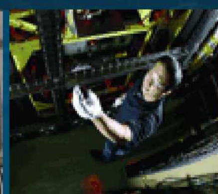
SAND2018-13741C

Brute force evaluation of Neumann series adequacy for electrically conductive fracture response in the presence of strong cultural artifacts



PRESENTED BY

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EM GEOPHYSICS: A NIGHTMARE SCENARIO

Electromagnetic geophysics in culturally cluttered environments is well known to be problematic:

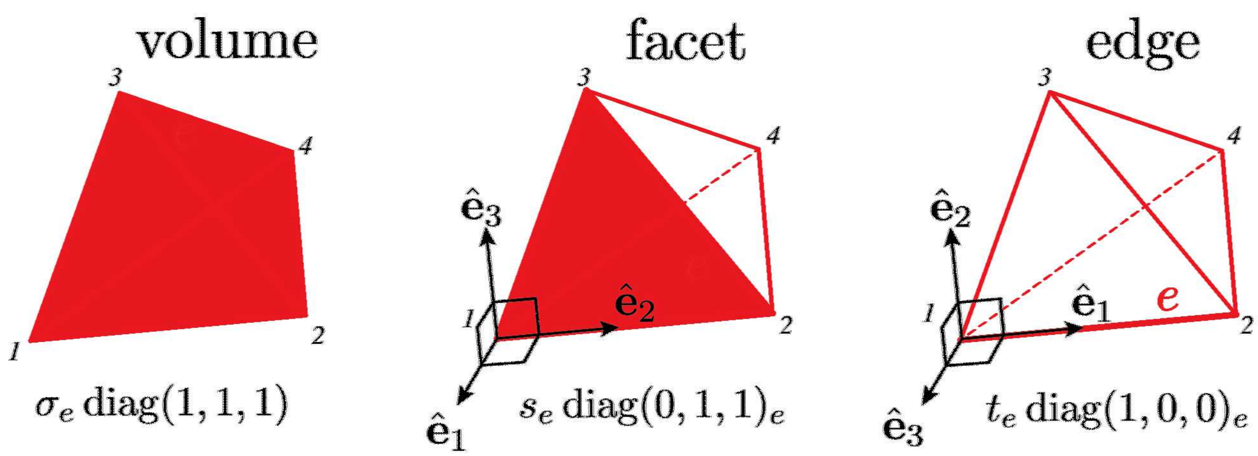
- Thin, strong conductors that are difficult to model
- Nuisance, active noise sources
- Complex coupling between target and clutter

Example: Kern River Oilfield

A HIERARCHICAL FINITE ELEMENT METHOD

Hanging the material properties on the tets, faces and edges of the unstructured tetrahedral mesh allows for thin conductors to be economically represented by facets and edges, rather than 100s of millions of tiny tets.

$$\boldsymbol{\sigma}(\boldsymbol{x}) = \sum_{e=1}^{N_V} \sigma_e \boldsymbol{\psi}_e^V(\boldsymbol{x}) + \sum_{e=1}^{N_F} s_e \boldsymbol{\psi}_e^F(\boldsymbol{x}) + \sum_{e=1}^{N_E} t_e \boldsymbol{\psi}_e^E(\boldsymbol{x})$$



Weiss, Geophysics, 2017

ASSEMBLY AND SOLUTION OF THE LINEAR SYSTEM

Variational formulation:

$$\int_\Omega \nabla v \cdot (\boldsymbol{\sigma} \cdot \nabla u)\ dx^3 = \int_\Omega vf\ dx^3$$

Weiss, Geophysics, 2017

Hierarchical model:

$$\boldsymbol{\sigma}(\boldsymbol{x}) = \sum_{e=1}^{N_V} \sigma_e \boldsymbol{\psi}_e^V(\boldsymbol{x}) + \sum_{e=1}^{N_F} s_e \boldsymbol{\psi}_e^F(\boldsymbol{x}) + \sum_{e=1}^{N_E} t_e \boldsymbol{\psi}_e^E(\boldsymbol{x})$$

3D inner products collapse to 2D and 1D inner products

$$\int_\Omega \nabla v \cdot \left[\sum_{e=1}^{N_V} \sigma_e \boldsymbol{\psi}_e^V(\boldsymbol{x})\right] \nabla u\ dx^3 = \sum_{e=1}^{N_V} \sigma_e \int_{V_e} \nabla v \cdot \nabla u\ dx^3 = \sum_{e=1}^{N_V} \sigma_e\, \mathbf{v}_e^{\mathrm{T}} \mathbf{K}_e^4 \mathbf{u}_e$$

$$\int_\Omega \nabla v \cdot \left[\sum_{e=1}^{N_F} s_e \boldsymbol{\psi}_e^F(\boldsymbol{x})\right] \nabla u\ dx^3 = \sum_{e=1}^{N_F} s_e \int_{F_e} \nabla_{23} v \cdot \nabla_{23} u\ dx^2 = \sum_{e=1}^{N_F} s_e\, \mathbf{v}_e^{\mathrm{T}} \mathbf{K}_e^3 \mathbf{u}_e$$

$$\int_\Omega \nabla v \cdot \left[\sum_{e=1}^{N_E} t_e \boldsymbol{\psi}_e^E(\boldsymbol{x})\right] \nabla u\ dx^3 = \sum_{e=1}^{N_E} t_e \int_{E_e} \nabla_1 v \cdot \nabla_1 u\ dx = \sum_{e=1}^{N_E} t_e\, \mathbf{v}_e^{\mathrm{T}} \mathbf{K}_e^2 \mathbf{u}_e$$

Global stiffness matrix is a sum of 3D, 2D and 1D element stiffness matrices.

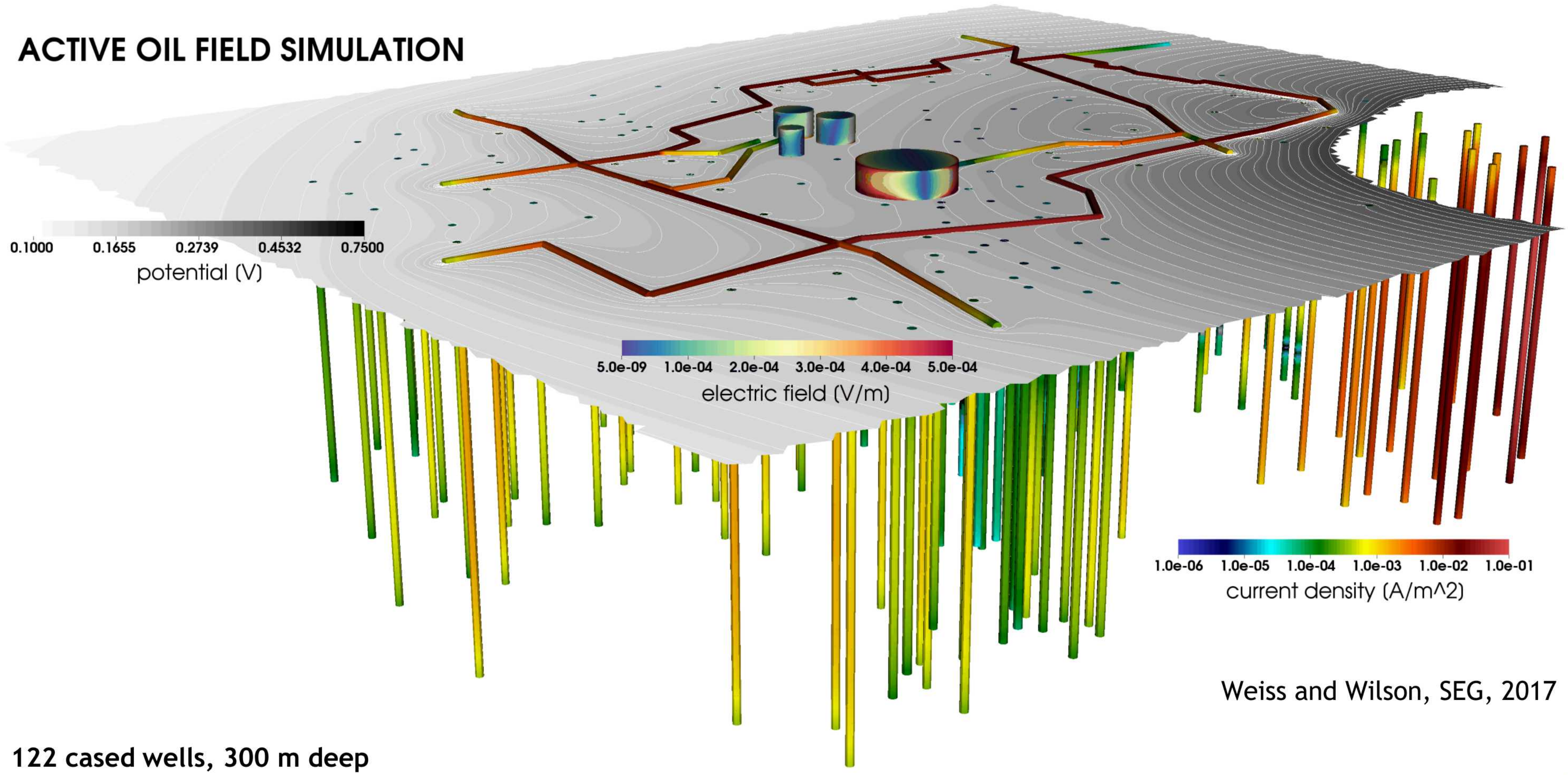
$$\mathbf{K}\mathbf{u} = \mathbf{b}$$

$$\mathbf{K} = \sum_{e=1}^{N_V} \sigma_e \mathbf{K}_e^4 + \sum_{e=1}^{N_F} s_e \mathbf{K}_e^3 + \sum_{e=1}^{N_E} t_e \mathbf{K}_e^2$$

Solve iteratively with Jacobi scaled conjugate gradients and on-the-fly matrix assembly (Weiss, 2001)

EXAMPLE: CASING AND SURFACE INFRASTRUCTURE

ACTIVE OIL FIELD SIMULATION



Weiss and Wilson, SEG, 2017

122 cased wells, 300 m deep
5 km surface pipes
~35 km pipeline/casing modeled at 10 m grid spacing: 3500 elements
Traditional FEM requires ~7e6 elements per km of pipline/casing.
HFEM decreases computational burden by ~4 orders of magnitude in this example (10 min vs 2 mo, estimated runtime)

NEUMANN SERIES EXPANSION: CONTINUOUS CASE

Choose σ_0 such that the Poisson Eq is easy to solve: $-\nabla \cdot \sigma_0 \nabla \phi_0 = \nabla \cdot \mathbf{J_s}$

Potential ϕ is therefore a sum of the zeroth order potential ϕ_0 and a residual, $\phi^{(1)}$:

$$\phi = \phi_0 + \phi^{(1)} \qquad -\nabla \cdot \sigma \nabla \phi^{(1)} = \nabla \cdot (\sigma - \sigma_0) \nabla \phi_0$$

Expand the residual $\phi^{(1)}$ as $\phi_1 + \phi^{(2)}$ such that $-\nabla \cdot \sigma_0 \nabla \phi_1 = \nabla \cdot (\sigma - \sigma_0) \nabla \phi_0$

Now $\phi^{(1)} = \phi_1 + \phi^{(2)} \mapsto \phi = \phi_0 + \phi_1 + \phi^{(2)}$ and $-\nabla \cdot \sigma \nabla \phi^{(2)} = \nabla \cdot (\sigma - \sigma_0) \nabla \phi_1$

In general . . .

$$-\nabla \cdot \sigma_0 \nabla \phi_i = \nabla \cdot (\sigma - \sigma_0) \nabla \phi_{i-1} \quad \forall \quad i = 1, 2, \ldots, N$$

$$\phi^{(N)} = \phi_N + \phi^{(N+1)} \mapsto \phi = \sum_{i=0}^{N} \phi_i + \phi^{(N+1)} \qquad -\nabla \cdot \sigma \nabla \phi^{(N+1)} = \nabla \cdot (\sigma - \sigma_0) \nabla \phi_N$$

If $\sigma - \sigma_0$ represents the change in state of the subsurface, the sum

$$\phi - \phi_0 = \sum_{i=1}^{N} \phi_i + \phi^{(N+1)}$$

represents the corresponding change in electric potential.

NEUMANN SERIES EXPANSION: DISCRETE CASE

Discrete form of the easy-to-solve model... $\mathbf{K}_0\boldsymbol{x}_0 = \boldsymbol{b}$

nodal values of $\phi_0 \mapsto \boldsymbol{x}_0$ $\boldsymbol{x} = \boldsymbol{x}_0 + \boldsymbol{x}^{(1)}$

Let $\mathbf{K}$ represent the discrete Poisson operator for the "full" model σ

Solve for residual, $\mathbf{K}\boldsymbol{x}^{(1)} = \boldsymbol{\delta}\mathbf{K}\,\boldsymbol{x}_0$ with $\boldsymbol{\delta}\mathbf{K} = \mathbf{K} - \mathbf{K}_0$

Next level of recursive iteration... $\mathbf{K}_0\boldsymbol{x}_1 = \boldsymbol{\delta}\mathbf{K}\,\boldsymbol{x}_0$

$\boldsymbol{x}^{(1)} = \boldsymbol{x}_1 + \boldsymbol{x}^{(2)} \mapsto \boldsymbol{x} = \boldsymbol{x}_0 + \boldsymbol{x}_1 + \boldsymbol{x}^{(2)}$ with $\mathbf{K}\boldsymbol{x}^{(2)} = \boldsymbol{\delta}\mathbf{K}\,\boldsymbol{x}_1$

i^{th} recursive term for discrete ϕ

$$\boldsymbol{x}_i = \left(\mathbf{K}_0^{-1}\boldsymbol{\delta}\mathbf{K}\right)\,\boldsymbol{x}_{i-1} = \left(\mathbf{K}_0^{-1}\boldsymbol{\delta}\mathbf{K}\right)^i\,\boldsymbol{x}_0$$

residual for i^{th} recursive term.

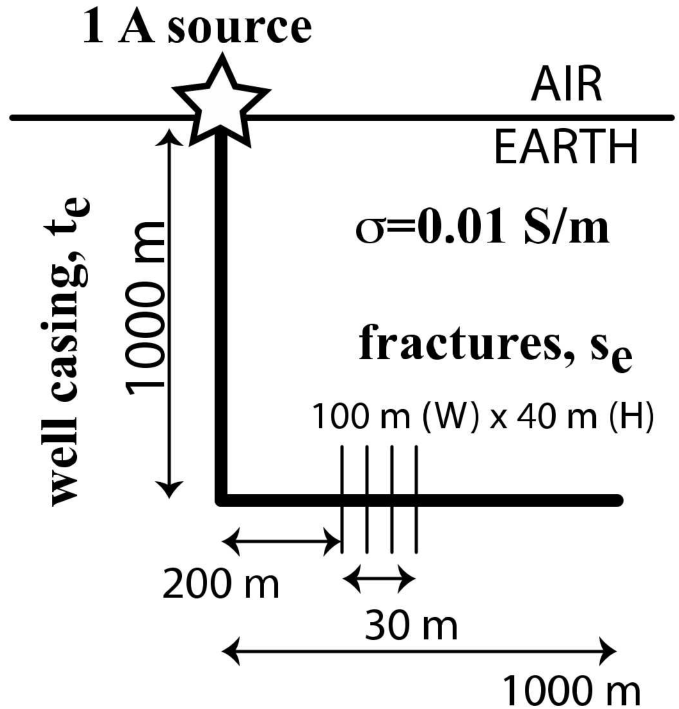
$$\boldsymbol{x}^{(i)} = \left(\mathbf{K}^{-1}\boldsymbol{\delta}\mathbf{K}\right)\,\boldsymbol{x}_{i-1} = \left(\mathbf{K}^{-1}\boldsymbol{\delta}\mathbf{K}\right)^i\,\boldsymbol{x}_0$$

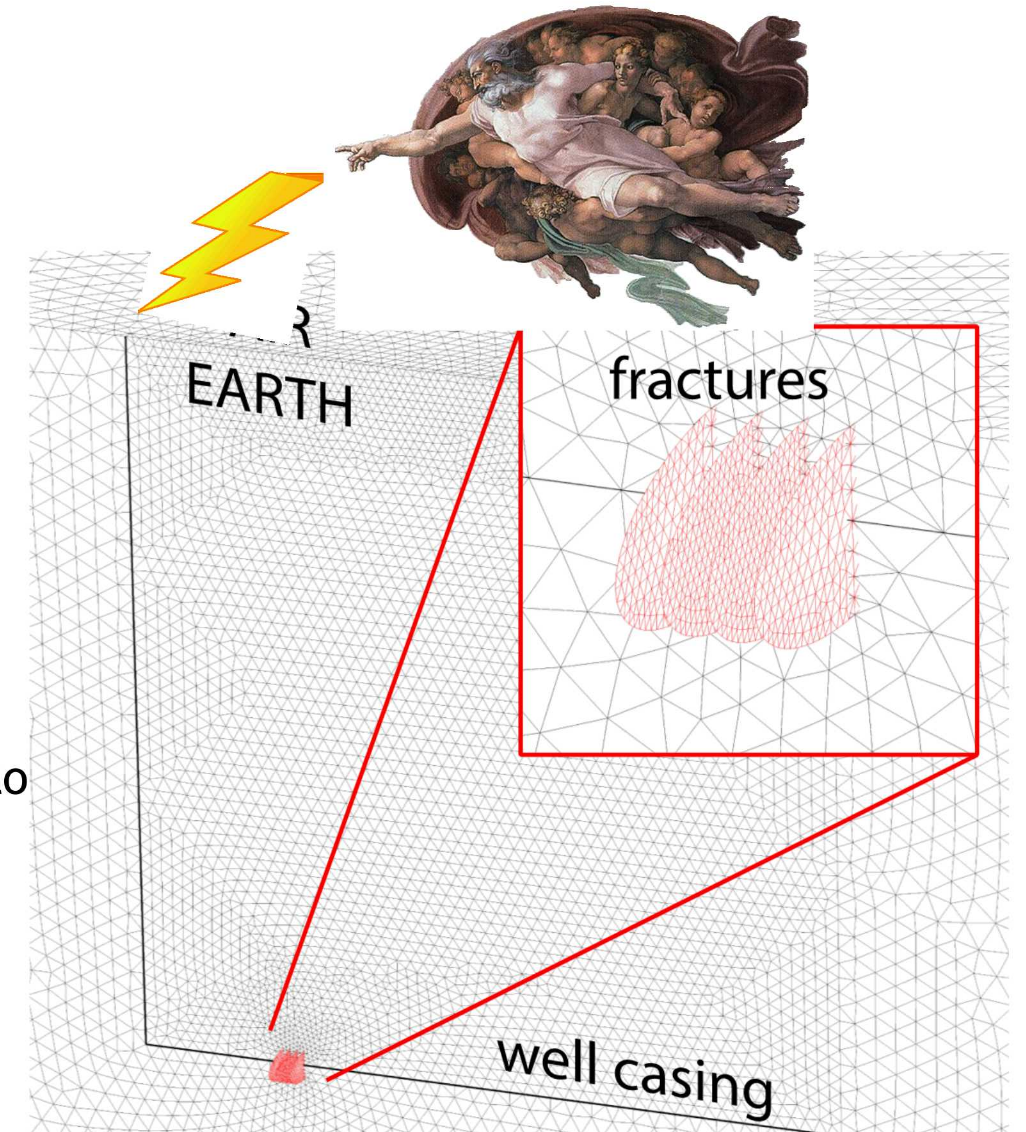
The N-term Neumann series...

$$\boldsymbol{x} = \boldsymbol{x}_0 + \left(\mathbf{T}_0 + \mathbf{T}_0^2 + \cdots + \mathbf{T}_0^N\right)\boldsymbol{x}_0 + \boldsymbol{x}^{(N+1)} \qquad \mathbf{T}_0 = \mathbf{K}_0^{-1}\boldsymbol{\delta}\mathbf{K}$$

$$\boldsymbol{x}^{(N+1)} = \left(\mathbf{T} + \mathbf{T}^2 + \cdots + \mathbf{T}^N\right)\boldsymbol{x}_0 \qquad \mathbf{T} = \mathbf{K}^{-1}\boldsymbol{\delta}\mathbf{K}$$

FE ENGINE TO COMPUTE TIME-LAPSE FRACTURE RESPONSE





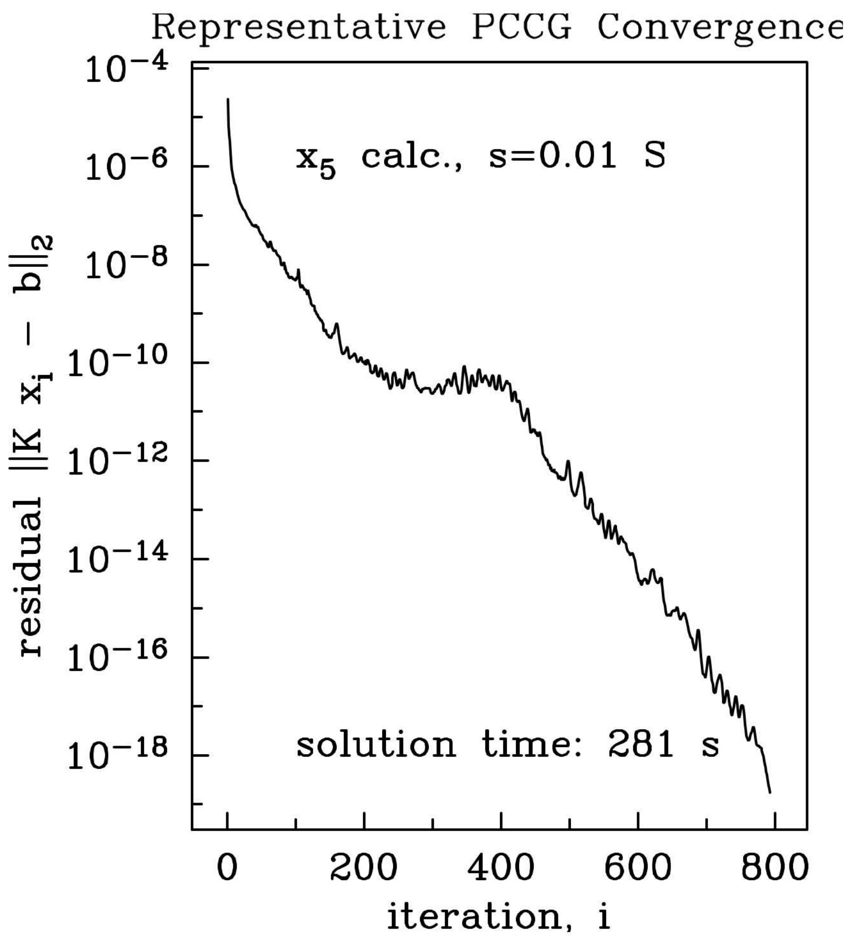
Weiss and van Bloemen Waanders, Geophysics, 2018

Choose the "easy-to-solve" model to be the EARTH+CASING.

Full model includes the fractures.

NS is therefore an expansion about the fracture conductivity anomaly.

FE ENGINE PERFORMANCE



Mesh Pre-Processing: (cubit.sandia.gov)

500k nodes
3.1M tets

105 edges for casing at 5e4 S.m

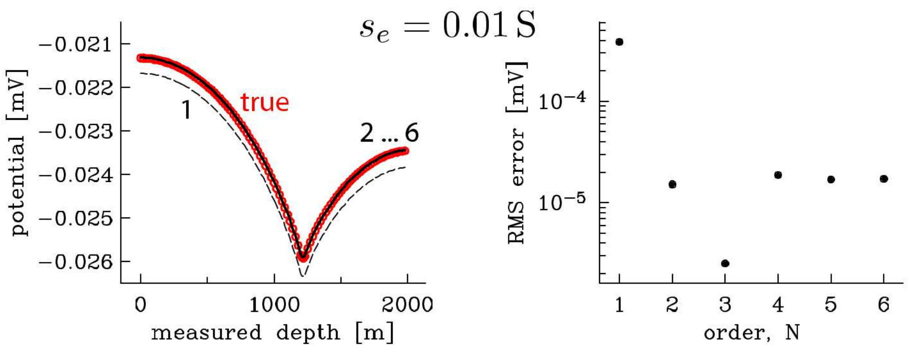
2616 facets at 0.01, 0.1 and 1.0 S

Linear solver: Preconditioned CG with Jacobi scaling.

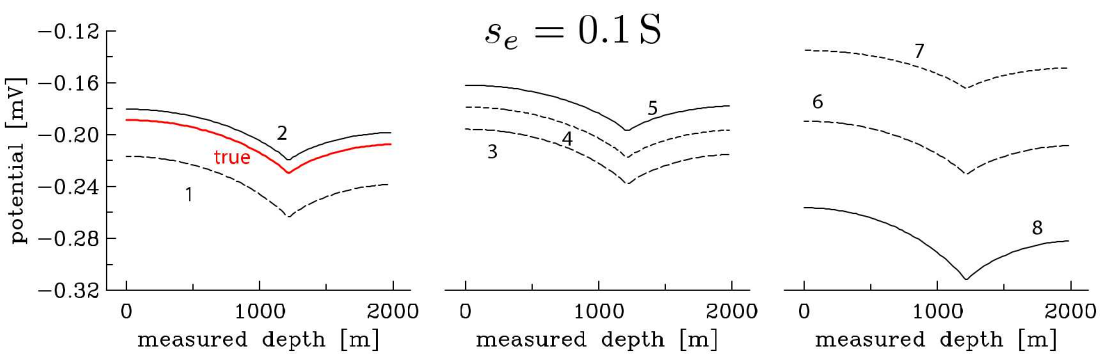
Weiss and van Bloemen Waanders, Geophysics, 2018

NS CONVERGENCE

Weak fracture anomaly

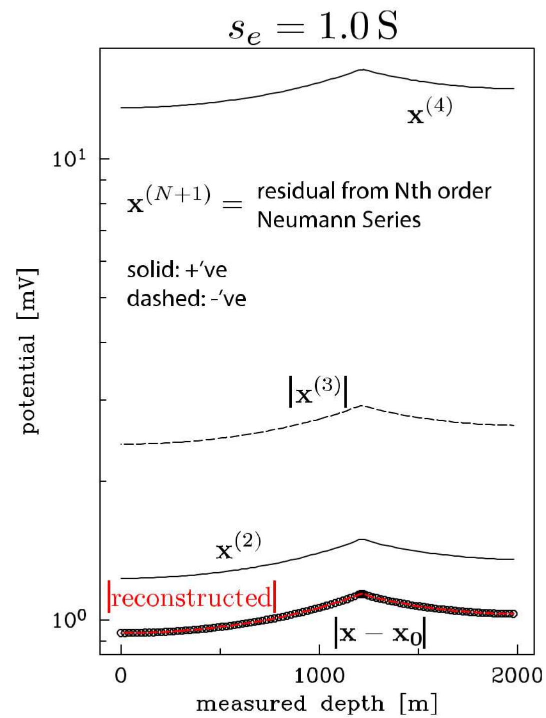


Moderate fracture anomaly



potentials along well casing

Strong fracture anomaly



residual along well casing

… small anomalies convergent
(so far, so good)

Weiss and van Bloemen Waanders, Geophysics, 2018

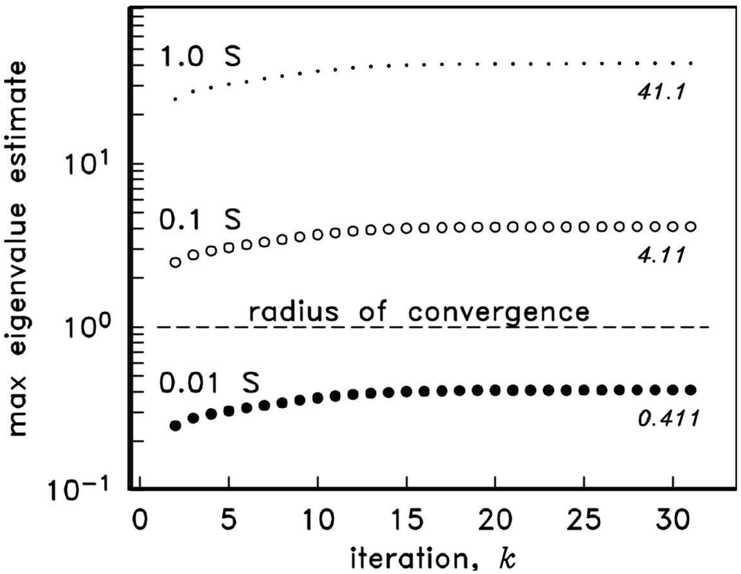
ESTIMATING SPECTRAL RADIUS OF THE NS

Neumann series is convergent if max eigenvalue $\lambda_{\max}$ for $\mathbf{T_0}$ is < 1.

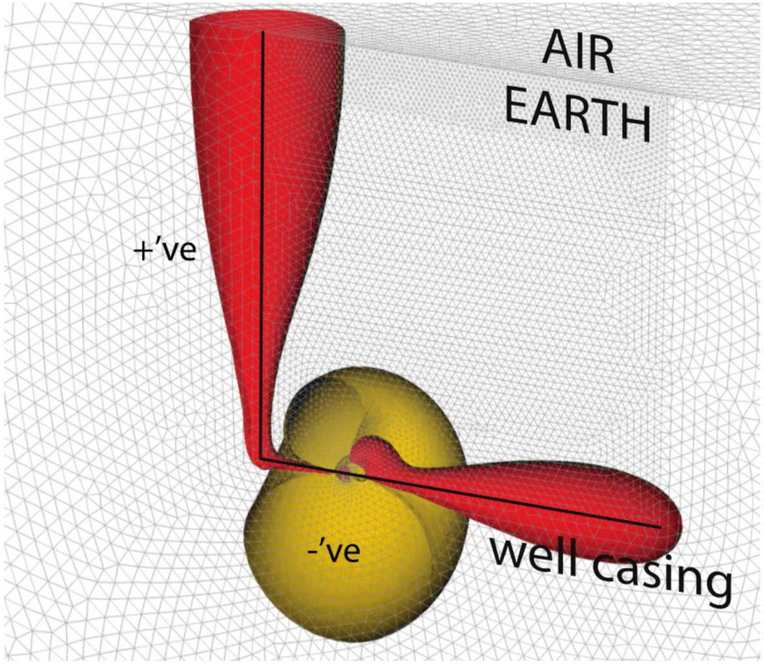
Use power method (Saad, 2011) to approximate eigenvector $\boldsymbol{v}$ associated with $\lambda_{\max}$.

$\boldsymbol{v}$ is approximated by successive iterates, $\boldsymbol{v}_k$

$$\boldsymbol{v}_{k+1} = \frac{\mathbf{T}_0 \boldsymbol{v}_k}{\|\mathbf{T}_0 \boldsymbol{v}_k\|_2} \qquad \lambda_{\max}^{(k)} = \frac{\boldsymbol{v}_k^T \mathbf{T}_0 \boldsymbol{v}_k}{\boldsymbol{v}_k^T \boldsymbol{v}_k}$$



dominant eigenvector, $\boldsymbol{v}$



strong coupling between fractures and casing!

Weiss and van Bloemen Waanders, Geophysics, 2018

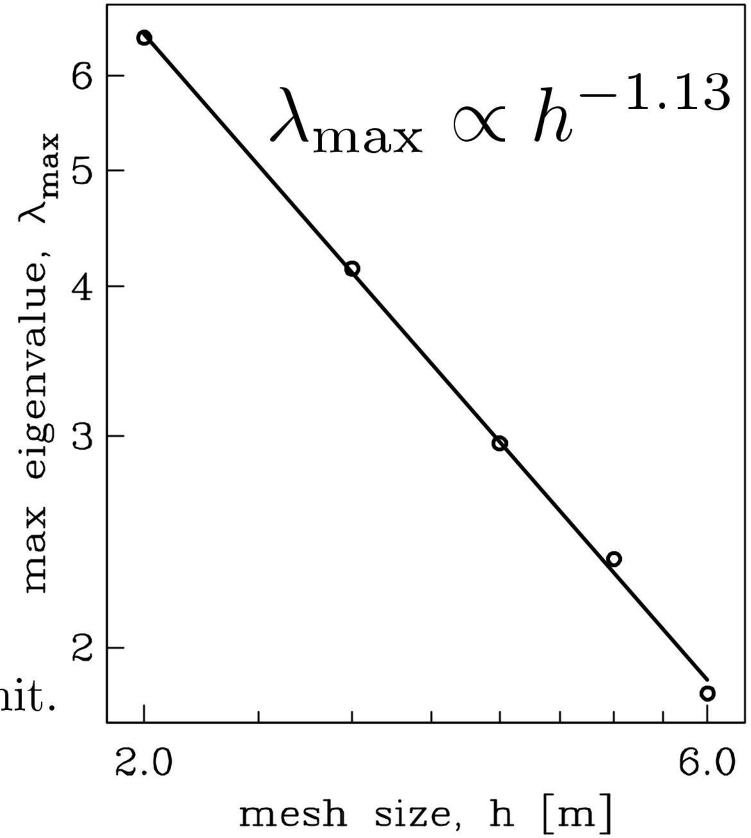
EFFECT OF MESH SIZE ON SPECTRAL RADIUS

Neumann series is convergent if max eigenvalue $\lambda_{\max}$ for $\mathbf{T_0}$ is < 1.

Use power method (Saad, 2011) to approximate eigenvector $\boldsymbol{v}$ associated with $\lambda_{\max}$.

$\boldsymbol{v}$ is approximated by successive iterates, $\boldsymbol{v}_k$

$$\boldsymbol{v}_{k+1} = \frac{\mathbf{T}_0 \boldsymbol{v}_k}{\|\mathbf{T}_0 \boldsymbol{v}_k\|_2} \qquad \lambda^{(k)}_{\max} = \frac{\boldsymbol{v}_k^T \mathbf{T}_0 \boldsymbol{v}_k}{\boldsymbol{v}_k^T \boldsymbol{v}_k}$$



Letting h denote mean node spacing for discretized fractures…

spectral radius is singular in the continuum limit.

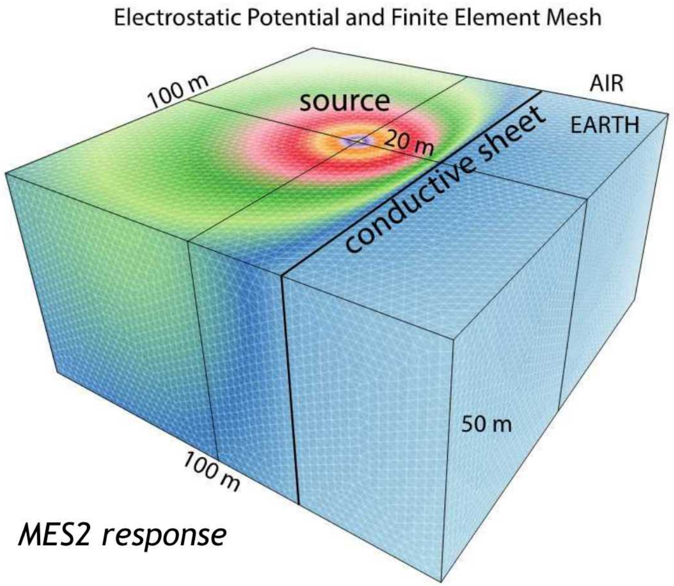
$\therefore$ NS is intrinsically divergent, regardless of fracture anomaly smallness.

Weiss and van Bloemen Waanders, Geophysics, 2018

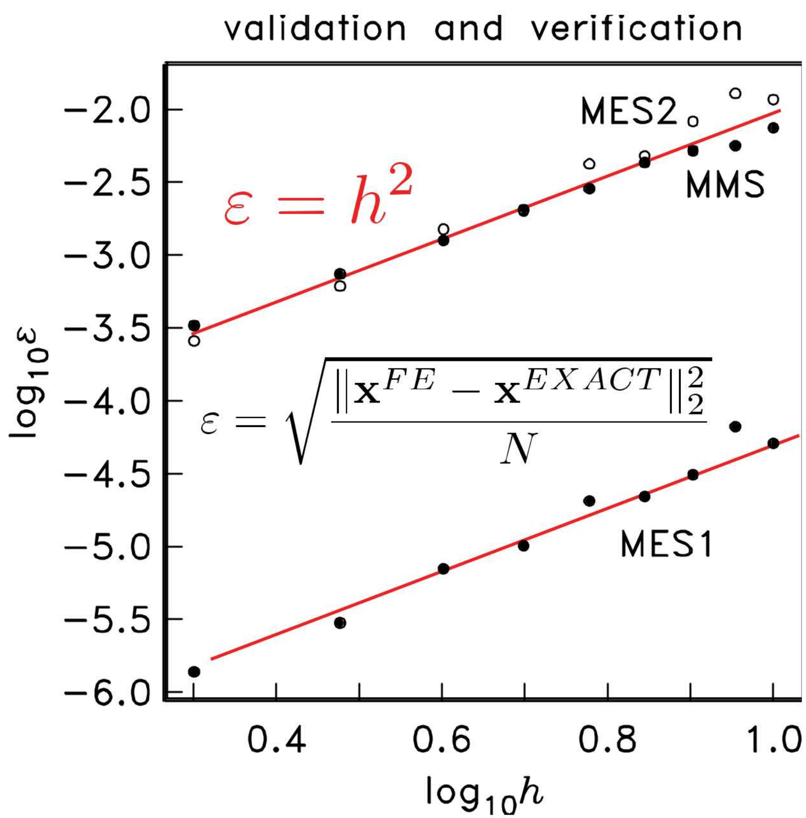
VALIDATION AND VERIFICATION (VnV)

Method of Exact Solution
When the exact solution is known for a given Earth model and source, compare it with FE solution.

MES1: dipole in a wholespace
MES2: dipole on a halfspace with a thin conductive sheet.



MES2 response



Method of Manufactured Solutions
Posit an analytic solution and then algebraically solve for the sourcing term. Compare it with FE solution.

MMS: $\text{choose: } \phi = \exp\left[-(r/a)^2\right] \text{ and } \sigma = \text{constant}$

Convergence Analysis: hierarchical FE error convergence consistent with classical FE.

Weiss and van Bloemen Waanders, Geophysics, 2018

CONCLUSIONS AND PATH FORWARD

We test the convergence of Neumann series as a potential method for rapid evaluation of electrostatic response of a fracture/infrastructure model.

Neumann series was computed using the hierarchical finite element method due to its ability to economically represent steel borehole casing and fractures.

Neumann series expansion was computed about the fracture anomaly, thus representing a time-lapse or change-detection modeling scenario.

For a fixed finite element discretization, the Neumann series was convergent if the fracture anomaly was sufficiently small…

… but this was proven to be a discretization artifact. Eigenvalue and VnV analyses show that, for this problem, the Neumann series is intrinsically divergent, regardless of anomaly smallness.

This effect was due to the strong coupling electrostatic between the fracture and steel borehole borehole casing.

Further investigation is required to quantify a “coupling threshold” below which the Neumann series could provide adequate responses.