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# A Discrete Element Approach to Rectified Bubble Motion

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Virtual Presentation Recorded By: Mark Ferraro, PhD

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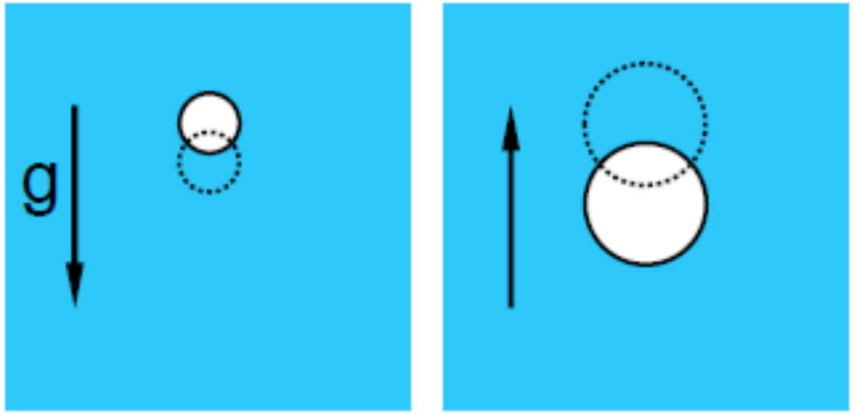
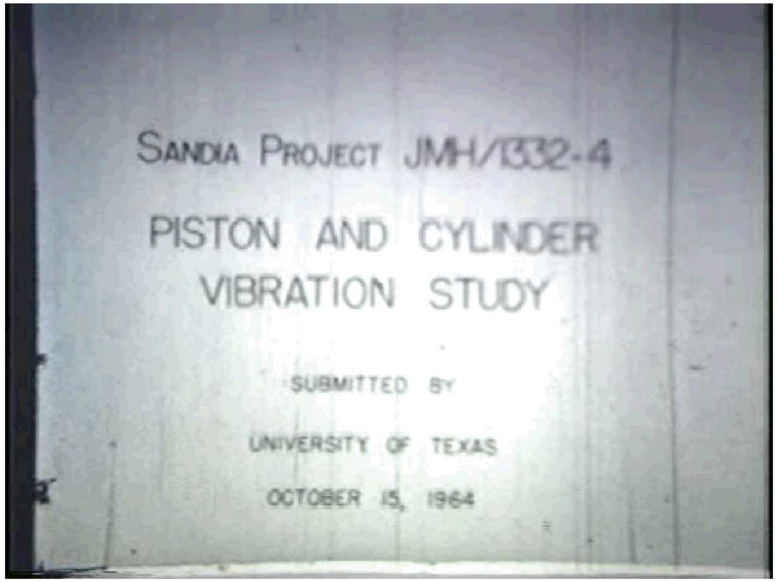
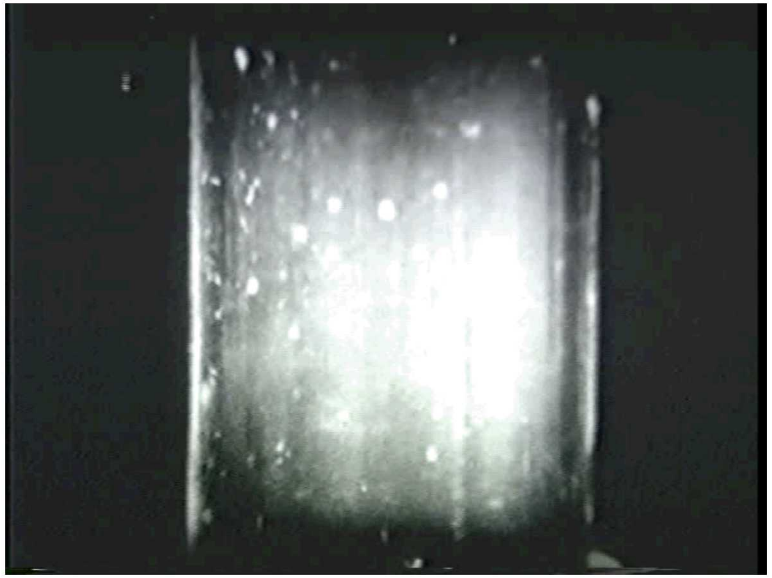
# Rectified Bubble Motion

- With vertical orientation, a counterintuitive result can occur
- Oscillations can be modeled as an alternating gravity field
- Asymmetry in upward and downward movement can push the bubble against gravity

$$g = g_0 + x_0\, \omega^2 \cos \omega t$$



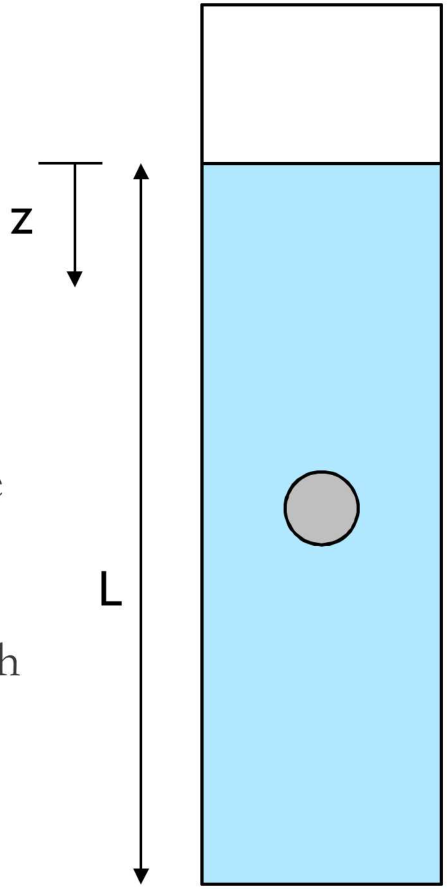



UT-Austin, 1964

This behavior can be seen through experiment, though predictive modeling capabilities are still needed

# The Model

- Simulation performed in LAMMPS
  - Assumption – Spherical bubbles of variable volume
- First look at a single bubble submerged in liquid with a single free surface above
- Pressure field will determine both force on the bubble and volume of the bubble (assuming ideal, isothermal)




$$Pressure\ Force = F = -V\frac{\partial P}{\partial z}$$

$$Buoyancy\ Force = F = \rho g\frac{4}{3}\pi R^3$$

$$Drag\ Force = F = -4\pi\mu R\frac{\partial z}{\partial t}$$

| Parameter | Value (unit) | Parameter | Value (unit) |
|---|---|---|---|
| $\rho_L$ | 950 kg/m$^3$ | $\rho_G$ | 1.2 kg/m$^3$ |
| $g$ | 9.81 m/s$^2$ | $c_G$ | 331 m/s |
| $L$ | 0.1 m | $c_L$ | 1450 m/s |
| $D_{bubble}$ | 1 mm | $\mu$ | 0.02 $Pa \cdot s$ |

# Single Bubble – Video Comparison

Frequency Range: 200Hz

Amplitude Range (n): 5-30G

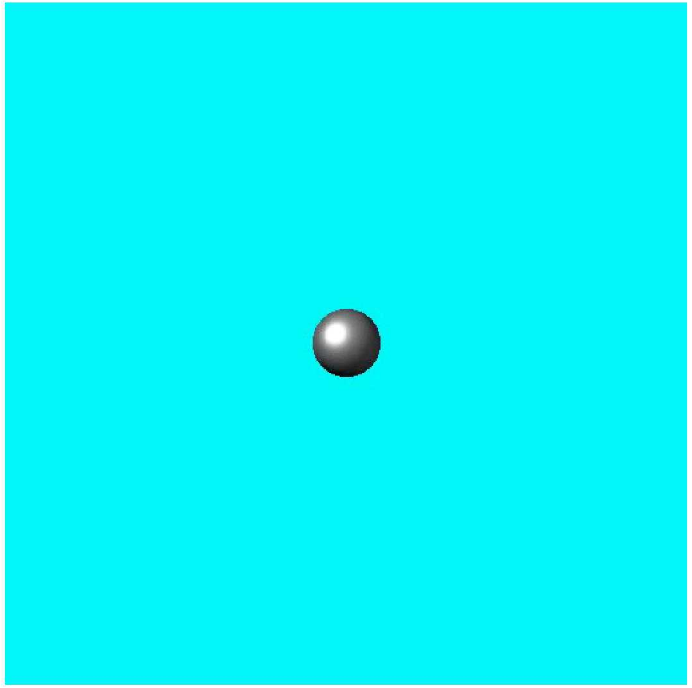
Bubble Diameter: 1mm

Column Height: 0.1m (~4”)

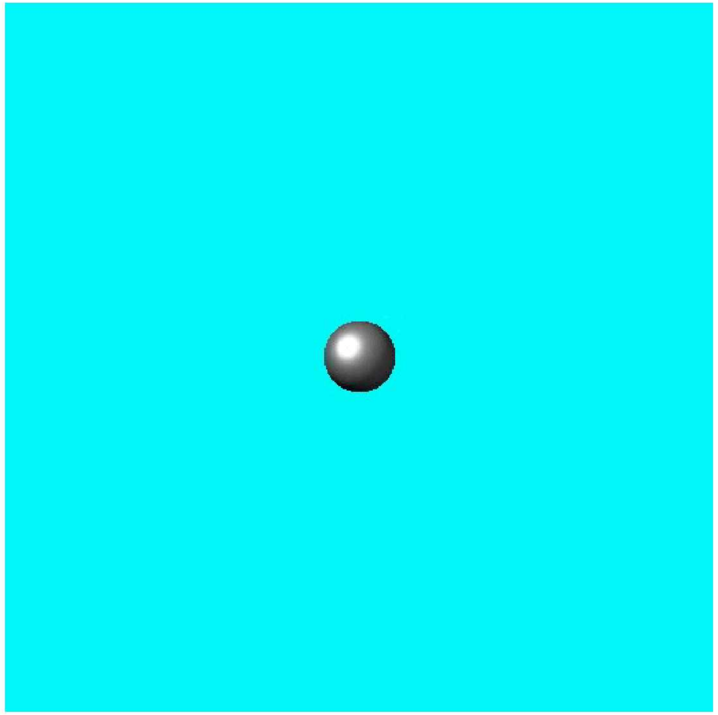
Forces: Bjerknes, Buoyancy, Stokes Drag

Bubble starts halfway down the column

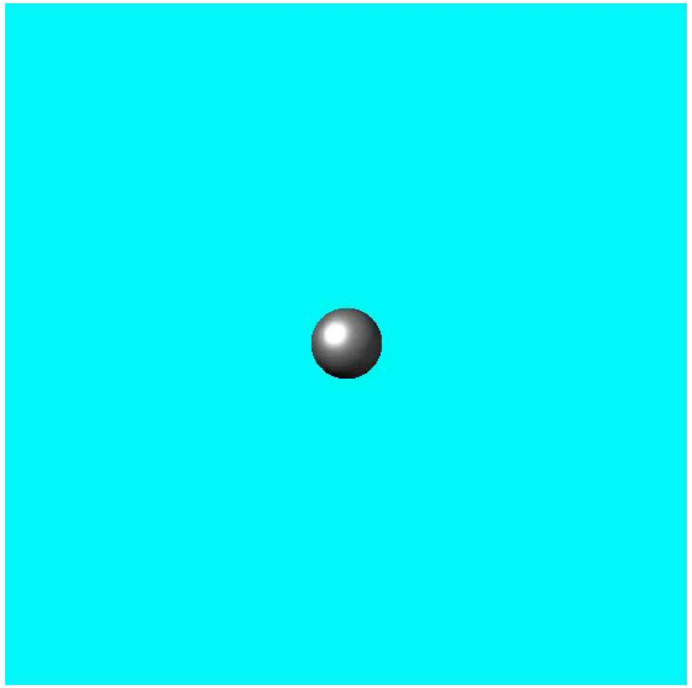
n = 0G



n = 10G



n = 30G



Bubble volume oscillates depending on local Pressure, assume bubbles are not coalescing and follow isothermal, ideal gas behavior

# Single Bubble – Parametric Study

Frequency Range: 50-250Hz

Amplitude Range: 5-30G

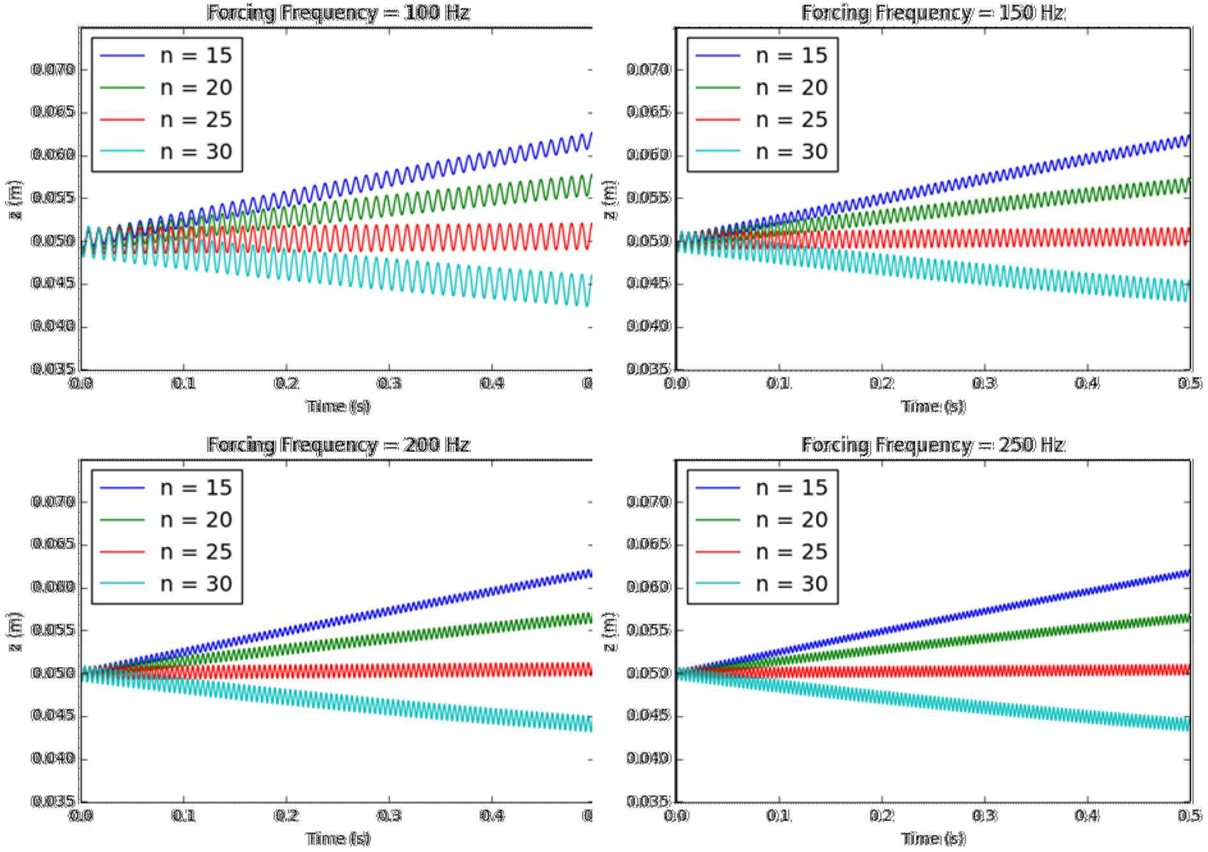
Bubble Diameter: 1mm

Column Height: 0.1m (~4”)

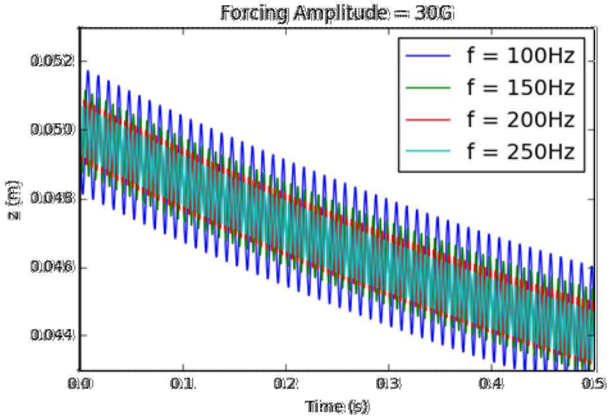
Forces: Bjerknes, Buoyancy, Stokes Drag

Bubble starts halfway down the column

Constant Vibration Frequency (f)
Varying Vibration Amplitude (n)




Constant Vibration Amplitude
Varying Vibration Frequency



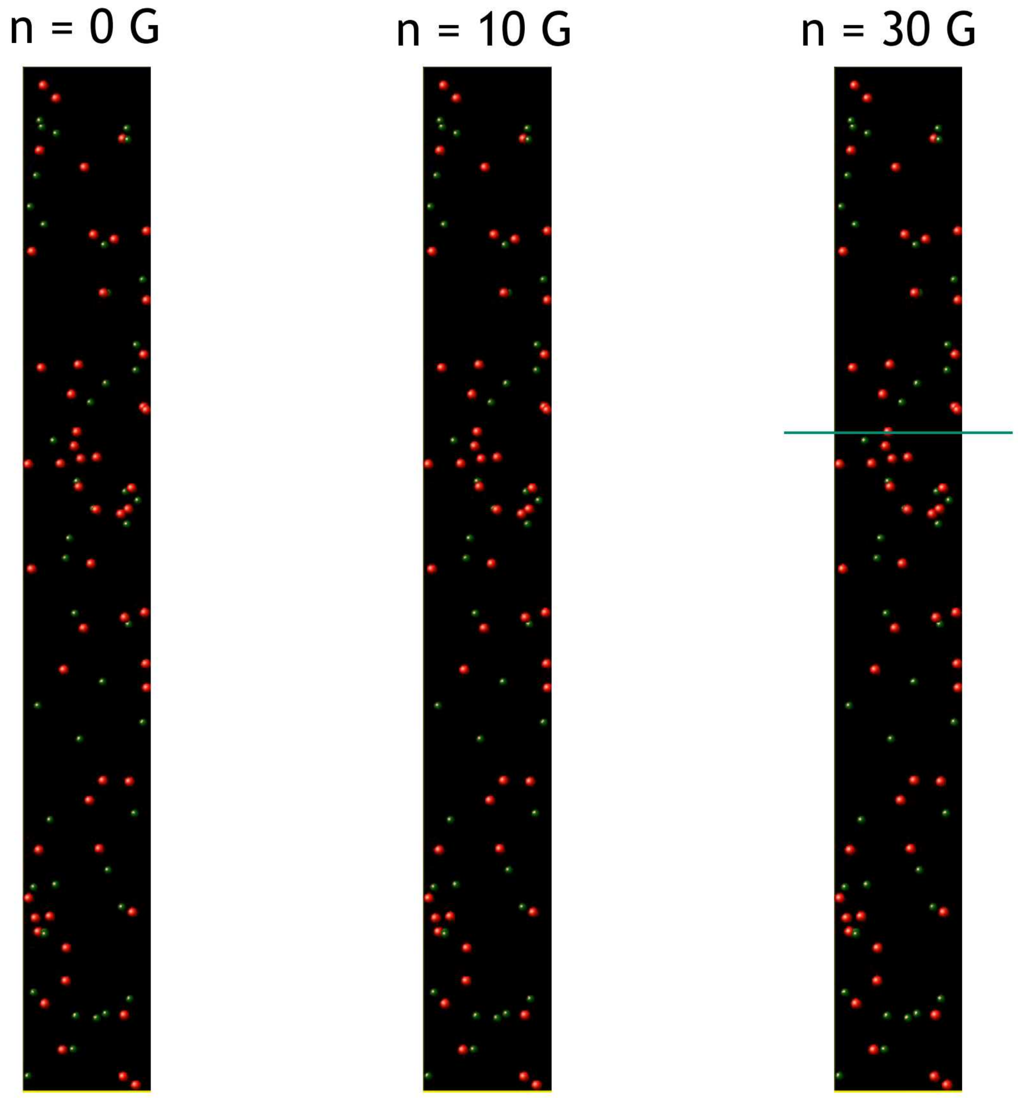
# Hydrostatic Components

- With a large bubble volume fraction, both hydrostatic force and wave speed are affected
- Density varies with volume fraction $\rho = \alpha_L \rho_L + \alpha_G \rho_G$
- Treat pressure as a perturbation $P = \bar{P} + \tilde{P}\cos(\omega t)$
- Wave equation derivation becomes: $$\frac{\partial}{\partial z} \cdot \frac{\partial \tilde{P}}{\partial z} + \frac{\omega^2}{c(z)^2}\tilde{P} = -\frac{\partial \rho(z)}{\partial z}\tilde{a}$$

where c(z) is the local sound speed, ω is the forcing frequency, $\rho$ is the local density, and $\tilde{a}$ is the forcing amplitude

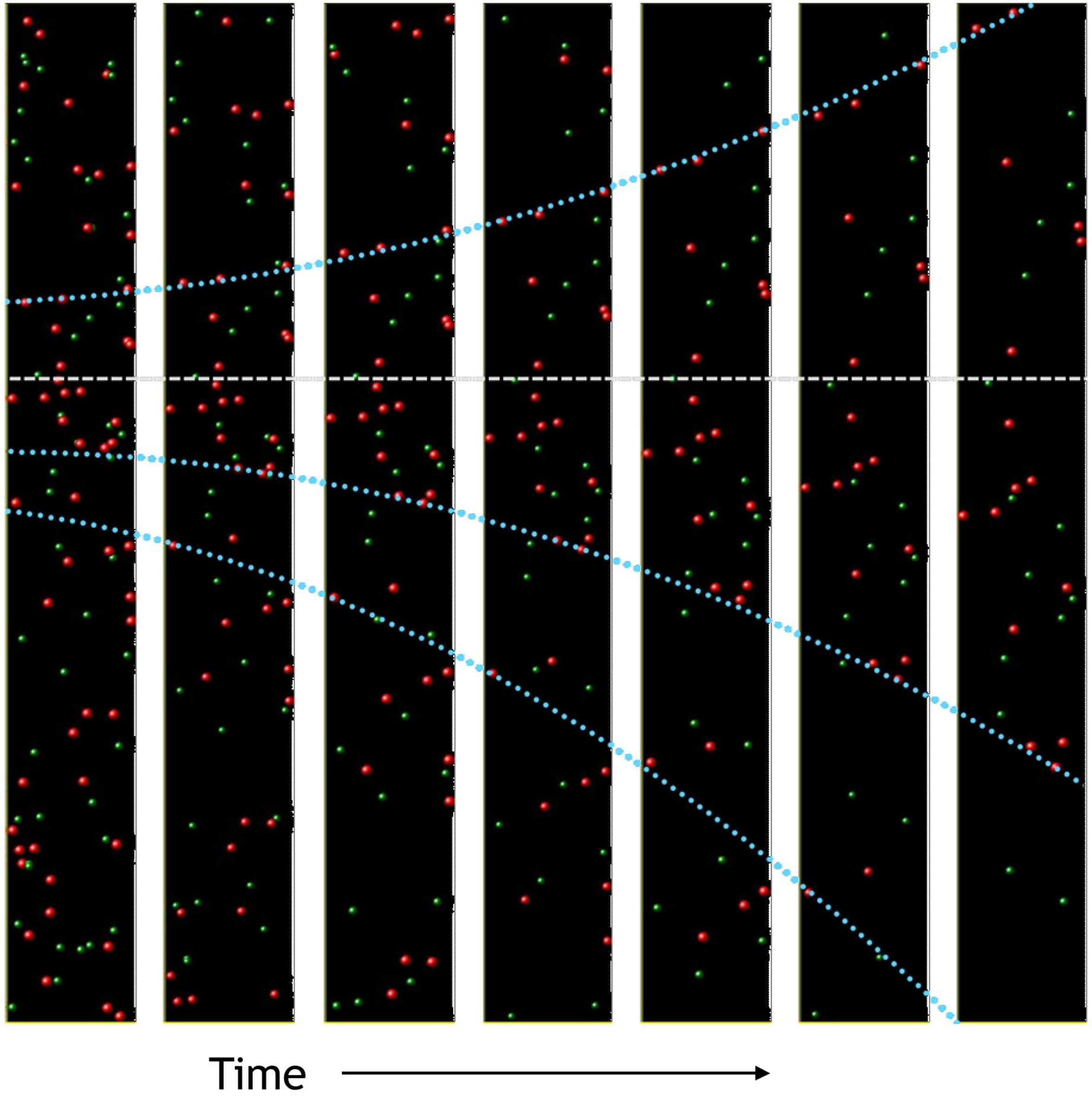
- Boundary condition set to match wall velocity
- A Runge-Kutta (RK4) algorithm provides a numerical solution for the pressure perturbation

# Multi-Bubble Simulation



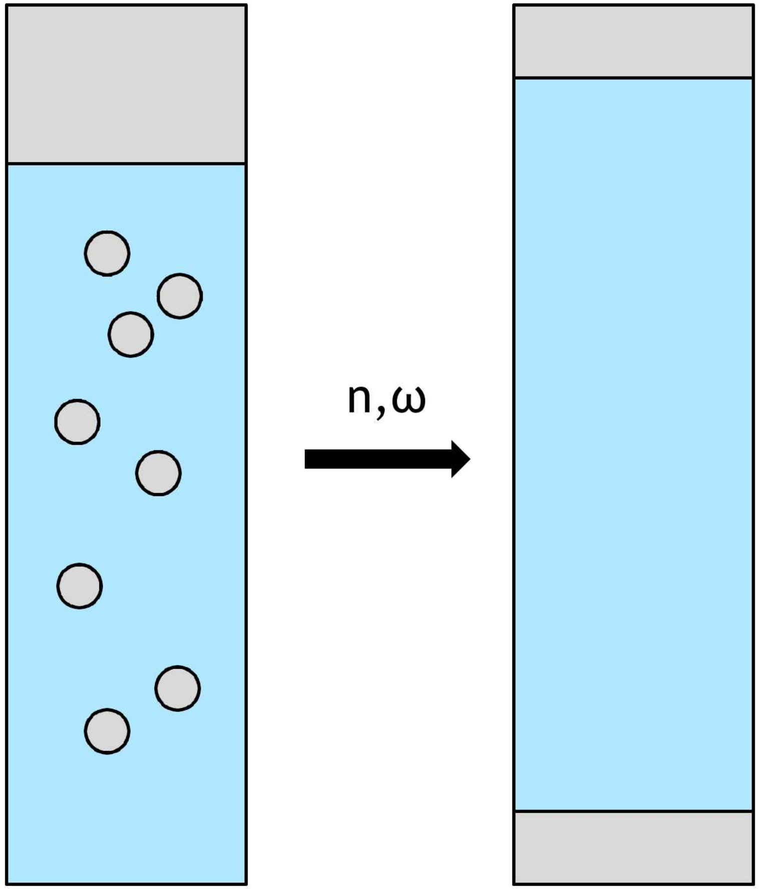

- With no forced oscillation (left), bubbles rise smoothly, with a speed determined by size and position.
- Light oscillation (middle) shows bubble vibration, but all net migration remains upward
- Heavy oscillation (right) displays both net upward and net downward motion, resulting in migration toward both ends of the tube
- There exists an equilibrium position which determines bubble movement

# Multi-Bubble Simulation

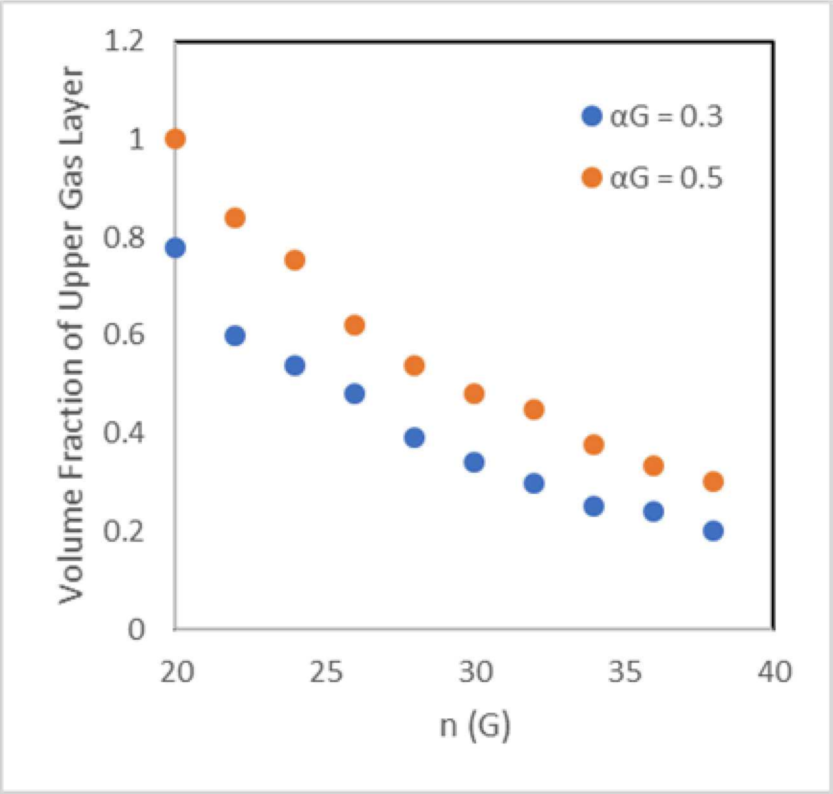



- Tracking individual bubbles shows trajectory over time
- Near the equilibrium point, bubbles hardly move
  - However this is unstable
- Moving further away from this line causes an increase in bubble velocity

# Multi-Bubble Gas Migration




- Bifurcation of upward and downward movement predicts experimentally observed "cap" formation
- Fraction of gas in each region is dependent on vibration conditions
- Initial fill fraction of gas = $\alpha_G$
- Higher gas fraction → More gas migrating upward
- Higher forcing amplitude → More gas migrating downward



# Conclusions

- A Discrete Element Approach can be a useful simulation tool for modeling rectified bubble migration
- Frequency of oscillation is not as significant as amplitude
- There is an equilibrium location which bubbles move away from dependent on their starting location

Future Work

- Compare simplified model with experiment
- Allow bubble accumulation/coalescence at interface
- Solve for the pressure field in 3-dimensions