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An Overview of the Conforming Reproducing Kernel (CRK) Method

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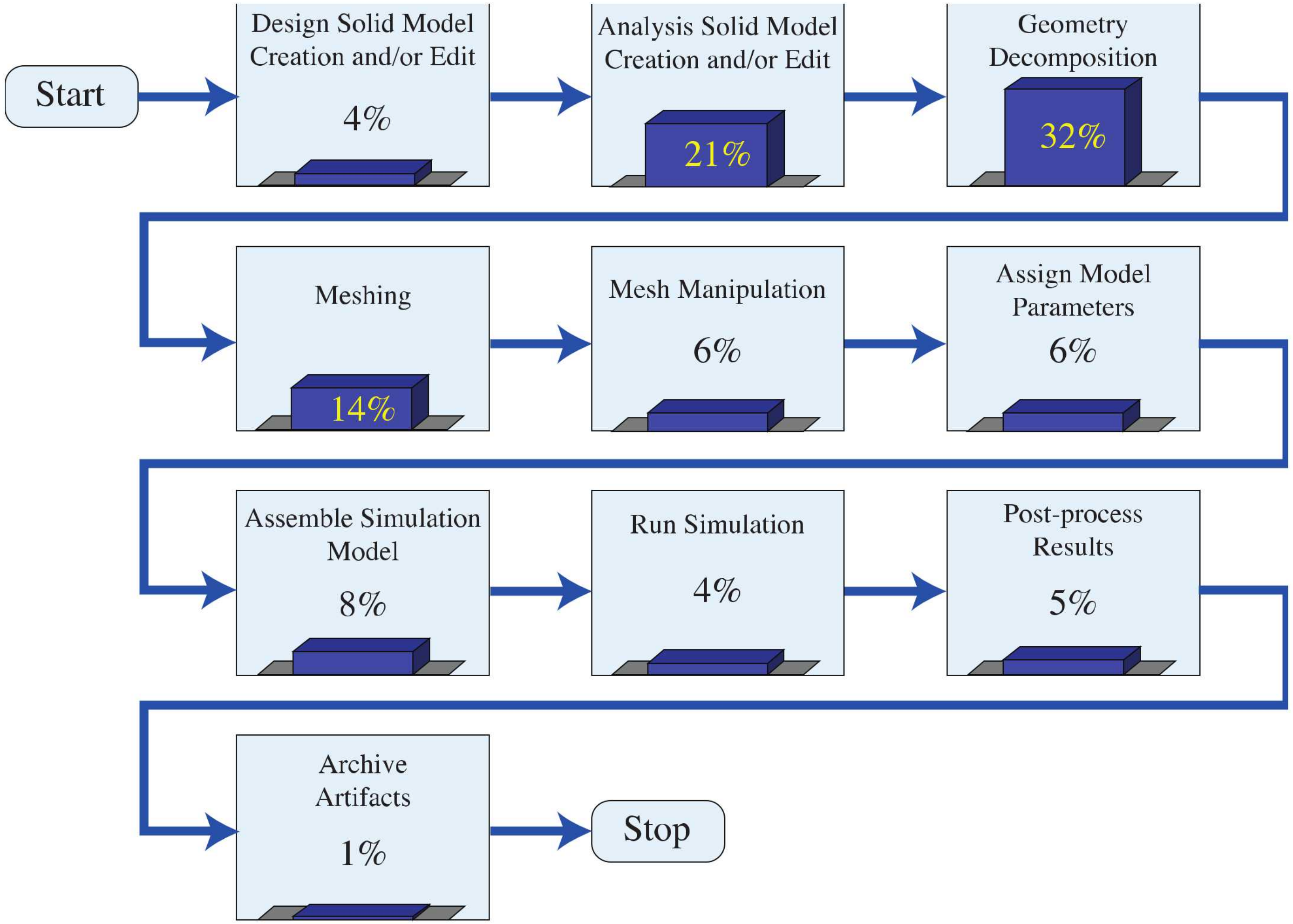
Engineering Science - Analysis Goal

A large portion of people using the finite element method are faced with a general task:

Deliver critical engineering analyses in a timeframe consistent with project requirements

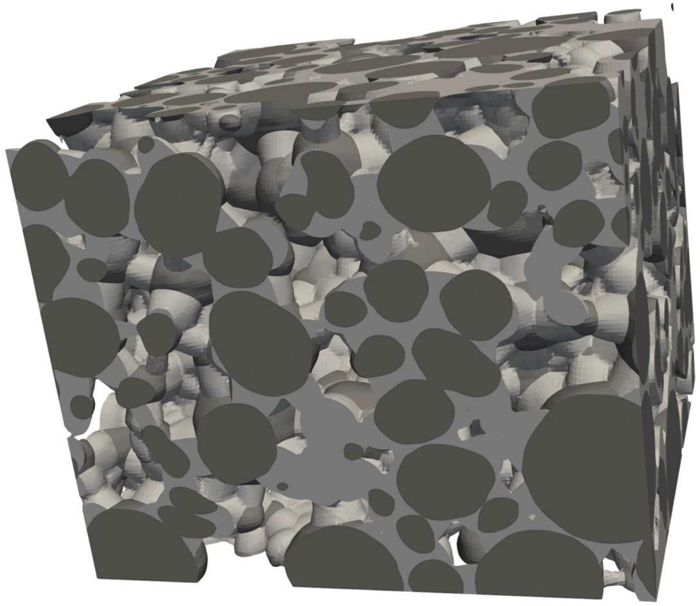
Engineering Analysis, Process Cost Breakdown[1]



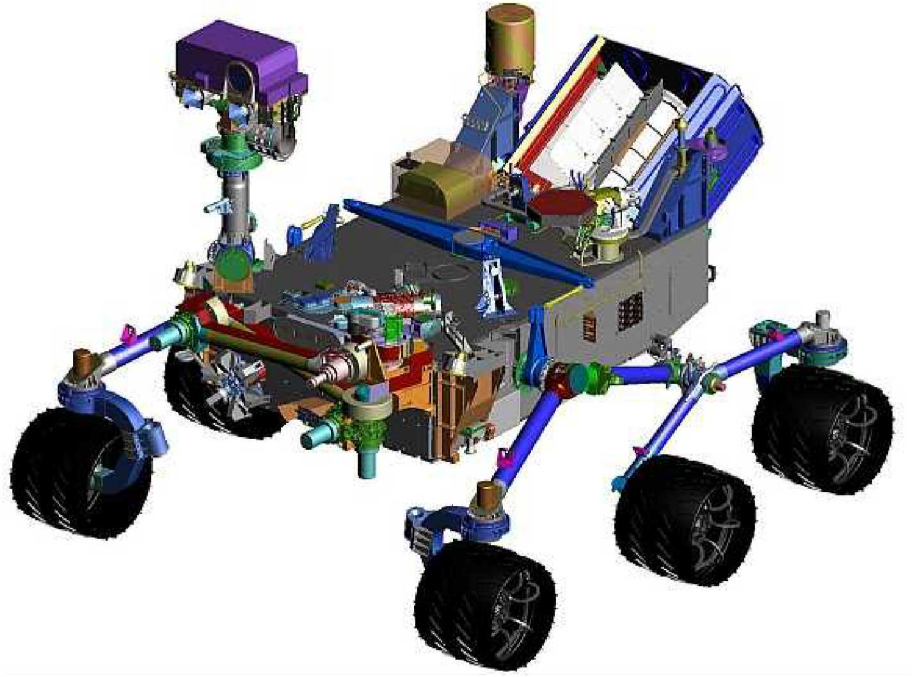
[1] M. F. Hardwick, R. L. Clay, P. T. Boggs, et al., "DART system analysis", Sandia National Laboratories, Tech. Rep. SAND2005-4647, 2005.

Application Spaces

Engineering analyses common at Sandia. Goal is to have a general solution, must address the more burdensome models: *multi-body /material, complex geometries, contact, nonlinear materials, dynamic loading*



Battery Microstructure



Mars Rover

Conforming Reproducing Kernel

The Conforming Reproducing Kernel (CRK) LDRD used meshfree ideas to improve design-to-simulation agility

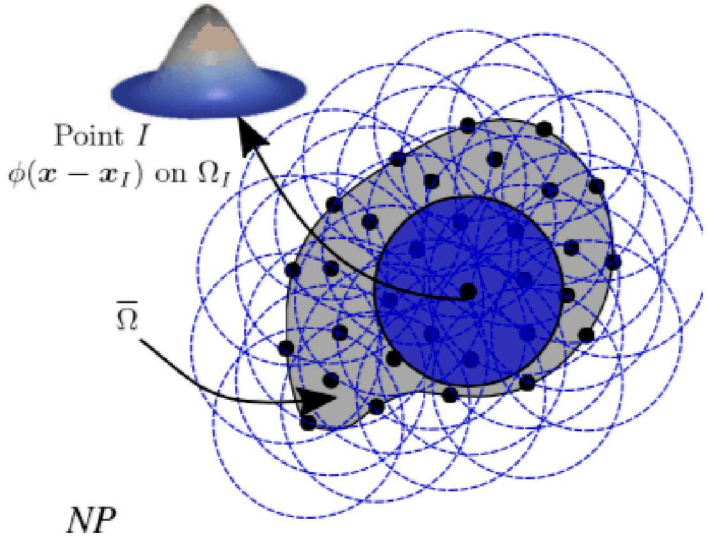
- Developed a means for handling boundary conditions and challenging geometries
- Designed efficient integration techniques for simulating nearly incompressible materials
- Brought bond-based fracture into a Galerkin framework
- Prototyped an efficient meshfree implementation using STK

CRK is in-between a mesh-based and meshfree method. It is designed to be robust, accurate and efficient on automatically generated meshes.

Reproducing Kernel Meshfree Approximation:

Approximate solutions are constructed over a meshfree discretization (point cloud). Shape functions are constructed as the product of a **kernel function** and a **correction function.**

$$u^h(x) = \sum_{I=1}^{NP} \Psi_I d_I; \qquad \Psi_I = C(x; x - x_I)\phi_a(x - x_I)$$



$$C(x; x - x_I) = \sum_{i=0}^{n} b_i(x)(x - x_I)^i \equiv \mathbf{H}^T(x - x_I)\mathbf{b}(x)$$

$$\mathbf{H}^T(x - x_I) = [\, 1, x - x_I, (x - x_I)^2, \ldots, (x - x_I)^n]$$

$\mathbf{b}(x)$ is obtained by imposing completeness requirement: $\sum_{I=1}^{NP} \Psi_I x_I^i = x^i,\ 0 \leqslant i \leqslant n$

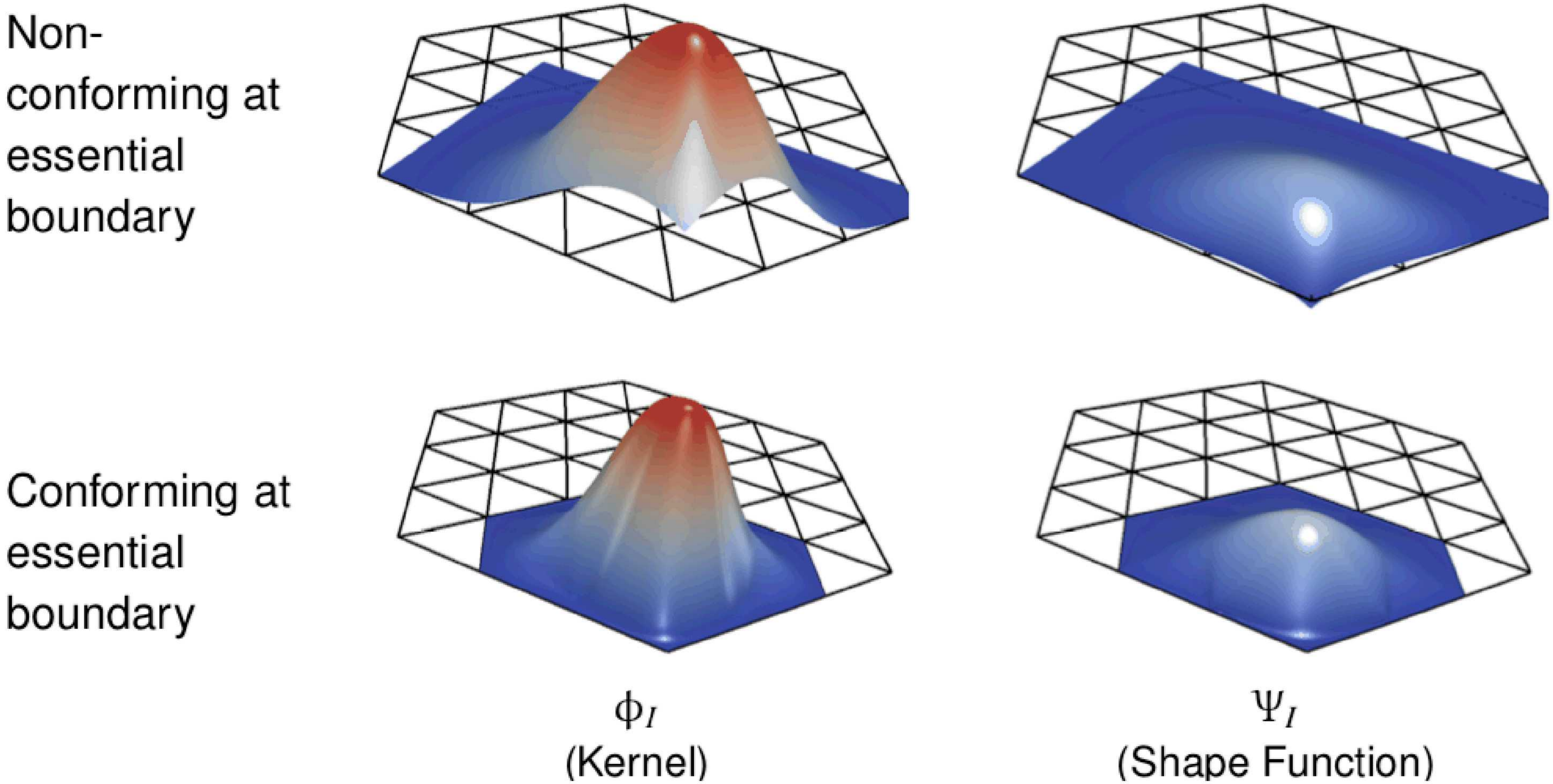
$$\mathbf{b}(x) = \mathbf{H}^T(0)\mathbf{M}^{-1}(x) \qquad \text{where} \qquad \mathbf{M}(x) = \sum_{I=1}^{NP} \mathbf{H}(x - x_I)\mathbf{H}^T(x - x_I)\phi_a(x - x_I)$$

- **Kernel function**: compact support, determines smoothness of the approximation
- **Correction function**: enforces polynomial reproducibility, allowing solution convergence

Liu, W. K., Jun, S., and Zhang, Y. F. Reproducing kernel particle methods. International Journal for Numerical Methods in Fluids 20:1081–1106, 1995.

Conforming Reproducing Kernel

Define kernel functions using, smooth spline spaces on triangles or tetrahedra (Bernstein-Bézier polynomials)



Elasticity Patch Test

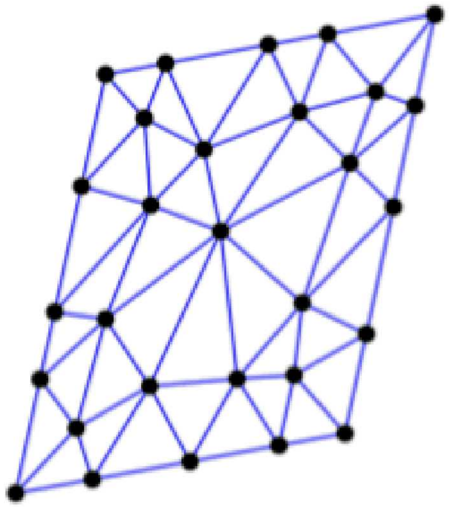


Figure: Deformed triangulation

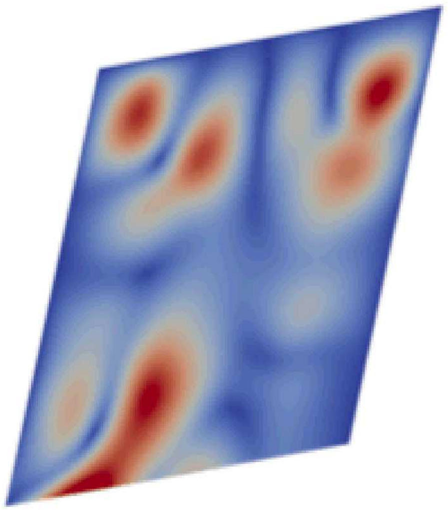


Figure: RKPM, transformation method

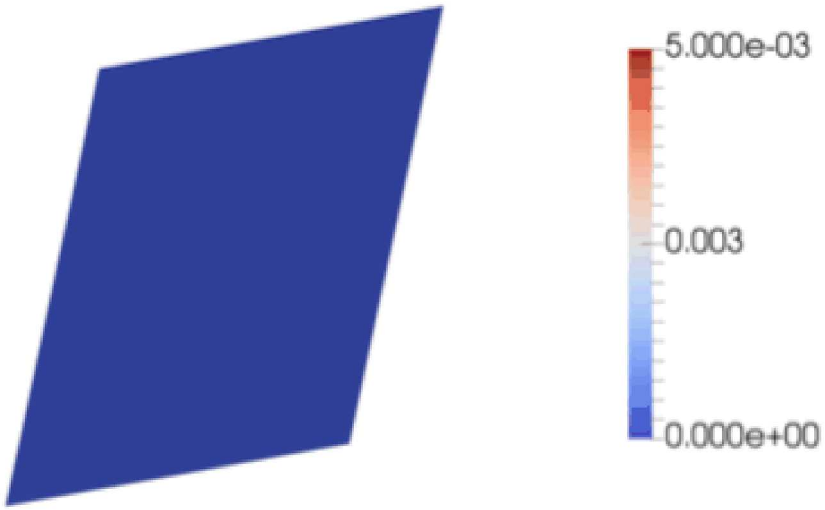


Figure: CRK, static condensation

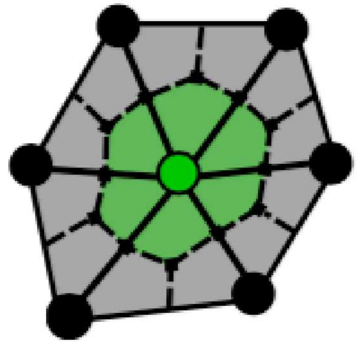
Method	L^2	H_1
RKPM with transformation method	2.05e-03	2.44e-02
Conforming window RK with static condensation	7.65e-17	1.04e-15

- Weak Kronecker-Delta → Kinematically Admissible Approximations
- Interpolatory along boundary: $u^h(\mathbf{x}_I) = d_I \rightarrow$ directly impose essential boundaries (like FEM)
- Conforming kernels also address issues with material interfaces and concavities

Integration for Meshfree Methods

Variationally consistent integration can be accomplished using a *smoothed gradient operator:*

$$\tilde{\nabla} f(\Omega_L) = \frac{1}{V_L}\int_{\Omega_L} \nabla f \, d\Omega, = \frac{1}{V_L}\int_{\Gamma_L} f\mathbf{n}\, d\Gamma$$



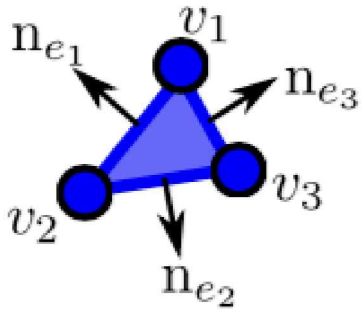
Example Nodal Domain (SCNI)

Where Ω_L is the smoothing volume surrounding corresponding to a material point L with boundary Γ_L, volume V_L and outward facing surface normal **n**

- Variationally consistent integration: passes patch test
- Efficient with a single material point per domain
- Robust

But, can have

- Low energy modes
- Pressure oscillations for nearly incompressible materials



Example Smoothing Cell

The advent of smoothed gradient integrations lead to the smoothed finite element method (SFEM) and inspired the virtual element method (VEM)

J.-S. Chen, C.-T. Wu, S. Yoon, and Y. You, "A stabilized conforming nodal integration for Galerkin mesh-free methods," International Journal for Numerical Methods in Engineering, vol. 50, no. 2, pp. 435–466, 2001.

New Integration Technique for Nearly Incompressible Materials

A method is proposed to address low energy modes and pressure oscillations. A multiplicative split is used to decompose the deformation gradient

$$\mathbf{F} = \mathbf{F}^{dil}\,\mathbf{F}^{dev}$$

$$\det \mathbf{F} = J = \det \mathbf{F}^{dil}, \quad \det \mathbf{F}^{dev} = 1, \quad \mathbf{F}^{dev} = J^{-1/3}\mathbf{F}, \quad \mathbf{F}^{dil} = J^{1/3}\mathbf{I}$$

The volumetric part is replaced with a projected value

$$\overline{\mathbf{F}} = \overline{\mathbf{F}}^{\,dil}\,\mathbf{F}^{dev}$$

with

$$\overline{\mathbf{F}}^{\,dil} = \pi(\mathbf{F}^{dil}) = \overline{J^{1/3}}\,\mathbf{I}$$

New Integration Technique for Nearly Incompressible Materials

Now for

$$\overline{\mathbf{F}} = \overline{\mathbf{F}}^{dil}\,\mathbf{F}^{dev}$$

we consider a smoothed deviatoric deformation gradient

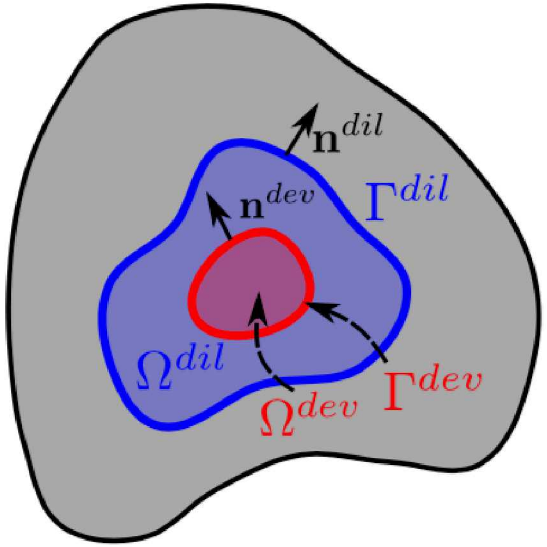
$$\mathbf{F}_L^{dev} = \widetilde{\mathbf{F}}(\Omega_L^{dev})$$

and a smoothed volumetric volumetric deformation gradient

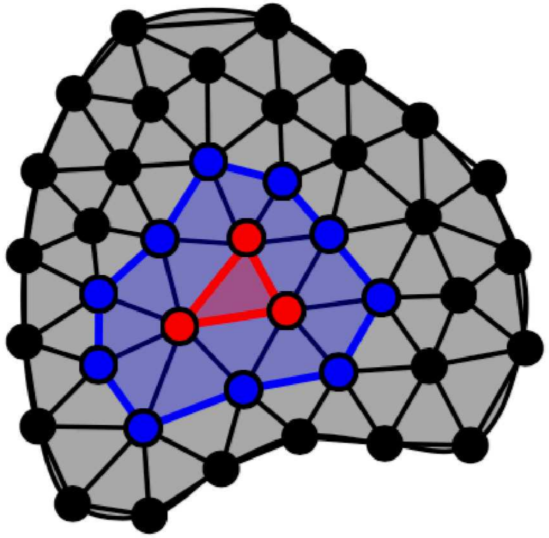
$$\overline{\mathbf{F}}_L^{dil} = \overline{J^{1/3}}(\Omega_L^{dil})\,\mathbf{I},$$

$$\bar{J}(\Omega_L^{dil}) = \frac{1}{V^{dil}} \sum_{c\in C} V_c \tilde{J}_c, \; V^{dil} = \sum_{c\in C} V_c$$

Where C is the number of smoothing subcells in the smoothing domain.

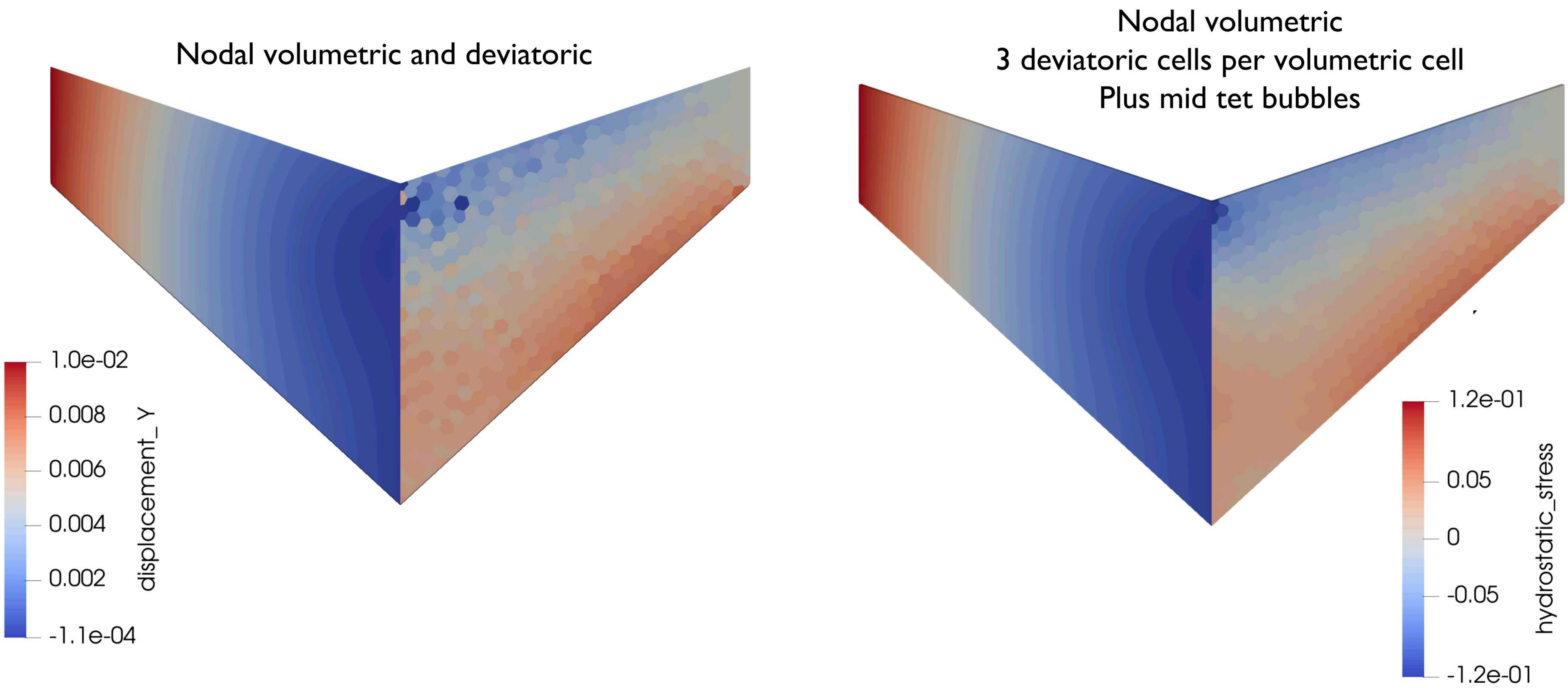


Nested smoothing domains



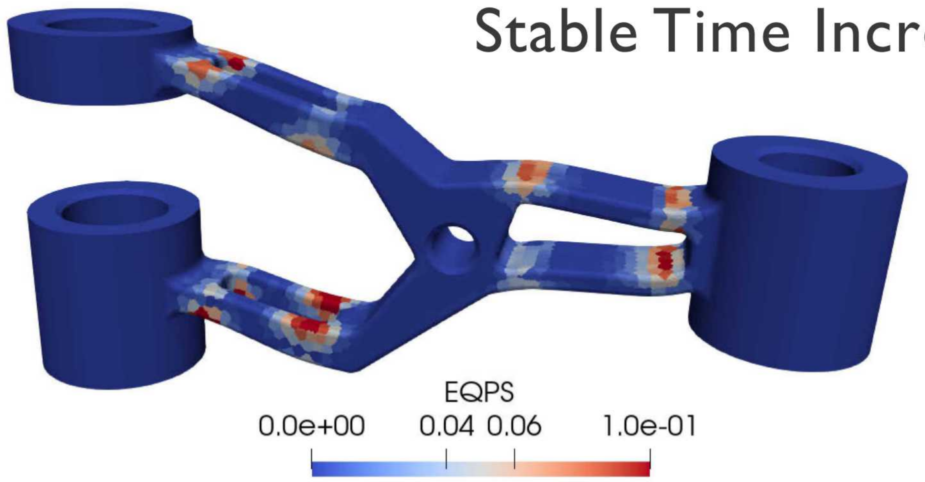
Meshed domains

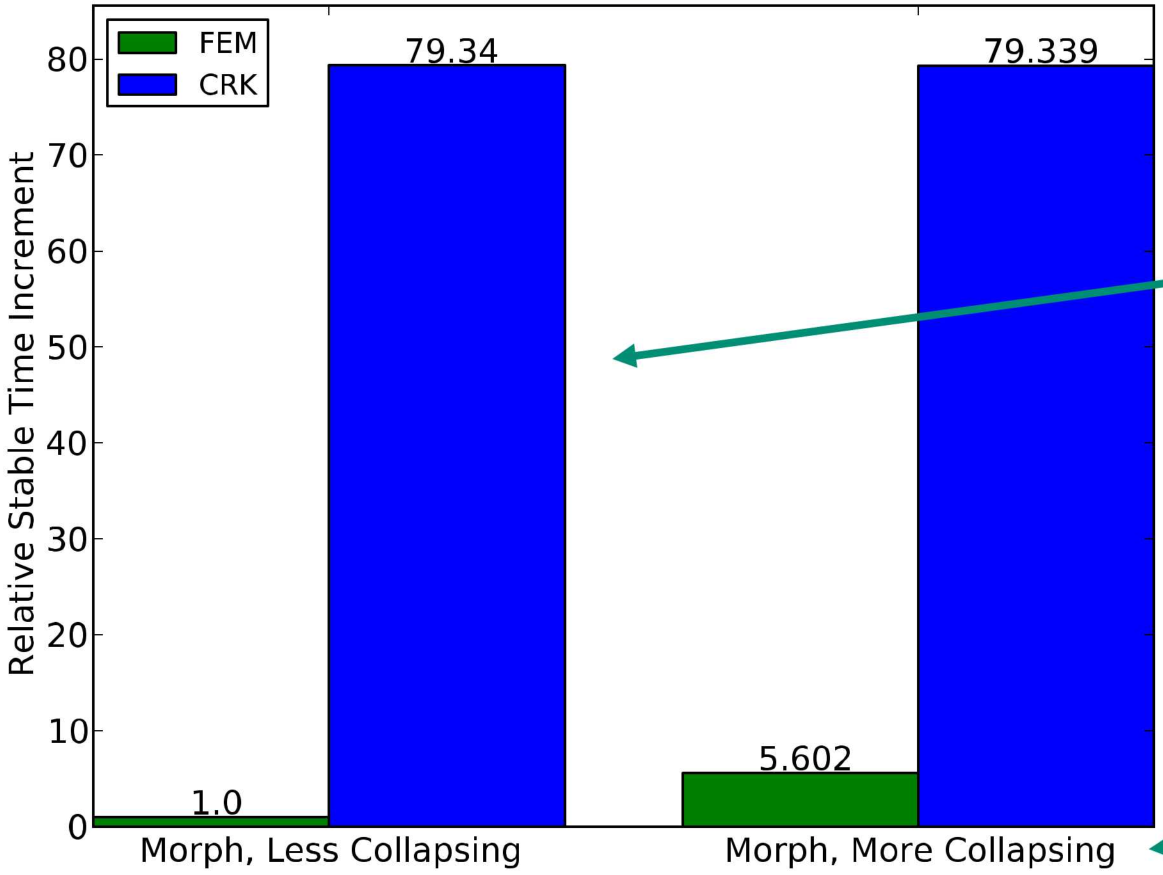
Example: Cook's Membrane, Poisson's Ratio = 0.499

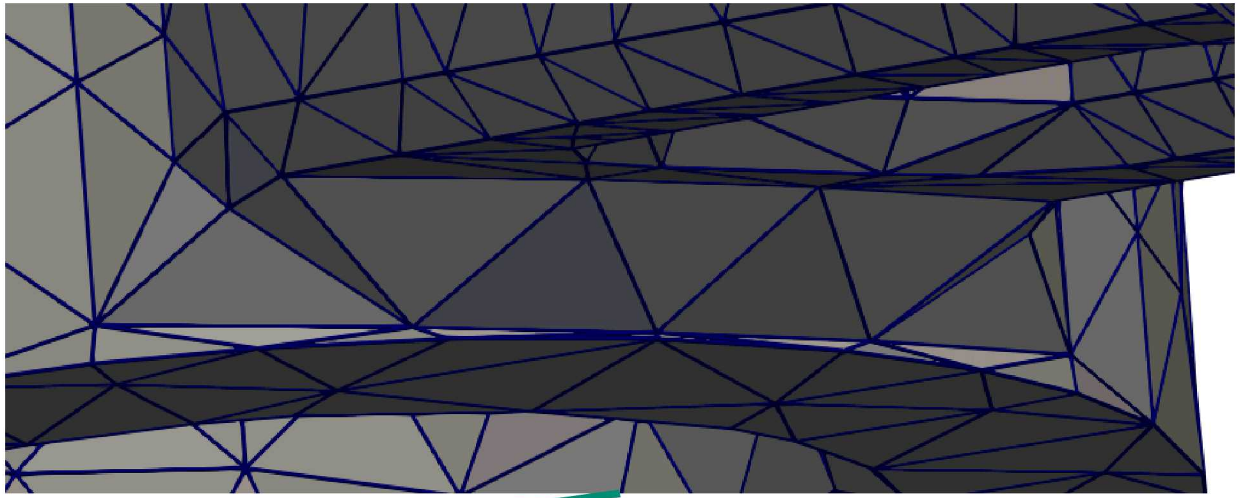


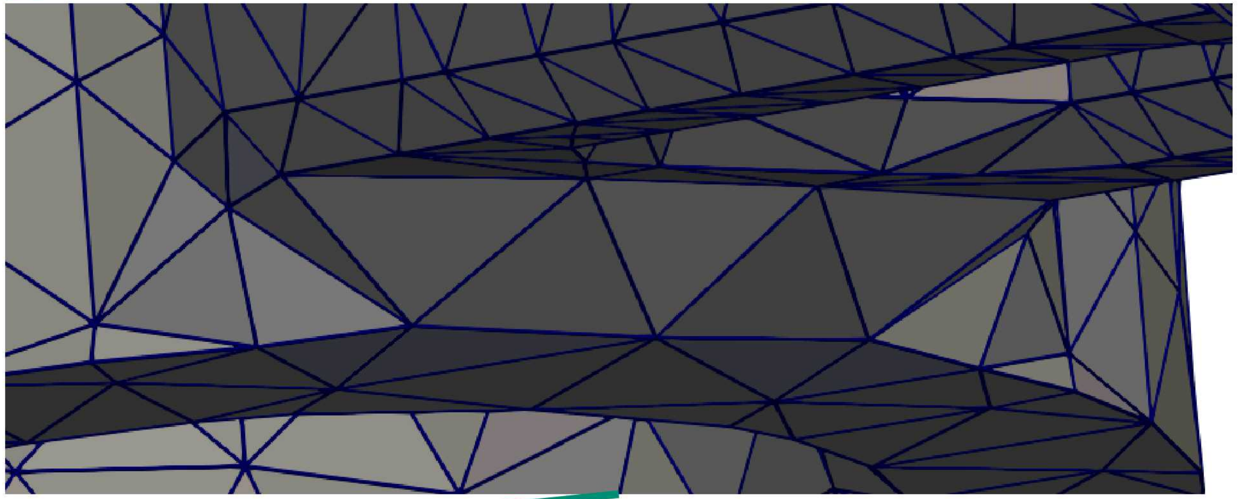
New integration method is displacement and integration stable

Stable Time Increment Comparison Using a Morph Mesh

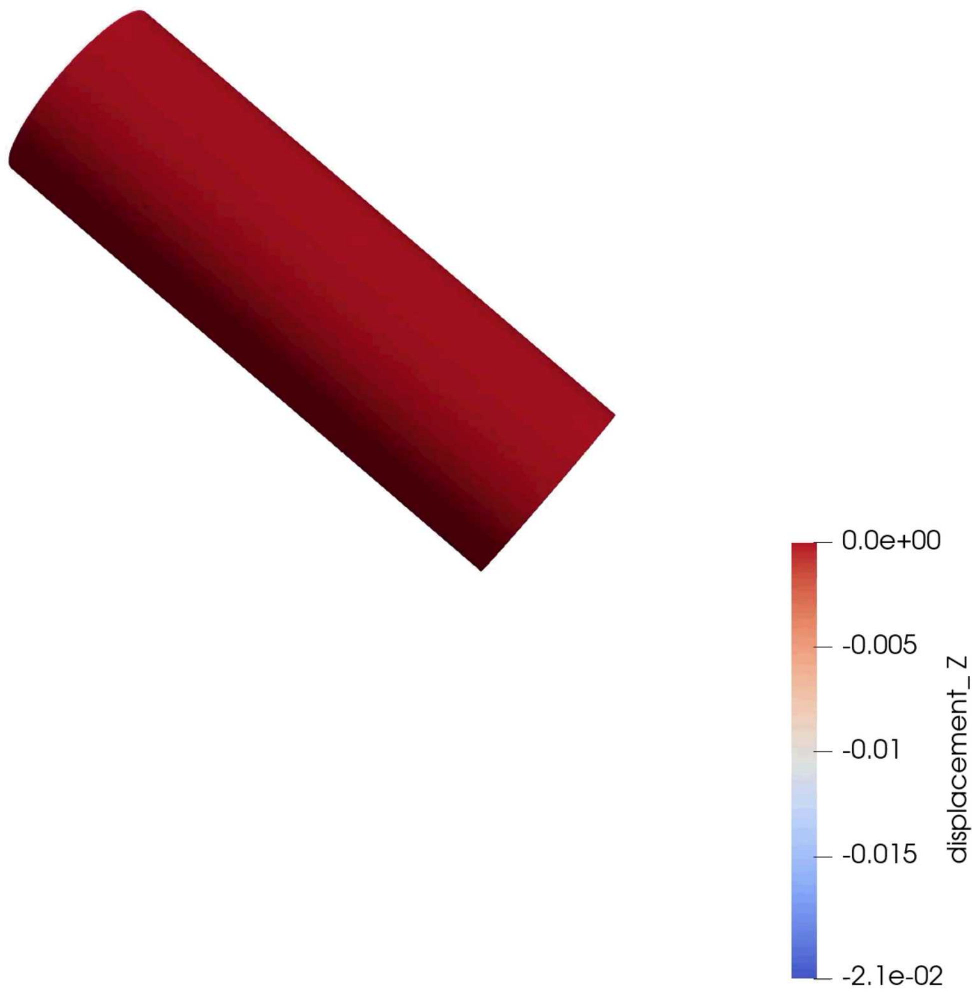




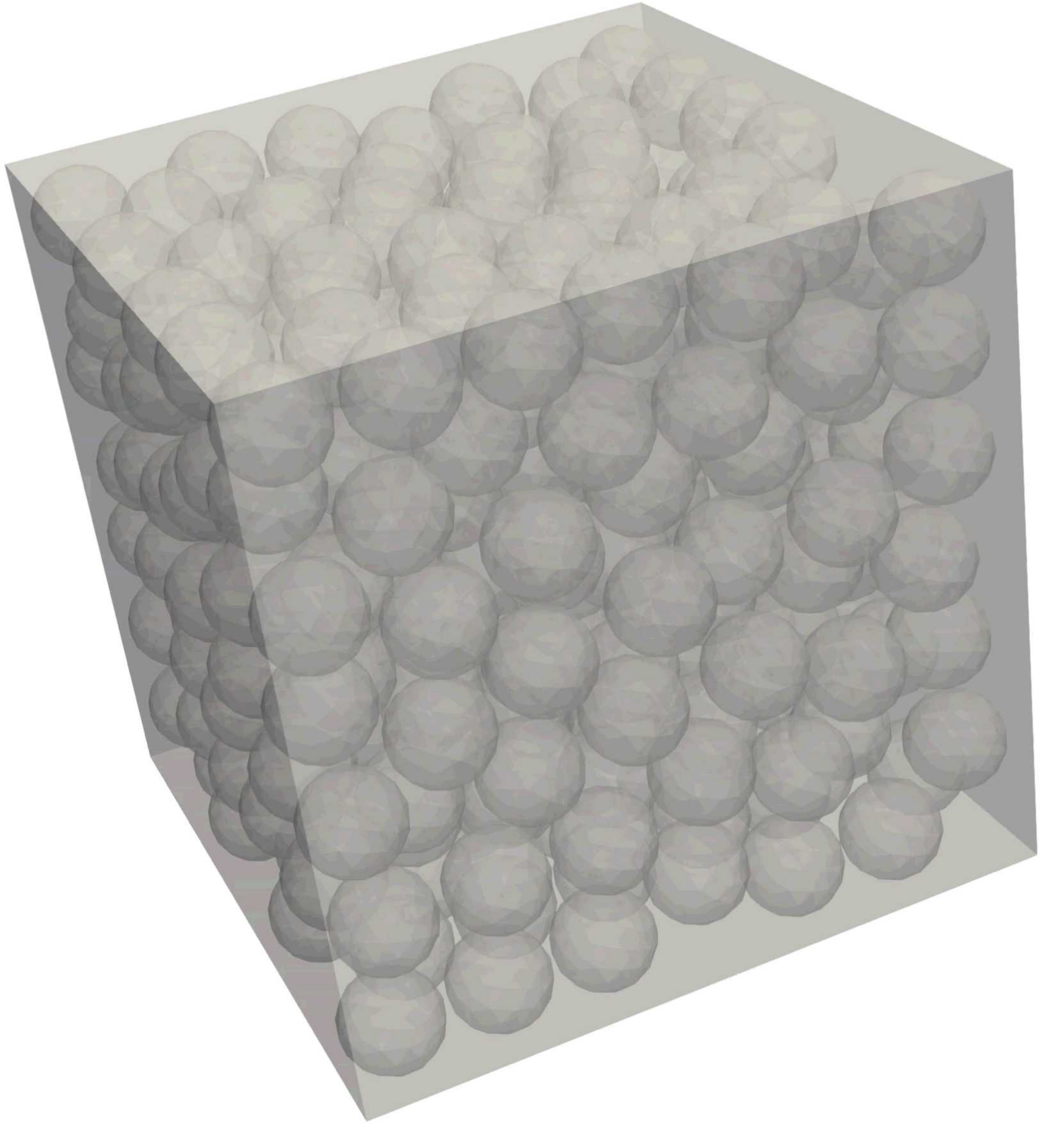




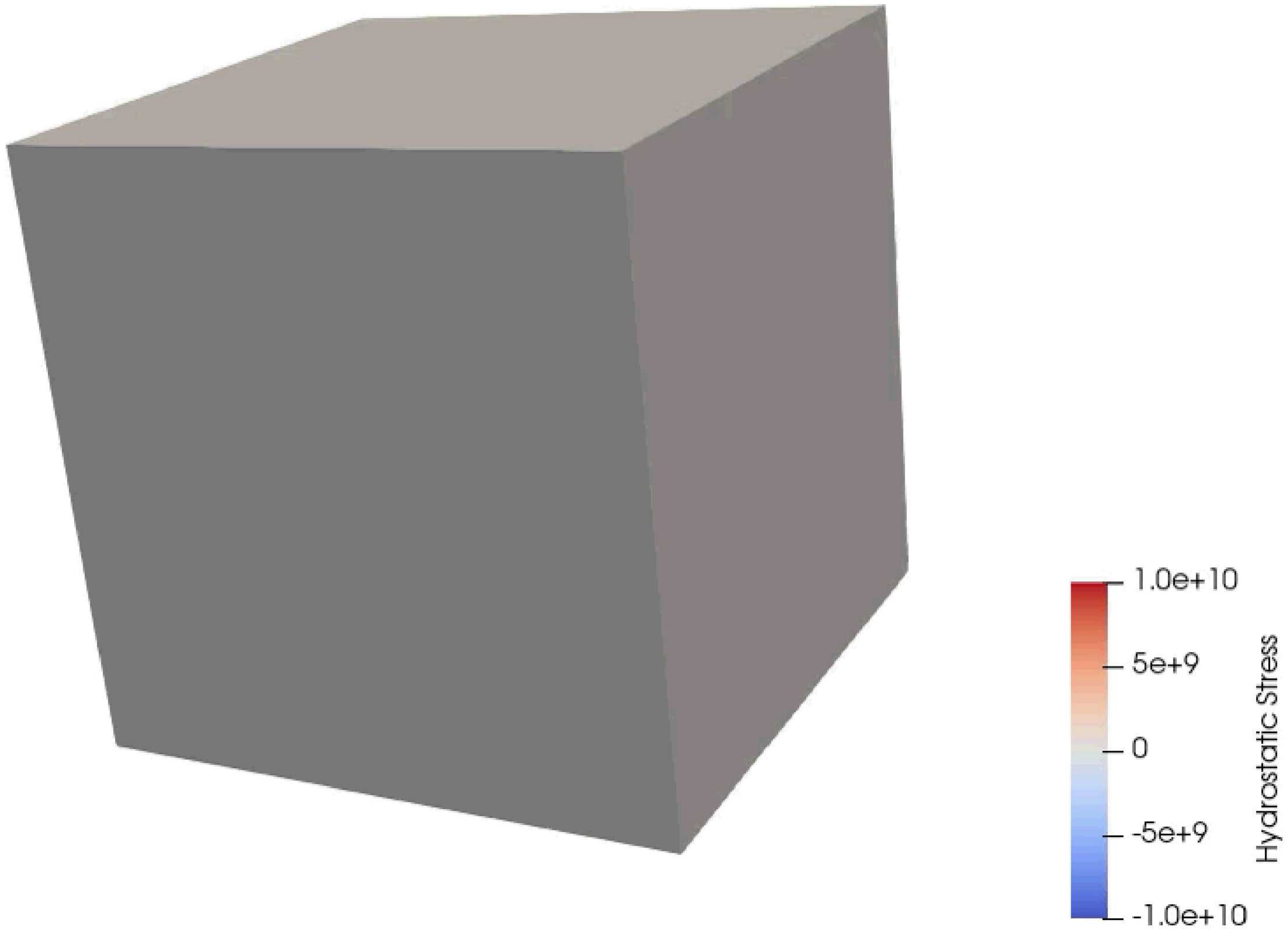
Taylor Bar Impact



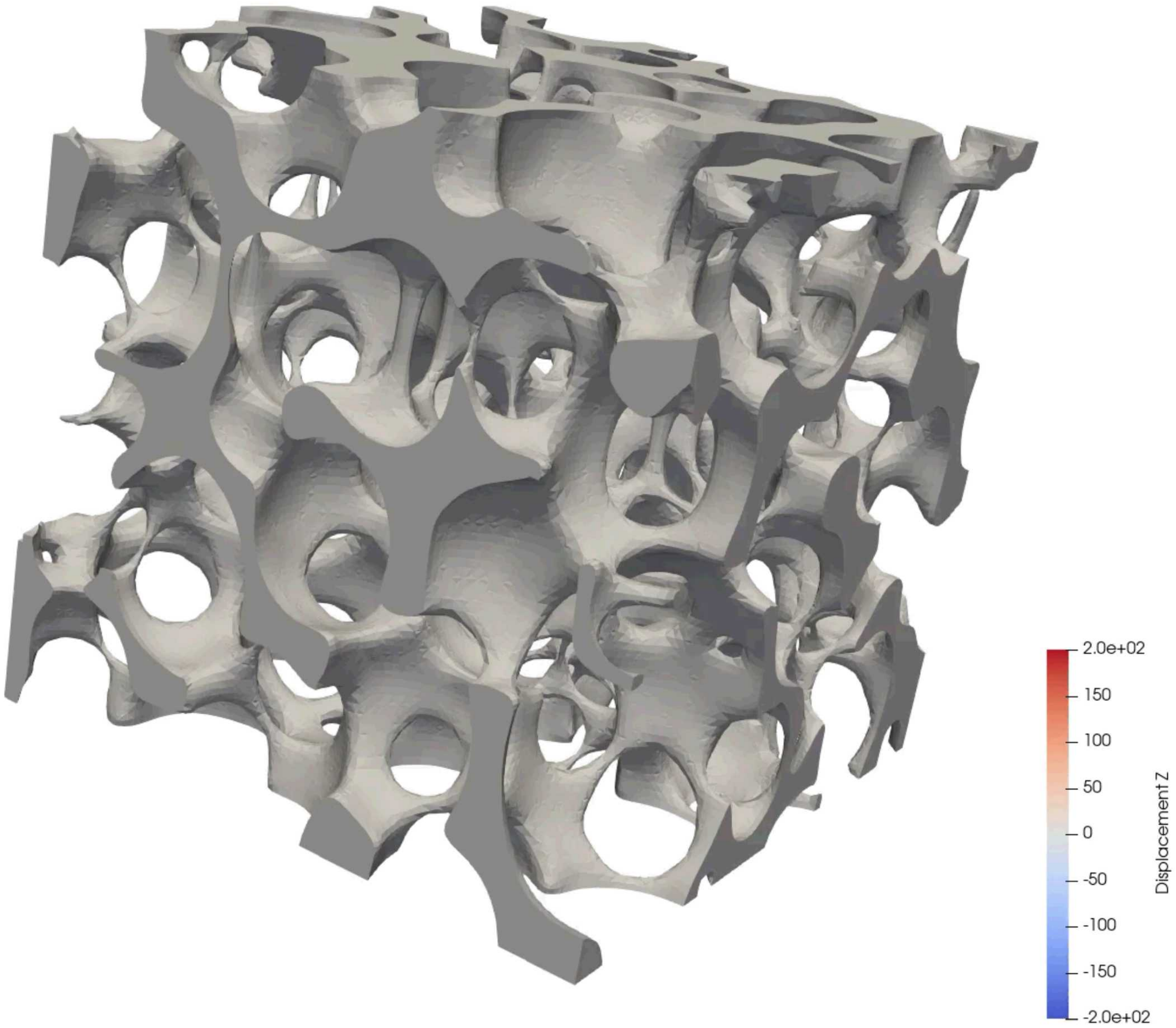
Foam Crush / Compression



Rubber Mesostructure in Shear



Foam Mesostructure



Additional Highlights and Conclusions

Realization of a multi-stage vision:

- Motivation came from analysis experience
- Core research completed in the doctoral study program and LDRD
- Prototyped in the LDRD
- In development in Sierra/SM
- Next: testing on components and systems, V&V, put into production

Funding:

- Past: doctoral study program, CRK LDRD (2 years)
- Current: WIPP, Goodyear, Foam LDRD, NGS, ASC/IC

Built on sound theories:

- It is a Galerkin method
- Integration properties are well understood
- Boundary condition enforcement is straight-forward
- Leverages many ideas from FEM to solve other challenges