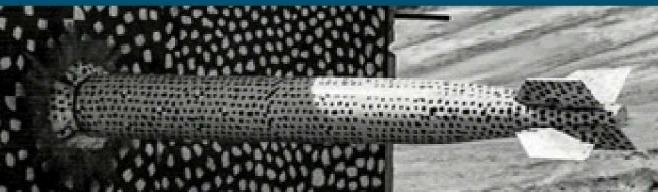


# Bayesian Data Assimilation for Pulsed Power Driven Fusion and HED Applications



*PRESENTED BY*

Patrick Knapp

2019 National Diagnostics Working Group

LLNL Livermore, CA



SAND2020-7512PE



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## Thanks to the many contributors that made this work possible

<sup>1</sup>M.E. Glinsky, <sup>1</sup>C. A. Jennings, <sup>1</sup>E. C. Harding, <sup>1</sup>A. J. Harvey-Thompson, <sup>1</sup>M. R. Gomez, <sup>1</sup>S. B. Hansen, <sup>1</sup>K.D. Hahn, <sup>1</sup>G.A. Chandler, <sup>5</sup>M. Evans, <sup>1</sup>M. Geissel, <sup>1,7</sup>C. Ruiz, G. Cooper, <sup>1</sup>S. A. Slutz, <sup>1</sup>R. W. Lemke, <sup>1</sup>M. R. Martin, <sup>1</sup>K.D. Cochrane, <sup>1</sup>I.C. Smith, <sup>1</sup>M. Schollmeier, <sup>1</sup>P.K. Rambo, <sup>1</sup>R. A. Vesey, <sup>4</sup>A. B. Sefkow, <sup>6</sup>R. D. McBride, <sup>1</sup>T. J. Awe, <sup>1</sup>P. F. Schmit, <sup>1</sup>D. B. Sinars, <sup>1</sup>M. E. Cuneo, <sup>2</sup>M. C. Herrmann, <sup>3</sup>C. W. Nakhleh, <sup>2</sup>B. E. Blue, <sup>4</sup>E. M. Campbell, <sup>1</sup>M. Jones, <sup>1</sup>J. L. Porter, <sup>1</sup>G. A. Rochau, <sup>1</sup>K. J. Peterson, <sup>1</sup>W. A. Stygar, <sup>1</sup>T.R. Mattsson, <sup>1</sup>M.P. Desjarlais, <sup>1</sup>J. S. Lash, <sup>1</sup>M. K. Matzen,<sup>1</sup> **and many more...**

<sup>1</sup>Sandia National Laboratories, Albuquerque, NM

<sup>2</sup>Lawerence Livermore National Laboratory, Livermore, CA

<sup>3</sup>Los Alamos National Laboratory, Los Alamos, NM

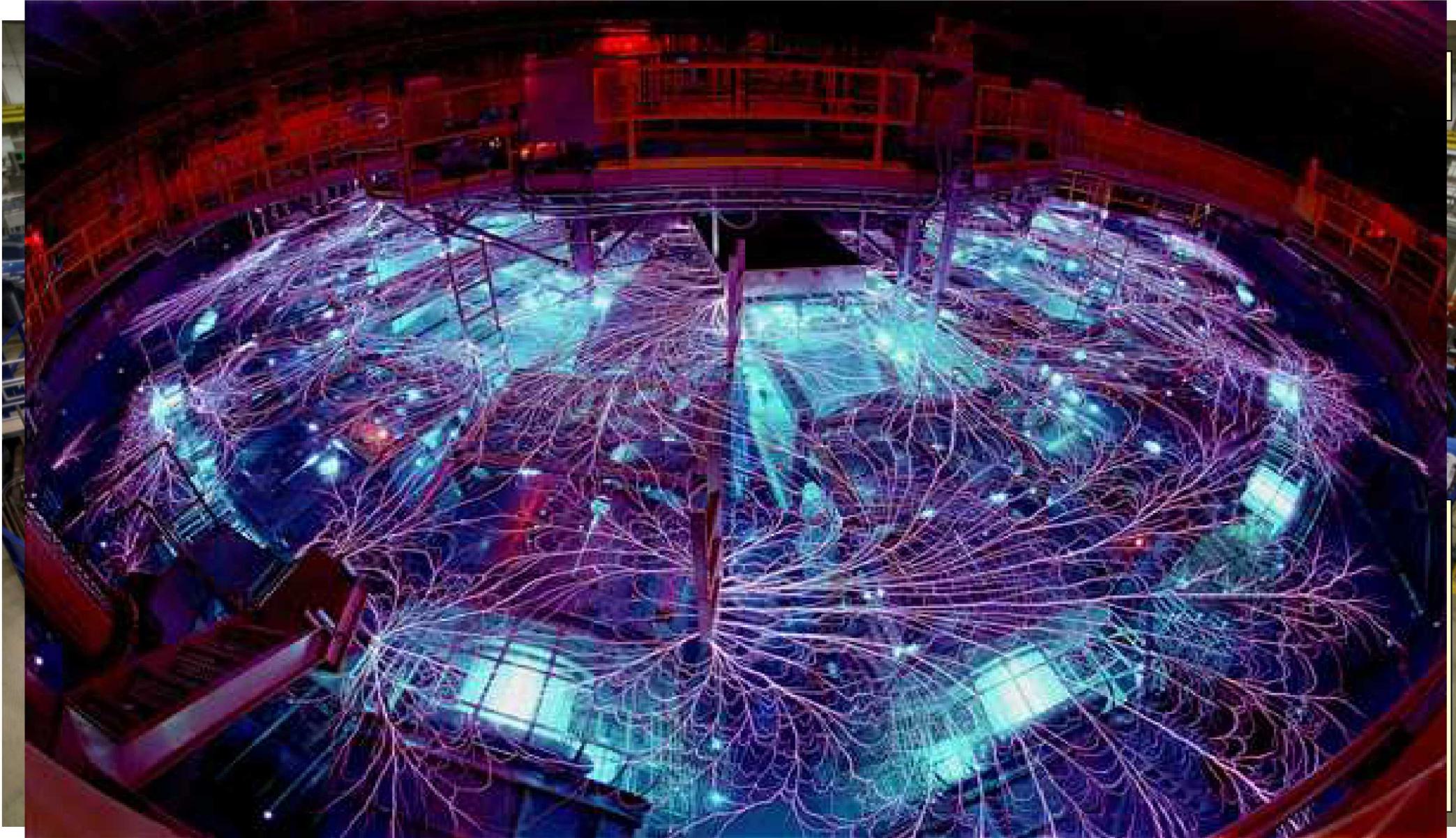
<sup>4</sup>Laboratory for Laser Energetics, University of Rochester, Rochester, NY

<sup>5</sup>Department of Physics and Astronomy, University of Rochester, Rochester, NY

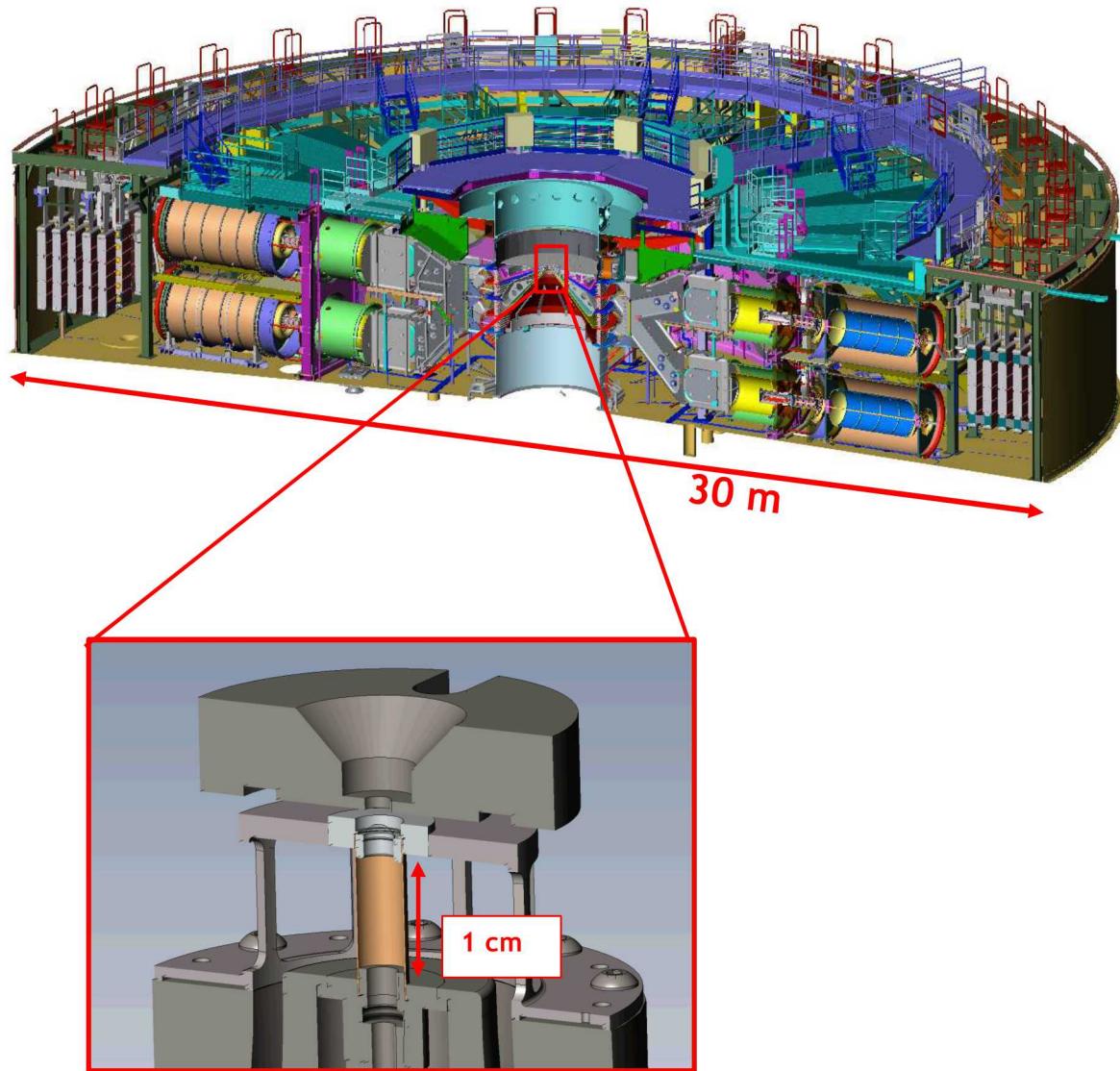
<sup>6</sup>Nuclear Engineering & Radiological Sciences Department, University of Michigan, Ann Arbor, MI

<sup>7</sup> Nuclear Engineering Department, University of New Mexico, Albuquerque, NM

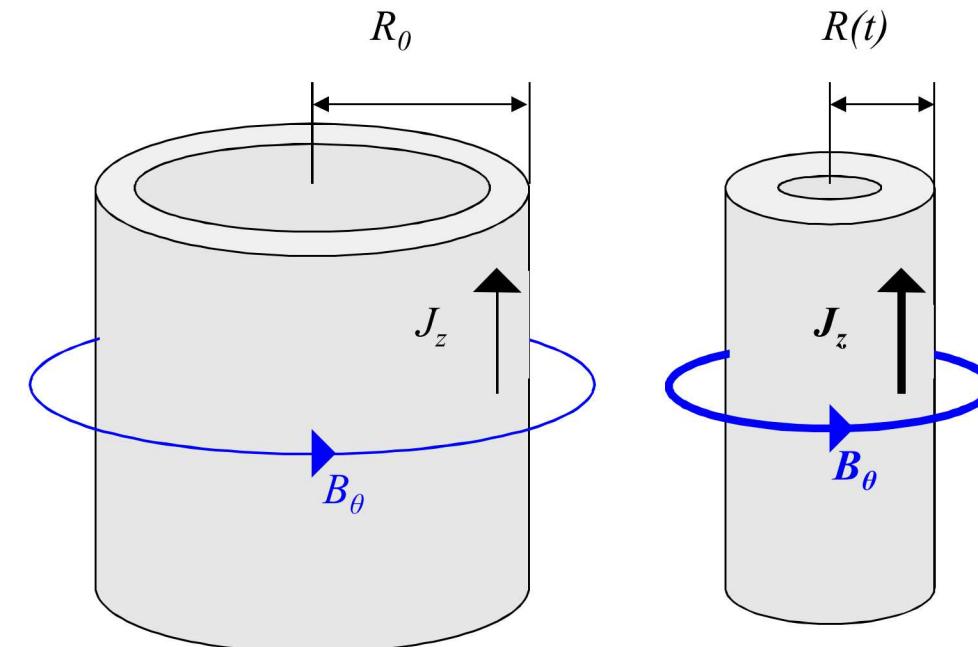
The Z facility combines the multi-MJ Z pulsed-power accelerator with the multi-kJ Z Beamlet Laser (ZBL)



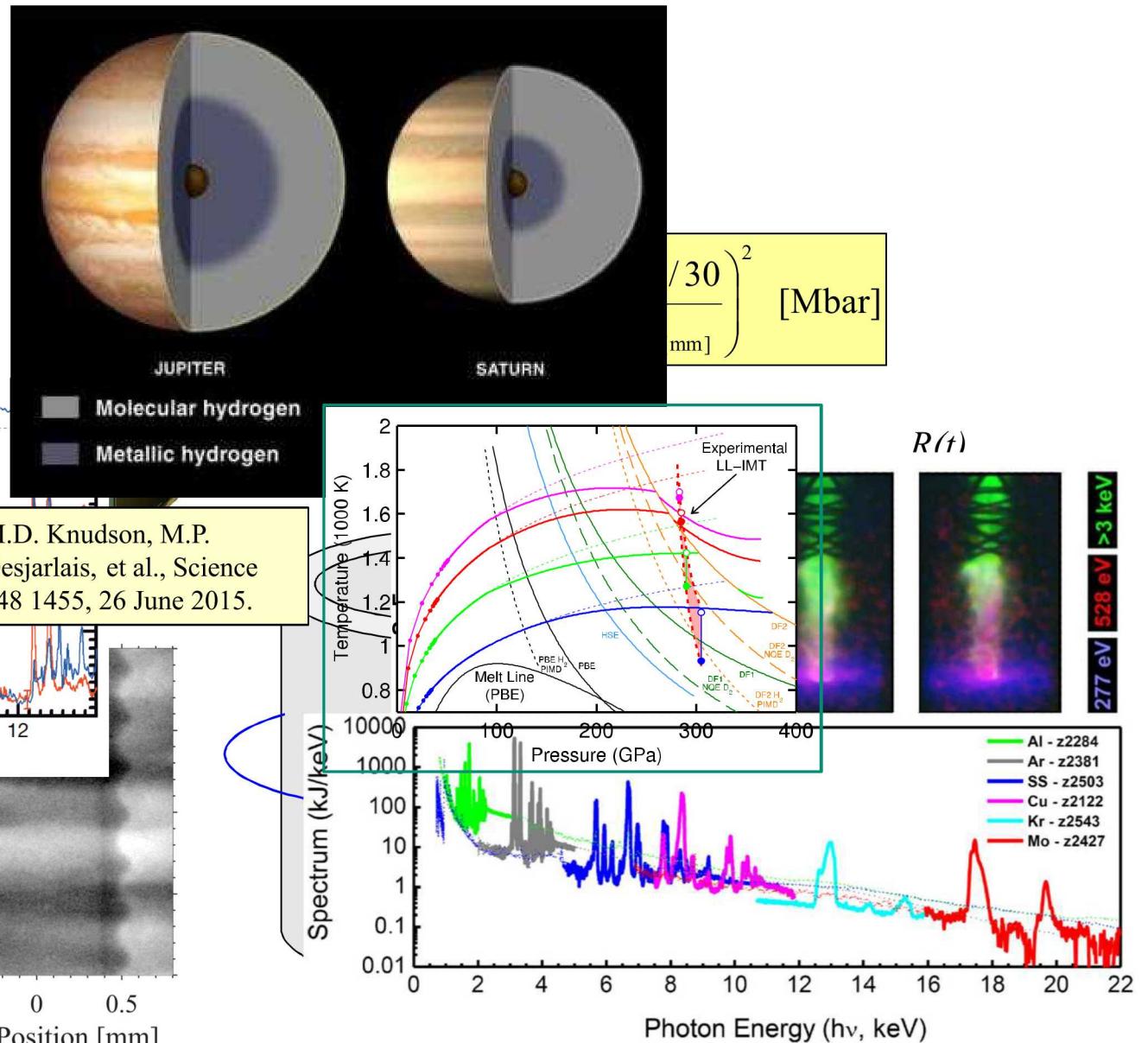
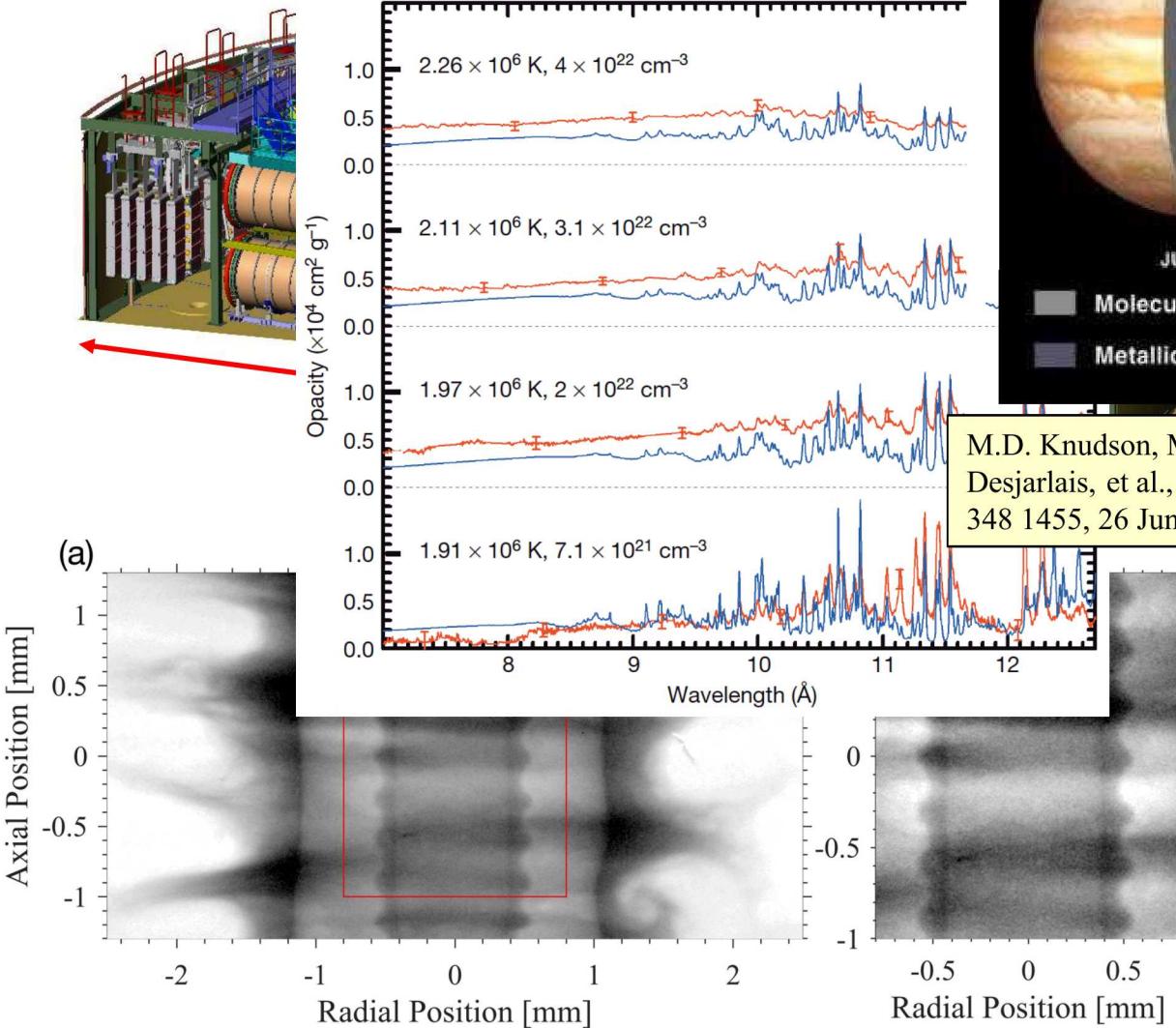
# Pulsed-power uses compression of electrical energy in both space and time to create HED conditions



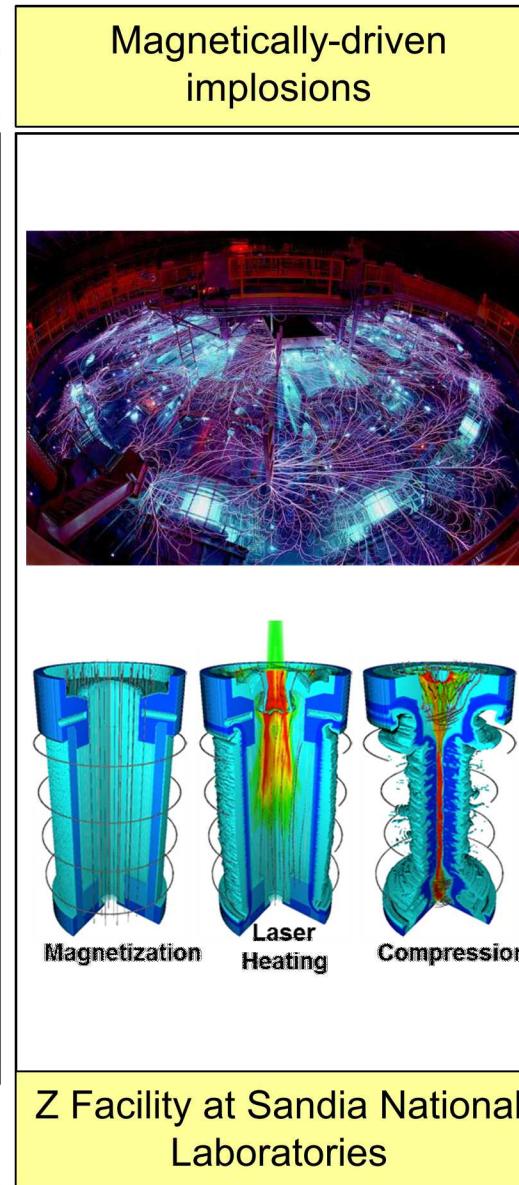
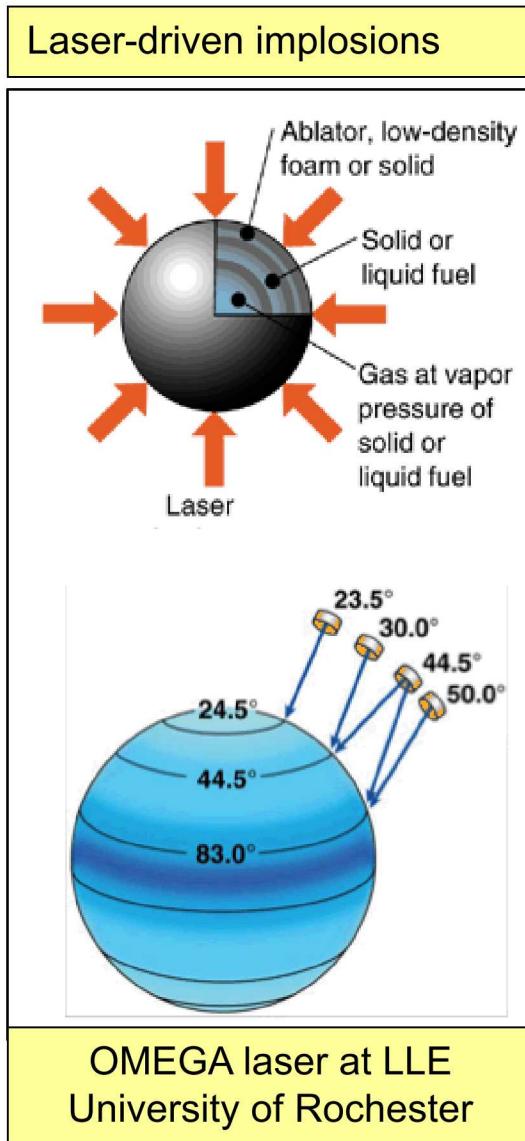
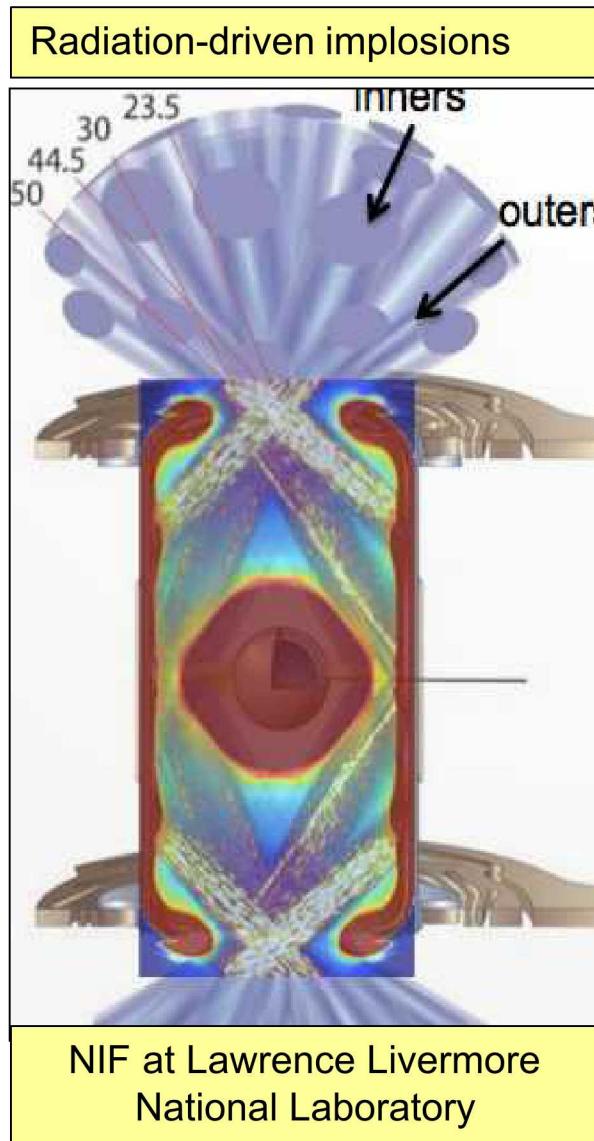
$$P = \frac{B^2}{2\mu_0} = 140 \cdot \left( \frac{I_{[\text{MA}]} / 30}{R(t)_{[\text{mm}]} \cdot 10^3} \right)^2 \text{ [Mbar]}$$



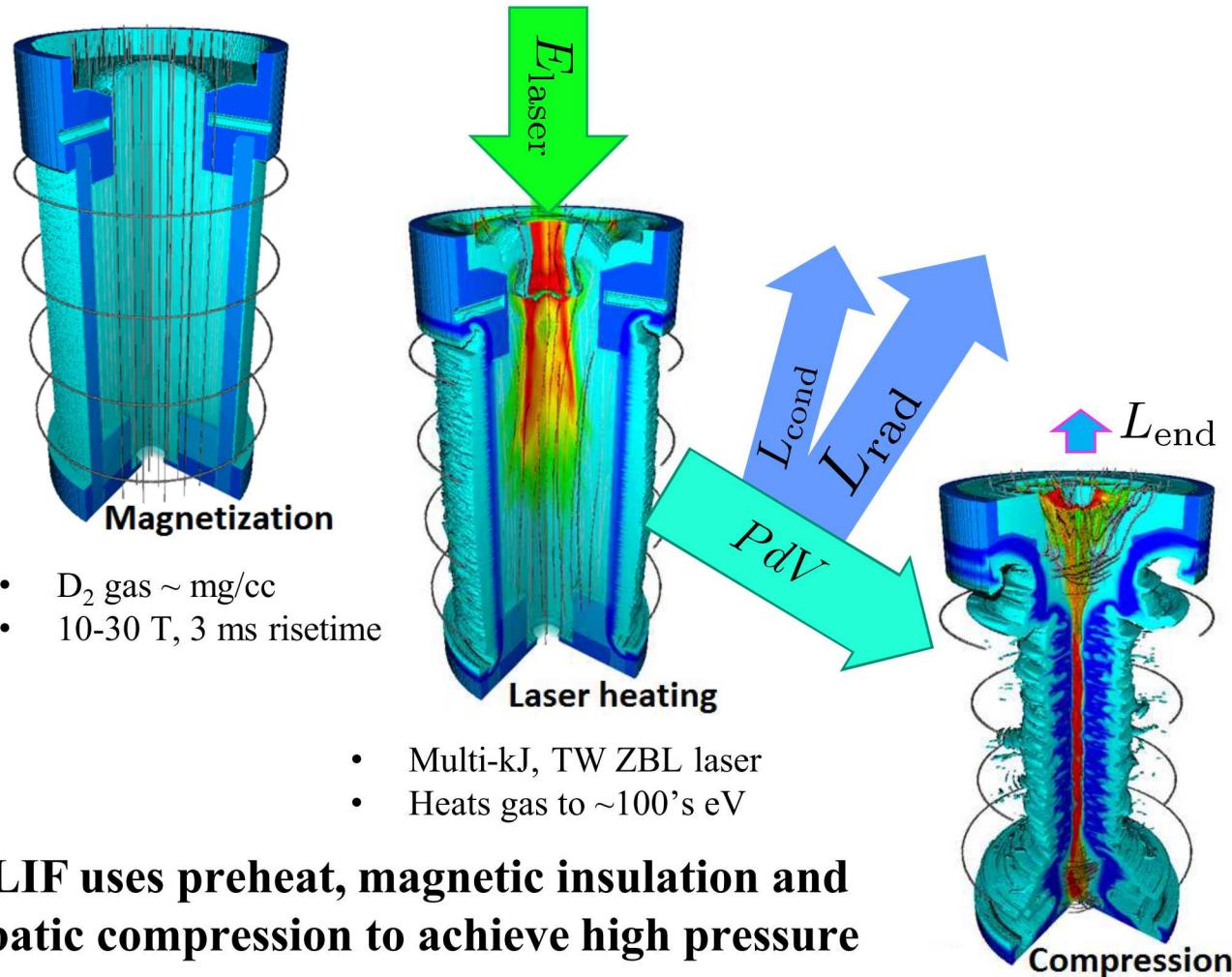
# We use this capability to perform a wide range of experiments



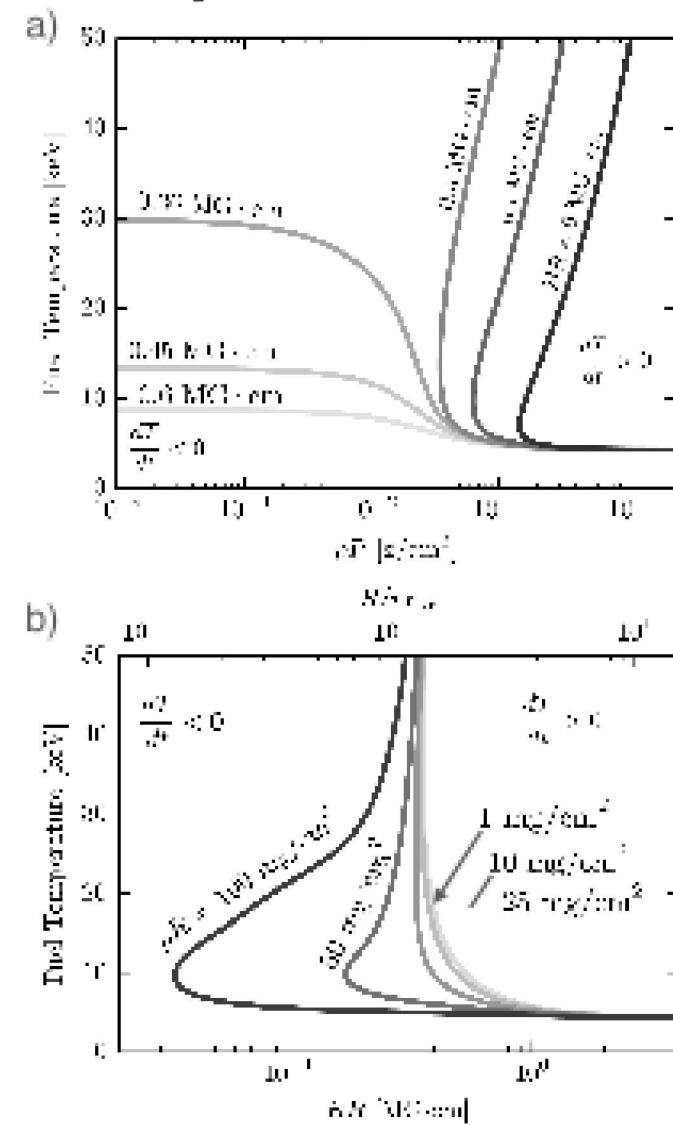
# ICF research using magnetic direct drive is part of the mainline national program



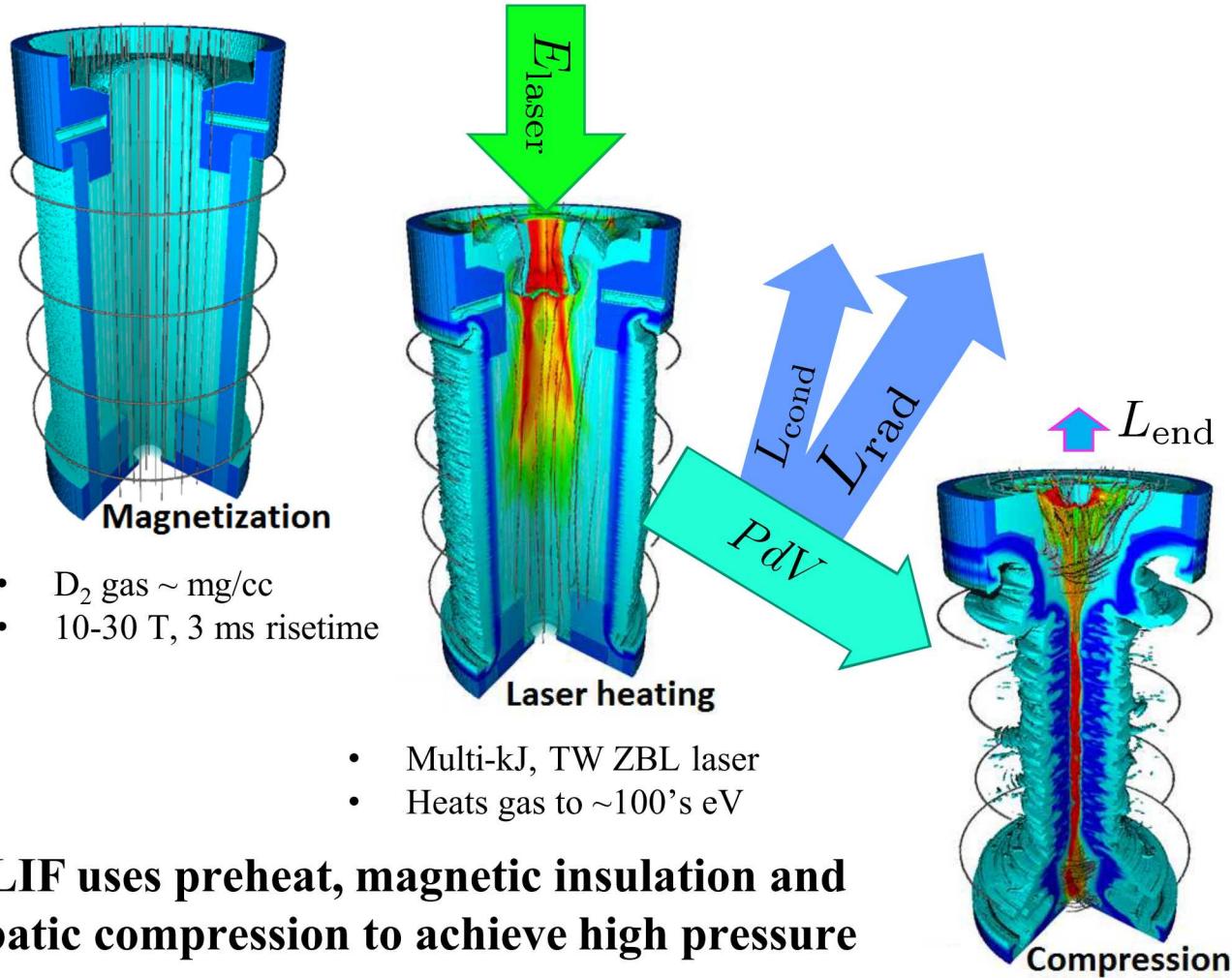
# MIF could provide a cheap, efficient path to high yield fusion, an enabling capability for stockpile stewardship sciences



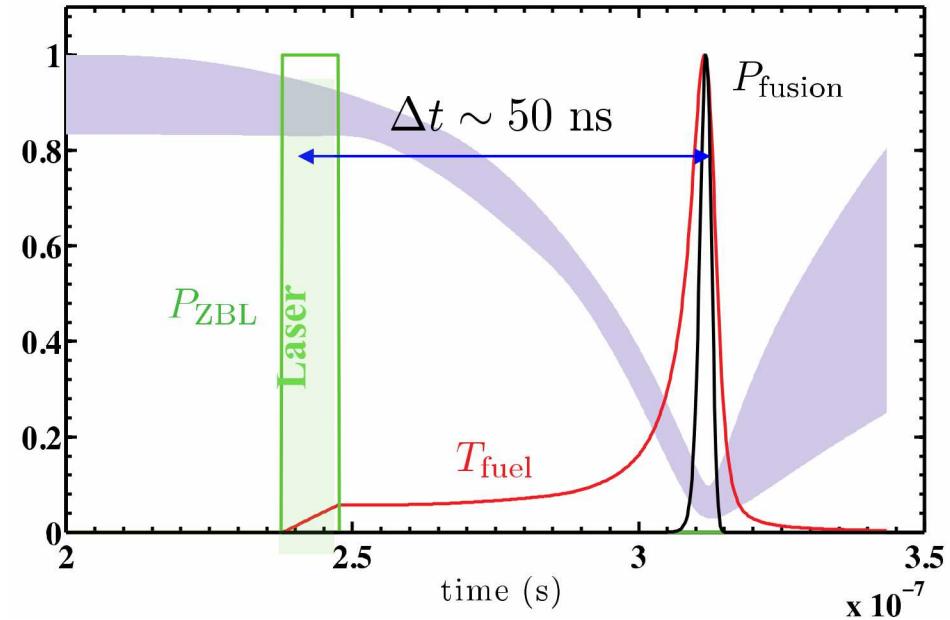
**MagLIF uses preheat, magnetic insulation and adiabatic compression to achieve high pressure**



# MIF could provide a cheap, efficient path to high yield fusion, an enabling capability for stockpile stewardship sciences

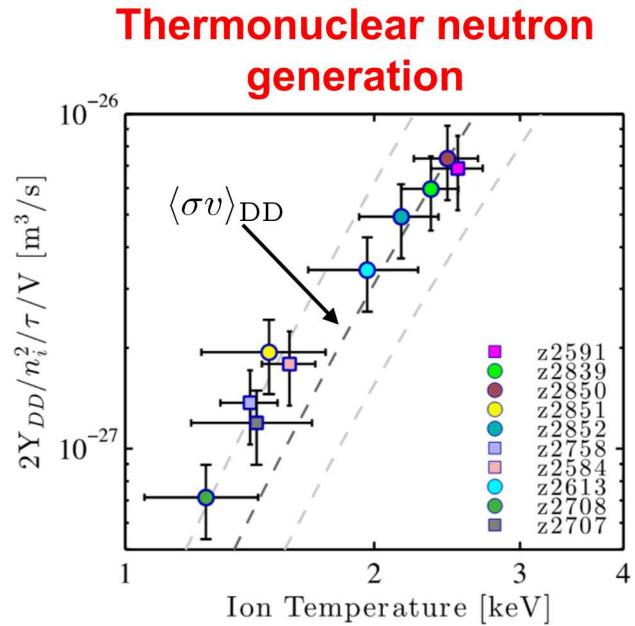


**MagLIF uses preheat, magnetic insulation and adiabatic compression to achieve high pressure**



- B-field confines fusion products with low fuel  $\rho R$
- Magnetic insulation keeps fuel hot
- Laser heating allows high pressures with the lower implosion velocities
- Calculations show scaling to high yield and gain

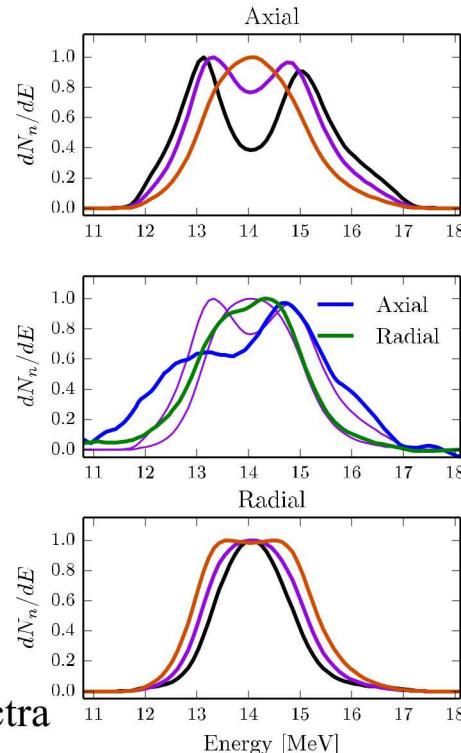
# MagLIF experiments have demonstrated key aspects of magneto-inertial fusion



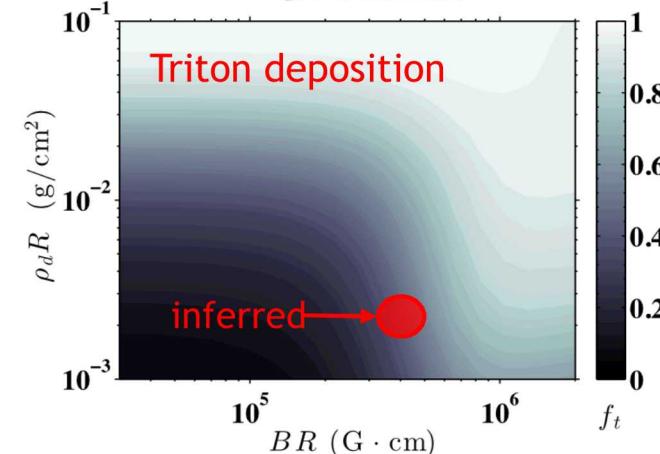
- Isotropic, Gaussian DD neutron spectra
- DD neutron yields =  $3 \times 10^{12}$
- Ion temps = 2.5-3 keV
- Electron temps = 3.1 keV (from x-ray spectroscopy)

M.R. Gomez et al., Phys. Rev. Lett. **113**, 155003 (2014)

## Magnetic flux compression



## Confinement of fusion products



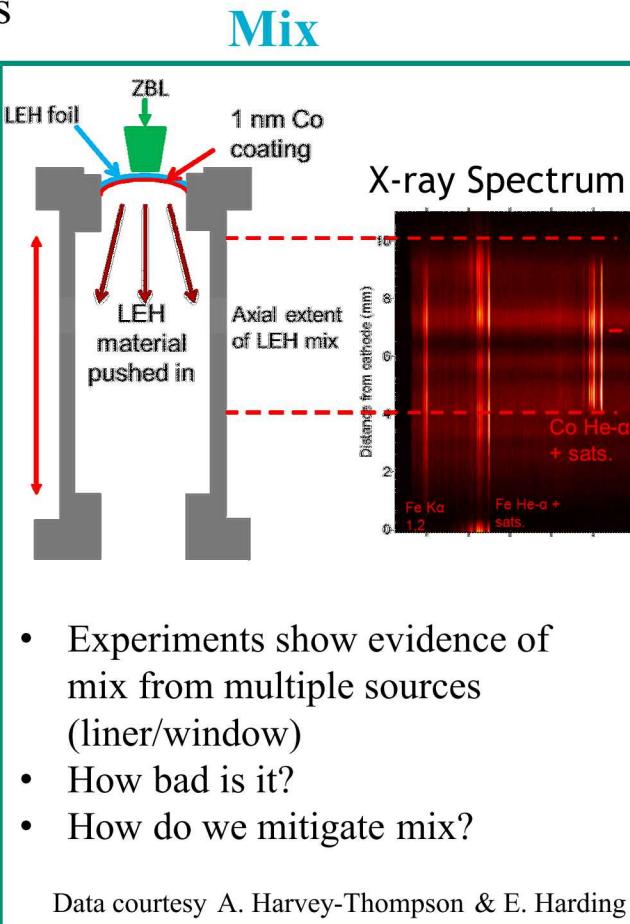
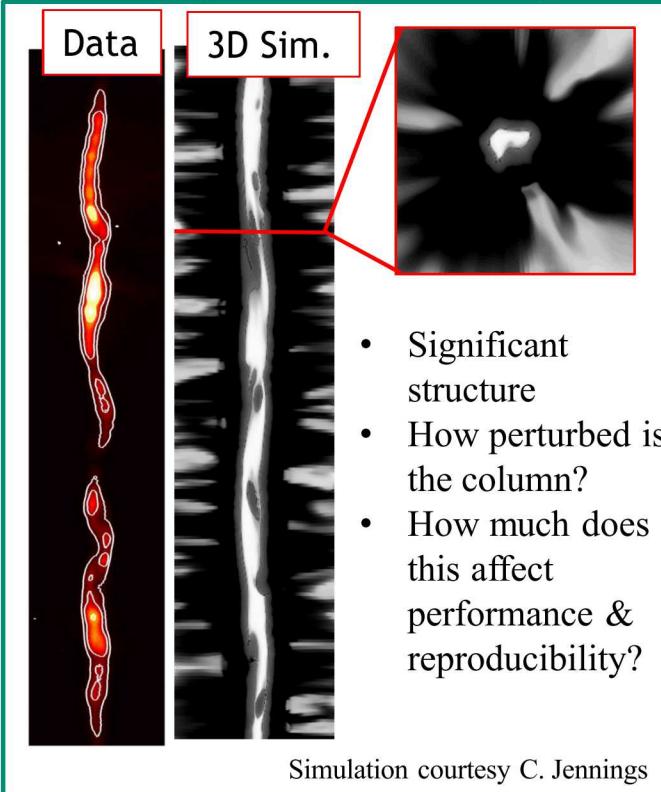
- $BR = 0.25\text{--}0.35 \text{ MG}^*\text{cm}$
- $R/R_{T,\alpha} \sim 1\text{--}2$
- Magnetized, trapped T's,  $\alpha$ 's!
- Important for scaling to ignition!

P.F. Knapp et al., Phys. Plasmas, **22**, 056312 (2015)  
P.F. Schmit et al., PRL 113, 155004 (2014)

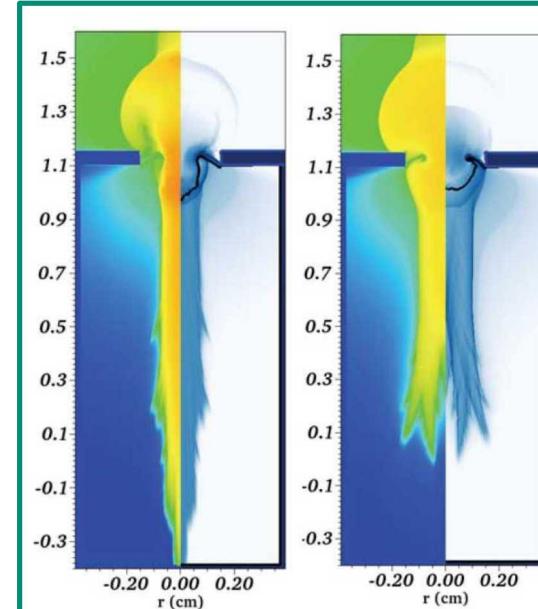
# Despite promising early results, questions and concerns remain

Target performance is not as high as predicted, what are the primary causes and how do we mitigate them?

## Implosion Stability



## Laser Energy Coupling



- How much energy?
- What is the optimum for good coupling w/ low mix
- How does performance scale w/ energy experimentally?

Simulations courtesy M. Glinsky

To address these concerns and improve our understanding of this platform we are developing a *Bayesian Data Assimilation* engine

## A canonical example: How do we measure the fuel pressure in fusion experiments?

$$P_{\text{HS}} = (1 + \langle Z \rangle) \sqrt{\frac{2Y_{\text{DD}}}{V\tau_b S(T)}}$$

$$S(T) = \frac{\langle \sigma v \rangle_{\text{DD}}}{T_i^2}$$

By assuming a uniform plasma in time and space we can estimate the pressure by inverting the yield equation

## A canonical example: How do we measure the fuel pressure in fusion experiments?

12

$$P_{\text{HS}} = (1 + \langle Z \rangle) \sqrt{\frac{2Y_{\text{DD}}}{V\tau_b S(T)}}$$

$$S(T) = \frac{\langle \sigma v \rangle_{\text{DD}}}{T_i^2}$$

Indium Activation measurement

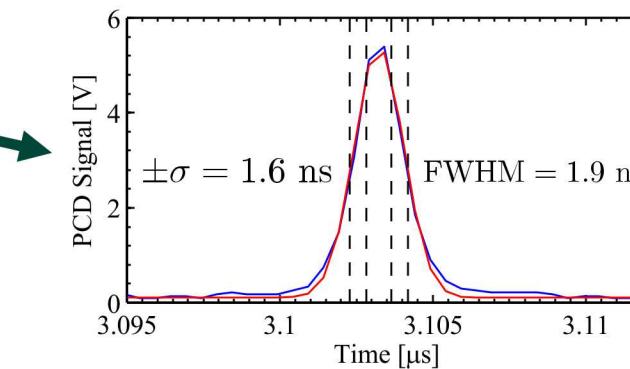
On Z, we measure the DD neutron yield using Indium activation

# A canonical example: How do we measure the fuel pressure in fusion experiments?

13

$$P_{\text{HS}} = (1 + \langle Z \rangle) \sqrt{\frac{2Y_{\text{DD}}}{V \tau_b} S(T)}$$

$$S(T) = \frac{\langle \sigma v \rangle_{\text{DD}}}{T_i^2}$$



The burn duration is measured using the x-ray power history as a surrogate

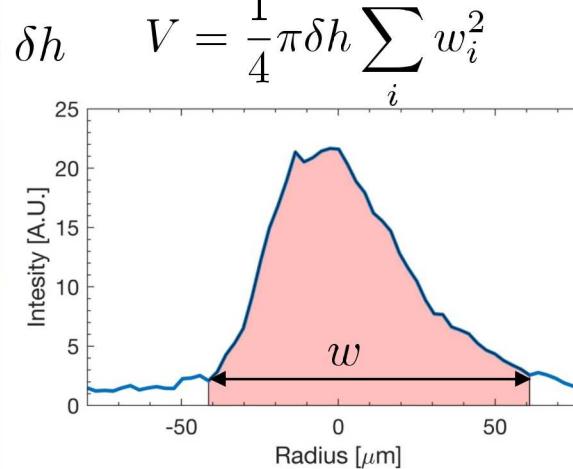
We assume the FWHM of the x-ray pulse is a good stand in

# A canonical example: How do we measure the fuel pressure in fusion experiments?

14

$$P_{\text{HS}} = (1 + \langle Z \rangle) \sqrt{\frac{2Y_{\text{DD}}}{V\tau_b S(T)}}$$

$$S(T) = \frac{\langle \sigma v \rangle_{\text{DD}}}{T_i^2}$$



We estimate the volume using x-ray imaging

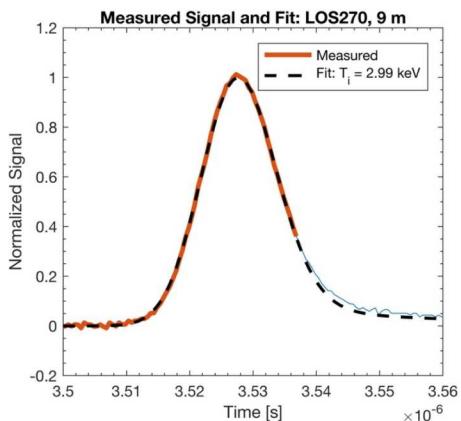
Assume the column is locally cylindrically symmetric and use the width containing 85% of the area under the curve to approximate the radius of the column

# A canonical example: How do we measure the fuel pressure in fusion experiments?

15

$$P_{\text{HS}} = (1 + \langle Z \rangle) \sqrt{\frac{2Y_{\text{DD}}}{V\tau_b S(T)}}$$

$$S(T) = \frac{\langle \sigma v \rangle_{\text{DD}}}{T_i^2}$$



The ion temperature is measured using neutron time of flight (nTOF) assuming no contribution to residual velocity

# A canonical example: How do we measure the fuel pressure in fusion experiments?

$$P_{\text{HS}} = (1 + \langle Z \rangle) \sqrt{\frac{2Y_{\text{DD}}}{V\tau_b S(T)}}$$

$$S(T) = \frac{\langle \sigma v \rangle_{\text{DD}}}{T_i^2}$$

The average ionization of the fuel is determined by mix

In this simple example we have no means of constraining this parameter

Assuming the mix species is fully ionized we have

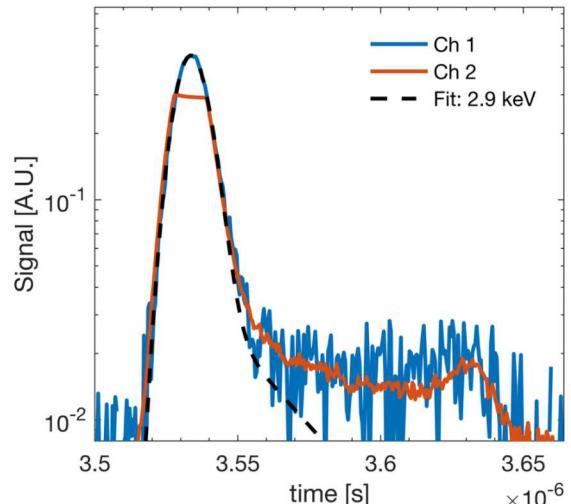
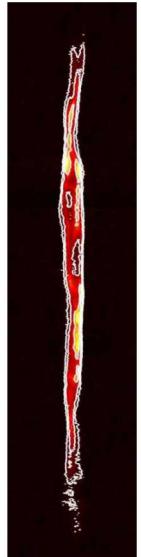
$$(1 + \langle Z \rangle) = 2 + f(Z_{\text{mix}} - 1)$$

So for 0%-10% mix of Be we get a +/-7% uncertainty in the pressure

# Putting this all together for two MagLIF experiment illustrates how this approach falls short

z3179: Uncoated AR9 Liner

$$Y_{DD} = 5.5e12$$



$$V = 8.06e-5 \text{ cm}^3$$

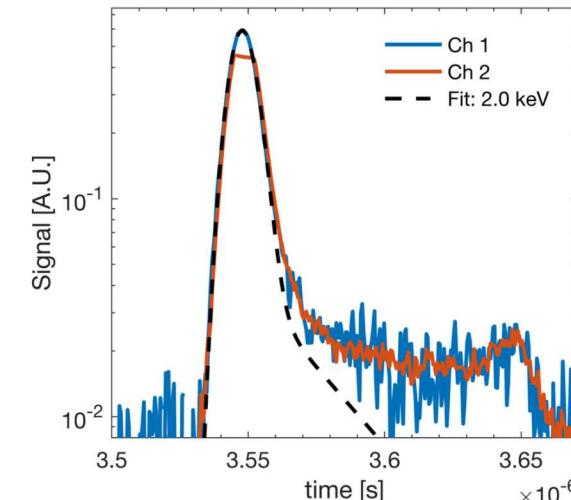
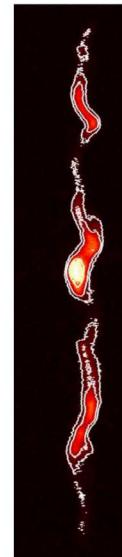
$$\tau_b = 1.8 \text{ ns}$$

$$T_i = 2.9 \text{ keV}$$

$$P = 0.7 \pm 0.17 \text{ Gbar}$$

z3303: Coated AR9 Liner

$$Y_{DD} = 3.5e12$$



$$V = 1.4e-4 \text{ cm}^3$$

$$\tau_b = 1.9 \text{ ns}$$

$$T_i = 2.4 \text{ keV}$$

$$P = 0.6 \pm 0.15 \text{ Gbar}$$

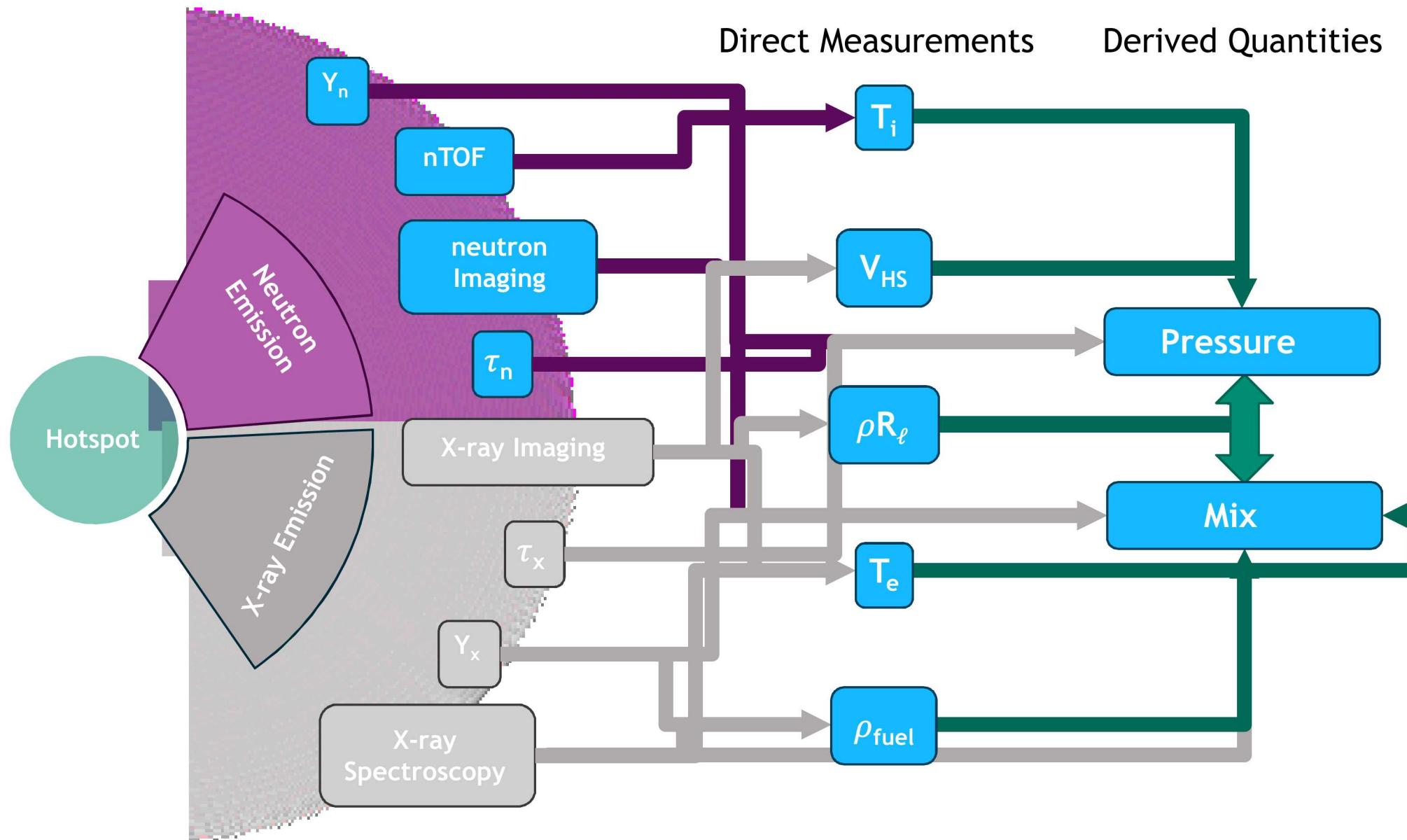
## Error Analysis

$$\sigma_P \approx \frac{P_o}{2} \sqrt{\frac{4\sigma_Z^2}{(1 + \langle Z \rangle)^2} + \frac{\sigma_Y^2}{Y^2} + \frac{\sigma_V^2}{V^2} + \frac{\sigma_\tau^2}{\tau^2} + (\eta - 2)^2 \frac{\sigma_T^2}{T^2}} \approx 25\%$$

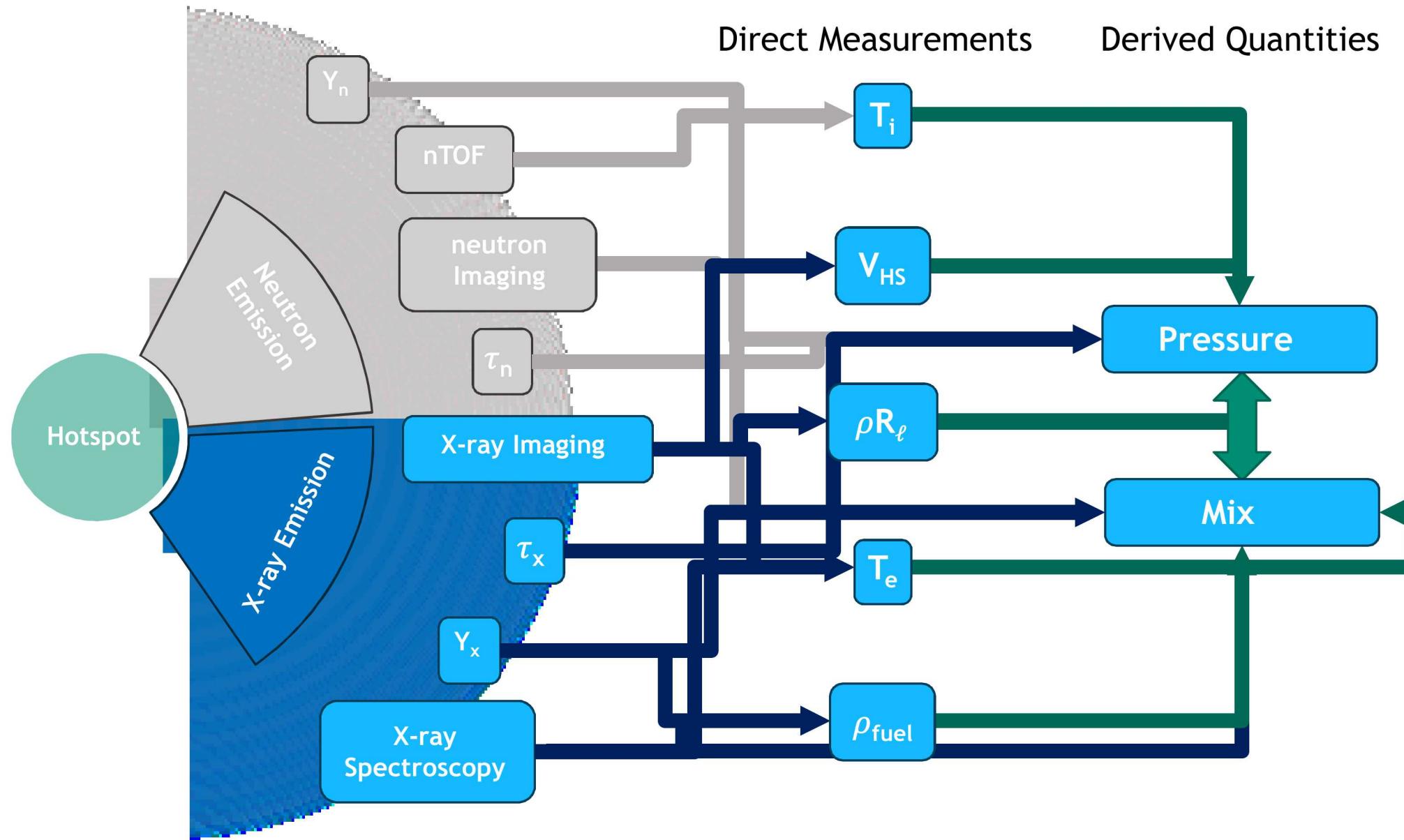
Dominant sources of error are mix and temperature

With the data available this technique is not able to distinguish between these two experiments despite a 2x difference in yield and dramatically different structure!

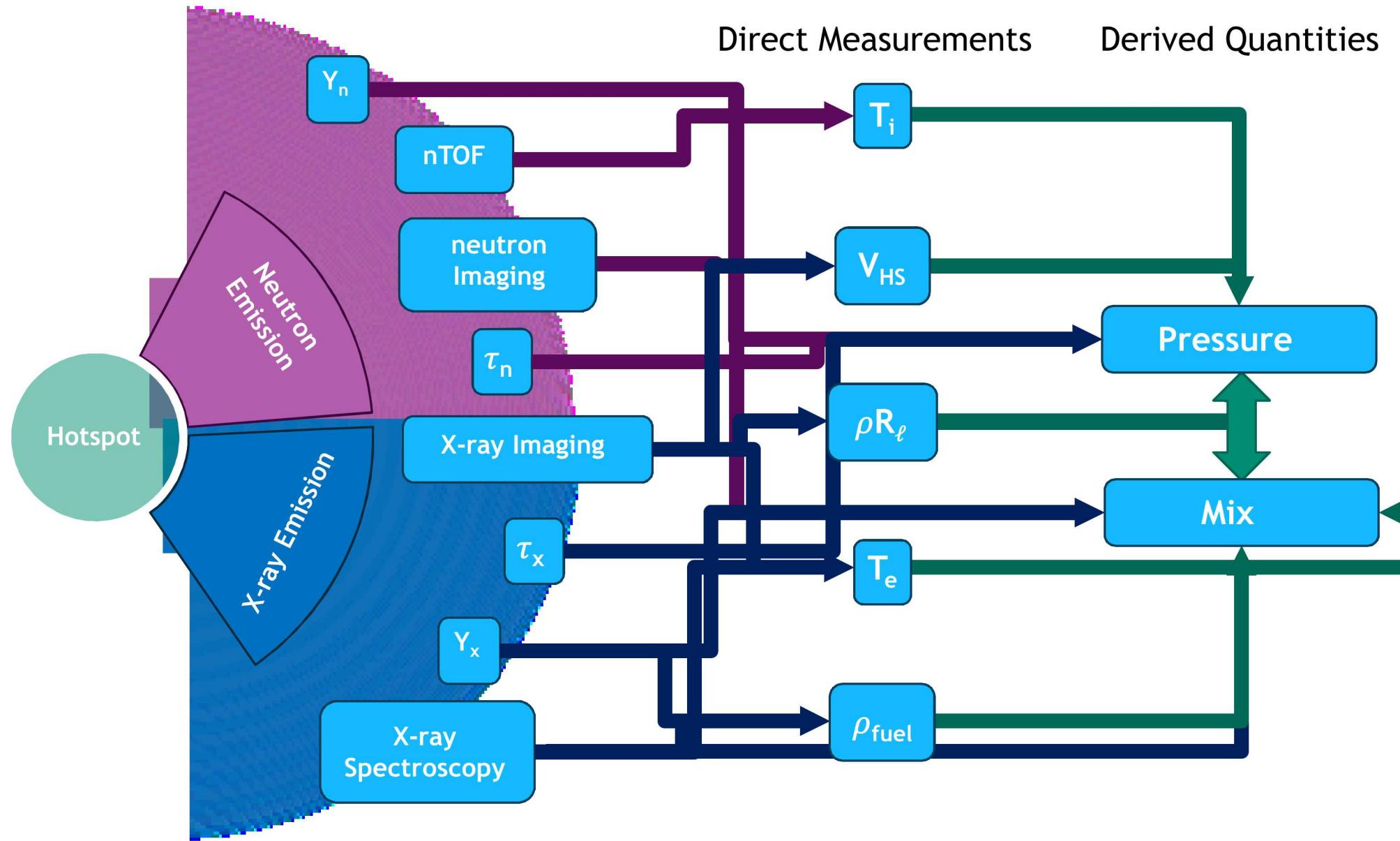
We can do better by leveraging the fact that all of our diagnostics are different transformations of the emission from the same plasma



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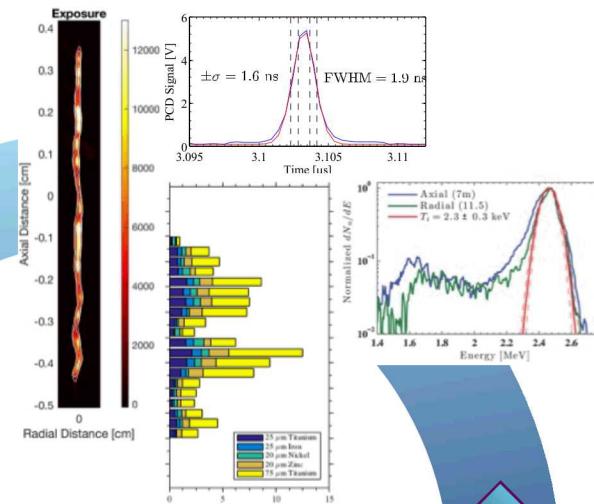


# Bayesian data assimilation provides a statistical framework with which to carry out this analysis

Bayes' Theorem

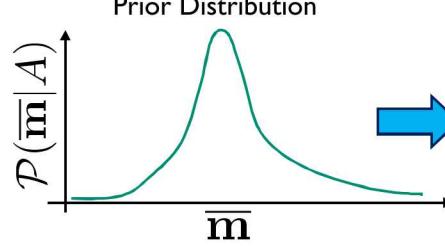
$$\mathcal{P}(\bar{\mathbf{m}}|\bar{\mathbf{d}}, A) = \frac{\mathcal{P}(\bar{\mathbf{d}}|\bar{\mathbf{m}}, A)\mathcal{P}(\bar{\mathbf{m}}|A)}{\mathcal{P}(\bar{\mathbf{d}}|A)}$$

Experimental Data



Likelihood

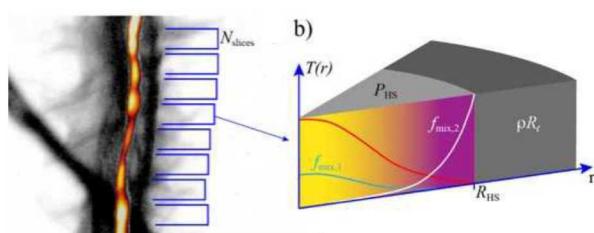
$$\mathcal{P}(\bar{\mathbf{x}}|\bar{\mathbf{m}}, A) \propto \prod_{i=1}^N \exp\left(-\frac{(\mathcal{F}_i(\bar{\mathbf{m}}) - x_i)^2}{2\sigma_i^2}\right)$$



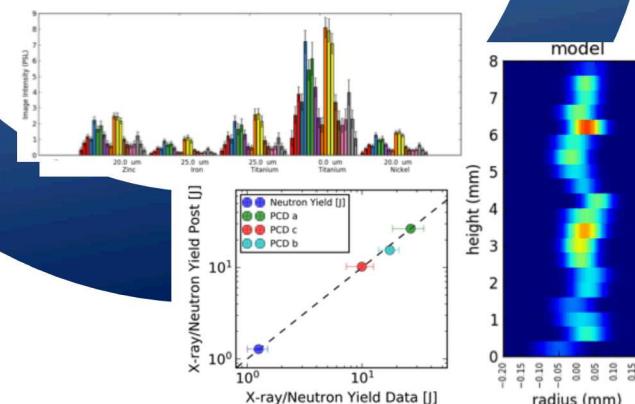
Model Parameters

$$\bar{\mathbf{m}} = \begin{cases} P_{\text{HS}} \\ T \\ f_{\text{mix}} \\ R_{\text{HS}} \\ \rho R_{\ell} \end{cases}$$

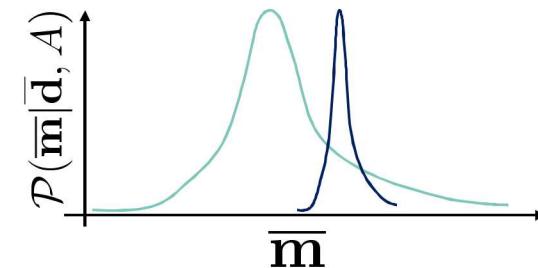
Proposed Stagnation Conditions



Synthetic Data



Posterior Distribution



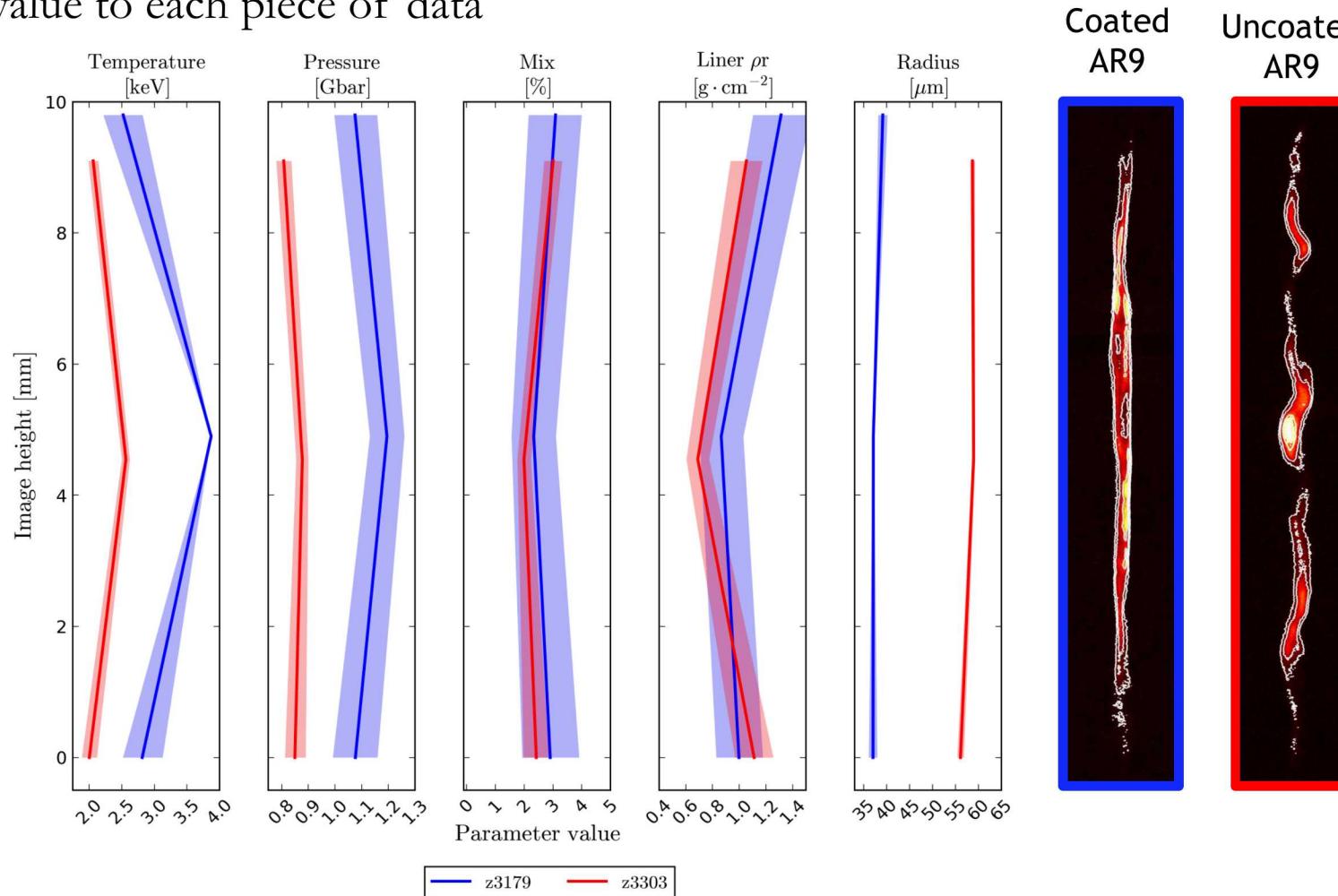
Outputs/Benefits:

- most likely parameter values
- confidence intervals
- correlations
- Value of information

# Comparing the same two experiments using our Bayesian model allows us to look deeper into the data set with more confidence

We are able to leverage more information from both x-ray and neutron diagnostics

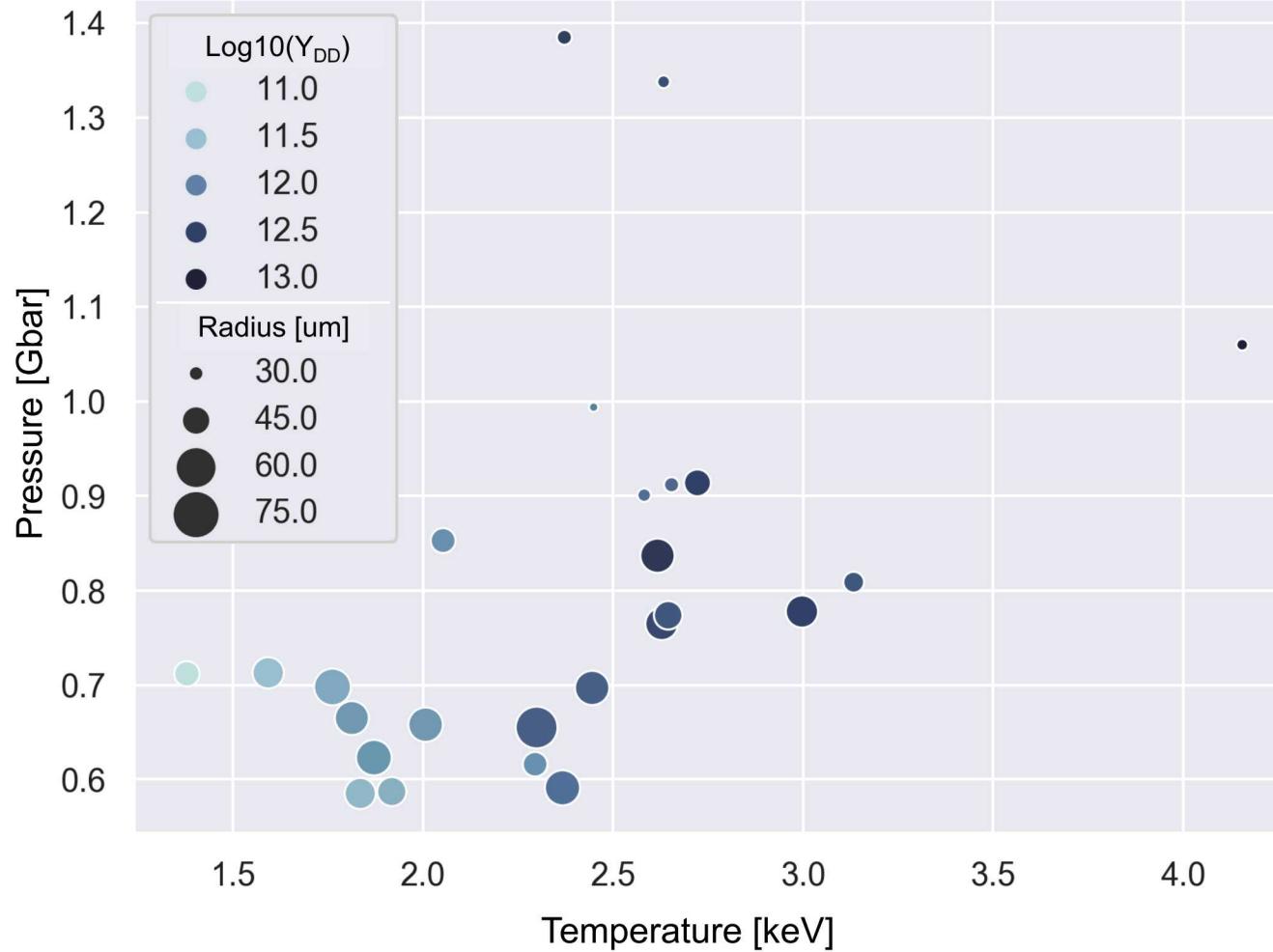
Our model requires consistency between x-rays and neutrons providing additional constraints and adding value to each piece of data



The amount of information we can get out is currently limited by the complexity of our model

- The plasma model assumes local cylindrical symmetry which limits the analysis to “bulk” properties and gross variations
- Capturing the morphology so that we can relate structure to conditions is the ultimate goal

# With this technique we begin mining data from a large database of MagLIF experiments



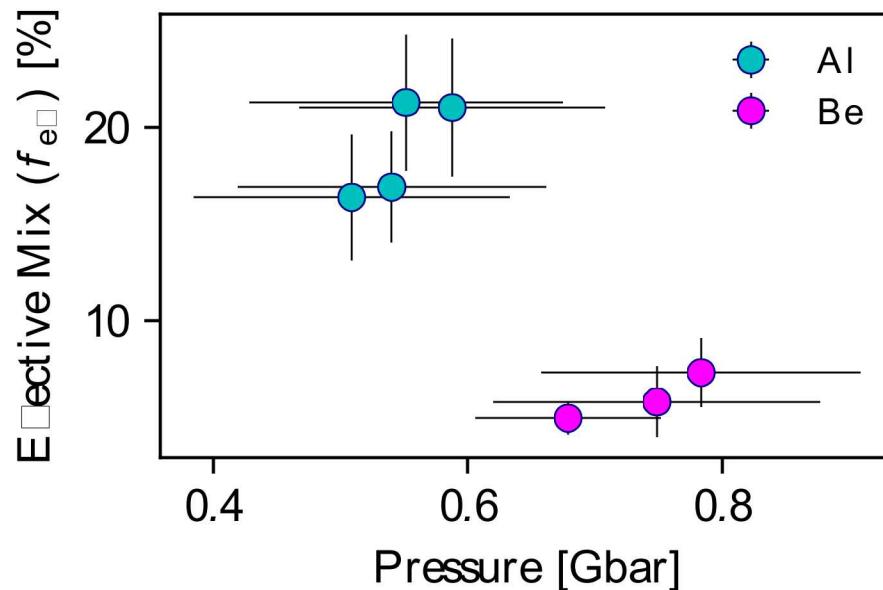
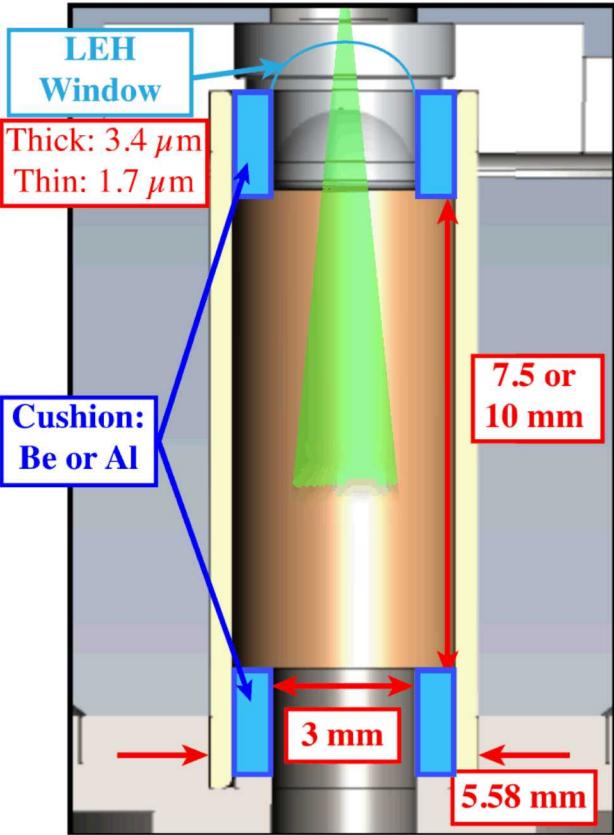
By determining all of the various model parameters simultaneously we can begin to examine trends across experiments

We see from this dataset that there are multiple ways to get the same yield e.g.

- moderate pressure, high temperature
- High pressure, moderate temperature

Central temperatures below  $\sim 2.3$  keV are always associated with low performance

We showed that this technique can be used to differentiate between high- and low-mix experiments and isolate the probable sources of mix

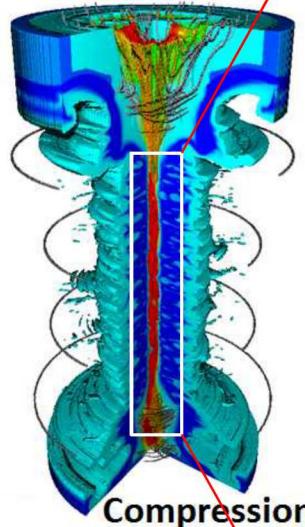


	Aluminum Cushion	Beryllium Cushion
Window	0.5%	0.5%
Cushion	0.57%	1.5%
Liner	2.6%	2.6%

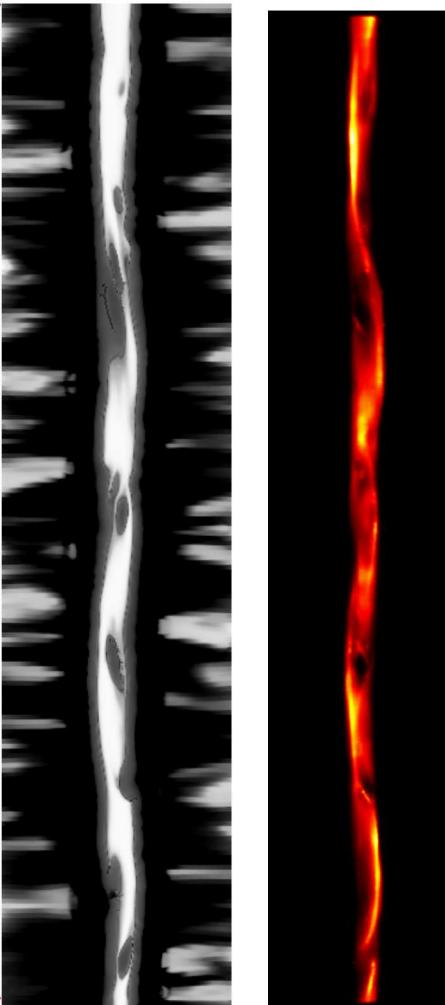
- This analysis determines the stagnation pressure and an *effective* mix fraction (assuming mix is 100% Be)
- The Be cushion shots have, on average
  - 3x less effective mix fraction
  - ~40% higher pressure
- The average hotspot energy is ~50% higher in the Be cushion experiments
- When cushions are made of Al, they overwhelmingly dominate the performance degradation
- Liner accounts for >50% of the mix (by atom)
- Simple  $Z^3$  scaling suggests liner mix and window mix are comparable in terms of losses at stagnation
- Window mix is almost certainly worse than liner mix since it is introduced earlier
- 100% mitigation of mix implies ~3x improvement in performance

## But, as always, there is a problem

Significant 3D structure  
is observed in simulation  
and experiment

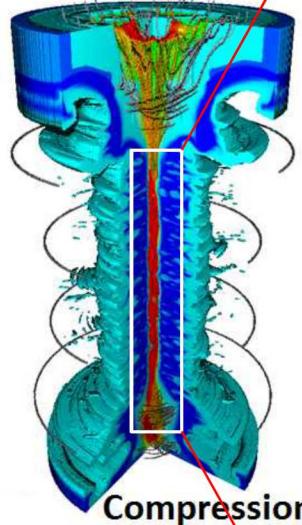


Density slice  
Synthetic image

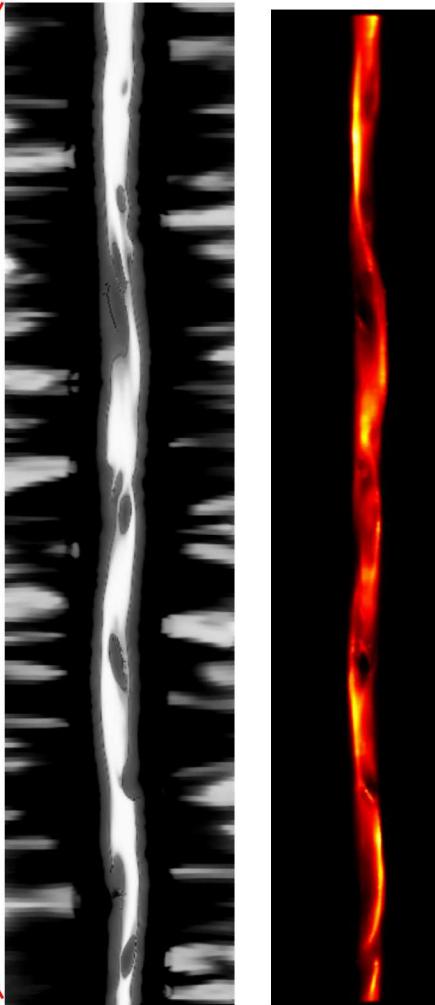


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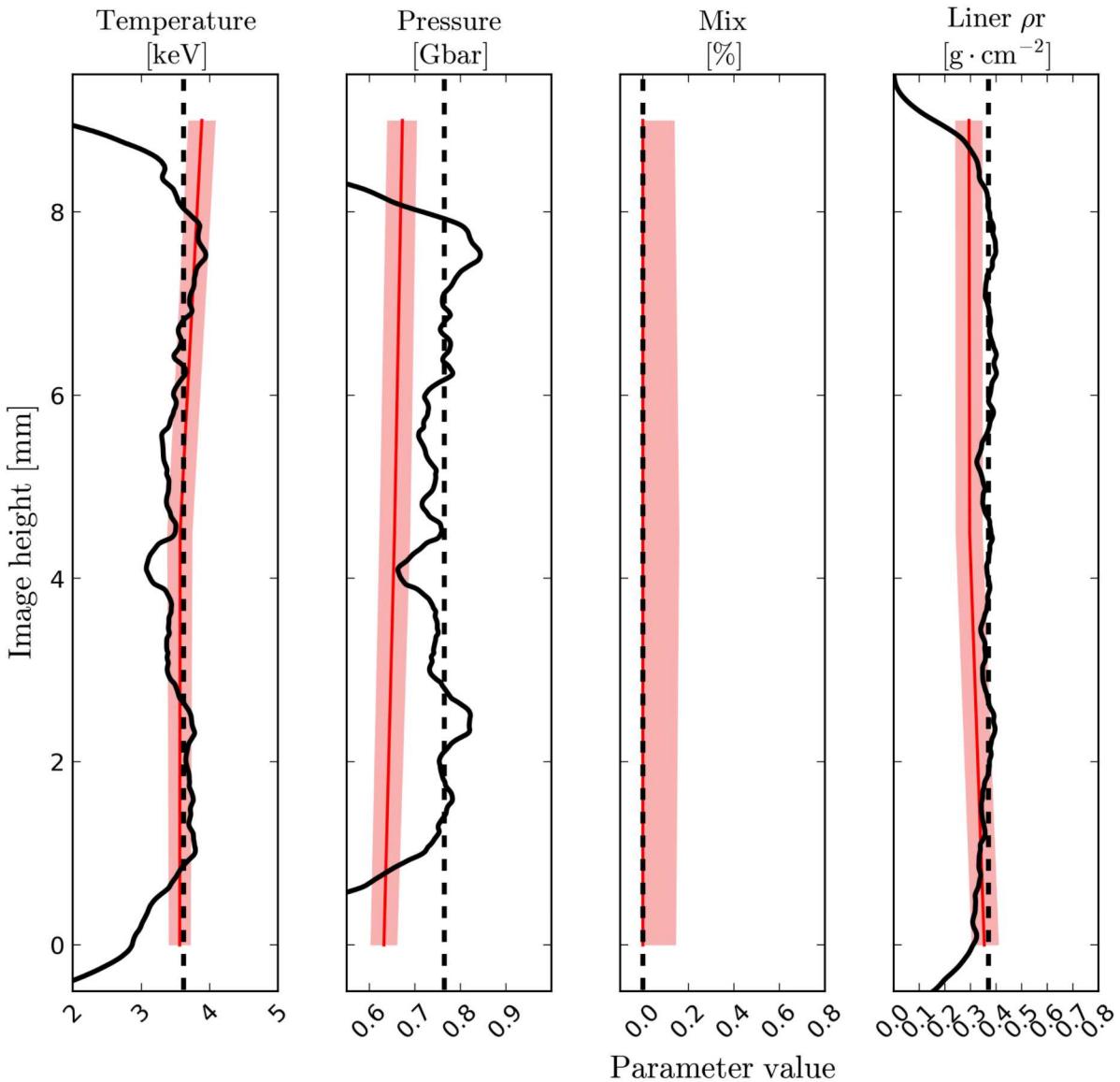
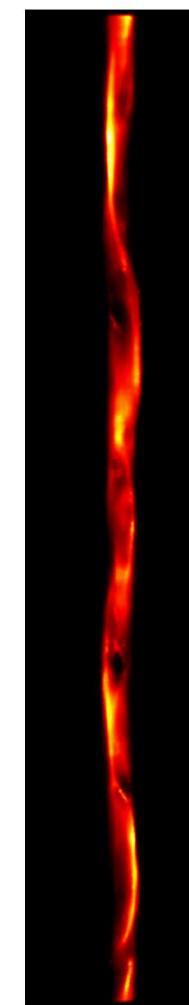
Significant 3D structure  
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Density  
slice

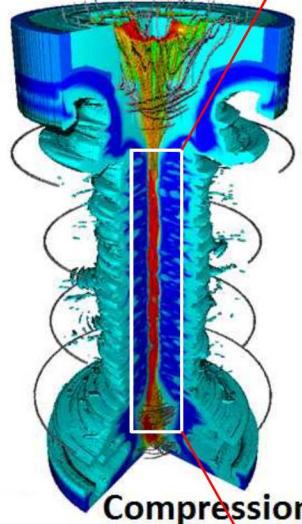


Synthetic  
image

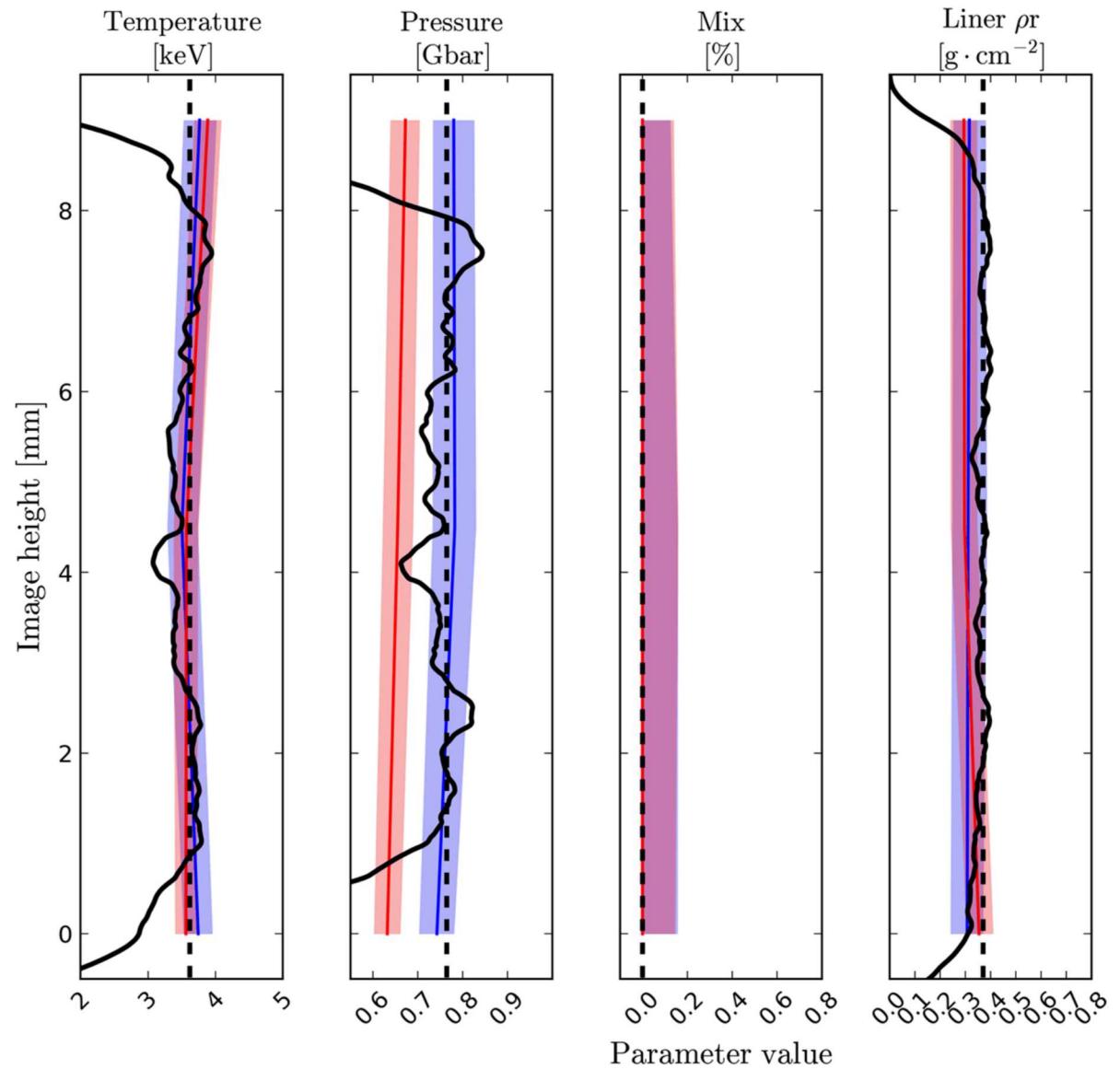
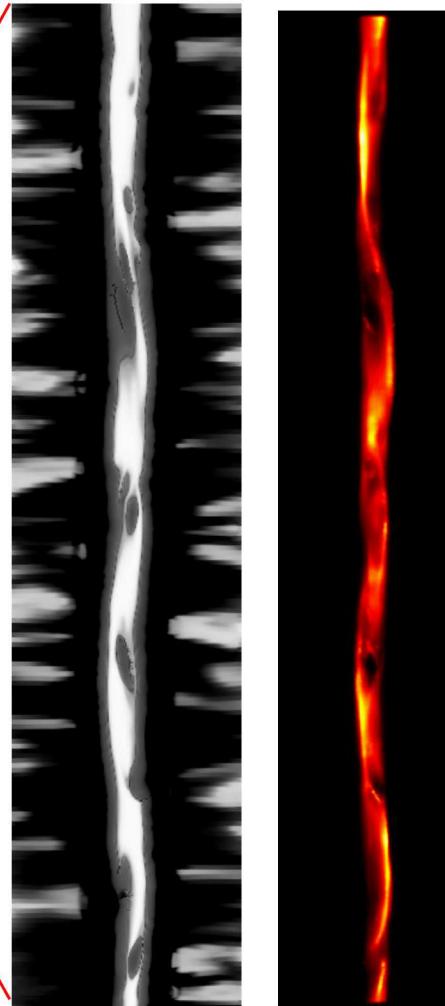


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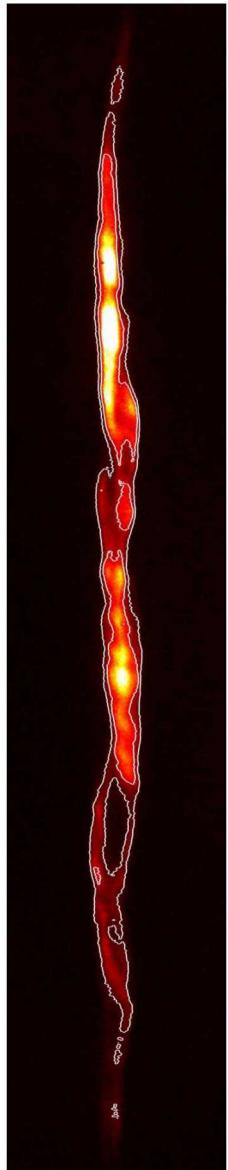
Significant 3D structure  
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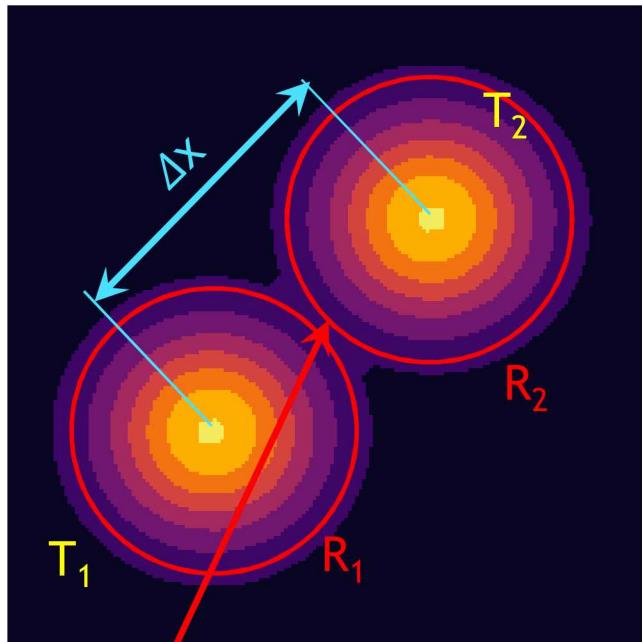
Density  
slice      Synthetic  
image



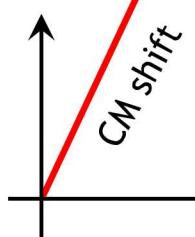
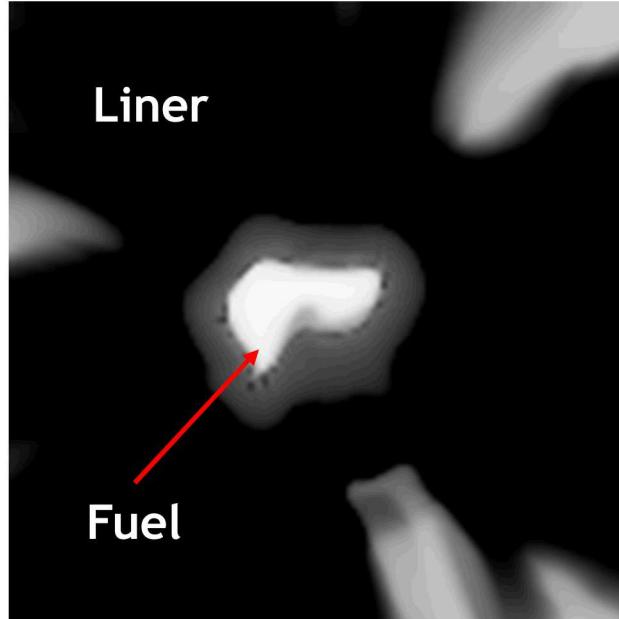
We are developing a model that will be able to handle the structure and hopefully provide deeper insights into confinement



KDE Expansion



Density Slice



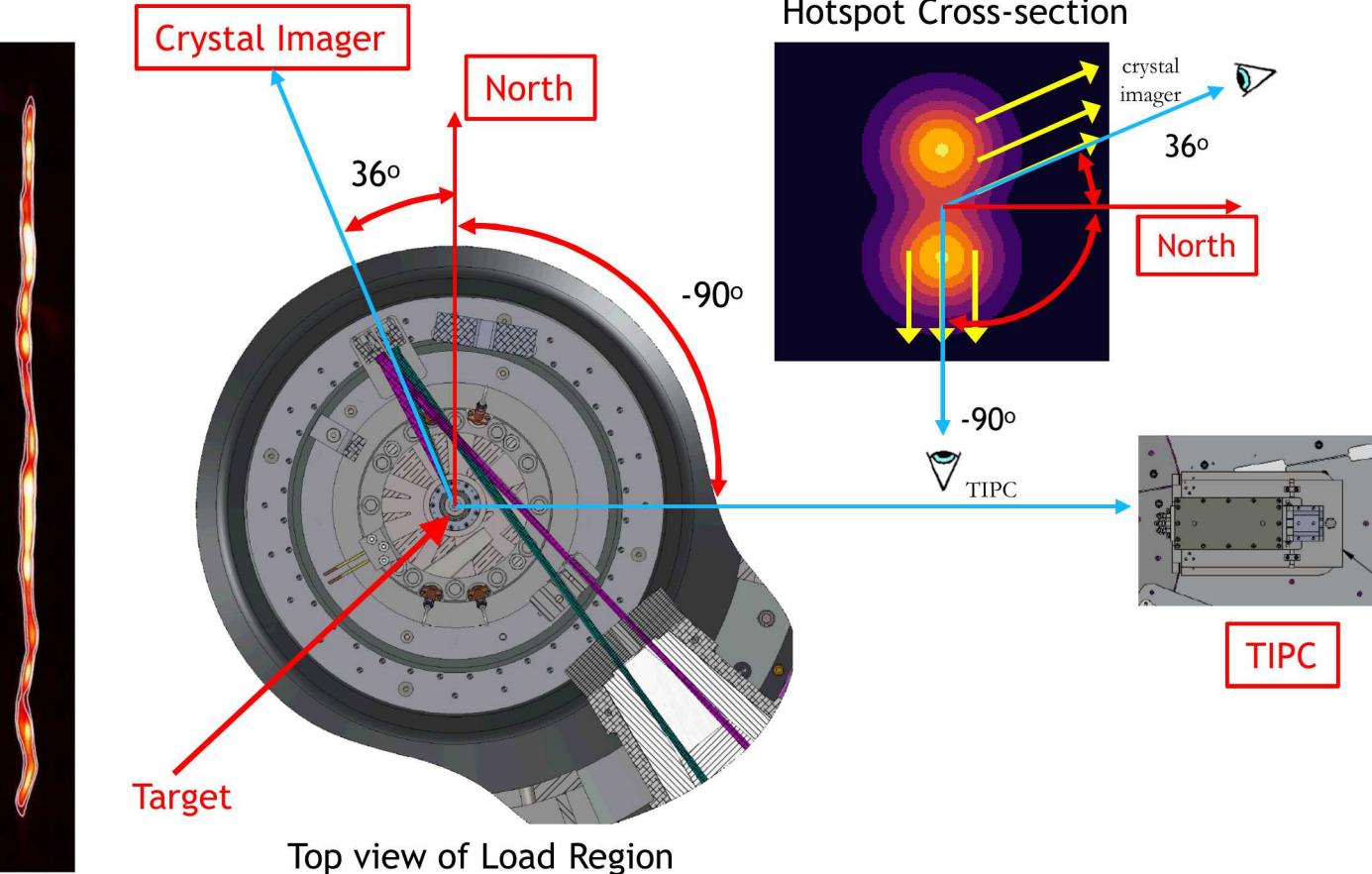
Super-Gaussian Temperature Kernel

$$K(\mathbf{x}|\mathbf{X}, R) = \exp \left( - \left( \frac{1}{2} \frac{(\mathbf{x} - \mathbf{X})^2}{\sigma^2} \right)^p \right)$$

$$\sigma = \frac{2}{3}R$$

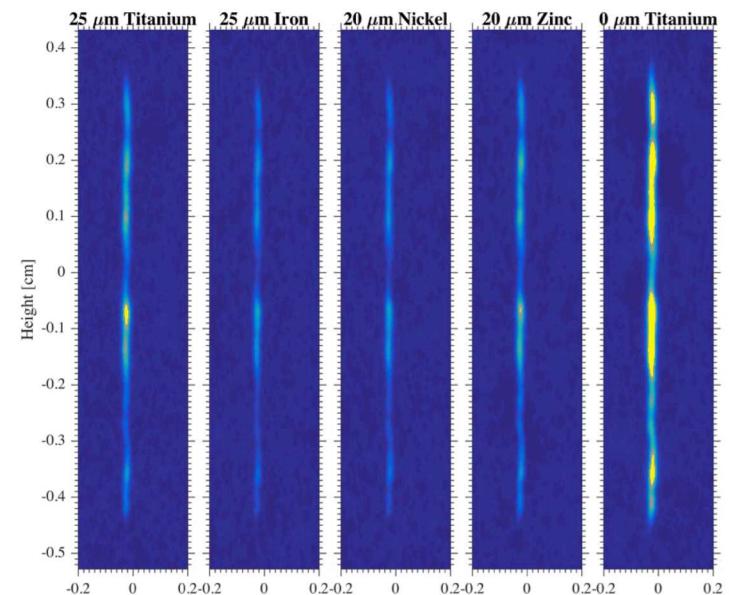
- We see asymmetric emission profiles, and occasional bifurcation of the emission column
- Implies non-cylindrical structure
- Reminiscent of a double helix
- Temperature parameters control the relative peaks of the two modes
- Radius parameters control the relative size of each mode
- Separation and CM shift parameters control spacing and location

# The addition of shape and CM shift parameters requires that we have an additional viewing angle in our diagnostics



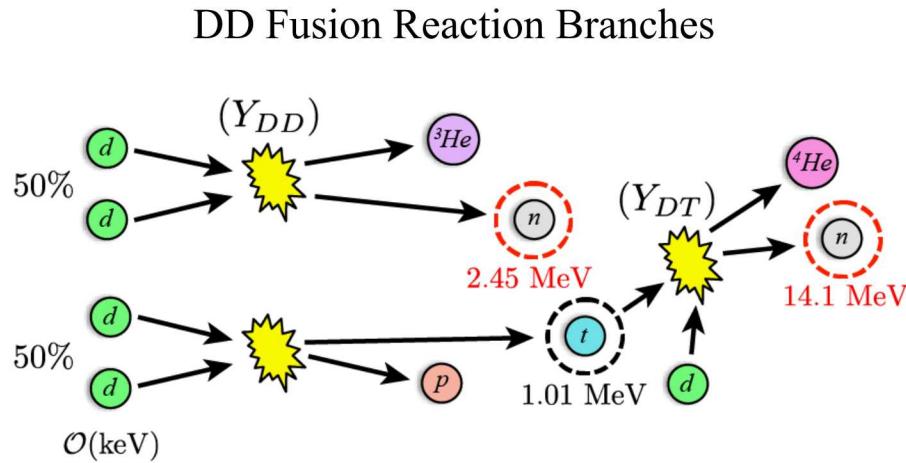
Original model treats TIPC as a 1D imager

Now, we exploit the full images as well as the viewing angles of the crystal imager and TIPC constrain the new parameters



But TIPC has much worse resolution than the crystal imager

# Modeling triton transport and reactions in magnetized plasmas



Probability of a triton reacting with a background deuteron:

$$P_i(\ell) = \int_0^{\ell} n_d(s) \sigma_{DT}(v_i(s)) ds \approx n_d \sigma_{DT} \ell$$

Unmagnetized

$$\frac{Y_{2n}^{DT}}{Y_{1n}^{DD}} \propto \rho R$$

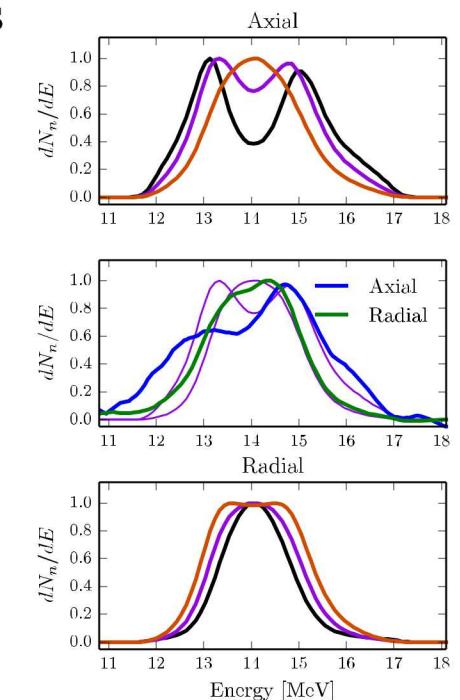
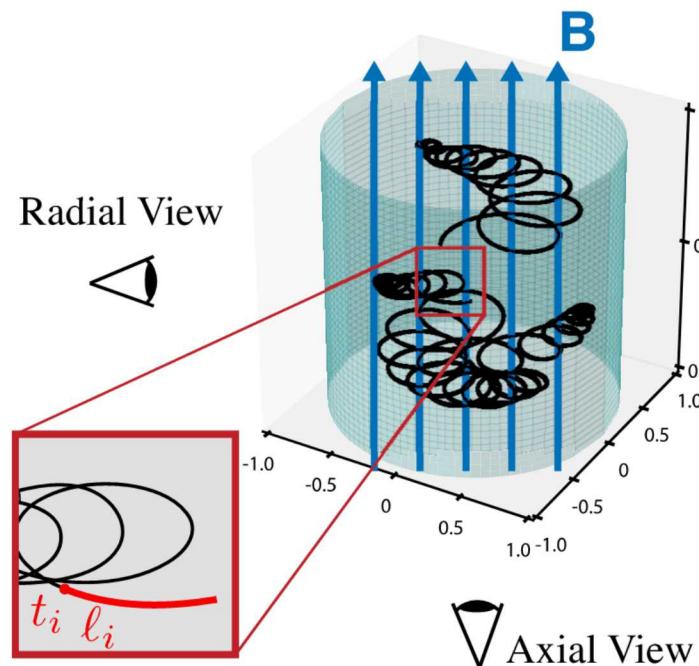
Magnetized

$$\frac{Y_{2n}^{DT}}{Y_{1n}^{DD}} \approx f(BR, \rho R)$$

In limit of low  $\rho R$ , increasing  $BR$  serves primarily to extend triton path length

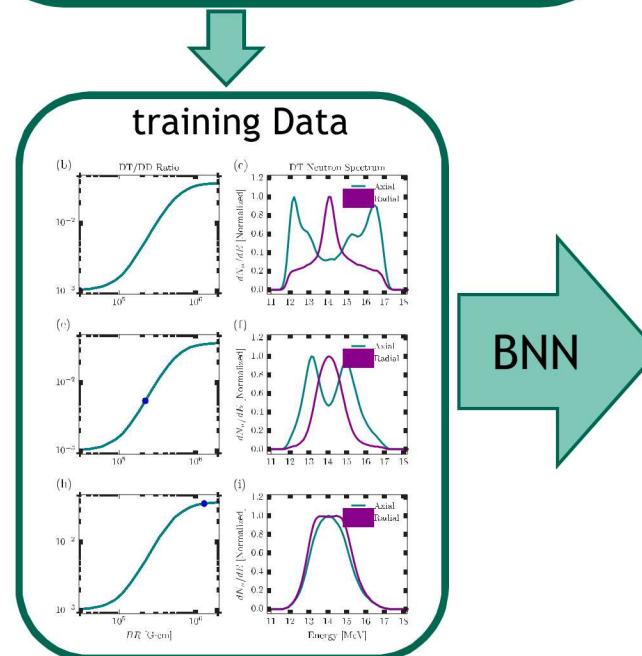
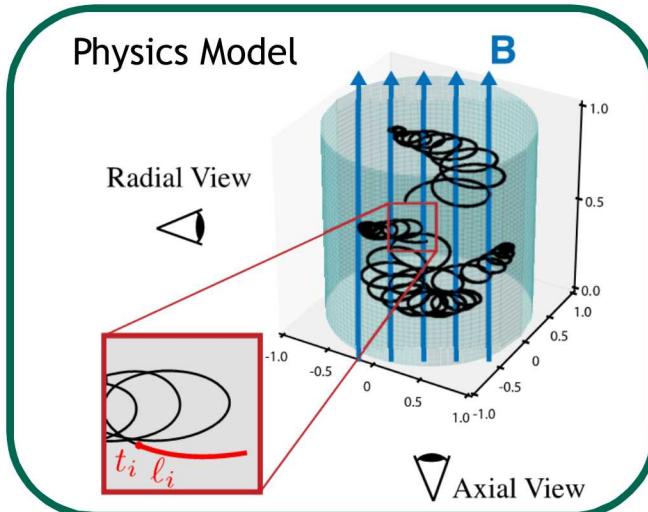
Magnetizing tritons effectively modifies the geometry they “see” as they travel through the fuel

This impacts the DT/DD yield ratio and the secondary DT neutron spectra, which we can use to determine BR, and therefore confinement, through the use of a kinetic model of the tritons

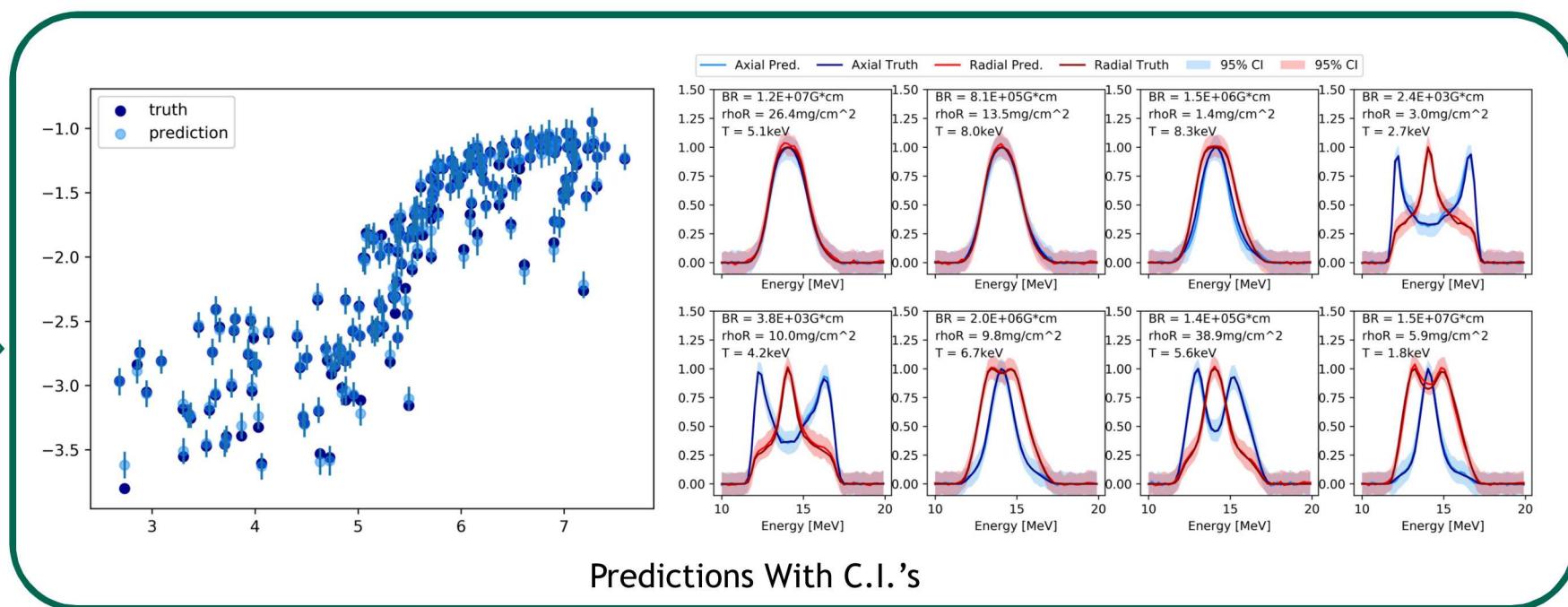


$$BR \approx 4(\pm 0.7) \times 10^5 \text{ G} \cdot \text{cm} \sim 17 \times (BR)_o$$

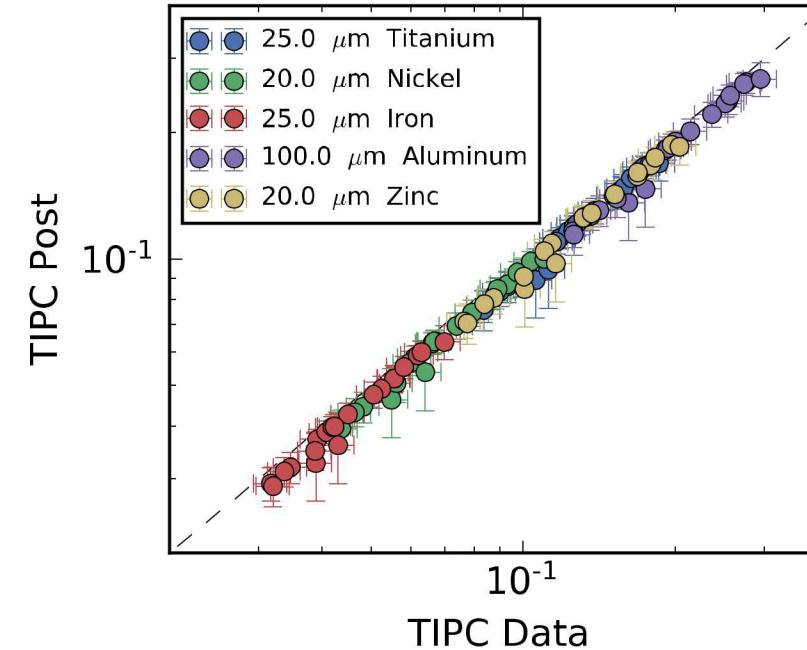
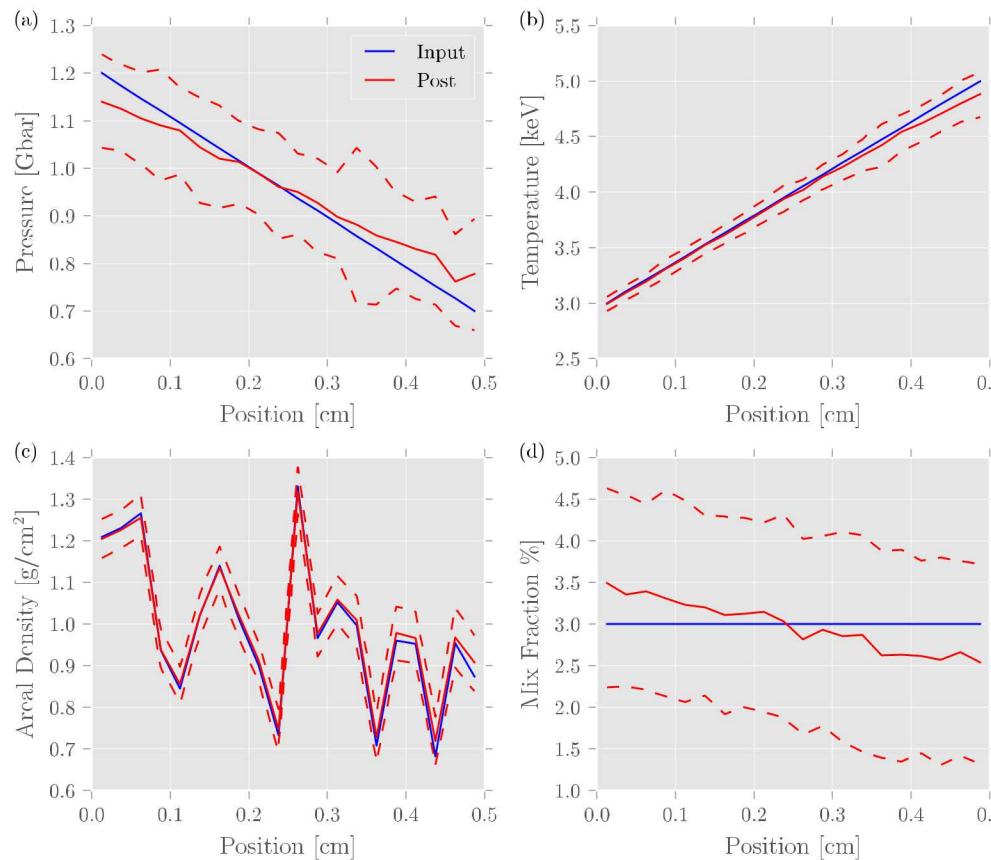
# We are developing proxy models to simplify complex calculations and incorporate more physics in the model



- BR (the magnetic field-radius product) is a critical burn parameter in MIF
- This can be measured via secondary DT neutron measurements, but the model is too expensive to implement inline
- We are training a Bayesian Neural Network to predict the DT/DD ratio and the DT spectra with uncertainties

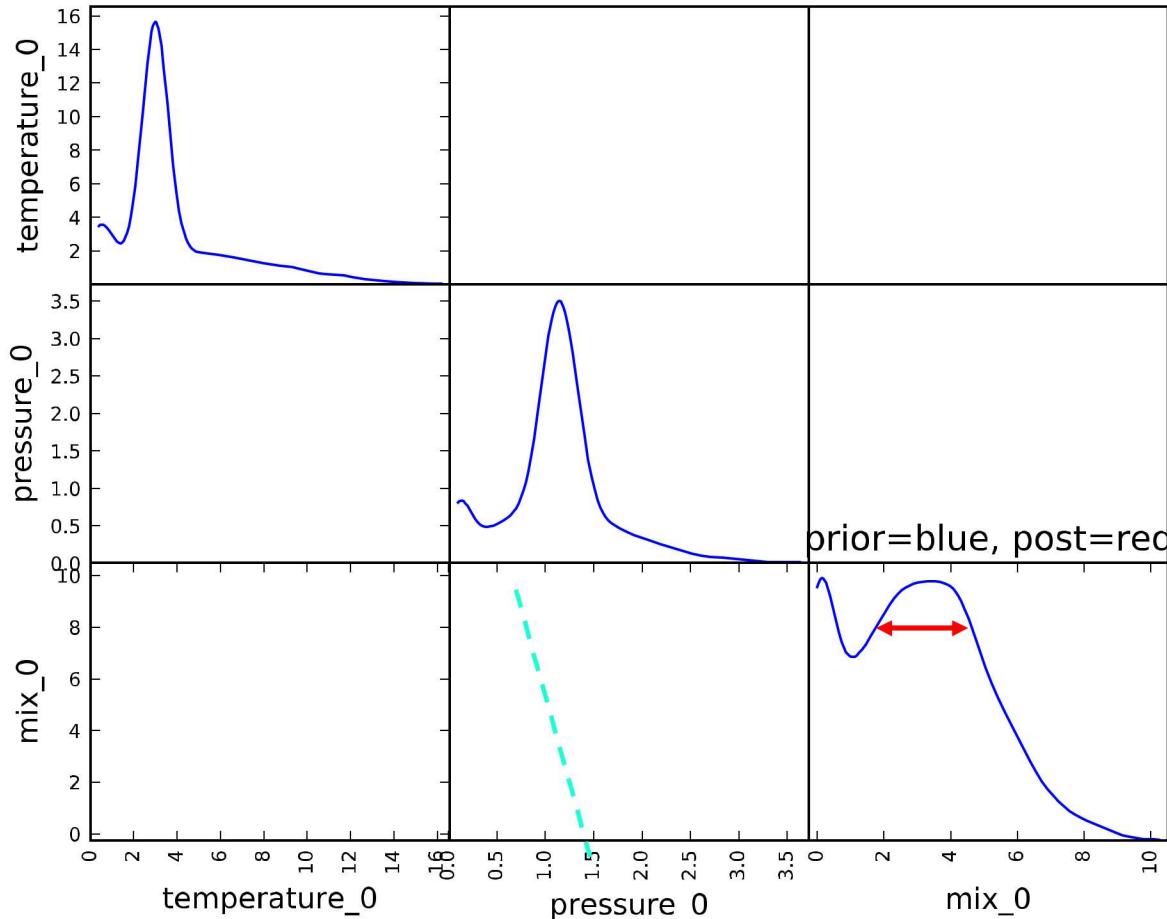


# Synthetic test cases show the algorithm works as intended and reveal correlations inherent in the system



- Near exact reconstruction of the diagnostic data
- High fidelity estimation of most of the input model parameters
- Ability to handle gradients and large swings

# The posterior pdf's reveal correlations between the model parameters



- Mix is relatively poorly determined
- Significant correlation with the Pressure

### X-ray Emission:

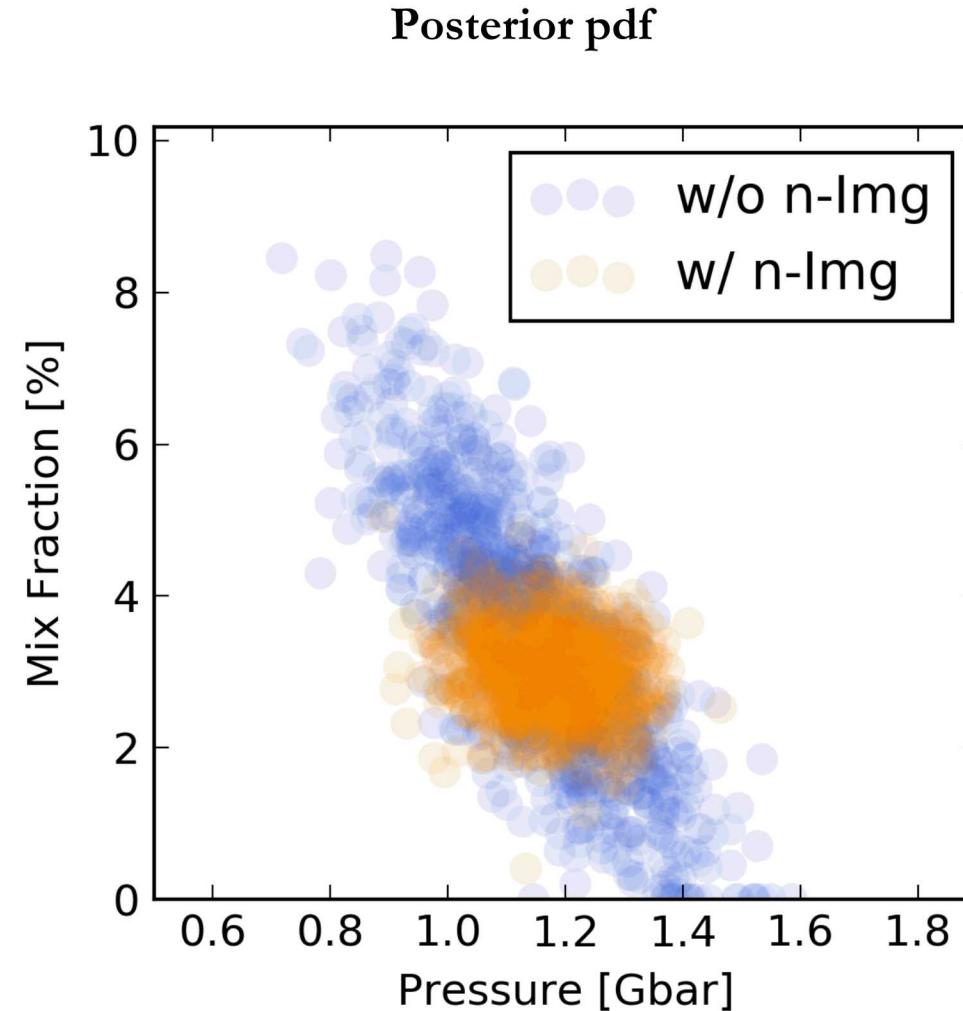
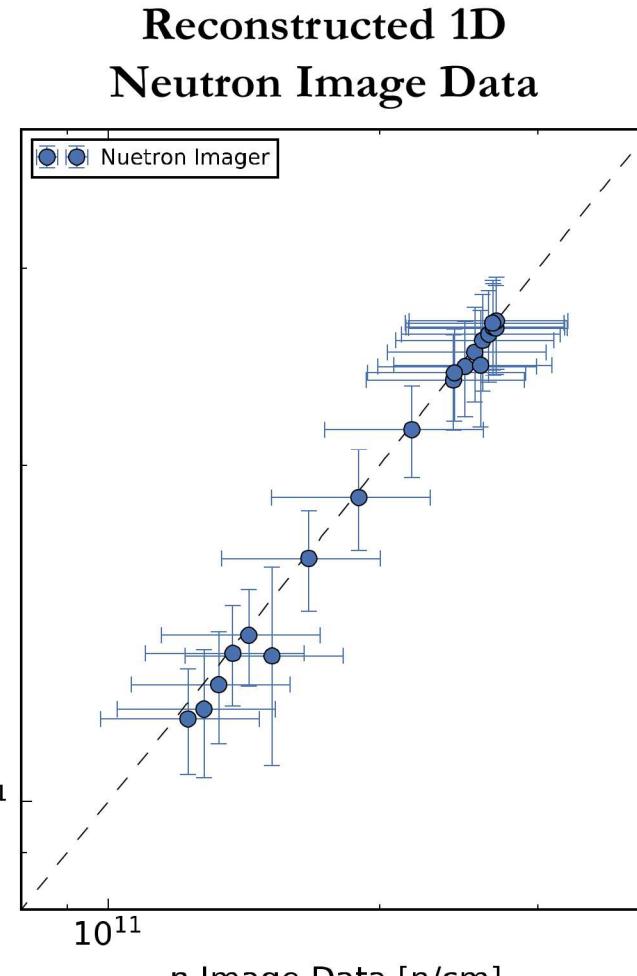
$$\epsilon_\nu = A_{f-f} e^{-\rho R_\ell \kappa_\nu} \tau_b P_{HS}^2 \frac{g_{FF} \langle Z \rangle}{(1 + \langle Z \rangle)^2} \sum_i f_i j_i \frac{e^{-h\nu/T}}{T^{5/2}}$$

### Neutron Emission:

$$\epsilon_E = \frac{P_{HS}^2 \tau_b}{1 + \delta_{1,2}} \frac{f_1 f_2 \langle \sigma v \rangle}{(1 + \langle Z \rangle)^2 T_i^2} I_o(E)$$

- Neutrons and x-rays have the same dependence on pressure, but not on mix
- We have local and global x-ray measurements, but only global neutron measurements...

# Adding new information can affect the correlations in the posterior pdf's, improving our ability to determine certain quantities



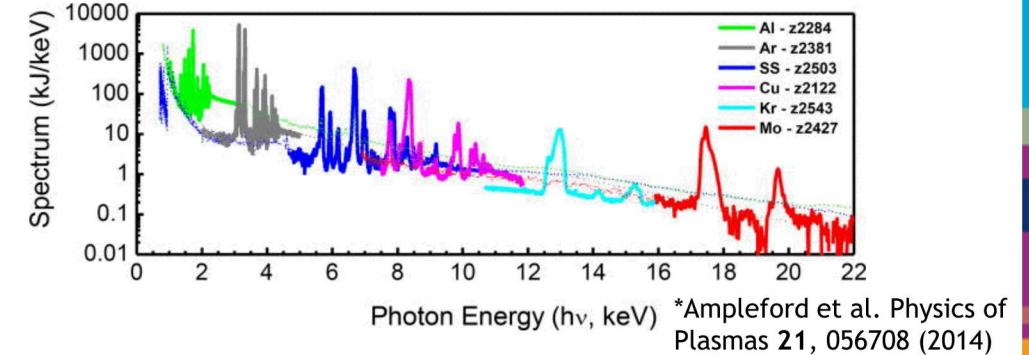
- Motivated by diagnostic developments and the previous observation, we implemented a simple 1D neutron image model\*
- This approach can be formalized and extended to the design and optimization of diagnostics\*\*

\*\*U. Von Toussaint, Rev. Mod. Phys.  
Vol. 83 (2011)

We are currently developing tools to bring the power of data assimilation to a variety of applications on Z

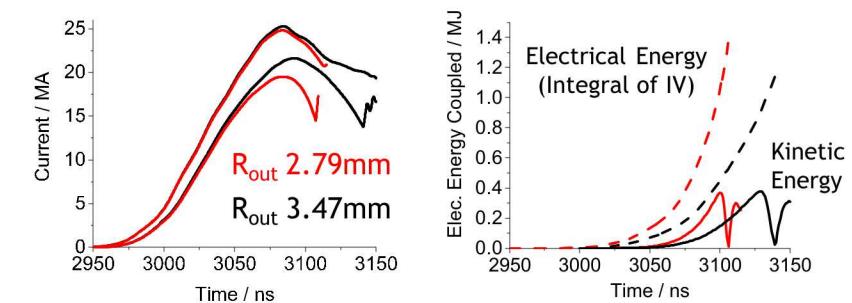
### Improving measurements of x-ray output on Z by integrating

- X-ray power detectors (PCD's, XRD's, etc.)
- Total x-ray yield measurements (Calorimeter, bolometer)
- X-ray spectra from multiple independent instruments
- Driven by Radiation Effects Sciences (RES) needs



### Use our knowledge of the Z circuit to constrain power delivery to the load and losses

- Electrical measurements at multiple points in the Z circuit
- Load current velocimetry constrained by the circuit model and implosion model
- Driven by a need for better post-shot simulation capability and understanding of powerflow for scaling



## Conclusions and future outlook

At Sandia we have developed a Bayesian data assimilation engine that is providing deeper insight into MagLIF experiments

- Currently limited by simple assumptions in the physics model and computational complexity of more physics-rich models
- We are expanding the data assimilation engine to other applications (x-ray output for RES, power coupling to loads on Z, physics-based decision making)

We are actively exploring learned surrogate models that can help us incorporate physics that is too expensive to calculate directly