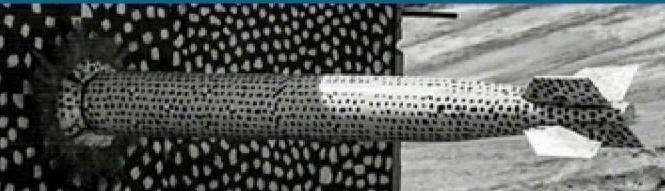


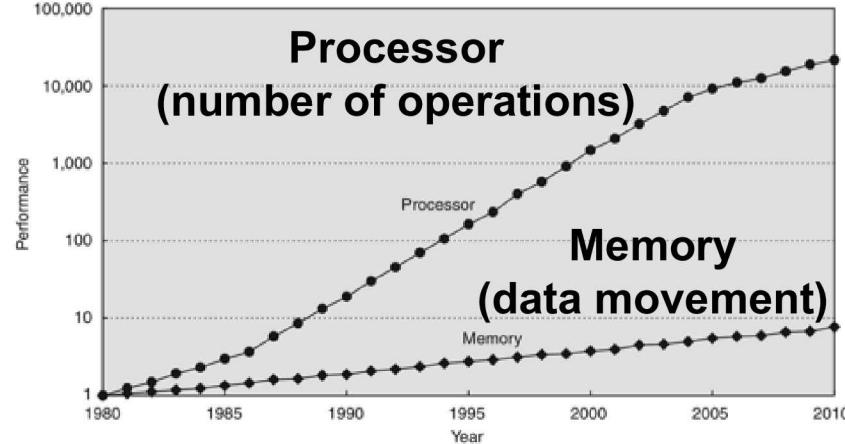
ALO-NMF: Accelerated Locality-Optimized Non-negative Matrix Factorization



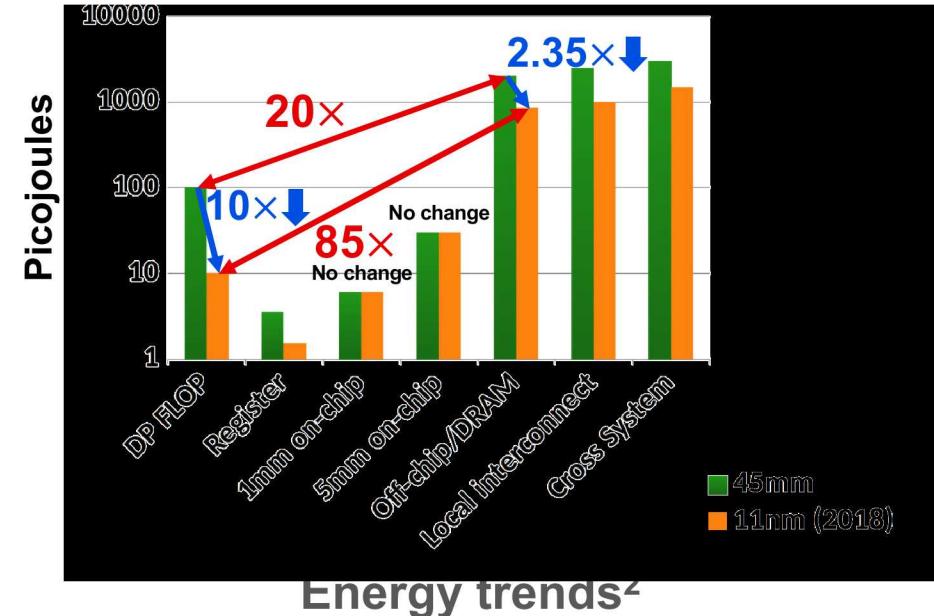
Architecture-aware Machine Learning

Machine Learning is becoming an integral part of everyday life

- How to achieve good performance on specialized architectures?



FLOPs vs. Data movement¹



Energy trends²

- FLOPs are free, but data movement is expensive
- Minimization of data movement overheads is increasingly critical

¹Source: John L. Hennessy (Stanford) and David A. Patterson (UC Berkeley)

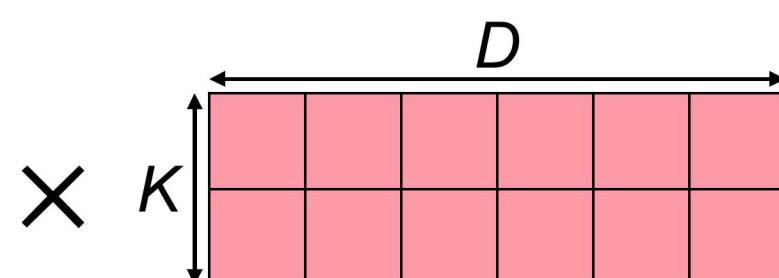
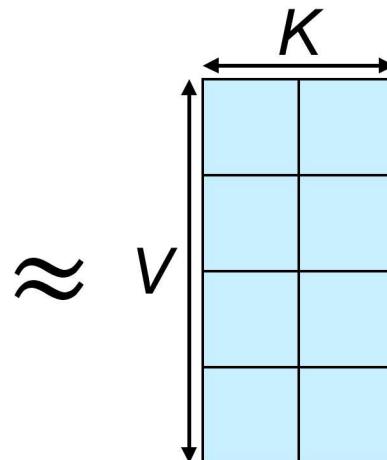
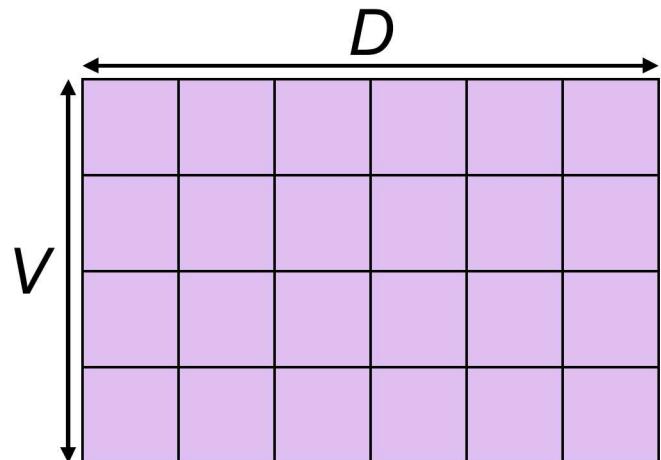
²Source: Jim Demmel (UC Berkeley) and John Shalf (LBL)

Non-negative Matrix Factorization (NMF)

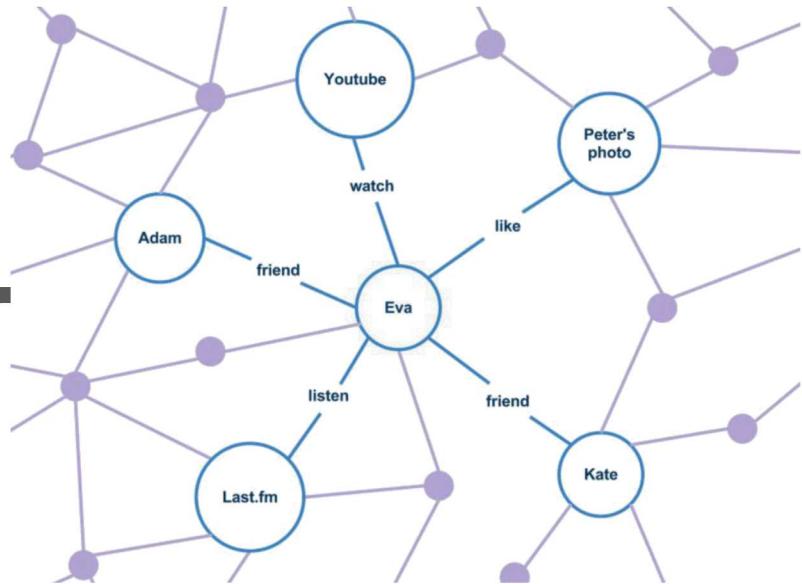
Given a matrix $\mathbf{A} \in \mathbb{R}_+^{V \times D}$ and latent variable $K \ll \min(V, D)$,

NMF estimates two rank- K matrices $\mathbf{W} \in \mathbb{R}_+^{V \times K}$ and $\mathbf{H} \in \mathbb{R}_+^{K \times D}$ such that,

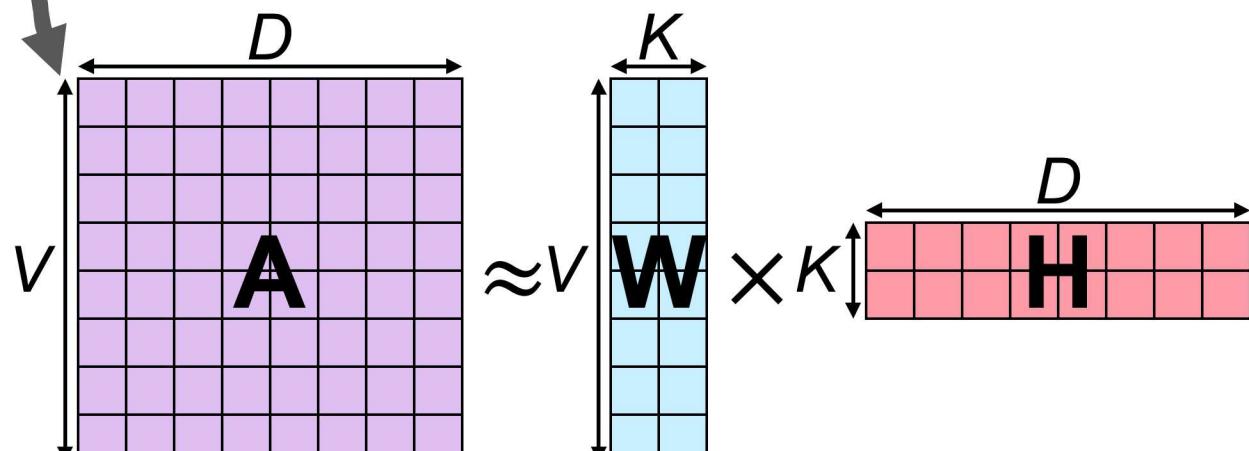
$$\mathbf{A} \approx \mathbf{WH}$$



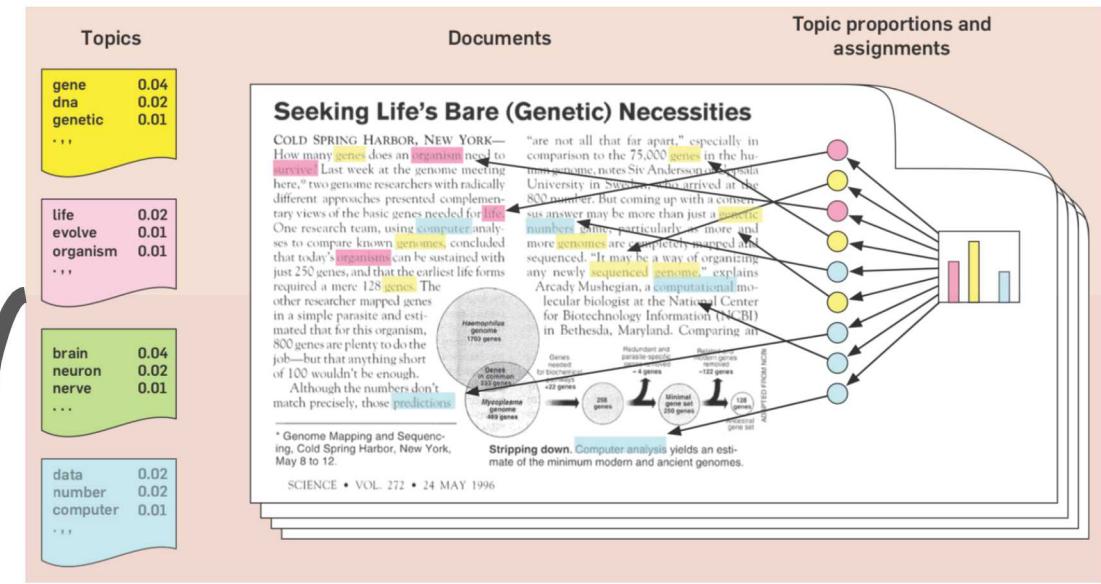
NMF Applications



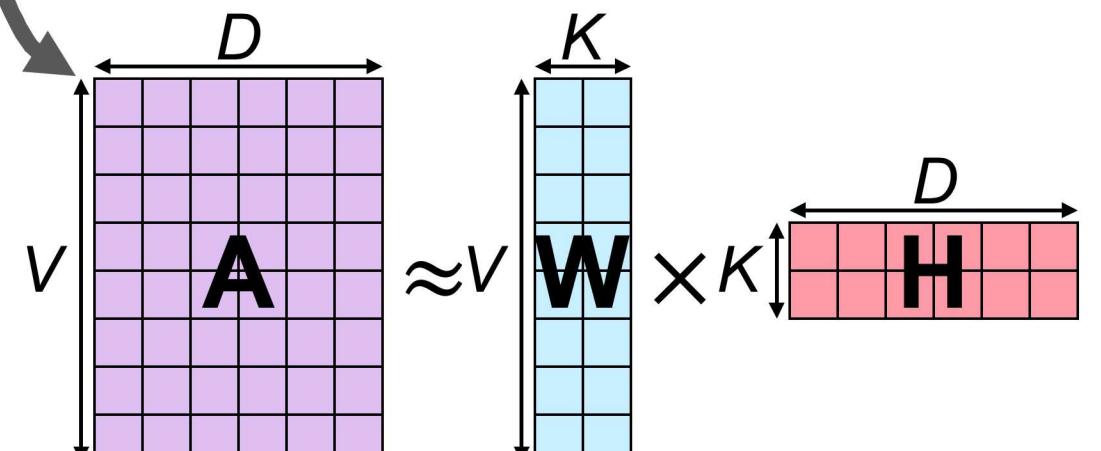
Node Embedding for Graph Mining



V: number of unique nodes
D: number of unique nodes



Topic Modeling for Text Mining*



V: vocabulary size
D: number of documents

*Source: David Blei. "Probabilistic Topic Models". (2012)

NMF Algorithms

Objective function

$$D_F(A||WH) = \frac{1}{2} ||A - WH||_F^2 = \frac{1}{2} \sum_{vd} (A_{vd} - (WH)_{vd})^2$$

Variants of NMF

- Multiplicative Update (MU)
- Additive Update (AU)
- Alternating Non-negative Least Squares (ANLS)
- Hierarchical Alternating Least Squares (HALS)

Performance Challenges in HALS-based NMF

Input: $A \in \mathbb{R}_+^{V \times D}$: non-negative input matrix, ε : machine epsilon

Initialize $W \in \mathbb{R}_+^{V \times K}$ and $H \in \mathbb{R}_+^{K \times D}$ with random non-negative numbers

repeat

Updating H {

- $R = A^T W$
- $S = W^T W$
- for** $k = 0$ **to** $K-1$ **do**
- $H_k = \max(\varepsilon, H_k + R_k - H^T S_k)$
- $P = A H^T$
- $Q = H H^T$
- for** $k = 0$ **to** $K-1$ **do**
- $W_k = \max(\varepsilon, W_k Q_{kk} + P_k - W Q_k)$
- $W_k = \frac{W_k}{\|W_k\|_2}$

Updating W {

until convergence

The main data movement overhead is associated with these k loops

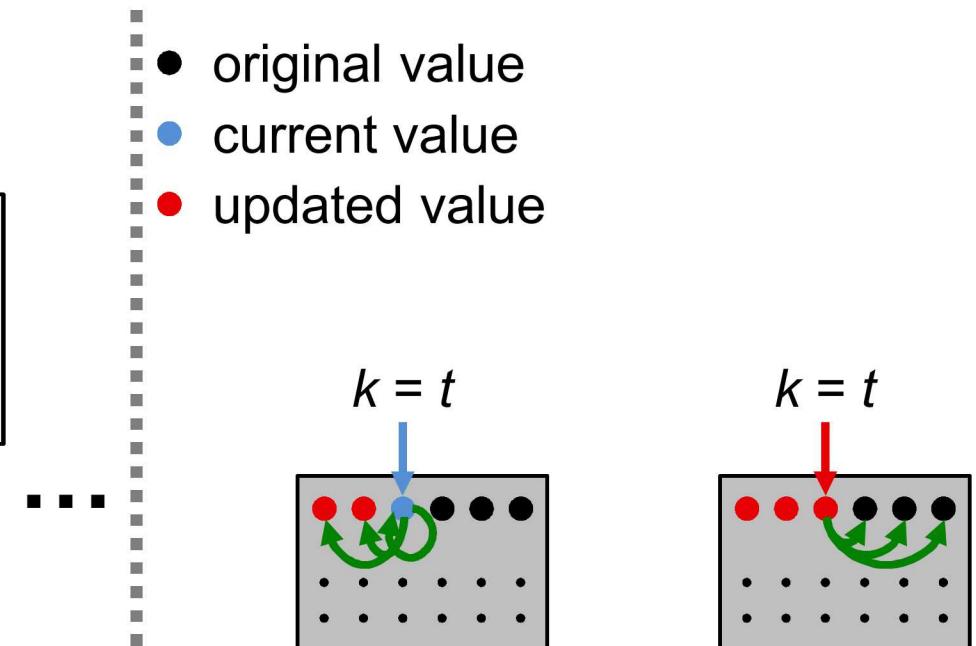
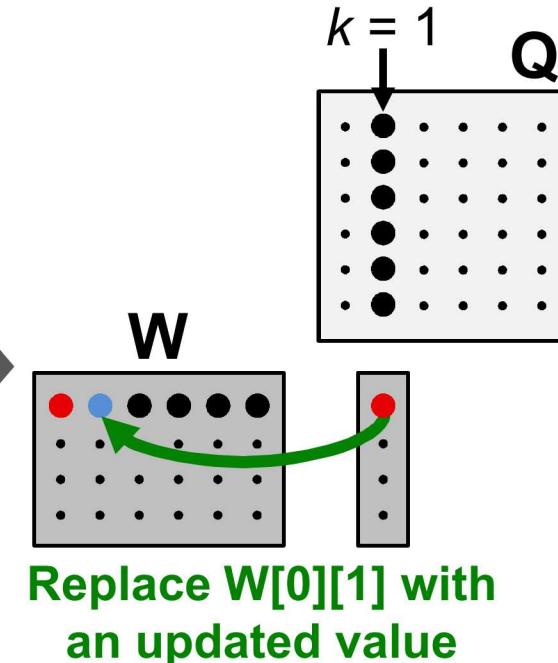
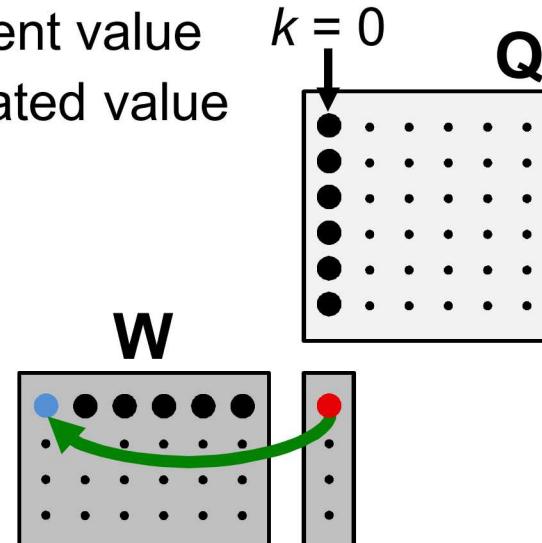
► 91% of the combined fractional data movement overhead

How to reduce data movement cost of these k loops?

Original HALS-based NMF

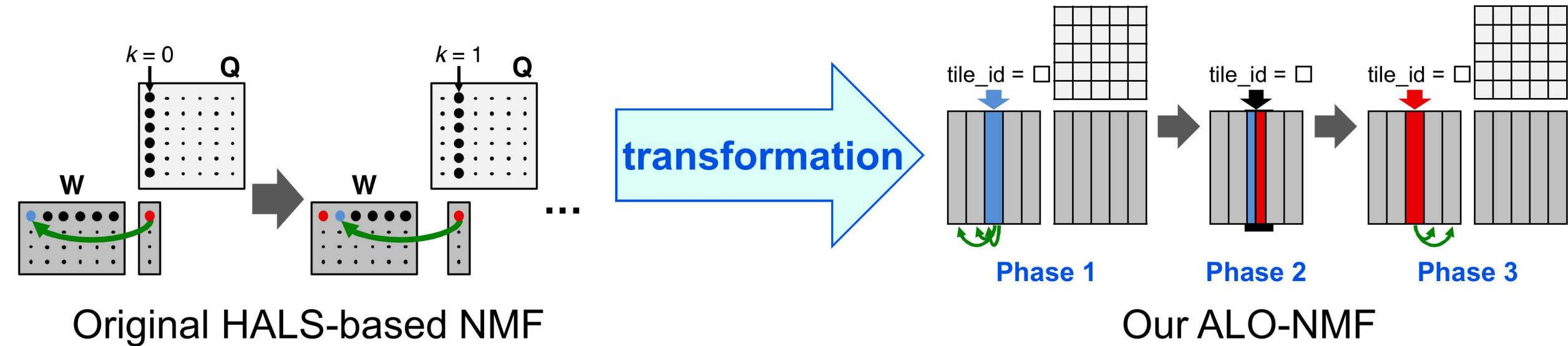
Interaction between different columns of \mathbf{W} with iterative matrix-vector multiplications

- original value
- current value
- updated value



Overview of Our Approach

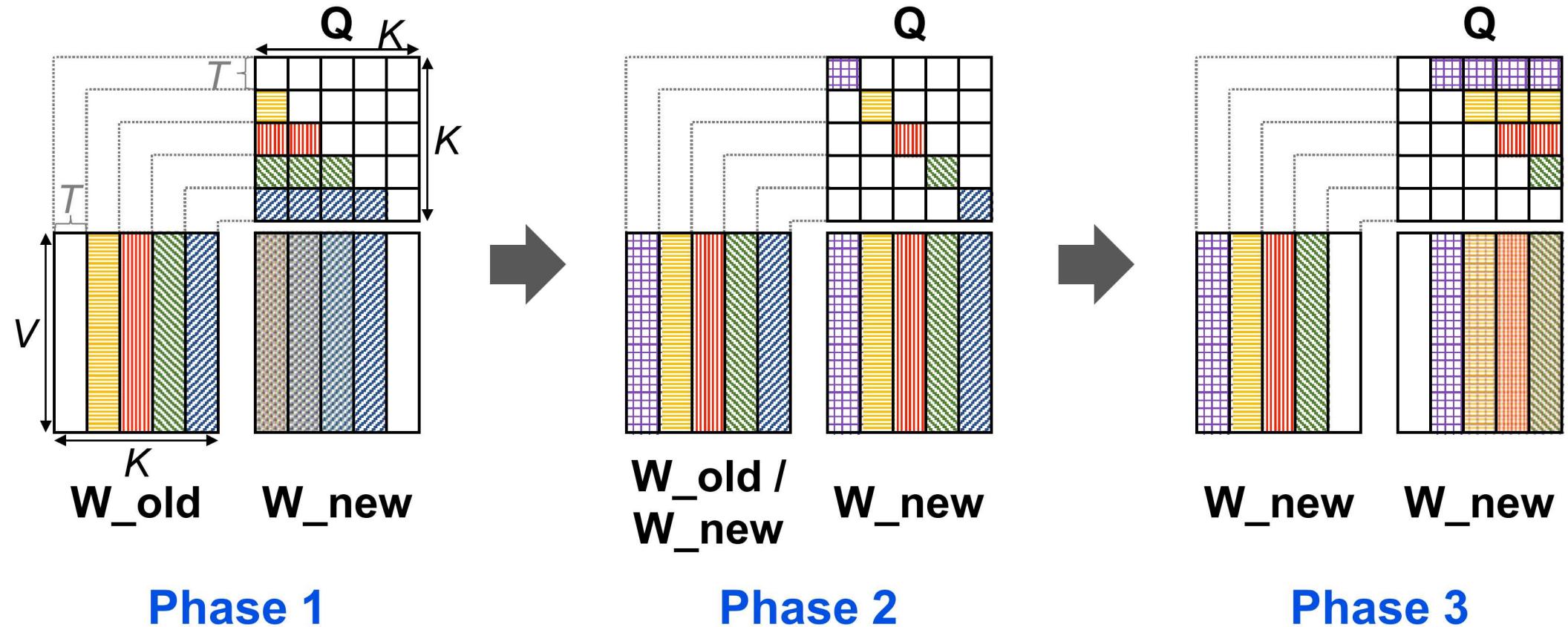
Our goal is to minimize data movement cost



How to reformulate the original iterative matrix-vector multiplications to matrix-matrix multiplication?

Brand New ALO-NMF (Accelerated Locality-Optimized NMF)

Updating \mathbf{W} with tiled matrix-matrix multiplications



Data Movement Comparison

Running on a PIE dense image dataset

V (# rows in W)	K (low rank)	T (tile size)	C (cache size)
11,554	256	16	33MB

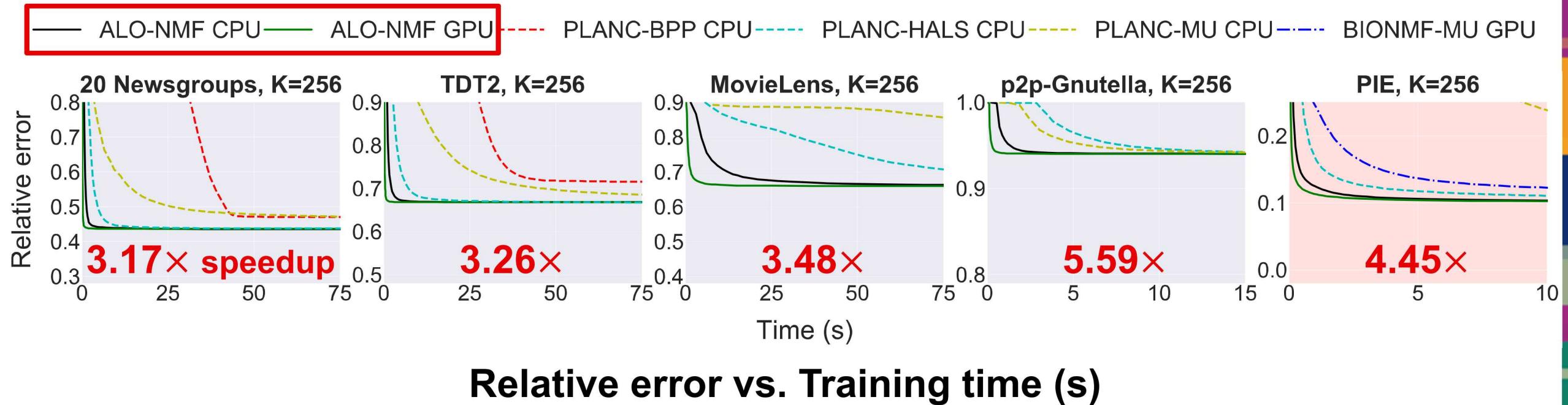
Data movement cost for updating W

Original HALS-based NMF (byte)	Our ALO-NMF (byte)
$K(VK + K + 6V + 1)$ $= 775,015,680$	$V \left(\frac{1}{T} + \frac{2}{\sqrt{C}} \right) (K^2 - KT) + KVT$ $= 338,840,256$

2.29 \times reduced

Performance Comparison: Speedup

ALO-NMF CPU/GPU achieved significant performance improvement over the existing state-of-the-art parallel NMF implementations



The lower the better

Summary and Conclusions

- Architecture-aware machine learning algorithm design is critical
- We focused on data locality optimizations for NMF
 - The associativity of addition is utilized to reorder additive contributions in updating elements of matrices **W** and **H**
- Our ALO-NMF achieved **2.29×** lower data movement and \sim **4.45×** speedup compared to the existing state-of-the-art parallel NMF implementations

Please check out our paper to learn more about this work.
Thank you. ☺