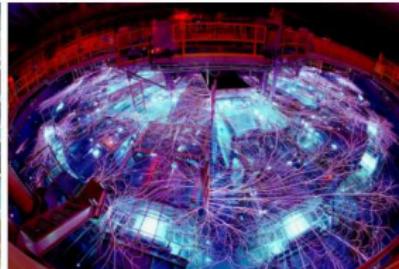


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SAND2020-7484PE



# What is Differential Privacy and Why Should I Care?

Evercita C. Eugenio

July 29, 2020



laboratories is a multimission laboratory managed and operated by National Technology & Engineering Solutions of Sandia, a Honeywell International Inc., for the U.S. Department of Energy's National Nuclear Security Administration under contract

# Motivation

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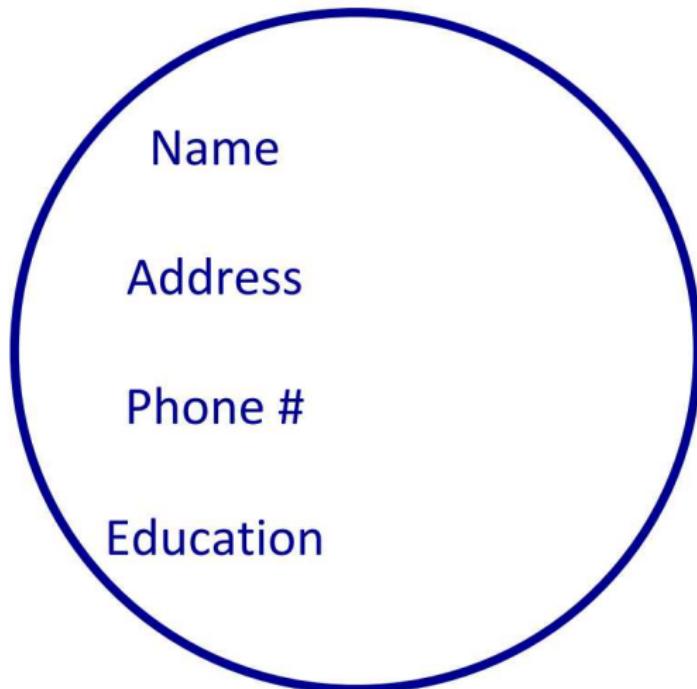
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- So handling all this data in a way that protects the confidentiality of the data subjects' identities and sensitive attributes while maintaining the statistical usability/accuracy of the data set has developed into a critical area of study.

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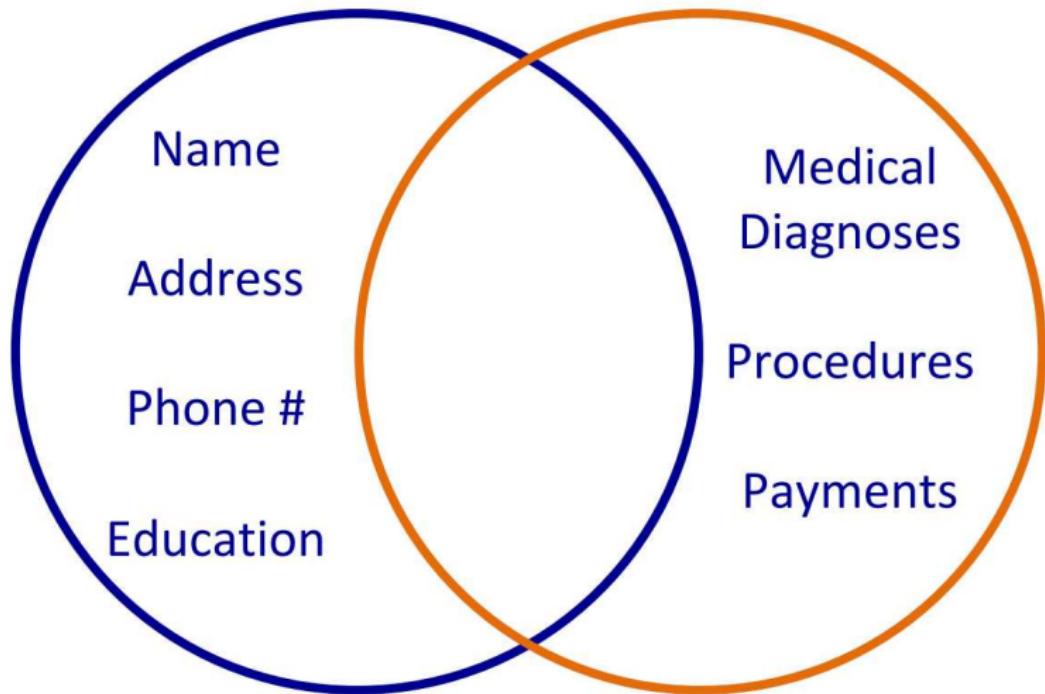
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- Government agencies, business, survey and research organizations, and medical institutions are constantly being asked to release and share more and more of their data for transparency and accountability.
- So handling all this data in a way that protects the confidentiality of the data subjects' identities and sensitive attributes while maintaining the statistical usability/accuracy of the data set has developed into a critical area of study.
- One of the most common ways to “protect” data is to simply anonymize the data (i.e. remove identifying information or sensitive characteristics).

# Motivation: Record Linkage

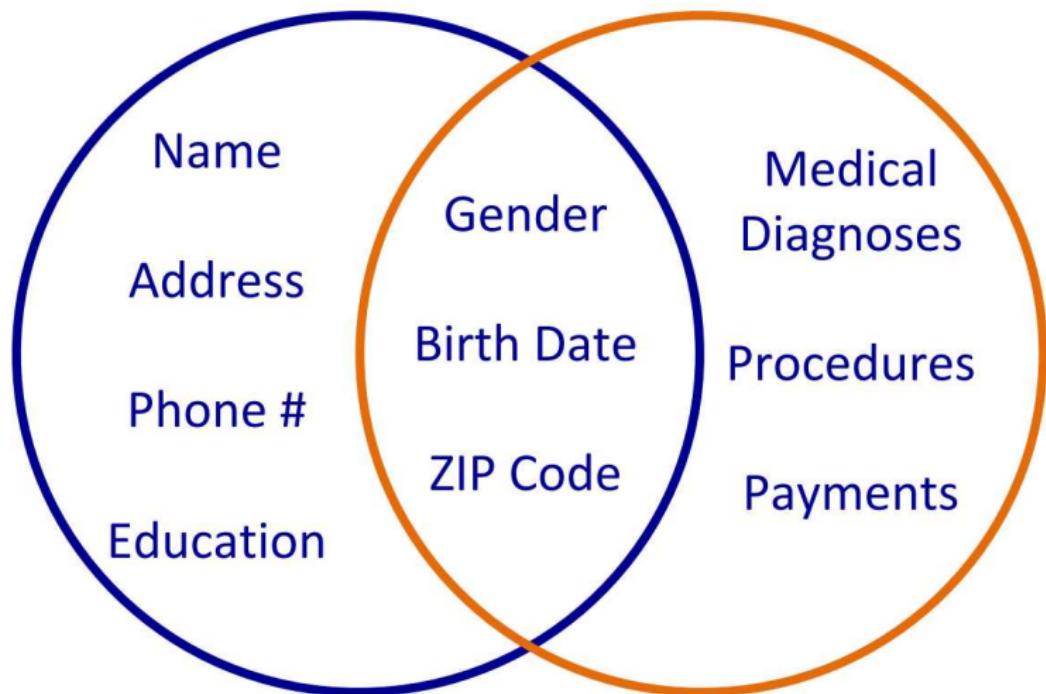
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- It has been shown that anonymized data can be linked with other publicly available datasets. This record linkage can possibly lead to re-identification.
- So instead of using the old techniques of anonymization, new privacy techniques have been formulated in the field of differential privacy.

# What is differential privacy?

# Differential Privacy

Differential privacy ensures that the addition or removal of a single database item does not substantially affect the outcome of any analysis.

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- A data curator manages queries sent to a data set.
- The curator can provide a response either using differential privacy or not.

## Differential Privacy (Dwork 2006)

Let  $\mathcal{K}$  be a mechanism (to be defined later), and let  $D_1$  and  $D_2$  be two databases that differ in at most one element. A randomized function  $\mathcal{K}$  gives  $\epsilon$ -differential privacy if for all data sets  $D_1$  and  $D_2$  differing on at most one element, and all  $S \subseteq \text{Range}(\mathcal{K})$ ,

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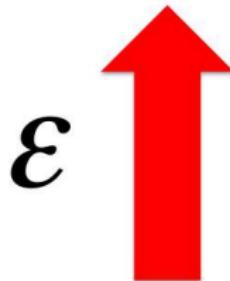
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$$\mathcal{E} = \begin{array}{l} \text{Amount of Privacy Used} \\ \text{or} \\ \text{Information Leaked} \end{array}$$

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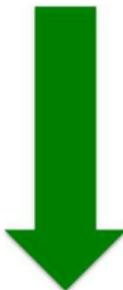
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Leak **MORE**  
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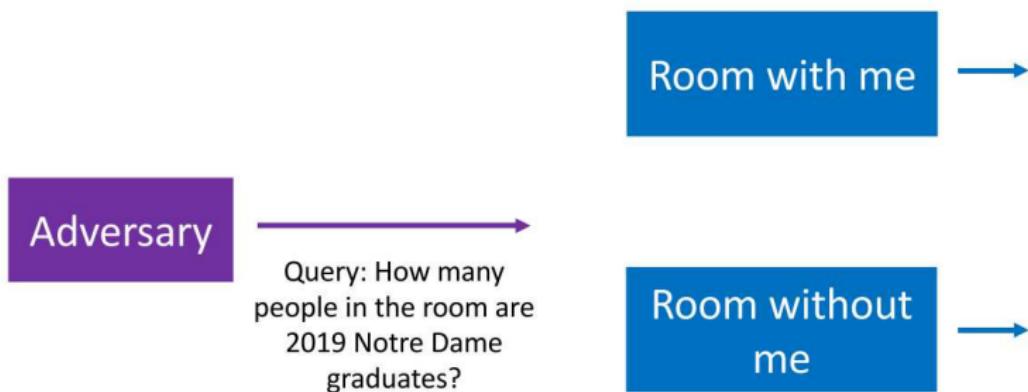


Query: How many  
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Room with me

Room without  
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# Differential Privacy Example: Notre Dame students



# Differential Privacy Example: Notre Dame students

Adversary



Query: How many  
people in the room are  
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Room with me

→ 1

Room without  
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→ 0

Can deduce that I am the  
only one who is a 2019 Notre  
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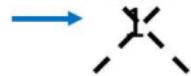
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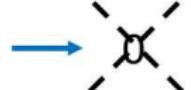


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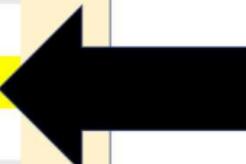
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- If the curator answered **WITH** differential privacy, then the risk to my privacy would not be substantially (as bounded by  $\epsilon$ ) increased as a result of participating in the statistical database.
- To apply differential privacy, we can add some noise using the mechanism  $\mathcal{K}$  to the result of a query on our dataset to ensure the formula for  $\epsilon$ -differential privacy holds.

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  - graphs and social networks.

# Laplace Mechanism

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Sensitivity (Dwork 2006)

For  $f : D \rightarrow R^k$ , the sensitivity of  $f$  is

$$\Delta f = \max_{D_1, D_2} \|f(D_1) - f(D_2)\|_1$$

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Captures how large a difference between the value of  $f$  on two databases (differing in a single element) must be hidden by the additive noise generated by the curator.

## Laplace Mechanism (Dwork 2006)

When the query is numeric, adding Laplace random noise independently to each of the components of  $f(X)$  guarantees  $\epsilon$ -differential privacy.

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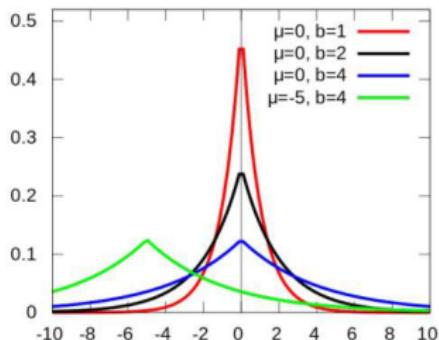


Image credit: [https://en.wikipedia.org/w/index.php?title=Laplace\\_distribution&oldid=697827002](https://en.wikipedia.org/w/index.php?title=Laplace_distribution&oldid=697827002)

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- Parallel Composition: If  $D_i$  are disjoint subsets of the original database and  $M_i$  provides differential privacy for each  $D_i$ , then the sequence of  $M_i$  provides differential privacy. The ultimate privacy guarantee only depends on the worst of the guarantees of each analysis, not the sum.

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- Can perform differentially private data synthesis, which yields differentially private synthetic data sets.

# Why should I care about differential privacy?

# Who uses differential privacy?

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The Google logo is displayed in its characteristic multi-colored, sans-serif font. The letters are arranged in a single row: 'G' is blue, 'o' is red, 'o' is yellow, 'g' is blue, 'l' is green, and 'e' is red.

Who uses differential privacy?

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# Uber

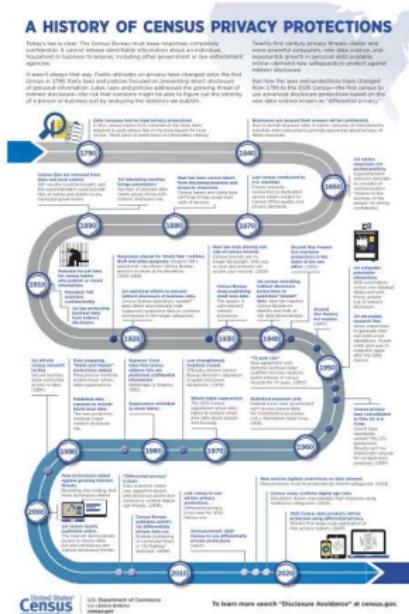
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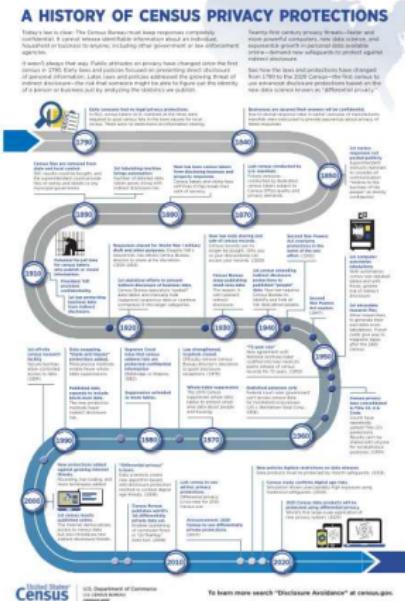


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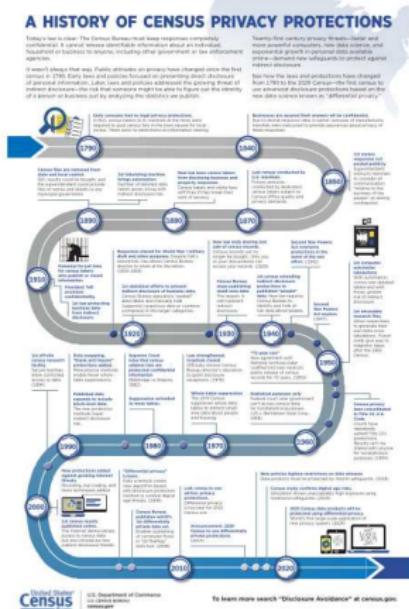
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Image credit:  
<https://www.census.gov/library/visualizations/2019/comm/history-privacy-protection.html>

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- Models and model parameters being shared across boundaries (federated learning).

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- Applying differential privacy to new applications areas.

# Questions?

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