a B field within the liner

The AutoMag field can be used to generate the opposite polarity field to form the FRC

No laser is required, but the plasma may need to be heated to 1-2 eV to partially freeze in the initial bias field.

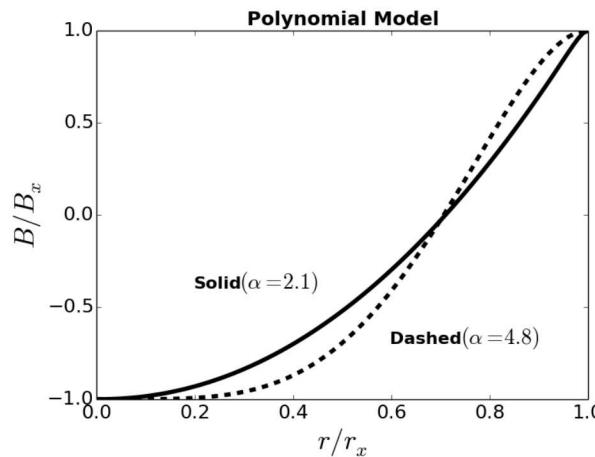
Radio frequency heating of spark arrays could be used to precondition the fuel

1D analytic FRC models are used as input to our gain model.



Rigid Rotator

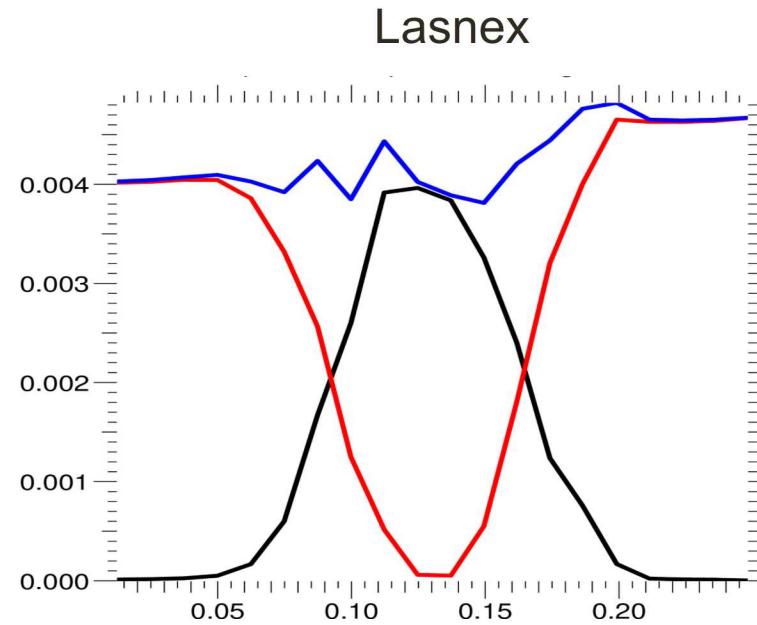
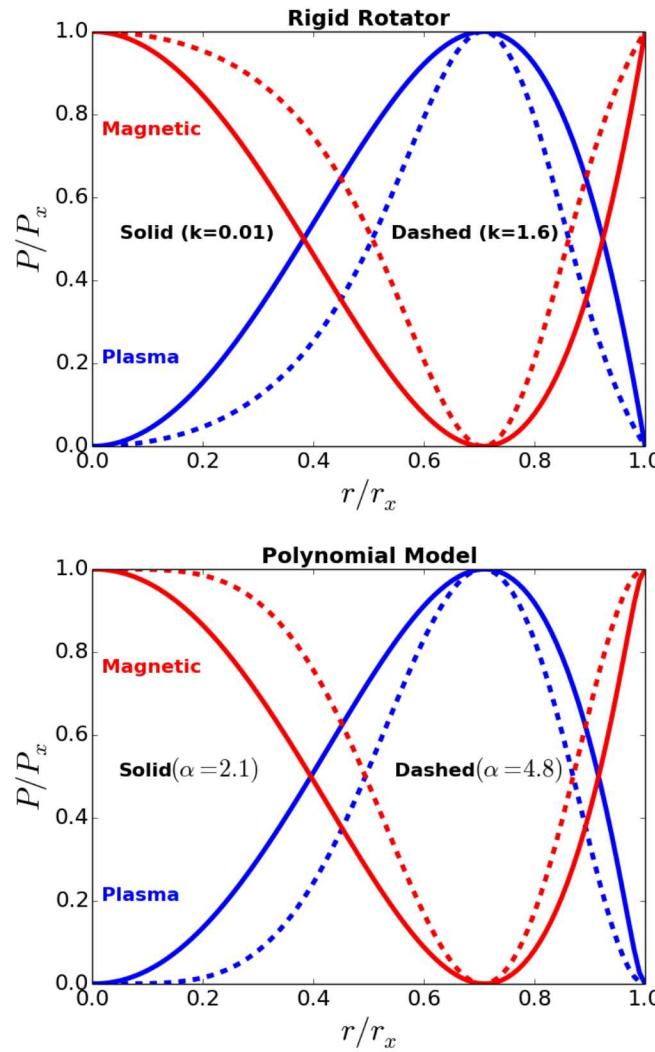
$B(r) = B_x \frac{\tanh(u)}{\tanh(K)}$, where $u = k(x^2 - 1)$,
 $x = r/r_x$, $r_x = \sqrt{2R}$
varying k gives a family of solutions



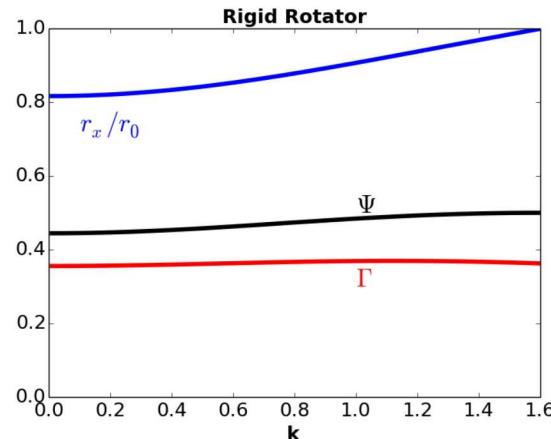
Polynomial Model

$B(x) = B_x b(x)$, $b(x) = ax^\alpha + cx^\beta + d$
Force $j(0) = j(1) = 0$ $a = \frac{2\beta}{\beta - \alpha}$ $\beta = \frac{2\alpha + 4}{\alpha - 2}$,
where $2 < \alpha < 4.83$.

The pressure profile of the analytic models are consistent with Lasnex simulations of FRC formation

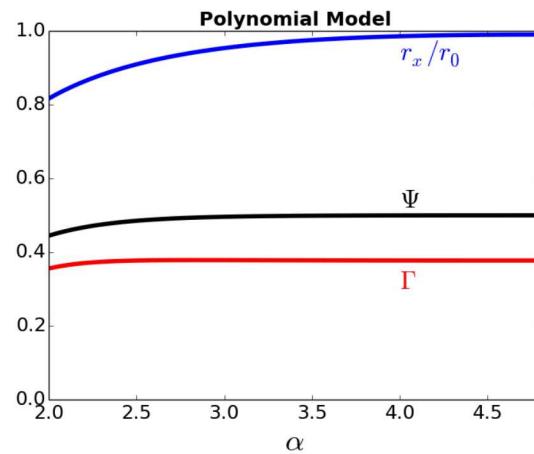


Gain model input parameters are insensitive to the analytic models



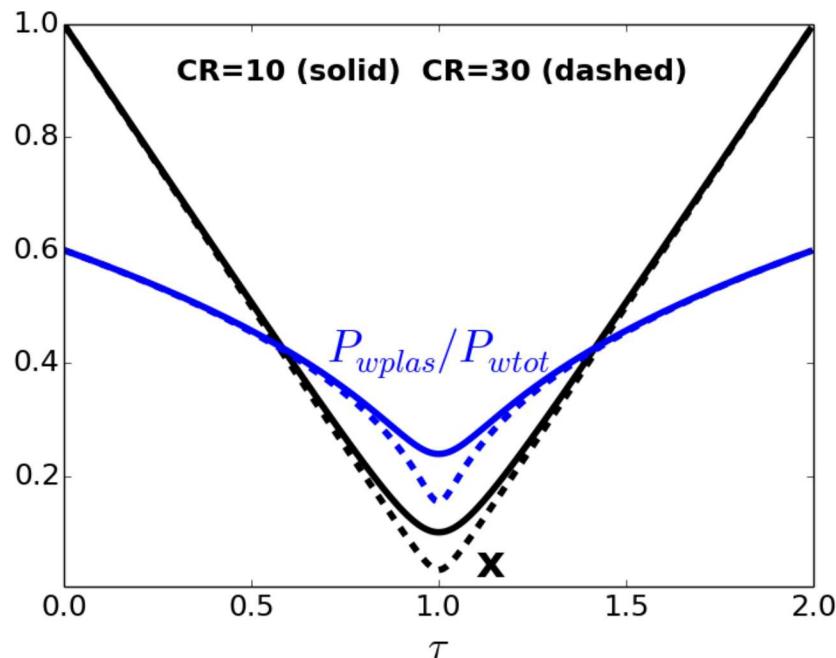
The fusion rate is proportional to $\langle \rho^2 \rangle = \Gamma \rho_x^2$. $\Gamma = 0.38$ for each model

The radiation losses and alpha particle trapping depend on $\langle \rho r \rangle = \Psi \rho_x r_0$. $\Psi \approx 0.5$



The liner motion is determined by the initial kinetic energy, the convergence ratio, and the FRC magnetic field strength

$P_0 = B_{x0}^2 / 2\mu_0$ B_{x0} is the initial maximum magnetic field strength



Assuming flux conservation, the total pressure is $P = P_0 x^{-4}$ $x = r/r_0$

$$x(\tau) = C_R^{-1} [1 + (C_R^2 - 1)(1 - \tau)^2]^{1/2}$$

$$\tau = t/t_{imp} \text{ and } t_{imp} = \frac{(C_R^2 - 1)^{1/2}}{\alpha^{1/2} C_R^2}$$

The fraction of the compressive work going into the plasma decreases with convergence because the length of the FRC decreases $L = L_0 x^{0.4}$. This limits the gain

The optimum fusion gain of adiabatic FRC implosions takes a simple form

The fusion gain $G = E_k^{-1} t_{imp} \int_0^2 A_f \langle \sigma v \rangle \langle \rho^2 V \rangle d\tau \langle \rho^2 V \rangle = \Gamma \rho_{x0}^2 V_0 x^{-2.4}$

$G = G_0 S(\theta_0, C_R)$, where $G_0 = \frac{\Gamma P_0^{1/6} (F_{AR} \rho_L)^{1/2} E_k^{1/3}}{(\pi L_{oR})^{1/3}}$ and

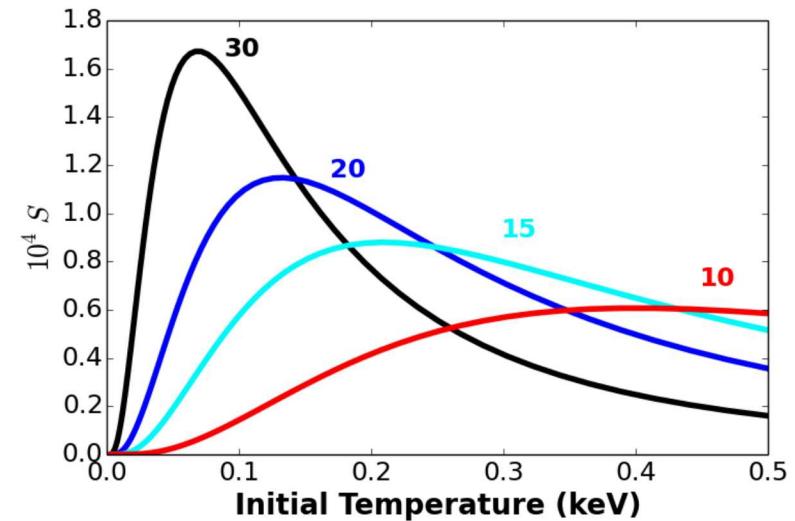
$$S(\theta_0, C_R) = \frac{A_f}{\sqrt{2} (C_R R \theta_0)^2 (C_R^2 - 1)^{-5/6}} \int_0^1 \frac{\langle \sigma v \rangle d\tau}{x^{2.4}}$$

$$\theta_{opt} = 15.93 C_R^{-1.6}$$

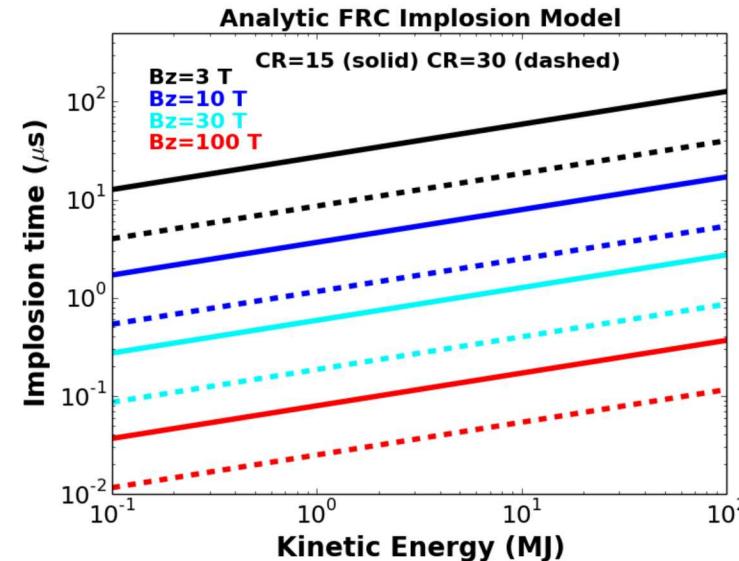
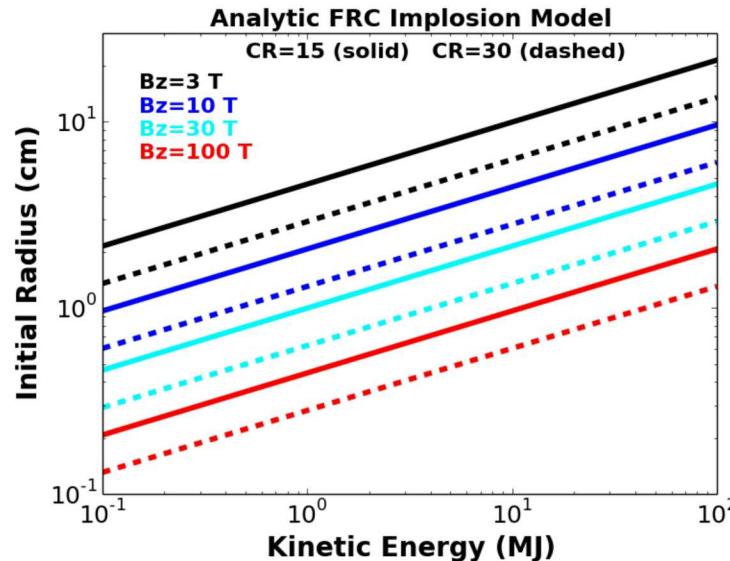
$$S_{opt} = 7.31 \times 10^{-6} C_R^{0.92}$$

Assuming a beryllium liner with an aspect ratio of 6 we find

$$G \cong 1.1 \left(\frac{B}{20} \right)^{1/3} \left(\frac{C_R}{20} \right)^{0.92} \left(\frac{E_k}{1 \text{ MJ}} \right)^{1/3}$$



The optimum initial radius and implosion time are simple functions of E_k , B , and C_R



The Z machine can deliver about 400 kJ to a liner implosion

The model can be modified to include radiation losses and alpha particle heating

The energy in the plasma of the FRC is $E = \frac{3}{2}PV_p$ where V_p is the plasma volume.

The rate of change is $\frac{dE}{dt} = \frac{3}{2} \left(P \frac{dV_p}{dt} + V_p \frac{dP}{dt} \right)$

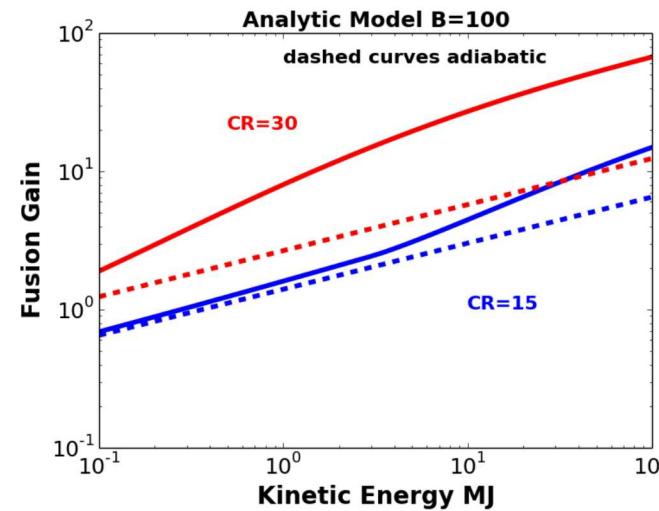
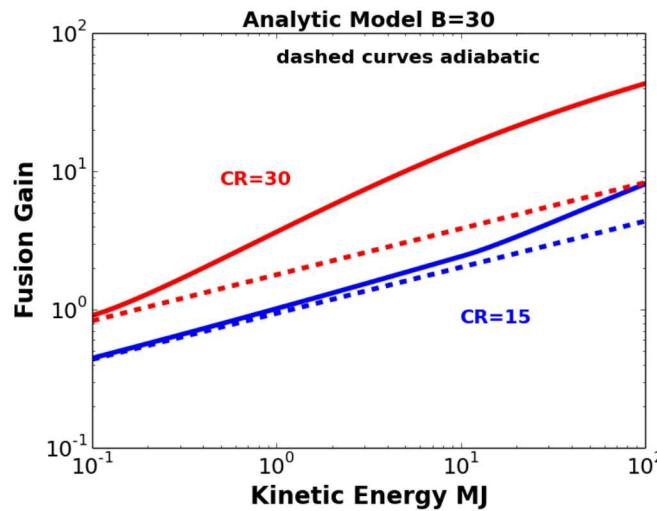
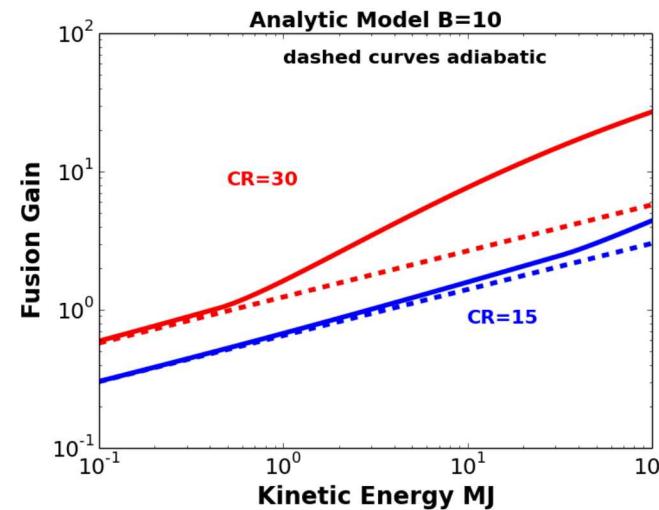
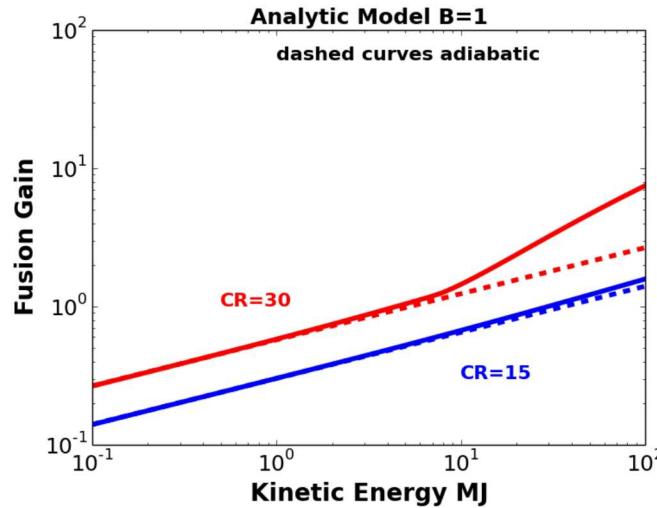
The power put into the FRC is $\frac{dW}{dt} = -P \frac{dV_p}{dt} + P_{na}$

where the first term is the rate of doing work on the FRC plasma and P_{na} is the nonadiabatic power alpha particle and radiation.

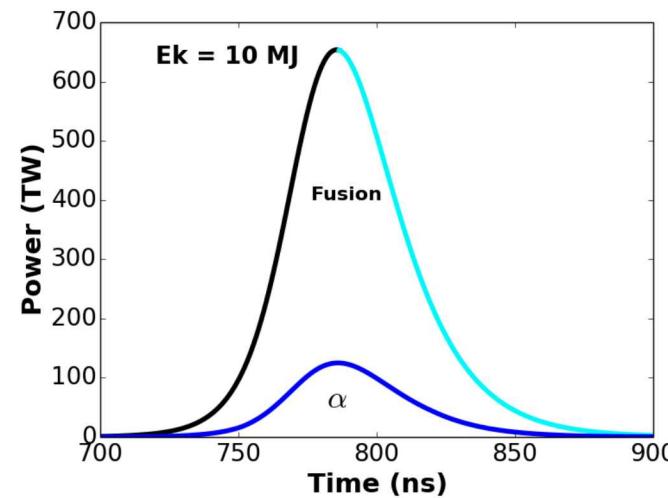
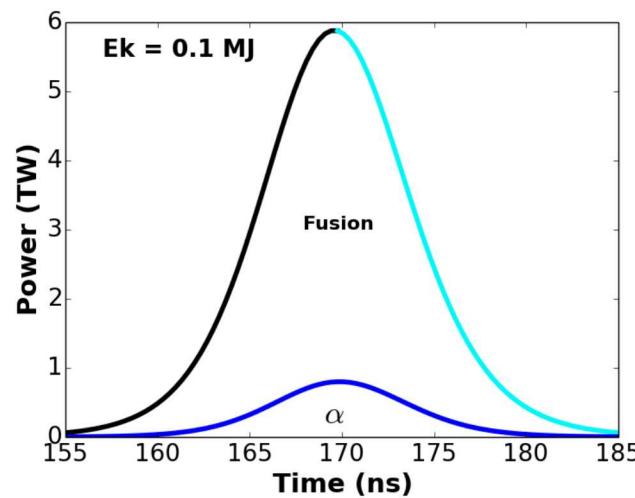
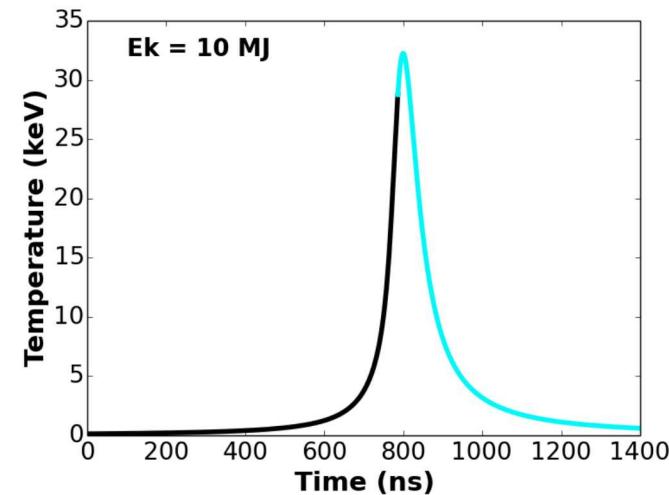
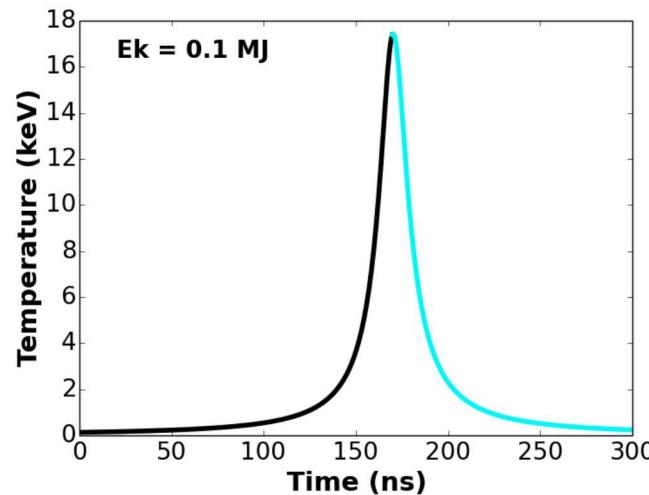
Setting $V_{p0} = \psi V_0$, $V_p = V_{g0} Y x^{2.4}$ $\frac{dY}{dt} = \frac{0.4x^{1.6}}{P_0 V_{p0}} P_{na}$

The parameter Y accounts for the change in the adiabat of the plasma. Defining an adiabat factor, F_A , such that the pressure of the plasma is $P_p = F_A P_0 \left(\frac{\rho}{\rho_0} \right)^{5/3}$ we find that $F_A = Y^{5/3}$.

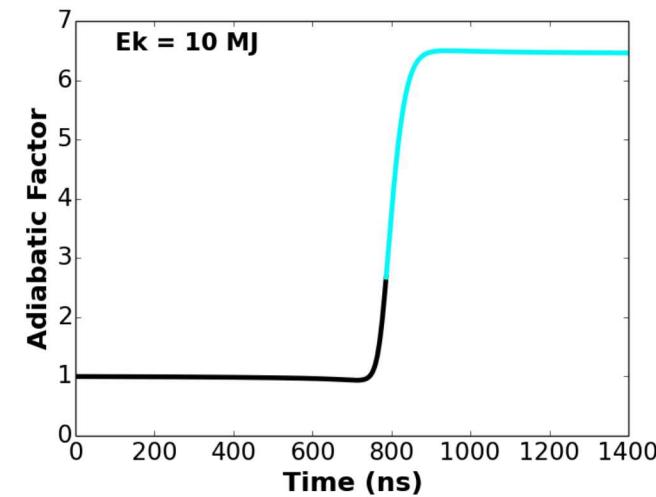
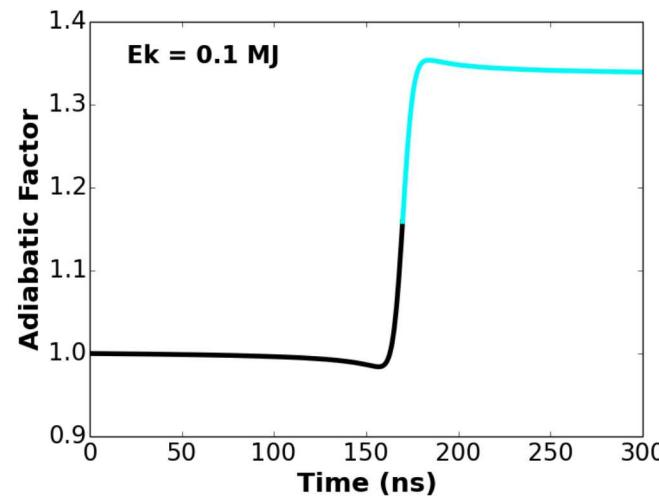
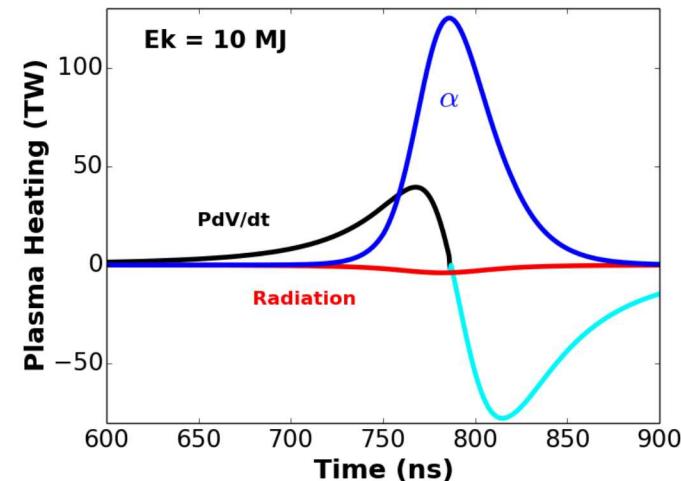
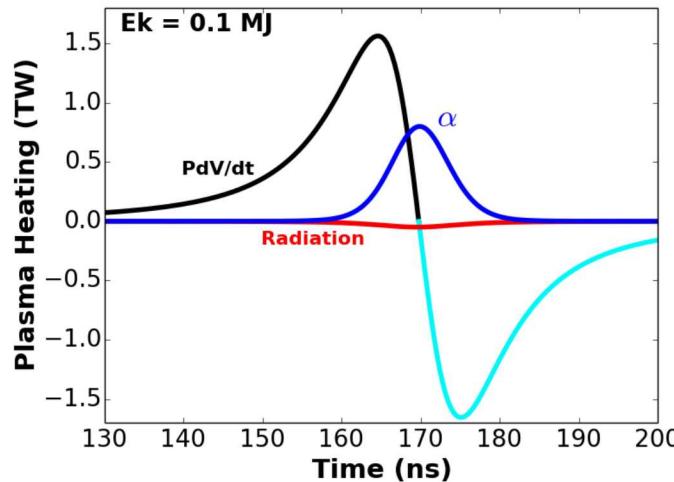
Radiation is unimportant and gain scales weakly with kinetic energy



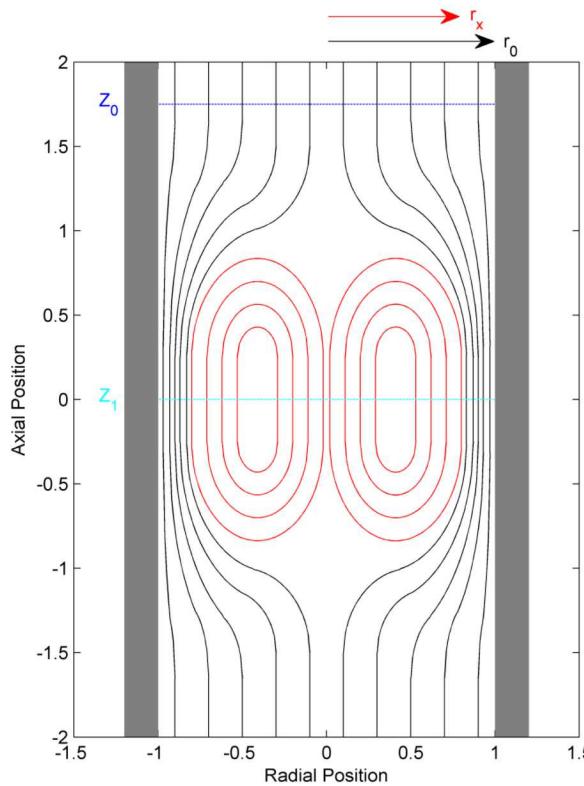
Alpha particle heating becomes important at high kinetic energy



Alpha particle heating becomes important at high kinetic energy



Liner driven implosions of Field Reversed Configuration (FRC) could produce significant fusion



FRC could be formed within a liner using exterior coils and

Weak scaling with implosion energy suggests interesting experiments on small machines

Magnetically confined fusion could be studied in a new parameter space (high density and magnetic fields)

Lasnex simulation have produced yields comparable to the model

Next steps:

- detail numerical simulations
- demonstrate the formation of an FRC
- perform implosion experiments