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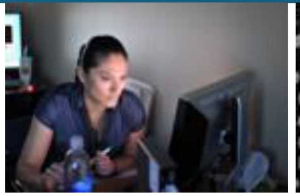
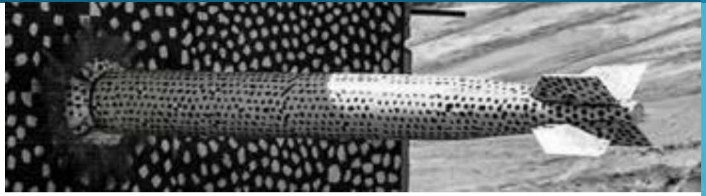
Structuring the Optimal Output Feedback Control Gain: A Soft Constraint Approach





PRESENTED BY

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Presentation outline

- Introduction & Motivation
- The Optimal Output Feedback Problem
- Fixing the Structure of the Optimal Gain
- Power Systems Example
- Conclusions

Introduction & Motivation

- Control of large scale systems is traditionally done at the local level (e.g. power systems)
- Advent of wide-area measurements has challenged this approach. However wide area measurements make available only certain information
- This work re examines the optimal output feedback control problem –where the idea is to find an optimal gain based on the outputs of the system
- Additionally this works addressed the problem of adding constraints to the optimal gain, two constraints are considered:
  - The sum of the rows of the optimal gain equals a specified value
  - Specific components of the optimal gain are set to zero

Optimal Output Feedback Control

- For a linear time invariant system described by

$$\dot{x}(t) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t)$$

$x \in \mathbb{R}^n$ states; $u \in \mathbb{R}^m$ control inputs; $y \in \mathbb{R}^p$ outputs

- The optimal output feedback control problem

$$u(t) = Ky(t)$$

- The problem is to find the optimal gain to minimize an index $\min_K J(K, \tau)$ with $K \in \mathbb{R}^{m \times p}$

- Typical cost function to minimize:

$$J(K, \tau) = \int_0^\tau x(t)^\top \left(Q + C^\top K^\top RKC\right) x(t)\, dt + x(\tau)^\top S_f x(\tau) \quad \textbf{(I1)}$$

finite time quadratic cost function

where

$R > 0 \in \mathbb{R}^{n \times n}$

$Q \geq 0 \in \mathbb{R}^{n \times n}$

The optimal gain K^* is assumed to exist and belong to: $\mathcal{S} \triangleq \{K \in \mathbb{R}^{m \times p} : \text{Re}\{\lambda(A + BKC)\} < 0\}$

Optimal Output Feedback Control

- Finding a solution on index **(I1)** is dependent on the initial conditions (not desirable). To circumvent this problem, an alternative index is proposed

$$\tilde{J} = \mathbb{E}\left\{\int_0^\tau x(t)^\top (Q + C^\top K^\top R K C) x(t) dt + x(\tau)^\top S_f x(\tau)\right\}$$

expected value → $\mathbb{E}\left\{x_0 x_0^\top\right\} \triangleq X_0$

- The problem is reformulated as $\min_K \; \tilde{J}(K,\tau)$

- By using $X = x(t)x(t)^\top$

- The problem is rewritten as

$$\min_K \; \tilde{J}(K,\tau) = \int_0^\tau \mathrm{tr}\left\{(Q + C^\top K^\top R K C) X\right\} \; dt + \mathrm{tr}\left\{x^\top S_f x\right\}$$

subject to $\dot{X}(t) = (A + BKC)X(t) + X(t)(A + BKC)^\top$

Optimal Output Feedback Control

- A Hamiltonian is needed to solve this problem

$$H(X,\Lambda,K) = \mathrm{tr}\{(Q + C^\top K^\top RKC)X\} + \mathrm{tr}\{\Lambda^\top[(A+BKC)X + X^\top(A+BKC)]\}$$

- The stationarity conditions are

$$\frac{\partial H}{\partial X} = -\dot{\Lambda} = Q + C^\top K^\top RKC + \Lambda(A+BKC) + (A+BKC)^\top \Lambda$$

$$\frac{\partial H}{\partial \Lambda} = \dot{X} = (A+BKC)X + X(A+BKC)^\top$$

$$\Lambda(\tau) = S_f \qquad X(0) = X_0$$

- In the infinite horizon case $\tau \to \infty$

$$0 = Q + C^\top K^\top RK + \Lambda(A+BKC) + (A+BKC)^\top \Lambda$$
$$0 = (A+BKC)P + P(A+BKC)^\top + X_0$$

Lyapunov type with $\int_0^\infty X\,dt = P$

$$0 = \frac{\partial \tilde{J}}{\partial K} = 2\left(RKC + B^\top \Lambda\right) PC^\top$$

$$K^* = -R^{-1}B^\top \Lambda PC^\top (CPC^\top)^{-1}$$

Structuring the Optimal Gain

- The optimal gain can be written as

$$K = \begin{bmatrix} k_{11} & k_{12} & \dots & k_{1p} \\ k_{21} & k_{22} & \dots & k_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ k_{m1} & k_{m2} & \dots & k_{mp} \end{bmatrix}$$

- Power systems **motivation** for structuring the optimal gain:

  In power systems the 'accesible' outputs (wide-area) are machine speeds, i.e. $y = [\omega_{a1} \ \dots \ \omega_{ap}]^\top$

  Note that in a power system whose synchronism is preserved as $t \to \infty$ then

$$\omega_{aj}(t) = \omega_F \qquad \forall j = 1 \dots p$$

- The sum of the rows of K represents the **droop gain (steady state action)** for a particular actuator.

Structuring the Optimal Gain

- This work proposes a method to fix the sum on the rows of the control gain while maintaining an optimal control approach to compute it.

Fixing the sum of the rows means: $K\mathbf{1}_p = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_m \end{bmatrix} \triangleq \bar{v}$

- To impose this constraint consider

$T_j \in \mathbb{R}^{p \times n}$ ⟵ matrix with all zeros except those in its jth column which are 1

- The penalty function to be included in the cost function is

$$g_2(K) = \gamma T_j^\top (K - \Upsilon)^\top (K - \Upsilon) T_j$$

with

$$\Upsilon \in \mathbb{R}^{m \times p}$$

$$\Upsilon = \bar{v} \frac{\mathbf{1}_p^\top}{p}$$

Parameter that reflect the importance of enforcing the constrain

Structuring the Optimal Gain

- An additional structure that this work imposes to the optimal gain is the possibility of making individual gains k_{ij} zero

- The idea is to penalize any individual component of the gain

using $E_j = [e_j \;\; \mathbf{0}]$ and $W_j = \mathrm{diag}(w_{1,j}, w_{2,j}, \ldots, w_{m,j})$

The term $E_j^\top K^\top W_j K E_j$ can be appropriately included in the cost function

and the element $w_{i,j}$ in W_j penalizes the component k_{ij} of the optimal gain

To penalize terms at all columns: $$g_1(K) = \sum_{j=1}^{p} E_j^\top K^\top W_j K E_j$$

Structuring the Optimal Gain

- The reformulation of the optimal output feedback control problem is

$$J(K,\tau) = \int_0^{\tau} x(t)^\top \left(Q + C^\top K^\top R K C + g_1(K) + g_2(K)\right) x(t)\, dt + x(\tau)^\top S_f x(\tau)$$

added penalty function for setting individual gains to zero

added penalty function for setting the rows to a selected value

- Note that this approach is a **soft constraint approach**

Again redefining the cost function to $\tilde{J}(K,\tau) = \mathbb{E}\{J(K,\tau)\}$

The problem is again $\min_K \tilde{J}(K,\tau)$

where K is assumed to exist and is chosen from the set $\mathcal{S}$ of stabilizing gains

$$\mathcal{S} \triangleq \{K \in \mathbb{R}^{m\times p} : \text{Re}\{\lambda(A+BKC)\} < 0\}$$

Structuring the Optimal Gain

- The solution of the reformulation of the OFC needs the following Hamiltonian function

$$H(X,\Lambda,K) = \mathrm{tr}\Big\{\Big(Q + C^\top K^\top RKC + \gamma T^\top (K-\Upsilon)^\top (K-\Upsilon)T + \sum_{j=1}^{p} E_j^\top K^\top W_j K E_j\Big)X\Big\}$$
$$+\,\mathrm{tr}\Big\{\Lambda^\top\Big((A+BKC)X + X^\top (A+BKC)\Big)\Big\}$$

- The stationarity conditions are then

$$\frac{\partial H}{\partial X} = -\dot{\Lambda} = Q + C^\top K^\top RKC + \sum_{j=1}^{p} E_j^\top K^\top W_j K E_j + \gamma T^\top (K-\Upsilon)^\top (K-\Upsilon)T + \Lambda(A+BK)$$
$$+\,(A+BK)^\top \Lambda$$

$$\frac{\partial H}{\partial \Lambda} = \dot{X} = (A+BKC)X + X(A+BKC)^\top$$

$$\Lambda(\tau) = S_f$$

$$X(0) = X_0$$

Structuring the Optimal Gain

- The solution of the reformulation of the OFC control problem when considering the infinite horizon case $\tau \to \infty$ and with the following definitions

$$X = x(t)x(t)^\top \qquad \int_0^\infty X\,dt = P \qquad S_f = 0$$

is obtained from the following equations:

$$0 = Q + C^\top K^\top R K + \sum_{j=1}^{p} E_j^\top K^\top W_j K E_j + \gamma T^\top (K - \Upsilon)^\top (K - \Upsilon) + \Lambda(A + BKC) + (A + BKC)^\top \Lambda \quad \textbf{(E1)}$$

$$0 = (A + BKC)P + P(A + BKC)^\top + X_0 \quad \textbf{(E2)}$$

Lyapunov like (E1, E2)

$$0 = \frac{\partial \tilde{J}}{\partial K} = 2\left(RKC + B^\top \Lambda\right) PC^\top + 2\sum_{j=1}^{p} W_j K E_j P E_j^\top + 2\gamma K T P T^\top - 2\gamma \Upsilon T P T^\top \quad \textbf{(E3)}$$

Structuring the Optimal Gain

- Formulation of the optimal output feedback control:

$$0 = \frac{\partial \tilde{J}}{\partial K} = 2\left(RKC + B^\top \Lambda\right) PC^\top + 2\sum_{j=1}^{p} W_j K E_j P E_j^\top + 2\gamma KTPT^\top - 2\gamma \Upsilon TPT^\top \quad \textbf{(E3)}$$

$$B^\top \Lambda PC^\top - \gamma \Upsilon TPT^\top + RKCPC^\top + \sum_{j=1}^{p} W_j K E_j P E_j^\top + \gamma KTPT^\top = 0$$

- This final equation is a linear matrix equation of the form

$$\sum_{i=1}^{n} \mathcal{A}_i \mathcal{X} \mathcal{B}_i = \mathcal{C}$$

which can be solved iteratively

Numerical Solution

- The implemented algorithm follows the form of a classic Anderson-Moore Algorithm

Algorithm 1

Step 0: Select an initial stabilizing feedback gain $K_k = K_0 \in \mathcal{S}$, set $k = 0$, and choose a small positive parameter δ.

Step 1: With $K = K_k$, solve the Lyapunov equation **(E2)** for $P \to P_k$

Step 2: With $K = K_k$ solve the Lyapunov equation **(E1)** for $\Lambda \to \Lambda_k$

Step 3: With $\Lambda = \Lambda_k$, $P = P_k$, solve the linear matrix equation in **(E3)** for the optimal gain $K \to \hat{K}_k$.

Step 4: Compute the descent direction $\Delta K_k = \hat{K}_k - K_k$.

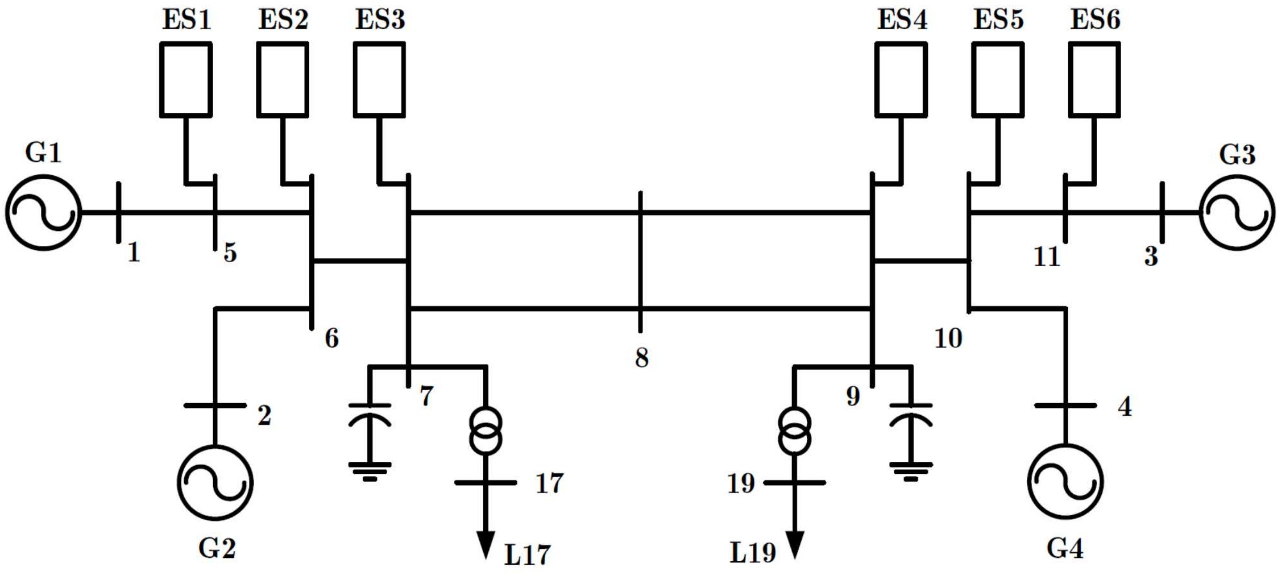
Step 5: Compute $\|\Delta K_k\|_F$, where $\|\cdot\|_F$ denotes the Frobenius norm. If $\|\Delta K_k\|_F \leq \delta$, stop the algorithm; otherwise, choose a value of α_k, $0 < \alpha_k \leq 1$, compute

$$K_{k+1} = K_k + \alpha \Delta K_k$$

set $k = k + 1$, and go to Step 1.

Power Systems Example

- Test system: two-area, four-machine power system with six energy storage devices



$$x \in \mathbb{R}^{42}$$

$y \in \mathbb{R}^4$ (machine speeds)

$u \in \mathbb{R}^6$ (actuators / energy storage devices)

- The power system has a dominant inter-area oscillation that is poorly damped

Power Systems Example

- Cases considered for control design

- **Case 0** no further constraints are imposed on K.
- **Case 1** imposes the constraint that the rows of K add up to zero (i.e. $\Upsilon = \mathbf{0}$).
- **Case 2** imposes the following structure in the gain K,

$$K = \begin{bmatrix} \bullet & 0 & \bullet & 0 \\ 0 & \bullet & 0 & \bullet \\ \bullet & 0 & 0 & \bullet \\ 0 & \bullet & 0 & \bullet \\ \bullet & 0 & \bullet & 0 \\ 0 & \bullet & \bullet & 0 \end{bmatrix}$$

Note that the desired structure assumes that only 2 measurements out of the 4 are available for each ES device.

- **Case 3** imposes the same structure as Case 2 but with the added constraint that the rows of K should add up to zero.

Power Systems Example

- The control gains for each of the cases considered

$$K_{c0} = \begin{bmatrix} 53.653 & 30.305 & -21.821 & -21.402 \\ 28.842 & 21.717 & -15.720 & -15.127 \\ 14.732 & 11.821 & -10.940 & -10.380 \\ -14.999 & -8.452 & 16.817 & 17.351 \\ -29.446 & -17.548 & 22.360 & 22.921 \\ -48.246 & -29.712 & 30.312 & 26.658 \end{bmatrix}$$

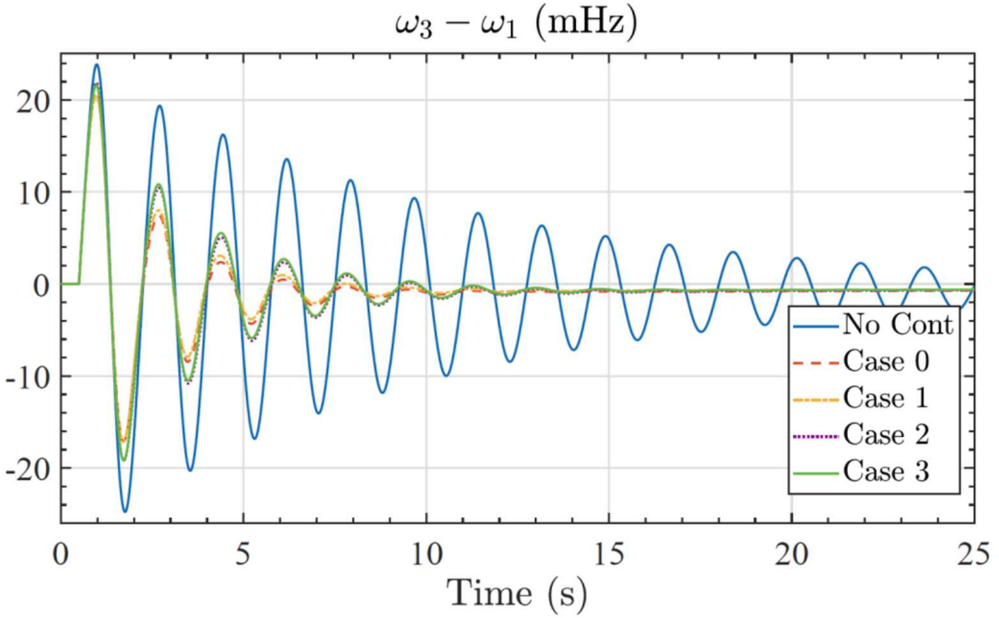
$$K_{c1} = \begin{bmatrix} 38.939 & 20.236 & -30.069 & -29.106 \\ 21.687 & 16.828 & -19.682 & -18.834 \\ 12.797 & 10.506 & -11.966 & -11.337 \\ -18.900 & -11.126 & 14.677 & 15.349 \\ -28.905 & -17.160 & 22.760 & 23.304 \\ -40.804 & -24.572 & 34.646 & 30.730 \end{bmatrix}$$

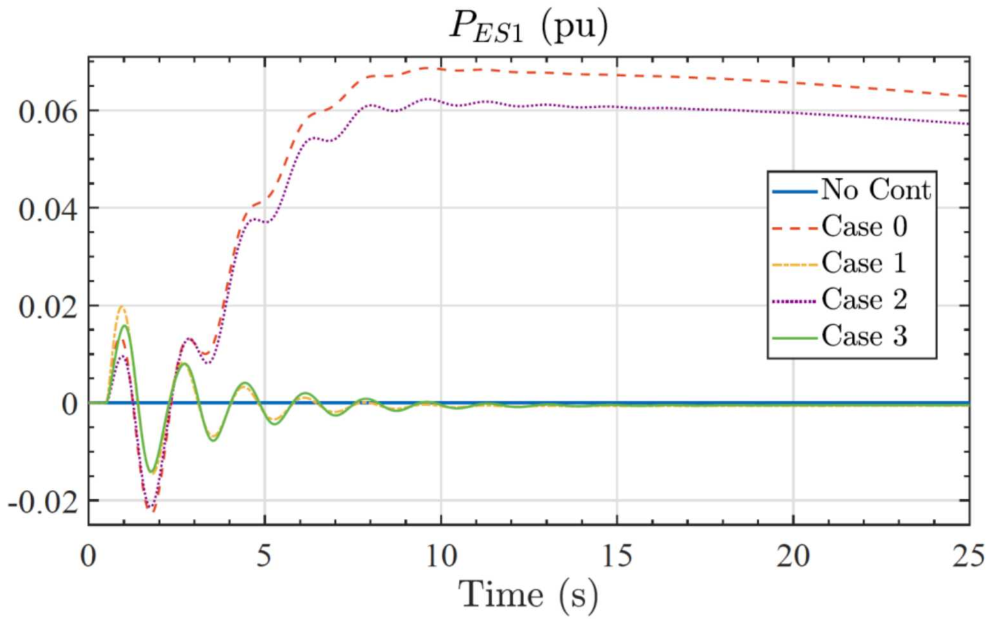
$$K_{c2} = \begin{bmatrix} 65.693 & \sim 0 & -29.304 & \sim 0 \\ \sim 0 & 31.547 & \sim 0 & -21.226 \\ 18.019 & \sim 0 & \sim 0 & -15.174 \\ \sim 0 & -12.884 & \sim 0 & 25.022 \\ -35.740 & \sim 0 & 31.286 & \sim 0 \\ \sim 0 & -45.389 & 38.487 & \sim 0 \end{bmatrix}$$

$$K_{c3} = \begin{bmatrix} 43.850 & \sim 0 & -43.850 & \sim 0 \\ \sim 0 & 25.742 & \sim 0 & -25.742 \\ 16.321 & \sim 0 & \sim 0 & -16.321 \\ \sim 0 & -19.505 & \sim 0 & 19.505 \\ -33.103 & \sim 0 & 33.103 & \sim 0 \\ \sim 0 & -41.402 & 41.402 & \sim 0 \end{bmatrix}$$

Power Systems Example

- Time domain simulations (of the nonlinear system)





- Every controller considered damps the inter-area oscillation properly
- In cases two and three (where half of the information is available) the performance is comparable to the other cases
- For cases 1 and 3 the output of the actuator is zero (as desired) with no decrease in performance

Conclusions

- The paper reconsiders the optimal output feedback control problem where only output measurements are available for control design
- The paper considers imposing a structure on the optimal gain
  - Some gains need to be zero as information from some output may be unavailable to certain actuators
  - The sum of the rows of the optimal gain matrix is equal to a desired set of values
- The structure was imposed using a soft constrain approach with the necessary conditions for the infinite horizon case yielding a set of matrix equations (two of them of the Lyapunov-type)
- The paper shows the application of imposing a structure of the optimal gain to a power system problem. For these kind of systems the sum of the rows of the control matrix is the droop (or steady state gain). For an application such as energy storage such a gain can be set to zero.

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Thank You!

Questions?