

Amnesia Radius Versions of Conditional Point Sampling for Radiation Transport in 1D Stochastic Media

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ABSTRACT

Conditional Point Sampling (CoPS) is a newly developed Monte Carlo algorithm for computing radiation transport quantities in stochastic media. CoPS has been demonstrated to produce results at a high degree of accuracy for 1D [1] and multi-D [2] problems, compute variance in outputs caused by material mixing [1], and is statistically errorless in 1D mixtures with Markovian mixing statistics [1]. However, the algorithm involves a growing list of point-wise material designations during simulation that causes potentially unbounded increases in runtime and memory, making the production of probability density functions (PDFs) computationally expensive. In this work, we use CoPS to produce PDFs for a set of benchmark problems by omitting material points used in the computation from being stored in persisting memory based on their proximity to neighboring material points already defined within a realization, and we conduct numerical studies to investigate trade-offs between accuracy, required computer memory, and computation time.

KEYWORDS: Monte Carlo, Conditional Point Sampling, stochastic media, amnesia radius

1. INTRODUCTION

Conditional Point Sampling (CoPS) is a Monte Carlo method developed for radiation transport in stochastic media that uses Delta tracking [3]. It has been demonstrated to produce mean results [1] as accurate as other well-known approximate methods for one-dimensional, binary, Markovian-mixed media such as the Atomic Mix (AM) approximation, Chord Length Sampling (CLS), Local Realization Preserving (LRP) [4], and Algorithm C (Alg. C) [5] for a set of benchmark problems of slab geometry defined in Ref. [6]. The algorithm easily extends to and is accurate for multi-D problems [2], and was demonstrated to accurately quantify the variance in mean results caused by random material mixing [1] using the Embedded Variance Deconvolution (EVADe) approach [7].

We are interested in characterizing stochastic media problems by producing probability density functions (PDFs) with Conditional Point Sampling (CoPS). This requires a collection of histories simulated on the same set of successively sampled material points that essentially are on the

same material “realization”; in Ref. [1] we began calling such a collection of histories a “cohort.” However, the CoPS algorithm involves a growing list of material points sampled during simulation, which directly increases the algorithm’s runtime and required computer memory, making the construction of a well-resolved PDF using larger cohorts impractical. In Ref. [8], recent memory versions of CoPS where only the sampled material assignments of the most recently visited locations by a history rather than that of every location were investigated in hopes of alleviating runtime and computer memory requirements while maintaining CoPS accuracy. In this work, we introduce an “amnesia radius” r_a , a user-defined parameter used to determine if a sampled material point is omitted from the composition of a realization, such that, if a sampled material point falls within the amnesia radius of a neighboring material point, it is not included in the growing list of points for the remainder of the particles in each cohort. Note that in the limit as $r_a \rightarrow 0.0$, the original version of CoPS is recovered, and memory of all sampled material points is retained.

In this paper, limited-memory versions of Conditional Point Sampling (CoPS) of varying cohort sizes and amnesia radii using a 2-point (CoPS2) and errorless 3-point (CoPS3PO) conditional probability function for binary, one-dimensional, Markovian-mixed media are used to perform transport calculations and construct PDFs. We present these results and benchmark them against reproduced benchmark results in Ref. [1] and results presented in Ref. [9]. We also present numerical studies investigating accuracy, memory requirement, and runtime trade-offs for each set of parameters to support future efforts in CoPS implementation on next-generation computing platforms.

2. THEORY AND CONDITIONAL POINT SAMPLING

Discussion on Markovian-mixing theory and the Conditional Point Sampling algorithm can be found in Ref. [10] and will be further described in the full paper.

3. RESULTS AND ANALYSIS

A set of problem parameters for planar geometries from the benchmark suite described in Ref. [6] is used. The problem parameters of Tables 10-18 in Ref. [6] are listed in Table 1, where $\Sigma_{t,j}$ is the total cross section, Λ_j is the average chord length, and c_j is the scattering ratio for each material $j \in \{0,1\}$. Only a slab length $L = 10$ is considered here. In this paper, a 2-point (CoPS2)

Table 1: Benchmark Set Parameters

Case Number	$\Sigma_{t,0}$	$\Sigma_{t,1}$	Λ_0	Λ_1	Case Letter	c_0	c_1
1	10/99	100/11	99/100	11/100	a	0.0	1.0
2	10/99	100/11	99/10	11/10	b	1.0	0.0
3	2/101	200/101	101/20	101/20	c	0.9	0.9

and errorless 3-point (CoPS3PO) conditional probability function for binary, one-dimensional, Markovian-mixed media are used in Conditional Point Sampling (CoPS) to perform transport calculations with cohort sizes of 2, 5, 25, 50, and 100 particles; amnesia radii of 0.0, 0.01, 0.1, and 1.0; and 1E6 particle histories.

Probability density functions (PDFs) for each benchmark case were produced using CoPS2 and CoPS3PO and cohort size of 100. Here, we present the PDF produced by CoPS3PO for Case 1a in Figure 1. This figure shows agreement between the benchmark results from Ref. [9] and CoPS3PO using amnesia radii of 0.0 and 0.01 with increased deviation using amnesia radii of 0.1 and 1.0. Note that CoPS3PO with $r_a = 0.0$ is errorless, and therefore it, like the benchmark approach, produces exact PDFs within statistical uncertainty. Figure 2 shows the average points

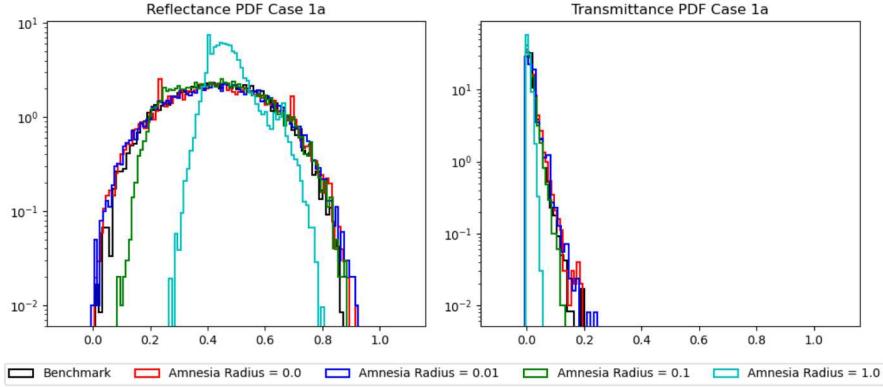


Figure 1: Case 1a reflectance and transmittance PDF using CoPS3PO.

remembered per realization and total runtime as a function of cohort size. When all points are remembered ($r_a = 0.0$), the increase in average points remember per realization and total runtime is unbound as a function of cohort size. For an amnesia radius $r_a \neq 0.0$, there is a limit on total runtime and required computer memory imposed, correlated to the maximum number of possible points allowed in a realization, $\text{int}(\frac{L}{r_a})$. For this set of benchmark problems, both memory and runtime for 1E6 particles plateau drastically at larger cohort sizes as amnesia radius increases with memory and runtime savings of a factor of over 10 for a cohort size of 100.

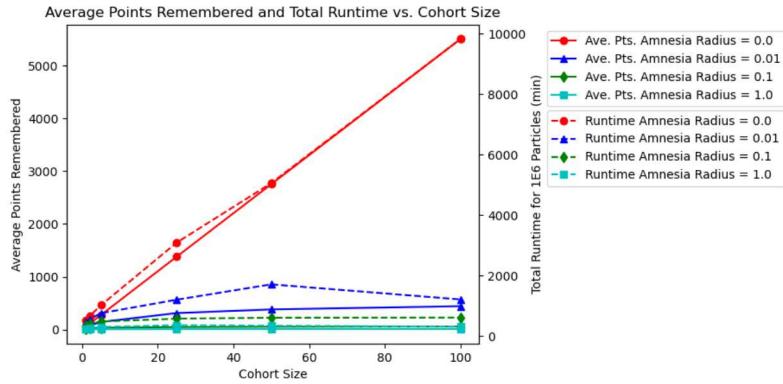


Figure 2: Average points remembered per realization and total runtime as a function of cohort size using CoPS3PO.

4. CONCLUSIONS

In the full paper, we plan to show accuracy comparisons between CoPS2 and CoPS3PO of varying amnesia radii, the convergence of PDFs produced using the benchmark approach and CoPS3PO as cohort size increases for small non-zero amnesia radii, and additional numerical studies showing CoPS3PO behavior in accuracy and runtime as a function of average points remembered per realization. In future publications, we hope to continue this investigation of limited-memory CoPS and study hybrid limited memory techniques including but not limited to amnesia radius and recent memory [8]. We would like to extend this to multi-dimension and multi-material CoPS.

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